On competition and banking

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Chapter 8

Asymmetric price adjustments with the mfc clause

8.1 Introduction

It is well known that prices may be rigid in the sense that cost changes are not immediately and/or fully reflected in prices. This is commonly attributed to the existence of imperfections like adjustment or menu costs. In the previous chapter, we argued that price rigidity can be asymmetric. In that case, there is relatively more downward (or upward) price rigidity. By 'more downward rigidity' we refer to the situation where for a cost change of given size, prices respond faster and/or more when the change is an increase than when it is a decrease. For examples, see Peltzman (2000) and the references therein. Most of the empirical studies of the phenomenon suggest that there is more downward price rigidity. Peltzman (2000) analyzes price rigidities for 77 consumer goods and 165 producer goods. His results show that there is more downward rigidity in more than two of every three markets[1] and he concludes that 'prices rise faster than they fall'. On the other hand there is some evidence of more upward rigidity as well. When Blinder (1994) asked businesspeople in a survey why they would adjust price with a lag after a cost change, they often replied that there was a fear to lose customers

[0]This chapter is a substantially revised version of Toolsema (2002a).

[1]For the other markets, Peltzman does not find significant asymmetries.

by being the first to raise price. This suggests more upward rigidity.

Despite the extensive empirical evidence for asymmetric price adjustments, the theoretical literature on this topic is limited. Several intuitive explanations exist for the phenomenon, though. We discussed these in detail in chapter 7. Summarizing, more downward price rigidity may be due to tacit collusion (with either asymmetric or symmetric information); consumer search or switching costs (possibly in combination with input price volatility); finite inventories and production lags; adjustment or menu costs (possibly in combination with input supply shocks); and varying markups over the business cycle. For a discussion of the intuition behind these explanations, see section 7.3. Only a few theoretical studies of asymmetric price adjustments exist. Hannan and Berger (1991) show that if menu costs (or the slope of the demand curve) differ between price increases and decreases, price adjustments will be asymmetric. Damania and Yang (1998) formalize the first explanation mentioned above, that of tacit collusion with asymmetric information (with respect to demand). In a model of an infinitely repeated duopoly they argue that the firms must punish with some probability when the competitor sets a low collusive price, and that firms may choose to forgo a price reduction when demand is low in order to avoid the punishment.

In this chapter, we present a theoretical model in which asymmetry in price adjustments may arise due to the most-favored-customer (mfc) policy. Here, the asymmetry refers to prices responding more (not faster) to cost changes in one direction than in the other. With this mfc policy, a firm guarantees its current customers that if it charges a lower price in the future (up to some specified date), they will be reimbursed the difference (Cooper, 1986; Tirole, 1988, pp. 330-332). Firms may choose to offer the mfc clause because it allows them to commit to a higher price, which increases their profits. The policy will be explained in more detail in the next section. We present a three-stage duopoly model, where the firms decide whether or not to offer an mfc clause at time zero. Then, the firms compete in prices for two periods. We consider the effects of a cost change in the second period. Note that the mfc clause can be interpreted as a kind of self-imposed, asymmetric adjustment cost: if the firm decreases its price in the second period, it will have to pay some endogenous amount of money. Intuitively, firms using this policy will be reluctant to decrease their price because of the costs of rebates incurred. This would suggest more downward rigidity. We show that this indeed will occur for most parameter values in our particular setup.

Surprisingly, however, there also exist cases in which the policy may lead to the opposite result, that is, to more upward rigidity. In particular, in our setup this may occur when two duopolist firms both offer the policy, and the cost change is large in absolute value.

The intuition behind these results comes from the fact that in a duopoly equilibrium prices are determined by the interaction of the two firms' price-setting behavior. Furthermore, the equilibrium price changes we refer to in the above are cost-change induced, *not* unilateral responses to price changes by the competitor. Indeed, if costs do not change but a competitor lowers its price, a firm offering the clause will unilaterally choose to keep its price constant because of the cost of rebates otherwise incurred (if the price reduction by the competitor is not too large). This is reflected in the shape of the reaction curves, which have a vertical segment below the initial equilibrium prices. However, if costs change, the reaction curves of the two firms shift. So the response of equilibrium prices to a cost change is described by the difference between the intersection of the old reaction curves and the intersection of the new reaction curves. Even though a firm is unilaterally reluctant to decrease its price when the other firm does so, this does not necessarily imply that prices are more rigid downward after a cost change. Furthermore, firms anticipate what will happen in the second period. Thus, the presence of an mfc clause and (the probability distribution of) the second-period cost change will affect first-period prices as well. Incorporating these effects, we argue below that price adjustments are indeed asymmetric in our model. This chapter thus provides an alternative explanation for asymmetric price adjustments, where the asymmetry may either refer to more downward or more upward rigidity.

The remainder of this chapter is structured as follows. In the next section we describe the mfc clause. Section 8.3 presents a simple linear duopoly model with heterogeneous goods, where firms decide at time zero whether or not to offer an mfc clause, and then compete in prices for two periods. In section 8.4 we derive the subgame-perfect Nash equilibrium of the game. We also show formally that, in this specific model, the policy will indeed be offered by at least one firm in equilibrium (provided that the cost change is not too small in absolute value), and discuss the effects of cost changes on price dynamics. We show that depending on the values of the parameters, there may be asymmetric price adjustments following a cost change, and show that this may result in either more downward rigidity or, in some cases, more upward rigidity.

