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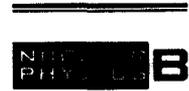
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# The super D9-brane and its truncations

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## Abstract

We consider two inequivalent truncations of the super D9-brane: the “Heterotic” and the “Type I” truncation. Both of them lead to an  $N = 1$  non-linear supersymmetrization of the  $D = 10$  cosmological constant. The propagating degrees of freedom in the Heterotic and Type I truncation are given by the components of a  $D = 10$  vector multiplet and a single Majorana–Weyl spinor, respectively. As a by-product we find that, after the Type I truncation, the Ramond–Ramond super ten-form provides an interesting reformulation of the Volkov–Akulov action. These results can be extended to all dimensions in which space-time filling D-branes exist, i.e.  $D = 3, 4, 6$  and  $10$ .

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## 1. Introduction

It is well known that the dynamics of  $Dp$ -branes [1] is described by a Dirac–Born–Infeld (DBI) action [2]. For super  $Dp$ -branes the kappa-symmetric generalization of the DBI action has been given in [3–7].<sup>5</sup> The case  $p = 9$  corresponds to a space-time filling brane, the super D9-brane. It leads to a supersymmetrization of the  $D = 10$  Born–Infeld (BI) action in a IIB supergravity background [5]. For a flat background

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<sup>5</sup> Super  $Dp$ -brane equations of motion with simultaneous world-volume and space-time supersymmetry have been constructed in Ref. [8].

the explicit supersymmetry rules, after gauge fixing the world-volume reparametrizations and the kappa symmetry, have been given in [7]. The super D9-brane has also been studied in [9]. Space-time filling branes in  $D = 6$  have recently been discussed in [10].

The D9-branes play an important role in the description of the Type I  $SO(32)$  superstring in the sense that they provide the Chan–Paton factors of the open superstring. In this description a truncation is performed using the world-sheet parity operator  $\Omega$  [11]. This leads to a specific “Type I” truncation of the IIB supergravity background. In the  $S$ -dual version the D9-branes are replaced by NS–9B branes, the world-sheet parity operator  $\Omega$  by the fermion number operator  $(-)^{F_L}$  and the Type I  $SO(32)$  superstring by the Heterotic  $SO(32)$  Superstring [12,13]. The truncation using  $(-)^{F_L}$  leads to a so-called “Heterotic” truncation of IIB supergravity. Both the Type I and Heterotic truncations preserve a linear  $N = 1$  supersymmetry of the supergravity fields.

It is the purpose of this paper to extend the Heterotic and Type I truncations of the IIB supergravity background to the action for a single super D9-brane.<sup>6</sup> We find that in both cases the surviving  $N = 1$  supersymmetry is non-linearly realized on the surviving world-volume fields. For the Heterotic (Type I) truncation these world-volume fields are given by the components of an Abelian  $D = 10$  vector multiplet (a single  $D = 10$  Majorana–Weyl fermion). The organization of this paper is as follows. In Section 2 we will first introduce a formulation of IIB supergravity in the string frame that will be needed to describe the super D9-brane action. A new feature is that we have added two 10-forms to the IIB supergravity multiplet on which the IIB supersymmetry algebra is realized. Their existence was already suggested in [12]. These 10-forms, which are also employed in [14], are needed to write down a gauge-invariant Wess–Zumino (WZ) term for the D9-brane (NS–9B brane). We will give the  $\mathbb{Z}_2$  symmetries that lead to the Heterotic and Type I truncation of the IIB supergravity multiplet. For later use we also include the flat background truncation, which actually preserves an  $N = 2$  global supersymmetry. In Section 3 we introduce the super D9-brane and extend the  $\mathbb{Z}_2$  symmetries of the IIB supergravity background to the full super D9-brane action. These  $\mathbb{Z}_2$  symmetries define the Heterotic and Type I truncations of the super D9-brane that will be discussed in Section 4. Finally, our Conclusions can be found in Section 5.

## 2. IIB supergravity and its truncations

Our starting point is the IIB supergravity multiplet whose supersymmetry rules, in the string frame, are given by<sup>7</sup>

$$\delta e_{\mu}{}^a = \bar{\epsilon} \Gamma^a \psi_{\mu},$$

<sup>6</sup> Only the Type I truncation of D9 (as well as the Heterotic truncation of NS–9B) have a natural origin in string theory. It would be interesting to see whether the Heterotic truncation of D9 (and the Type I truncation of NS–9B) also have a natural place in string theory.