Section 8.5 concludes.

8.2 The most-favored-customer clause

With the most-favored-customer (mfc) clause, a firm guarantees its current customers that if it will charge a lower price in the future - up to some specified date - they will be reimbursed the difference. Cooper (1986) has analyzed the effects of this policy on competition and prices. He argues that in a model of price competition among two firms producing heterogeneous goods, it may be beneficial to a firm if it can somehow commit to charge a minimum price above the Bertrand equilibrium price (Cooper, 1986, p. 381). By doing so the firm can earn at most the profits that a Stackelberg leader would earn. The competing firm benefits from this commitment as well, because setting a higher price reduces competition. Cooper shows that the mfc policy as described above effectively enables the firm offering the policy to commit to such a minimum price, softening price competition. Examples include the mfc policies offered by two manufacturers of turbine generators from the early 1960s to the mid 1970s (see Cooper, 1986, pp. 385-386), in the pharmaceutical industry (Scott Morton, 1997a and 1997b), and in long-term natural gas contracts (Crocker and Lyon, 1994).

Cooper (1986) suggests several reasons why the mfc clause may be difficult to implement. He mentions the costs of maintaining records of customers and communicating price changes. The above references indicate that they do occur, however. Furthermore, we believe the policy to be more general than this argument suggests, for two reasons.

First, imagine buying an expensive durable good, say a new tv set, at your local store. If you notice shortly after the transaction that the price of this particular tv set has declined dramatically, you may well step in and ask for a rebate. In fact, the store owner may even give you some rebate, out of fear to lose you as a future customer or to damage his reputation. This type of implicit mfc policy may be widespread.

Second, we believe that (a variant of) the policy applies to various markets in which consumers can invite offers by suppliers. An example is the Dutch mortgage market, discussed in the previous chapter. As we argued there, the mortgage offer resembles an option to obtain a mortgage at the specified rate. We also mentioned in that chapter that this offer policy resembles the mfc clause to some extent. In fact, for those clients who do indeed accept their offer and obtain a mortgage from the bank, everything is as if they already decided to accept when they invited the offer, combined with an mfc clause that guarantees them the lowest mortgage interest rate offered by the bank in the 'future', i.e. the weeks in between the invitation and the acceptation of the offer. Thus, at least for those clients who obtain a mortgage via an offer, the offer policy is closely related to the mfc policy. A similar argument holds for other products and services for which consumers can invite this kind of option-like offers (e.g. in constructing).

8.3 The model

Below we present a simple linear model that allows us to formally describe the effects of cost changes in the presence of the mfc clause. We require a highly stylized model with many fixed parameter values in order to keep the analysis tractable and obtain interesting results. Using other parameters, one would find other (possibly qualitatively different) results. However, the model presented below allows us to analyze asymmetric price adjustments. Not only does it confirm our intuition that prices may rise more than they fall with an mfc clause; also, it illustrates that the opposite case (more upward rigidity) might occur as well.

Consider the following three-stage model of price competition with differentiated goods and an mfc policy (based on Cooper, 1986). There are two firms, A and B, that face identical cost and demand conditions. In the first stage of the model, say at time zero, the firms decide whether or not to offer the mfc policy. This choice is indicated by the policy variable M^i which takes on the value 1 if firm i, $i = A, B$, adopts the policy, and the value 0 otherwise. Thus $M = (M^A, M^B)$ represents the policy choices of the two firms. Given the choices M, the next two stages of the model are a competition subgame in which firms compete for two periods. In these periods 1 and 2, they compete by setting prices. However, if at time zero a firm has chosen to offer the mfc policy and it sets a second-period price below its first-period price, it has to reimburse the difference to its first-period customers. Although it is usually not true in the real world, we assume for simplicity that all first-period consumers claim their refund. If this is not true and only a part of the total rebate is claimed, the effects derived below will be weaker but qualitatively similar. We choose to abstract from this complicating factor and thus do not model (possibly heterogeneous) consumer behavior explicitly.

Additionally, we assume that after the first competition period the firms' marginal costs change, and we analyze the response of equilibrium prices to this cost change. Initially, in period 1, we have marginal cost $c = \frac{1}{2}$ for both firms. The marginal cost faced by the two firms changes to the new level $c = \frac{1}{2} + \Delta c$, where we require $-\frac{1}{2} < \Delta c < \frac{1}{2}$, in the second period. We assume that the firms are risk neutral, and anticipate the possible future cost changes. Let us assume the following ex ante probability distribution for the cost change Δc:

$$\Pr\{\Delta c = x\} = \begin{cases} \frac{1}{2} & \text{for } x = \delta \\ \frac{1}{2} & \text{for } x = -\delta \end{cases} \tag{8.1}$$

where $0 < \delta < \frac{1}{2}$. We ignore the possibility that marginal cost remains at the first-period level in order to keep the analysis tractable. Additionally, we require below that consumers themselves do not anticipate possible future (price) changes, or at least do not condition their behavior on such anticipated changes. That is, the demand functions of periods 1 and 2 are independent. This type of behavior obtains for example when the firm's products are perishable and intertemporal substitution is not possible.

Each of the two stages of the competition subgame in our framework is based on the following simple one-period model of price competition with product differentiation. The two firms, $i = A, B$, produce heterogeneous goods, with inverse demand for each firm given by

$$\begin{aligned} p^A &= 1 - q^A - \frac{1}{2}q^B, \\ p^B &= 1 - q^B - \frac{1}{2}q^A. \end{aligned}$$

Note that we impose specific parameter values in the demand functions, again to keep the analysis tractable. In direct form, demand can be written as

$$\begin{aligned} q^A &= \frac{2}{3}\left(1 + p^B - 2p^A\right), \\ q^B &= \frac{2}{3}\left(1 + p^A - 2p^B\right). \end{aligned}$$

Let $0 < c < 1$ denote the constant marginal cost. Firm i's maximization problem is given by

$$\max_{p^i} \quad \Pi^i = \left(p^i - c\right) q^i.$$

The standard reaction functions are given by

$$\begin{aligned} p^A &= r_c^A\left(p^B\right) = \frac{1}{4}\left(1 + p^B + 2c\right), \\ p^B &= r_c^B\left(p^A\right) = \frac{1}{4}\left(1 + p^A + 2c\right). \end{aligned} \tag{8.2}$$

Solving for prices, in equilibrium we have

$$p^A = p^B = \widetilde{p}_c = \frac{1}{3}(1 + 2c) \tag{8.3}$$

and

$$q^A = q^B = \widetilde{q}_c = \frac{4}{9}(1 - c),$$

and equilibrium profits are

$$\Pi^A = \Pi^B = \widetilde{\Pi}_c = \frac{4}{27}(1 - c)^2,$$

where we use a tilde to refer to the Bertrand equilibrium value of a variable, and the subscript c refers to the cost level c used here.

8.4 Solution of the model

In this section, we derive the subgame-perfect Nash equilibrium (SPNE) of the game presented above. We focus on pure strategy equilibria, and consider only symmetric equilibria (i.e. in which prices are the same for both firms) for the symmetric cases in which either no or both firms offer the policy, i.e. $M = (0, 0)$ and $M = (1, 1)$. Using backward induction, we first discuss the price-setting behavior of the firms in the competition subgame (periods 1 and 2). We do so for the various possible policy choices M, skipping the case $M = (0, 1)$ because it is symmetrical to that of $M = (1, 0)$. We assume above that the firms anticipate the cost changes. Thus, the firms might wish to accommodate their first-period behavior to likely second-period developments. Therefore, the probability distribution (8.1) should be taken into account when deriving first-period and second-period equilibrium prices. As we will show below, equilibrium prices depend on the parameter δ, and with respect to asymmetric price adjustments different cases may arise, also depending on the value of this parameter. For example, for some values of δ, prices may rise (after a cost increase) but not fall (after a cost decrease), and for others prices may both rise and fall. In order to derive the precise equilibrium prices for a given parameter value, we need to analyze and compare all possible cases. For that reason, we also discuss the effects of an actual cost change on second-period prices in this section. Finally, we turn to the policy choice itself, and determine the SPNE of the whole game.

8.4.1 No firm offers the mfc clause: $M = (0, 0)$

If no firm offers the mfc policy, periods 1 and 2 are simply repetitions of the benchmark competition model described in section 8.3. The equilibrium in each period of the competition subgame is the same as that of the standard single-period model, substituting the actual cost level. Thus, in equilibrium the firms charge the Bertrand prices $\widetilde{p}_{\frac{1}{2}} \equiv \widetilde{p}$ in period 1, and either $\widetilde{p}_{\frac{1}{2}+\delta} \equiv \widetilde{p}_{+}$ or $\widetilde{p}_{\frac{1}{2}-\delta} \equiv \widetilde{p}_{-}$ in period 2. From (8.3), clearly second-period price adjustments are symmetric. A firm's expected total profits are given by

$$\widetilde{\Pi}_{\frac{1}{2}} + \frac{1}{2}\widetilde{\Pi}_{\frac{1}{2}+\delta} + \frac{1}{2}\widetilde{\Pi}_{\frac{1}{2}-\delta} = \frac{2}{27}\left(1 + 2\delta^{2}\right).$$

8.4.2 Only firm A offers the mfc clause: $M = (1, 0)$

Now suppose that only firm A has chosen to offer the policy. Using backward induction, we first discuss what happens in the second period of the competition subgame, taking the first-period prices p_1^A and p_1^B as given. Firm B will always be on its standard reaction curve r^B (see (8.2)). This is evident since firm B simply maximizes second-period (i.e. single-period) profits. For firm A we have the following. In period 2, if $c_2 = c + \delta \equiv c_+$, firm A may increase its price in period 2 if it wants to, and it will be on its standard reaction curve (corresponding to the cost increase) then. In that case, both firms would charge the Bertrand price $\widetilde{p}_+$. However, firm A may not want to do this, since its first-period price p_1^A may *exceed* the second-period Bertrand price $\widetilde{p}_+$, in which case firm A will continue to charge p_1^A. If $c_2 = c - \delta \equiv c_-$, firm A may reduce its price, but has to incur the rebate $\left(p_1^A - p_2^A\right) q \left(p_1^A, p_1^B\right)$ when doing so. Thus, this amount should be substracted from second-period profits. From this expression, an adjusted reaction curve for A can be derived, taking the rebate into account. This reaction curve is valid for prices below p_1^A. This is illustrated in figure 8.1.