<sup>7</sup> We ignore terms bilinear or higher order in the fermions.

$$\begin{aligned}
 \delta\psi_\mu &= \mathcal{D}_\mu \epsilon - \frac{1}{8} \not{\ell}_\mu \sigma^3 \epsilon + \frac{1}{16} e^\varphi \sum_{n=1}^6 \frac{1}{(2n-1)!} \not{\mathcal{G}}^{(2n-1)} \Gamma_\mu \mathcal{P}_n \epsilon, \\
 \delta\mathcal{B}_{\mu\nu} &= 2\bar{\epsilon} \sigma^3 \Gamma_{[\mu} \psi_{\nu]}, \\
 \delta\mathcal{B}^{(10)}_{\mu_1 \dots \mu_{10}} &= e^{-2\varphi} \bar{\epsilon} \sigma^3 (10 \Gamma_{[\mu_1 \dots \mu_9} \psi_{\mu_{10}}] - \Gamma_{\mu_1 \dots \mu_{10}} \lambda), \\
 \delta C^{(2n-2)}_{\mu_1 \dots \mu_{2n-2}} &= -(2n-2) e^{-\varphi} \bar{\epsilon} \mathcal{P}_n \Gamma_{[\mu_1 \dots \mu_{2n-3}} \left( \psi_{\mu_{2n-2}} - \frac{1}{2(2n-2)} \Gamma_{\mu_{2n-2}} \lambda \right) \\
 &\quad + \frac{1}{2} (2n-2)(2n-3) C^{(2n-4)}_{[\mu_1 \dots \mu_{2n-4}} \delta\mathcal{B}_{\mu_{2n-3} \mu_{2n-2}}], \\
 \delta\lambda &= (\not{\partial} \varphi - \frac{1}{12} \not{\ell} \sigma^3) \epsilon + \frac{1}{4} e^\varphi \sum_{n=1}^6 \frac{(n-3)}{(2n-1)!} \not{\mathcal{G}}^{(2n-1)} \mathcal{P}_n \epsilon, \\
 \delta\varphi &= \frac{1}{2} \bar{\epsilon} \lambda,
 \end{aligned} \tag{2.1}$$

where

$$\mathcal{P}_n = \begin{cases} \sigma^1, & \text{even} \\ i\sigma^2, & \text{odd} \end{cases} \tag{2.2}$$

The IIB field strengths are defined in form language by [15]

$$\begin{aligned}
 \mathcal{H} &= 3d\mathcal{B}, \\
 G^{(2n+1)} &= dC^{(2n)} - \mathcal{H}C^{(2n-2)}.
 \end{aligned} \tag{2.3}$$

These curvatures are invariant under the gauge transformations

$$\begin{aligned}
 \delta_\Sigma \mathcal{B} &= d\Sigma_{(\text{NS})}, \\
 \delta_\Sigma C^{(2n)} &= d\Sigma_{(\text{RR})}^{(2n-1)} - \Sigma_{(\text{RR})}^{(2n-3)} \mathcal{H}.
 \end{aligned} \tag{2.4}$$

In component language the curvatures are given by

$$\begin{aligned}
 \mathcal{H} &= 3\partial\mathcal{B}, \\
 G^{(2n-1)} &= (2n-1) \left\{ \partial C^{(2n-2)} - \frac{1}{2} (2n-2)(2n-3) \partial\mathcal{B} C^{(2n-4)} \right\},
 \end{aligned} \tag{2.5}$$

where all indices (not shown explicitly) are assumed to be completely antisymmetrized with weight one. The RR potentials  $C^{(2n-2)}$  are independent for  $n = 1, 2, 3$ . The potentials for  $n = 4, 5$  are related to those of  $n = 2, 1$  via the relations

$$G^{(7)} \equiv -^*G^{(3)}, \quad G^{(9)} \equiv ^*G^{(1)}, \tag{2.6}$$

where  $^*G$  is the Hodge dual. The curvature  $G^{(5)}$  is self-dual.

Note that we have introduced two 10-forms  $\mathcal{B}^{(10)}$  and  $C^{(10)}$ . Their 11-form curvatures are identically zero and they do not describe any dynamical degree of freedom. They are needed in order to write down a gauge-invariant and supersymmetric WZ term in the action of a super D9-brane (NS–9B brane).

Note that IIB supergravity is *not* invariant under a target-space parity transformation, which takes the self-dual, chiral IIB supergravity into the anti-self-dual anti-chiral IIB

supergravity. Under target-space parity transformations, only  $C^{(0)}$ ,  $C^{(2)}$  and  $C^{(4)}$  are pseudotensors and get an extra minus sign while the remaining RR potentials, which are defined via Hodge duality, are tensors and do not get any extra signs.

The IIB supersymmetry rules (2.1) realize the following IIB supersymmetry algebra (bosonic symmetries only)

$$[\delta(\epsilon_1), \delta(\epsilon_2)] = \delta_{\text{gct}}(a^\mu) + \delta(\Sigma_{(\text{RR})}) + \delta(\Sigma_{(\text{NS})}), \tag{2.7}$$

where the transformation parameters on the right-hand side are given by

$$\begin{aligned} a^\mu &= \bar{\epsilon}_2 \Gamma^\mu \epsilon_1, \\ \Sigma_{(\text{RR})}^{(2n-1)}{}_{\mu_1 \dots \mu_{2n-1}} &= a^\rho C_{\mu_1 \dots \mu_{2n-1} \rho}^{(2n)} + e^{-\varphi} \bar{\epsilon}_2 \mathcal{P}_n \Gamma_{\mu_1 \dots \mu_{2n-1}} \epsilon_1 \\ &\quad - (2n-1) C_{\mu_1 \dots \mu_{2n-2}}^{(2n-2)} \bar{\epsilon}_2 \sigma^3 \Gamma_{\mu_{2n-1}} \epsilon_1, \\ (\Sigma_{(\text{NS})})_\mu &= a^\rho \mathcal{B}_{\mu\rho} - \bar{\epsilon}_2 \sigma^3 \Gamma_\mu \epsilon_1, \\ (\Sigma_{(\text{NS})})_{\mu_1 \dots \mu_9} &= a^\rho \mathcal{B}_{\mu_1 \dots \mu_9 \rho}^{(10)} - e^{-2\varphi} \bar{\epsilon}_2 \sigma^3 \Gamma_{\mu_1 \dots \mu_9} \epsilon_1. \end{aligned} \tag{2.8}$$

Note that on the ten-form potentials the general coordinate transformation cancels against a field-dependent gauge transformation.