Figure 8.1 shows the second-period reaction functions of the firms in the case where only firm A has decided to offer the mfc policy for a given cost level c. For now, we ignore the possibility of a cost change and focus on the shape of firm A's reaction curve in the presence of an mfc clause, and the resulting equilibrium prices in period 2. The second-period prices of firms A and B are denoted by p^A and p^B, respectively. Firm B's reaction curve $p^B\left(p^A\right)$ is simply the standard reaction curve, as we argued above, because it is assumed not to offer an mfc clause. Suppose firm A charged the price p_1^A in the first period. Firm A's second-period reaction function $p^A\left(p^B\right)$ equals the standard reaction function for any price *above* the first-period price p_1^A. However, because firm A offers the mfc policy its second-period reaction function shifts outward for any price below its first-period price p_1^A, because of the cost of the rebate. Thus, as soon as firm A's reaction function hits p_1^A, at point F in the figure, $p^A\left(p^B\right)$ becomes vertical. For these levels of p^B the rebates to first-period customers outweigh the benefits of reducing price below p_1^A. However, as firm B's price becomes extremely low, say below the level indicated by point D in the figure, firm A will set a second-period price below p_1^A even though it has to rebate the difference to its first-period customers. In that case, the positive effect on profits of charging a lower price and getting a greater share of the market dominates the negative effect of the rebate. Firm A's second-period reaction curve thus has a flat (vertical) segment at firm A's first-period price, the length of which is positively related to the firm's first-period sales (which indicates the number of customers that can claim a refund in the second period).

Point E denotes the resulting second-period equilibrium, taking first-period prices p_1^A and p_1^B as given. Note that in equilibrium both firms charge a price above the Bertrand equilibrium price $\widetilde{p}$, which is indicated by point **B** in figure 8.1. We focus on the case where the reaction curves intersect in the flat segment of firm A's reaction curve. Alternatively, there could be an equilibrium where the intersection is in the lower segment of that curve.[2]

If only firm A offers the mfc clause, i.e. $M = (1, 0)$, we may obtain asymmetric price adjustments in this setup. Intuitively, the flat segment of the reaction curve suggests that a firm offering an mfc policy is reluctant to lower its price. However, when discussing asymmetries in price adjustments, we concentrate on price adjustments following a

[2] The other equilibrium, where the intersection is in the lower part of A's reaction curve (the part with the rebate) turns out to be irrelevant in our simple model.

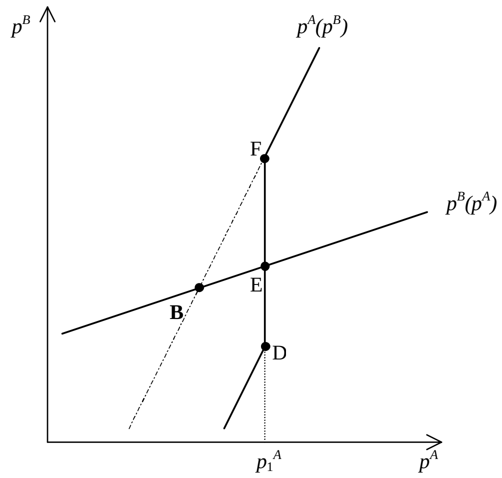

Figure 8.1: Reaction functions of firms A and B when only firm A offers the mfc policy.

cost change. This refers to a *shift* of the reaction curves. Since the firms are assumed to face identical cost and demand conditions, we focus on an industry-wide cost change, which is the same for both firms. Figures 8.2 and 8.3 illustrate the effects of a cost increase and a cost decrease, respectively, in this case. We use these figures to show that in our setup, with respect to asymmetric price adjustments, four situations or cases can be distinguished. When the cost change is an increase, equilibrium price may or may not increase as well; and when the cost change is a decrease, equilibrium price may or may not decrease. For any given (absolute) value of a cost change δ this determines the (a)symmetry in price adjustment.

In figure 8.2 the dashed lines illustrate the effects of a small cost increase in the second competition period. Note that we continue to take first-period prices as given here. The general effect of such a cost increase is to shift reaction functions outward in the direction of the arrows. Firm A's reaction curve remains vertical at p_1^A, as shown in figure 8.2. For the particular cost change described in the figure, we see that firm A will continue to charge the old equilibrium price p_1^A, whereas firm B will set a higher price than before. However, had the

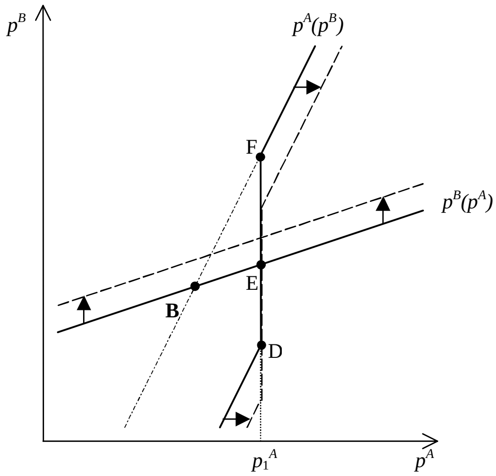

Figure 8.2: Reaction functions of firms A and B when only firm A offers the mfc policy: the effects of a cost increase (curves shift from solid to dashed lines as indicated).

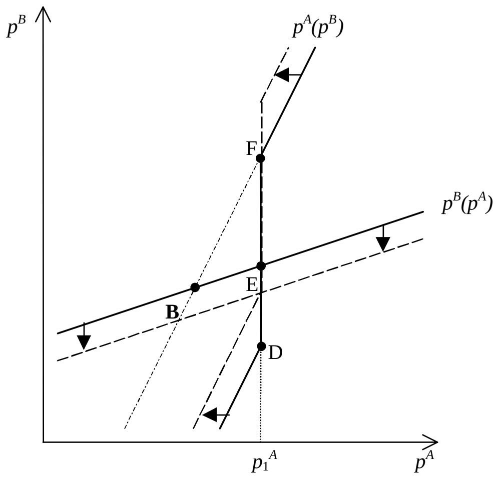

Figure 8.3: Reaction functions of firms A and B when only firm A offers the mfc policy: the effects of a cost decrease (curves shift from solid to dashed lines as indicated).

line segment EF been shorter (or the cost increase larger), firm A might have increased its price as well, and in that case the increase for firm B would have been larger (in an absolute as well as in a relative sense). Note that $\widetilde{p}_+$ is determined by the intersection of the new standard reaction functions. In the figure, this intersection lies to the left of p_1^A. So, if δ is small enough, we might have $p_1^A > \widetilde{p}_+$, as predicted above.

Similarly, figure 8.3 describes the effects of a cost decrease of the same size. The dashed lines illustrate the second-period reaction curves after the cost decrease. The general effect of such a cost decrease is to shift reaction functions inward. Firm A's reaction curve will again remain vertical at p_1^A. For the particular cost change described in the figure, we see that firm A will just continue to charge the old equilibrium price, whereas firm B will set a lower price. However, had the line segment DE been slightly shorter (or the cost decrease slightly larger), firm A would have decreased its price as well, and for firm B the price decrease would have been larger (in an absolute as well as in a relative sense).

In the particular example in figures 8.2 and 8.3, there is more upward rigidity. Since the line segment EF is longer than the line segment DE, prices are more rigid upward than downward. For a cost change of given size, firm B will always adjust its price. Firm A will not adjust its price for small cost changes; it will decrease its price for intermediate cost decreases but it will not increase its price for corresponding cost increases; and it will adjust its price for large cost changes in either direction. Furthermore, the price change by firm B will be relatively small for the cases in which firm A does not adjust its price. Alternatively, if the line segment DE is longer than the line segment EF, there will be more downward rigidity. Note that the length of the segment DF is determined by the size of the rebate to first-period consumers, and the relative lengths of the two segments DE and EF are determined by the point at which firm B's reaction curve intersects the flat segment. Thus, for example, if the rebate is relatively large, firm A will be more reluctant to decrease its price. This may imply more downward rigidity. The precise effects also depend on the firms' first-period equilibrium prices, which were simply taken as given here.

We now return to our formal model. Firm i's operating profits in period t with marginal cost c_t are given by

$$\Pi_t^i \left(p_t^i, p_t^j, c_t \right) \equiv \left(p_t^i - c_t \right) q_t^i \left(p_t^i, p_t^j \right), \quad i \in \{A, B\}, i \neq j, t = 1, 2.$$

We write $\Pi_1^i\left(p_1^i, p_1^j, \frac{1}{2}\right) \equiv \Pi_1^i\left(p_1^i, p_1^j\right)$. A firm offering the mfc policy incurs a rebate whenever its second-period price is below the first-period price. Therefore, we define

$$\widehat{\Pi}_2^i\left(p_1^i, p_1^j, p_2^i, p_2^j, c_2\right) \equiv \Pi_2^i\left(p_2^i, p_2^j, c_2\right) - \left(p_1^i - p_2^i\right) q_1^i\left(p_1^i, p_1^j\right) I_{p_1^i > p_2^i}$$

for firm i if $M^i = 1$, where $i \in \{A, B\}, i \neq j$, and $I_{p_1^i > p_2^i}$ is an indicator function such that

$$I_{p_1^i > p_2^i} = \begin{cases} 1 & \text{if } p_1^i > p_2^i, \\ 0 & \text{otherwise.} \end{cases}$$

Note that via the rebate term, $\widehat{\Pi}_2^i(\cdot)$ depends on first-period prices as well, unlike $\Pi_2^i(\cdot)$. In solving the period 2 subgame, p_1^i and p_1^j should be taken as given, and c_2 is known. Therefore, we can treat p_1^i, p_1^j, and c_2 as parameters and write $\widehat{\Pi}_2^i\left(p_1^i, p_1^j, p_2^i, p_2^j, c_2\right) = \widehat{\Pi}_2^i\left(p_2^i, p_2^j\right)$ in that stage.

If the mfc clause is offered by at least one firm, it is convenient to use the following definition for the period 2 equilibrium (P2E).