There are five  $\mathbb{Z}_2$  symmetries of the IIB supermultiplet.<sup>8</sup> The first two are called “ $\sigma^3$ ” symmetries and are given by

$$\left\{ \begin{array}{l} f \rightarrow \pm \sigma^3 f, \\ C^{(2n-2)} \rightarrow -C^{(2n-2)}, \quad \forall n, \end{array} \right. \tag{2.9}$$

where  $f$  stands for any fermion doublet. The next two  $\mathbb{Z}_2$  symmetries are called “ $\sigma^1$ ” symmetries and read

$$\left\{ \begin{array}{l} f \rightarrow \pm \sigma^1 f, \\ C^{(2n-2)} \rightarrow (-1)^n C^{(2n-2)}, \end{array} \right. \quad \left\{ \begin{array}{l} \mathcal{B} \rightarrow -\mathcal{B}, \\ \mathcal{B}^{(10)} \rightarrow -\mathcal{B}^{(10)}. \end{array} \right. \tag{2.10}$$

Finally, the fifth discrete symmetry is just the  $\mathbb{Z}_2$  grading of the  $N = 2B$  superspace which reverses the sign of all fermions. It is associated to a truncation from  $N = 2B$  to  $N = 0$  supersymmetry in which only the bosonic fields remain. We will not discuss this grading any further.

The  $\mathbb{Z}_2$  symmetries lead to truncations that are defined by setting to zero all fields that change sign under the corresponding  $\mathbb{Z}_2$ . The surviving supersymmetry is determined by the requirement that the fields that have been set equal to zero remain zero under supersymmetry. This leads to two possibilities. The first is, in both the  $\sigma^3$  and the  $\sigma^1$  cases, to keep local  $N = 1$  supersymmetry. The second possibility is to keep two global supersymmetries, but to set more fields than those that change sign under the  $\mathbb{Z}_2$  transformation, equal to zero. In fact, in this last case only a flat zehnbein remains and supersymmetry is realized in a trivial way. Nevertheless, this flat case is relevant later when the IIB multiplet appears as a background for the D9-brane.

<sup>8</sup> These symmetries correspond to involutions of the IIB superalgebra [16].

The truncations with local  $N = 1$  supersymmetry will be referred to as the “Heterotic” truncations ( $\pm\sigma^3$ ), and the “Type I” truncations ( $\pm\sigma^1$ ). We summarize the truncations below.

*Flat background truncation:*

$$e_\mu{}^a = \delta_\mu{}^a \quad (\text{all other fields zero}). \tag{2.11}$$

There are two unbroken supersymmetries with *constant* parameter  $\epsilon$ . Note that for simplicity we have set the dilaton  $\varphi$  equal to zero.

*Heterotic truncations:*

$$\begin{cases} C^{(2n-2)} = 0, & n = 1, \dots, 6, \\ (1 \mp \sigma^3) f = 0. \end{cases} \tag{2.12}$$

The transformation rules of the resulting  $N = 1$  Heterotic supergravity theory are

$$\begin{aligned} \delta e_\mu{}^a &= \bar{\epsilon} \Gamma^a \psi_\mu, \\ \delta \psi_\mu &= \mathcal{D}_\mu \epsilon \mp \frac{1}{8} \mathcal{H}_\mu \epsilon, \\ \delta \mathcal{B}_{\mu\nu} &= \pm 2 \bar{\epsilon} \Gamma_{[\mu} \psi_{\nu]}, \\ \delta \mathcal{B}^{(10)}_{\mu_1 \dots \mu_{10}} &= \pm \bar{\epsilon} \Gamma_{[\mu_1 \dots \mu_9} \psi_{\mu_{10}]}, \\ \delta \lambda &= (\not{\partial} \varphi \mp \frac{1}{12} \mathcal{H}) \epsilon, \\ \delta \varphi &= \frac{1}{2} \bar{\epsilon} \lambda. \end{aligned} \tag{2.13}$$

The unbroken supersymmetry parameter satisfies  $\epsilon = \pm\sigma^3 \epsilon$ .

These two multiplets are associated to the effective field theory of the Heterotic string based on right-handed and left-handed world-sheet fields, respectively. They are related by the duality transformation  $\mathcal{B} \rightarrow -\mathcal{B}$ ,  $\mathcal{B}^{(10)} \rightarrow -\mathcal{B}^{(10)}$ . We observe that in the Heterotic frame  $\mathcal{H}$  occurs as torsion, i.e. it can be combined with the spin connection in  $\mathcal{D}_\mu \epsilon$ . As in the IIB supergravity theory, the Heterotic supergravity multiplet is not invariant under target-space parity transformations which interchange the chirality of spinors.

*Type I truncations:*

$$\begin{cases} C^{(2n-2)} = 0, & n = 1, 3, 5, \\ \mathcal{B} = 0, \\ \mathcal{B}^{(10)} = 0, \\ (1 \mp \sigma^1) f = 0. \end{cases} \tag{2.14}$$

The transformation rules are

$$\begin{aligned} \delta e_\mu{}^a &= \bar{\epsilon} \Gamma^a \psi_\mu, \\ \delta \psi_\mu &= \mathcal{D}_\mu \epsilon \pm \frac{1}{8 \cdot 3!} e^\varphi \mathcal{G}^{(3)} \Gamma_\mu \epsilon, \end{aligned}$$

$$\begin{aligned}
 \delta C^{(2)}{}_{\mu\nu} &= \mp 2 e^{-\varphi} \bar{\epsilon} \Gamma_{[\mu} (\psi_{\nu]} - \frac{1}{4} \Gamma_{\nu]} \lambda) , \\
 \delta C^{(10)}{}_{\mu_1 \dots \mu_{10}} &= \mp 10 e^{-\varphi} \bar{\epsilon} \Gamma_{[\mu_1 \dots \mu_9} (\psi_{\mu_{10}} - \frac{1}{20} \Gamma_{\mu_{10}} \lambda) , \\
 \delta \lambda &= \left( \not{\partial} \varphi \mp \frac{1}{2 \cdot 3!} e^{\varphi} \not{\mathcal{F}}^{(3)} \right) \epsilon , \\
 \delta \varphi &= \frac{1}{2} \bar{\epsilon} \lambda ,
 \end{aligned} \tag{2.15}$$

where the unbroken supersymmetry parameter satisfies  $\epsilon = \pm \sigma^1 \epsilon$ .

These two  $N = 1$  supergravity multiplets are associated to the effective field theories of Type I strings. Observe that in the Type I frame  $G^{(3)} = 3\partial C^{(2)}$  does *not* occur as torsion. These two  $N = 1$  theories are related by the duality transformation  $C^{(2)} \rightarrow -C^{(2)}$ ,  $C^{(10)} \rightarrow -C^{(10)}$ .

### 3. The super D9-brane

In this section we introduce the action for the super D9-brane in curved IIB super-space. Using the notation of Ref. [5], the world-volume fields are the supercoordinates and the Born–Infeld vector

$$\{Z^M, V_i\}, \quad Z^M = (x^\mu, \theta^{\alpha I}), \tag{3.1}$$

where  $\mu = 0, \dots, 9, i = 0, \dots, 9, \alpha = 1, \dots, 32, I = 1, 2$  and the string-frame background superfields are

$$\{\varphi, E_M^A, \mathcal{B}_{MN}, \mathcal{B}_{M_1 \dots M_{10}}^{(10)}, C^{(2n-2)}{}_{M_1 \dots M_{2n-2}}\}, \quad n = 1, \dots, 6. \tag{3.2}$$

The action is given by

$$S^{(D9)} = S_{\text{DBI}}^{(D9)} + S_{\text{WZ}}^{(D9)}, \tag{3.3}$$

with Dirac–Born–Infeld (DBI) action

$$S_{\text{DBI}}^{(D9)} = - \int_{M^{10}} d^{10} \xi e^{-\varphi} \sqrt{|g_{ij} + \mathcal{F}_{ij}|}, \tag{3.4}$$

and Wess–Zumino (WZ) term

$$S_{\text{WZ}}^{(D9)} = \int_{M^{10}} C e^{\mathcal{F}}, \tag{3.5}$$

where we have set the D9-tension equal to one. The tensor  $\mathcal{F}$  is given by

$$\mathcal{F}_{ij} = F_{ij} - B_{ij}, \quad F_{ij} = 2\partial_{[i} V_{j]}, \quad B_{ij} = \partial_i Z^M \partial_j Z^N \mathcal{B}_{NM}. \tag{3.6}$$

Furthermore

$$g_{ij} = E_i^a E_j^b \eta_{ab}, \quad E_i^A = \partial_i Z^M E_M^A, \quad A = a, \alpha, I, \tag{3.7}$$

where  $a = 0, \dots, 9$  and

$$C = \sum_{n=1}^6 C^{(2n-2)},$$

$$C^{(2n-2)} = \frac{1}{(2n-2)!} dZ^{M_1} \dots dZ^{M_{(2n-2)}} C^{(2n-2)}_{M_{(2n-2)} \dots M_1}. \tag{3.8}$$

Note that the superfield  $\mathcal{B}^{(10)}$  does not occur in the action (3.3).