Definition 8.1 *A second-period equilibrium (P2E) of the game described in section 8.3, given the policy choice $M \in \{(1,0), (1,1)\}$, first-period prices p_1^A, p_1^B, and the cost level c_2, is defined as follows: $\left(p_2^{A*}, p_2^{B*}\right)$ is a P2E iff.*

- $p_2^{A*} = \arg\max \widehat{\Pi}_2^A\left(p_2^A, p_2^{B*}\right)$, *and*
- $p_2^{B*} = \begin{cases} \arg\max \Pi_2^B\left(p_2^B, p_2^{A*}\right) & \text{if } M = (1,0), \\ \arg\max \widehat{\Pi}_2^B\left(p_2^B, p_2^{A*}\right) & \text{if } M = (1,1). \end{cases}$

Note that the P2E is simply an equilibrium of the second-period subgame. It is a function of first-period prices, as well as of the cost level prevailing in period 2, c_2. From the graphical analysis, for $M = (1,0)$, it is easy to see that this yields a unique second-period equilibrium with the following properties. If $c_2 = c_+$, firm A raises its price to $\widetilde{p}_+$ if $\widetilde{p}_+ > p_1^A$; otherwise, firm A charges the first-period price p_1^A again. For the case $c_2 = c_-$, we denote by $\widehat{p}_-^A$ and $\widehat{p}_-^B$ the prices of firms A and B, respectively, at the intersection of firm A's reaction function *with the rebate* and firm B's standard reaction function. If $c_2 = c_-$, firm A lowers its price if $\widehat{p}_-^A < p_1^A$. Otherwise, A continues to charge p_1^A. Evidently, for $M = (1, 1)$ we will have a similar result: the price rises to $\widetilde{p}_+$ if this exceeds the first-period price, and the price falls to the level corresponding to the intersection of the reaction functions with the rebate, say $\widehat{p}_-$, if this is below the first-period price.

For any set of first-period prices, the P2E allows us to derive the corresponding second-period equilibrium prices. Thus, using backward induction and applying the P2E, in the analysis of period 1 we can write the second-period profits $\widehat{\Pi}_2^i$ in terms of first-period prices only. Similarly, Π_2^i can be expressed in terms of first-period prices as well. We use a bold Π in this notation; that is, $\widehat{\mathbf{\Pi}}_2^i$ ($\mathbf{\Pi}_2^i$) refers to second period profits $\widehat{\Pi}_2^i$ (Π_2^i) expressed in terms of first-period prices only, where corresponding second-period prices follow uniquely from the definition of the P2E. Thus, $\widehat{\mathbf{\Pi}}_2^i = \widehat{\mathbf{\Pi}}_2^i \left(p_1^i, p_1^j\right)$ and $\mathbf{\Pi}_2^i = \mathbf{\Pi}_2^i \left(p_1^i, p_1^j\right)$.

We now define the equilibrium concept for the competition subgame of periods 1 and 2 for $M = (1, 0)$, that is, assuming that firm A has chosen to offer the policy, but firm B has not. We denote a strategy for firm i by $\left(p_1^i, p_+^i, p_-^i\right)$, where p_1^i refers to the price charged in period 1; p_+^i refers to the price charged in period 2 if $c_2 = c_+$; p_-^i refers to the price charged in period 2 if $c_2 = c_-$. From the discussion above, we know that four different cases might arise in period 2: firm A may not adjust its price at all; it may raise its price if $c_2 = c_+$ but not lower its price if $c_2 = c_-$; conversely it may not raise its price if $c_2 = c_+$ but lower its price if $c_2 = c_-$; and finally it may adjust its price in either situation. We distinguish these four possibilities in the definition below.

Definition 8.2 *For $M = (1, 0)$, an SPNE of the competition subgame consists of a strategy $\left(p_1^{A*}, p_+^{A*}, p_-^{A*}\right)$ for firm A, and a strategy $(p_1^{B*}, p_+^{B*}, p_-^{B*})$ for firm B, such that one of the four sets of conditions below is satisfied.*

- *Type I equilibrium: $\left(p_1^{A*}, p_1^{A*}, p_1^{A*}\right)$ and $\left(p_1^{B*}, p_+^{B*}, p_-^{B*}\right)$ is a type I equilibrium if*

  1. *$\left(p_1^{A*}, p_+^{B*}\right)$ is a P2E if $c_2 = c_+$;*
  2. *$\left(p_1^{A*}, p_-^{B*}\right)$ is a P2E if $c_2 = c_-$;*
  3. *$p_1^{A*} = \arg\max_{p_1^A} \Pi_1^A \left(p_1^A, p_1^{B*}\right) + \mathrm{E}\widehat{\mathbf{\Pi}}_2^A \left(p_1^A, p_1^{B*}\right)$ and $p_1^{B*} = \arg\max_{p_1^B} \Pi_1^B \left(p_1^B, p_1^{A*}\right) + \mathrm{E}\mathbf{\Pi}_2^B \left(p_1^B, p_1^{A*}\right)$.*

- *Type II equilibrium:* $\left(p_1^{A*}, \widetilde{p}_+, p_1^{A*}\right)$ *and* $\left(p_1^{B*}, \widetilde{p}_+, p_-^{B*}\right)$ *is a type II equilibrium if*

  1. $(\widetilde{p}_+, \widetilde{p}_+)$ *is a P2E if* $c_2 = c_+$;
  2. $\left(p_1^{A*}, p_-^{B*}\right)$ *is a P2E if* $c_2 = c_-$;
  3. $p_1^{A*} = \arg\max_{p_1^A} \Pi_1^A\left(p_1^A, p_1^{B*}\right) + \mathrm{E}\widehat{\Pi}_2^A\left(p_1^A, p_1^{B*}\right)$ *and* $p_1^{B*} = \arg\max_{p_1^B} \Pi_1^B\left(p_1^B, p_1^{A*}\right) + \mathrm{E}\Pi_2^B\left(p_1^B, p_1^{A*}\right)$.