The super D9-brane action (3.3) has the following symmetries:

*World-volume reparametrizations:*

$$\begin{cases} \delta_\eta Z^M = \eta^i \partial_i Z^M, \\ \delta_\eta V_i = \eta^j \partial_j V_i + (\partial_i \eta^j) V_j. \end{cases} \tag{3.9}$$

*$\kappa$ -symmetry transformations:*

$$\begin{cases} \delta_\kappa Z^M E_M^a = 0, \\ \delta_\kappa Z^M E_M^{\alpha I} = [\bar{\kappa} (1 + \Gamma)]^{\alpha I}, \\ \delta_\kappa Z^M = [\bar{\kappa} (1 + \Gamma)]^{\alpha I} E_{\alpha I}^M, \\ \delta_\kappa V_i = E_i^A \delta_\kappa E^B B_{BA}, \end{cases} \tag{3.10}$$

where  $\Gamma$  is defined by

$$\begin{cases} \Gamma = \frac{\sqrt{|g|}}{\sqrt{|g + \mathcal{F}|}} \sum_{n=0}^5 \frac{(-1)^n}{2^n n!} \gamma^{j_1 k_1 \dots j_n k_n} \mathcal{F}_{j_1 k_1} \dots \mathcal{F}_{j_n k_n} \mathcal{P}_n \otimes \Gamma_{(0)}, \\ \Gamma_{(0)} = \frac{1}{10! \sqrt{|g|}} \epsilon^{i_1 \dots i_{10}} \gamma_{i_1 \dots i_{10}} = \frac{E}{|E|} \Gamma_{11}, \quad E = \det E_i^a, \quad \Gamma_{(0)}^2 = \mathbf{1}_{32 \times 32}, \\ \gamma_i = E_i^a \Gamma_a, \end{cases} \tag{3.11}$$

and satisfies

$$\Gamma^2 = \mathbf{1}_{64 \times 64}. \tag{3.12}$$

The presence of  $\mathcal{P}_n$  (see Eq. (2.2)) in the definition of  $\Gamma$  ensures that  $\Gamma$  can be written as [7]

$$\Gamma = \begin{pmatrix} 0 & \gamma \\ \tilde{\gamma} & 0 \end{pmatrix} \tag{3.13}$$

with

$$\gamma \tilde{\gamma} = \tilde{\gamma} \gamma = \mathbf{1}_{32 \times 32}. \tag{3.14}$$

Note that  $\kappa$ -symmetry requires that the IIB supergravity fields satisfy their equations of motion. This does not lead to any restriction on  $C^{(10)}$ .<sup>9</sup>

<sup>9</sup> Observe that  $C^{(10)}$  is a background field. We are not supposed to take the equation of motion that follows from varying  $C^{(10)}$  in the super D9-brane action. Clearly this equation of motion would be inconsistent.

*Target-space super-reparametrizations:*

All superspace fields transform as supertensors under the super-reparametrization (including general coordinate transformations)

$$\begin{cases} \delta_K Z^M = -K^M(Z), \\ \delta_K V_i = \Delta_i, \end{cases} \quad (3.15)$$

where  $\Delta_i$  is defined through the supersymmetry transformation of  $B_{ij}$

$$\delta_K B_{ij} = 2\partial_{[i}\Delta_{j]}. \quad (3.16)$$

In a flat background we have

$$K^\mu = a^\mu - \frac{1}{2}\bar{\epsilon}\Gamma^\mu\theta, \quad K^{\dot{\alpha}l} = \epsilon^{\dot{\alpha}l}. \quad (3.17)$$

$\mathbb{Z}_2$  transformations:

All  $\mathbb{Z}_2$  symmetries of the IIB supergravity background discussed in the previous section can be extended to the full super D9-brane action as follows:

(1)  $\sigma^3$ -symmetries: The  $\sigma^3$ -symmetry must be supplemented by

$$\theta \rightarrow \pm\sigma^3\theta, \quad (3.18)$$

and by a world-volume parity transformation  $\Omega_{D9}$ . We would like to stress the fact that  $\theta$  does not change parity under world-volume parity transformations because it is a world-volume (anticommuting scalar and a target-space spinor).

Note that the action of the  $\mathbb{Z}_2$  symmetry on the pulled-back superfields is in form the same as the action (2.9) on the component background fields.

(2)  $\sigma^1$ -symmetries: The action (2.10) on the background fields must be supplemented by the following action on the world-volume fields:

$$\begin{cases} \theta \rightarrow \pm\sigma^1\theta, \\ V \rightarrow -V. \end{cases} \quad (3.19)$$

Again, the action of the pulled-back superfields is in form the same as the action (2.10) on the component background fields.

#### 4. Truncations of the super D9-brane action

In the previous section we have seen that the  $\mathbb{Z}_2$  symmetries of the IIB background can be extended to  $\mathbb{Z}_2$  symmetries of the full super D9-brane action. In this section we will discuss the corresponding truncations. In particular, we will investigate what happens with the fermionic symmetries ( $\kappa$ -symmetry and target-space super-reparametrizations) in each case.

For the explicit results (action and transformation rules) we will limit ourselves to a flat background. This will lead to two different  $N = 1$  globally supersymmetric field theories. Our calculations ensure that these can be extended to the full  $N = 1$  local

supersymmetry. To compare we have also included the flat background truncation which leads to an  $N = 2$  global supersymmetry.

4.1. Flat background truncation

This case has been discussed in [7]. The flat background truncation is defined by

$$e_\mu^a = \delta_\mu^a \quad (\text{all other background fields zero}) . \tag{4.1}$$

The (bosonic and fermionic) embedding coordinates remain non-zero. There are two unbroken supersymmetries with constant parameter  $\epsilon$ . In Appendix A we have given the expansion of the superfields (up to terms linear in  $\theta$ ) corresponding to flat IIB superspace.