- *Type III equilibrium:* $\left(p_1^{A*}, p_1^{A*}, \widehat{p}_-^A\right)$ *and* $\left(p_1^{B*}, p_+^{B*}, \widehat{p}_-^B\right)$ *is a type I equilibrium if*

  1. $\left(p_1^{A*}, p_+^{B*}\right)$ *is a P2E if* $c_2 = c_+$;
  2. $\left(\widehat{p}_-^A, \widehat{p}_-^B\right)$ *is a P2E if* $c_2 = c_-$;
  3. $p_1^{A*} = \arg\max_{p_1^A} \Pi_1^A\left(p_1^A, p_1^{B*}\right) + \mathrm{E}\widehat{\Pi}_2^A\left(p_1^A, p_1^{B*}\right)$ *and* $p_1^{B*} = \arg\max_{p_1^B} \Pi_1^B\left(p_1^B, p_1^{A*}\right) + \mathrm{E}\Pi_2^B\left(p_1^B, p_1^{A*}\right)$.

- *Type IV equilibrium:* $\left(p_1^{A*}, \widetilde{p}_+, \widehat{p}_-^A\right)$ *and* $\left(p_1^{B*}, \widetilde{p}_+, \widehat{p}_-^B\right)$ *is a type II equilibrium if*

  1. $(\widetilde{p}_+, \widetilde{p}_+)$ *is a P2E if* $c_2 = c_+$;
  2. $\left(\widehat{p}_-^A, \widehat{p}_-^B\right)$ *is a P2E if* $c_2 = c_-$;
  3. $p_1^{A*} = \arg\max_{p_1^A} \Pi_1^A\left(p_1^A, p_1^{B*}\right) + \mathrm{E}\widehat{\Pi}_2^A\left(p_1^A, p_1^{B*}\right)$ *and* $p_1^{B*} = \arg\max_{p_1^B} \Pi_1^B\left(p_1^B, p_1^{A*}\right) + \mathrm{E}\Pi_2^B\left(p_1^B, p_1^{A*}\right)$.

Note that if A finds it optimal to raise price, it raises its price to the level $\widetilde{p}_+$; if it finds it optimal to lower its price the equilibrium prices $\left(\widehat{p}_-^A, \widehat{p}_-^B\right)$ are determined by the intersection of A's reaction function with the rebate, and B's standard reaction function. Also, note that we might have multiple equilibria, if more than one of the sets of conditions can be satisfied. This may occur in particular for values of δ close to the boundary between two different cases. For example, if in equilibrium the price rises if marginal cost rises, and it falls if marginal cost falls, it might also be an equilibrium to set a slightly lower first-period price and avoid the price decrease in period 2 (see the next subsubsection for an illustration). For expositional convenience, we focus here on the SPNE that yields the highest expected total profits $\Pi_1^A + E\widehat{\Pi}_2^A$ to firm A (but not necessarily to firm B), for given value of δ. This allows us to derive results with respect to asymmetric price adjustments in equilibria in which firms do indeed have an incentive to offer the mfc policy. Further, when analyzing whether or not a firm wants to offer the mfc policy, this maximum-payoff SPNE appears to be the relevant alternative. Also, although we did not model it explicitly, the firm that has decided to offer the policy might somehow be able to coordinate and determine which equilibrium occurs. In that case, the maximum-payoff equilibrium type must result. Note that if we do not use this simplication, the results below do not change qualitatively; the maximum-profits SPNE may not be the unique SPNE in that case, however.

Using this equilibrium concept, we can derive the following proposition.

Proposition 8.1 *If $M = (1, 0)$, that is only one firm offers the mfc clause, equilibrium price adjustments may be asymmetric. In particular, in our model, we have the following. Suppose firm A is the firm offering the policy. For small cost changes δ ($0 < \delta \leq \frac{1}{230}$) firm A's price is sticky and does not adjust at all (neither rises nor falls) in period 2. Firm B's price does adjust, but the adjustment is symmetric in this case. For intermediate values of δ ($\frac{1}{230} < \delta \leq \frac{47}{150}$), firm A's price rises but does not fall, so prices are more rigid downward. For large δ ($\frac{47}{150} < \delta < \frac{1}{2}$), firm A's price adjusts in either direction, but rises more than it falls, and again prices are more rigid downward.*

Proof The proof of this proposition consists of six steps. In the first four steps, we derive strategies that satisfy the conditions for the four types of equilibrium. The second-order conditions (SOCs) for a maximum can easily be checked for each of the equilibrium prices derived below, and are not presented. Then, in the fifth step, we compare firm A's expected total profits in each of the four cases to see which equilibrium type constitutes the SPNE of the competition subgame. Finally, we examine the relative price changes in period 2 for cost increases and decreases, respectively, and show that prices always rise more than they fall. Taken together, this proves the proposition.