The Born–Infeld part of the action (3.4) takes the form

$$\mathcal{L} = -\sqrt{|M_{ij}|} , \tag{4.2}$$

where

$$M_{ij} = \eta_{ij} + F(V)_{ij} + \bar{\theta} \Gamma_{(i} \partial_{j)} \theta + \bar{\theta} \sigma^3 \Gamma_{[i} \partial_{j]} \theta + \frac{1}{4} \bar{\theta} \Gamma^a \partial_i \theta \bar{\theta} \Gamma_a \partial_j \theta + \frac{1}{4} \bar{\theta} \sigma^3 \Gamma^a \partial_i \theta \bar{\theta} \Gamma_a \partial_j \theta . \tag{4.3}$$

The Wess–Zumino term (3.5) retains its form, but now with flat background RR superfields.

The symmetry transformations are

$$\begin{aligned} \delta \bar{\theta} &= -\bar{\epsilon} + \bar{\kappa} (1 + \Gamma) + \eta^j \partial_j \bar{\theta} , \\ \delta x^\mu &= \frac{1}{2} \bar{\epsilon} \Gamma^\mu \theta + \frac{1}{2} \bar{\kappa} (1 + \Gamma) \Gamma^\mu \theta + \eta^j \partial_j x^\mu , \\ \delta V_i &= -\frac{1}{2} \bar{\epsilon} \sigma^3 \Gamma_\mu \theta \partial_i x^\mu - \frac{1}{24} (\bar{\epsilon} \sigma^3 \Gamma^a \theta \bar{\theta} \Gamma_a \partial_i \theta + \bar{\epsilon} \Gamma^a \theta \bar{\theta} \sigma^3 \Gamma_a \partial_i \theta) \\ &\quad - \frac{1}{2} \bar{\kappa} (1 + \Gamma) \Gamma_\mu \sigma^3 \theta \partial_i x^\mu - \frac{1}{8} \bar{\kappa} (1 + \Gamma) \Gamma^a \theta \bar{\theta} \Gamma_a \sigma^3 \theta \\ &\quad - \frac{1}{8} \bar{\kappa} (1 + \Gamma) \Gamma^a \sigma^3 \theta \bar{\theta} \Gamma_a \theta + \eta^j \partial_j V_i + (\partial_i \eta^j) V_j . \end{aligned} \tag{4.4}$$

Note that the flat background truncation preserves the  $\kappa$ -transformations. At this stage these  $\kappa$ -transformations can be gauge fixed as described in [7]. We will see that the Heterotic and Type I truncation correspond, in a flat background, to truncations of (4.2) and (4.4) in which the  $\kappa$ -transformations are automatically eliminated. These truncations are described below.

4.2. Heterotic truncation

The Heterotic truncation is defined by the discrete symmetry of the D9-brane generated by  $\pm \sigma^3$ :

$$C^{(2n-2)} = (1 \mp \sigma^3) f = 0 , \quad n = 1, \dots, 6 . \tag{4.5}$$

The  $N = 1$  Heterotic superspace is defined by the truncation

$$(1 \mp \sigma^3)\theta = 0. \tag{4.6}$$

The action after truncation is given by

$$S_{\text{Heterotic}} = - \int_{M^{10}} d^{10}\xi e^{-\varphi} \sqrt{|g_{ij} + \mathcal{F}_{ij}|}, \tag{4.7}$$

where it is understood that all superfields are  $N = 1$  Heterotic, i.e.  $\sigma^3$ -truncated IIB superfields. On-shell they describe the Heterotic supergravity multiplet (2.13). The Wess–Zumino term vanishes in this truncation.

We now take a flat background and choose the sign for the  $\sigma^3$  truncation such that  $\theta_2 = 0$ .<sup>10</sup> Then the truncation of the IIB background fields implies that  $\epsilon_2 = 0$ . The remaining symmetry parameters have to be constrained by the condition  $\delta\theta_2 = 0$ , which means that  $\bar{\kappa}_2 + \bar{\kappa}_1\gamma = 0$ . This implies that  $\bar{\kappa}(1 + \Gamma)$  vanishes. At this stage it is convenient to go to the static gauge, i.e. to choose  $x^\mu = \delta_i^\mu \xi^i$ . This requires a compensating world-volume reparametrization with parameter

$$\eta^\mu = -\frac{1}{2}\bar{\epsilon}_1\Gamma^\mu\theta_1. \tag{4.8}$$

The result of this truncation in a flat background is given by an action of the form (4.2), now with

$$M_{\mu\nu} = \eta_{\mu\nu} + F(V)_{\mu\nu} + \bar{\chi}\Gamma_\mu\partial_\nu\chi + \frac{1}{4}\bar{\chi}\Gamma^a\partial_\mu\chi\bar{\chi}\Gamma_a\partial_\nu\chi, \tag{4.9}$$

where we have set  $\chi \equiv \theta_1$ . This action is invariant under the supersymmetry transformations ( $\epsilon \equiv \epsilon_1$ )

$$\begin{aligned} \delta\bar{\chi} &= -\bar{\epsilon} + \eta^\mu\partial_\mu\bar{\chi}, \\ \delta V_\mu &= -\frac{1}{2}\bar{\epsilon}\Gamma_\mu\chi - \frac{1}{12}\bar{\epsilon}\Gamma^a\chi\bar{\chi}\Gamma_a\partial_\mu\chi + \eta^\rho\partial_\rho V_\mu + (\partial_\mu\eta^\rho)V_\rho, \end{aligned} \tag{4.10}$$

with  $\eta$  given by (4.8).

Note that the Heterotic  $\mathbb{Z}_2$  also involves a world volume parity transformation  $\Omega_{D9}$ . This acts only on the Wess–Zumino term, and in the form of  $\Gamma$  (see the  $\kappa$ -transformation rules). However, the truncation sets the Wess–Zumino term equal to zero, and eliminates the  $\kappa$ -transformations, as we have seen above. Therefore the world volume parity transformation becomes irrelevant. This is important in going to the static gauge, which turns the D9-theory into an ordinary  $D = 10$  field theory. Before truncation the static gauge would have been inconsistent with  $\Omega_{D9}$ , since this gauge choice identifies  $\Omega_{D9}$  with the target space parity transformation, which is not a symmetry of the IIB multiplet. Therefore taking the static gauge before truncation would break the  $\mathbb{Z}_2$  symmetry of the D9-brane, while after truncation there is no problem.

<sup>10</sup>The other sign leads to the same result with  $\theta_1 = 0$ .

The Heterotic truncation implies that a locally (non-linear) supersymmetric extension of the  $D = 10$  Maxwell theory exists which contains a cosmological constant.<sup>11</sup> As far as we know the existence of such an  $N = 1$  supersymmetrization has not been noted before in the literature.

### 4.3. Type I truncation

The Type I truncation is associated to the discrete symmetry of the D9-brane generated by  $\pm\sigma^1$ :

$$\mathcal{B} = C^{(2n-2)} = (1 \pm \sigma^1) f = V_i = 0, \quad n = 1, 3, 5. \tag{4.11}$$

The Type I superspace is defined by the truncation

$$(1 \pm \sigma^1)\theta = 0. \tag{4.12}$$

The action after truncation is given by

$$S_{\text{Type I}} = - \int_{M^{10}} d^{10}\xi \left\{ e^{-\varphi} \sqrt{|g_{ij}|} + C^{(10)} \right\}. \tag{4.13}$$

It is understood that all superfields are  $N = 1$  Type I, i.e.  $\sigma^1$ -truncated IIB superfields. On-shell they describe the Type I supergravity multiplet (2.15).

We now consider a flat background. Note that in this truncation we have set  $V_i = 0$ .<sup>12</sup> This simplifies the  $\kappa$ -transformations considerably, since now  $\Gamma = \sigma^1 \otimes \Gamma^{(0)}$ . In particular, in the flat background which we will consider below  $\Gamma^{(0)} = \Gamma_{11}$ . Therefore

$$\bar{\kappa}(1 + \Gamma) \rightarrow \bar{\kappa} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}. \tag{4.14}$$

Let us choose the truncation  $\theta_1 - \theta_2 = 0$ ,<sup>13</sup> and set  $\chi = \theta_1/\sqrt{2}$ . The truncation of the IIB background requires also  $\epsilon = \epsilon_1/\sqrt{2} = \epsilon_2/\sqrt{2}$ . The remaining supersymmetry transformations are, in the static gauge and in a flat background

$$\delta\bar{\chi} = -\bar{\epsilon} + \eta^\mu \partial_\mu \bar{\chi}, \tag{4.15}$$

with  $\eta$  as in (4.8). This leaves invariant the action (4.13), with  $\varphi = 0$  and

<sup>11</sup> Since there is no superalgebra including the bosonic group  $SO(2, 10)$  this statement needs some explanation. If the Einstein  $R$  term is added to the action, the equations of motion of the theory allow for a domain wall solution whose metric, in an appropriate “dual” frame, is identical to  $AdS_{10}$  space-time and therefore has the isometry group  $SO(2, 10)$ . The same happens for the standard D8-brane solution [17,18]. However, as opposed to the D3-brane, the solution has a non-trivial dilaton (this is the dilaton that multiplies the cosmological constant) that which is not invariant under the  $AdS_{10}$  isometries which are not symmetries of the full background.

<sup>12</sup> The elimination of the BI vector is natural from the string theory point of view where orientifolding the Type IIB superstring with  $\Omega$  changes the Chan–Paton factors from  $U(N)$  to  $SO(N)$ . In our case, for a single D9, we have  $N = 1$  and hence no BI vector field. We thank B. Schellekens for pointing this out to us.

<sup>13</sup> The other truncation,  $\theta_1 + \theta_2 = 0$ , leads to a surviving kappa-symmetry. The action, however, in this truncation becomes zero (see below).

$$g_{ij} \rightarrow g_{\mu\nu} = \eta_{\mu\nu} + \bar{\chi} \Gamma_{\mu} \partial_{\nu} \chi + \frac{1}{4} \bar{\chi} \Gamma^a \partial_{\mu} \chi \bar{\chi} \Gamma_a \partial_{\nu} \chi. \quad (4.16)$$

This action is the Volkov–Akulov action [19] generalized to ten dimensions.<sup>14</sup>

We observe that the Wess–Zumino term in the original D9-brane action is separately invariant under supersymmetry transformations: only  $\kappa$ -invariance, which in this truncation disappears, requires a collaboration of the Born–Infeld and the Wess–Zumino term. Therefore the Born–Infeld and the Wess–Zumino term should be separately invariant under the transformation (4.15). The existence of two Volkov–Akulov invariants appears unlikely, and indeed, a closer look at the expansion in  $\chi$  shows that, in a flat background, they are equal up to an additive constant:<sup>15</sup>

$$\sqrt{|g_{\mu\nu}|} - \sqrt{|\eta_{\mu\nu}|} = C^{(10)}. \quad (4.17)$$

Thus the Wess–Zumino term, expressed in  $C^{(10)}$  only, provides an interesting reformulation of the Volkov–Akulov action. This Wess–Zumino construction of a Volkov–Akulov theory can be done in any dimension which allows space-time filling D-branes, i.e.  $D = 3, 4, 6$  and  $10$ .

## 5. Conclusions

In this paper we have discussed two inequivalent truncations of the super D9-brane. Only one of them, the Type I truncation, has a natural string theory origin. It is the truncation that is triggered by the world-sheet parity operator  $\Omega$  of the fundamental string. We only considered the truncation of a single super D9-brane and this leads to a  $D = 10$  Volkov–Akulov action in terms of a single Majorana–Weyl fermion.

As a by-product we found the following interesting reformulation of the  $N = 1, D = 10$  Volkov–Akulov action:<sup>16</sup>

$$S_{\text{Volkov–Akulov}} = \int_{M^{10}} d^{10}x C^{(10)}, \quad (5.1)$$

where  $C^{(10)}$  is the pull-back of a 10-form superfield in the Type I truncated flat superspace. Its 11-form curvature  $G^{(11)} = dC^{(10)}$  has non-zero components given by

$$G_{\alpha I \beta J a_1 \dots a_n}^{(11)} = \frac{i}{2} (\Gamma_{a_1 \dots a_n} (1 + \Gamma_{11}))_{\alpha\beta} (\sigma^1)_{IJ}. \quad (5.2)$$

Such superfields have explored in the past [22] and here we find a nice application of them.

<sup>14</sup> The occurrence of a Volkov–Akulov action, after truncation of the BI vector was also mentioned in [20]. Volkov–Akulov actions also occur in truncated and gauge-fixed versions of non-space-time-filling branes [21].  
<sup>15</sup> Note that in the other truncation,  $\theta_1 + \theta_2 = 0$ ,  $C^{(10)} \rightarrow -C^{(10)}$  and the kinetic and WZ term cancel against each other.

<sup>16</sup> Similar results hold in any space-time dimension in which space-time-filling D-branes exist, i.e.  $D = 3, 4, 6$  and  $10$ .

It would be interesting to extend our work to the non-Abelian case and make contact with the supersymmetric low-energy effective action of the Type I  $SO(32)$  superstring. The (truncated version of the) world-volume action of 32 coincident D9-branes should be supplemented with a contribution from a single orientifold O9-plane to give the Type I effective action

$$S_{32 \text{ D9}} + S_{O9} = S_{IO_{32}} . \tag{5.3}$$

Some of the coupling terms in  $S_{O9}$  have been calculated [23], for more recent results, see Ref. [24]. Apart from this we need to find a kappa-symmetric non-Abelian generalization of the super D9 brane action. It would be very interesting to find such a generalization.

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**Appendix A. Expansion of flat superfields**

In this appendix we collect the value of the relevant superfields in a flat background. The target-space super-reparametrization parameter  $K^M$  is determined by

$$K^\mu = a^\mu - \frac{1}{2} \bar{\epsilon} \Gamma^\mu \theta , \quad K^{\dot{\alpha}I} = \epsilon^{\dot{\alpha}I} . \tag{A.1}$$

The supervielbein  $E_M^A$  takes on the form

$$\begin{aligned} E_\mu^a &= \delta_\mu^a , & E_{\dot{\alpha}I}^a &= -\frac{1}{2} (\bar{\theta} \Gamma^a)_{\dot{\alpha}I} , \\ E_\mu^{\alpha I} &= 0 , & E_{\dot{\alpha}I}^{\alpha J} &= \delta_{\dot{\alpha}I}^{\alpha J} . \end{aligned} \tag{A.2}$$

The superfields  $\mathcal{B}$  and  $C^{(2n-2)}$  are more complicated. For our purposes we only need the terms linear in  $\theta$  which are given by

$$\begin{aligned} \mathcal{B}_{\mu\dot{\alpha}I} &= -\frac{1}{2} (\bar{\theta} \sigma^3 \Gamma_\mu)_{\dot{\alpha}I} , \\ C^{(2n-2)}_{\mu_1 \dots \mu_{2n-3} \dot{\alpha}I} &= \frac{1}{2} (\bar{\theta} \mathcal{P}_n \Gamma_{\mu_1 \dots \mu_{2n-3}})_{\dot{\alpha}I} . \end{aligned} \tag{A.3}$$

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