1. First consider the type I equilibrium. In order to derive the strategies $\left(p_1^{A*}, p_1^{A*}, p_1^{A*}\right)$ and $\left(p_1^{B*}, p_+^{B*}, p_-^{B*}\right)$ that satisfy the three corresponding conditions, we proceed as follows. The first-period prices uniquely determine the P2E. Conditions 1 and 2 specify the type of P2E we require for a type I equilibrium to occur (i.e. for A

not to raise nor lower its price in period 2). Substituting this into the first-period maximization problem of period 1, we can derive first-period prices. However, this only yields necessary (but not sufficient) conditions for an equilibrium. So, given these first-period prices, we should check whether indeed firm A does not want to raise or lower its price later. If it does, the only possible type I equilibrium is given by a corner solution, such that firm A's behavior will be as assumed (that is, it does not have an incentive either to increase or to decrease its price). That is, first-period prices are derived by maximizing

$$\Pi_1^A\left(p_1^A, p_1^B\right) + \frac{1}{2}\Pi_2^A\left(p_1^A, p_2^B\left(p_1^A\right), c_+\right) + \frac{1}{2}\Pi_2^A\left(p_1^A, p_2^B\left(p_1^A\right), c_-\right)$$

with respect to p_1^A, substituting the reaction function of firm B into second-period profits Π_2^A. Note that no rebate is involved, since A does not lower its price in period 2. Firm B has a similar maximization problem. Deriving the first-order conditions (FOCs) and combining, we find that firm A's equilibrium price in each period is given by $p_1^{A*} = \frac{39}{58}$, and firm B's equilibrium price is given by $p_1^{B*} = \frac{155}{232}$. Expected profits to firm A in this case can be shown to equal $\frac{125}{1682}$. We require $\delta < \frac{1}{116}$ for $p_1^{A*} > \widetilde{p}_+$, i.e. for firm A to have no incentive to charge a higher price in period 2. Furthermore, we need to check whether firm A has no incentive to lower price if $c_2 = c_-$. We argued above that this holds whenever $p_1^{A*} < \widehat{p}_-^A$. Therefore we now determine the price $\widehat{p}_-^A$ as a function of p_1^A and p_1^B. For this, we require firm A's second-period reaction function taking the rebate into account. That is, firm A maximizes $\widehat{\Pi}_2^A\left(p_2^A, p_2^B\right)$ for $c_2 = c_-$, over p_2^A. This yields the reaction function

$$p_2^A = \frac{1}{4}\left(3 + p_2^B - 2p_1^A - 2\delta + p_1^B\right)$$

for firm A. Combining this reaction function with the standard second-period reaction function of B, we obtain $\widehat{p}_-^A = \frac{1}{15}(-8\delta + 14 - 8p_1^A + 4p_1^B)$. It can be shown that the condition $p_1^{A*} < \widehat{p}_-^A$ requires $\delta < \frac{35}{232}$. If the condition $\delta < \frac{1}{116}$ is violated, $\left(p_1^{A*}, p_1^{A*}, p_1^{A*}\right)$ and $\left(p_1^{B*}, p_+^{B*}, p_-^{B*}\right)$ with $p_1^{A*} = \frac{39}{58}$ and $p_1^{B*} = \frac{155}{232}$ do not satisfy the conditions for a type I equilibrium. In that case, we might find a corner solution for p_1^{A*} and p_1^{B*} which does satisfy the conditions. However, such a corner solution would evidently yield expected total profits below $\frac{125}{1682}$, and we will argue below (in step 5) that the condition $\delta < \frac{1}{116}$ is satisfied for any value of δ for which the type I equilibrium may arise.

2. Using a similar approach as in step 1, for a type II equilibrium we maximize firm A's expected total profits

$$\Pi_1^A\left(p_1^A, p_1^B\right) + \frac{1}{2}\Pi_2^A\left(\widetilde{p}_+, \widetilde{p}_+, c_+\right) + \frac{1}{2}\Pi_2^A\left(p_1^A, p_2^B\left(p_1^A\right), c_-\right)$$

with respect to p_1^A. Taking the FOC and combining it with that of firm B (which is the same as before) we obtain $p_1^{A*} = \frac{59}{88} - \frac{9}{44}\delta$ and $p_1^{B*} = \frac{235}{352} - \frac{9}{176}\delta$. This yields expected total profits to firm A of $\frac{62\,063}{836\,352} + \frac{6067}{209\,088}\delta + \frac{25\,127}{209\,088}\delta^2$. We require $p_1^{A*} < \widetilde{p}_+$, that is $\delta > \frac{1}{230}$. Also, we should have $p_1^{A*} < \widehat{p}_-^A$, that is $\delta < \frac{5}{14}$. If this condition is not satisfied, we obtain a corner solution, yielding lower profits, however. Again, we will argue below that the conditions are satisfied for any value of δ for which this type II equilibrium may arise.

3. For a type III equilibrium we maximize

$$\Pi_1^A\left(p_1^A, p_1^B\right) + \frac{1}{2}\Pi_2^A\left(p_1^A, p_2^B\left(p_1^A\right), c_+\right) + \frac{1}{2}\left[\Pi_2^A\left(\widehat{p}_-^A, p_2^B\left(\widehat{p}_-^A\right), c_-\right) - \left(p_1^A - \widehat{p}_-^A\right) q_1^A\left(p_1^A, p_1^B\right)\right]$$

with respect to p_1^A, with $\widehat{p}_-^A$ as derived above. Combining the resulting FOC with that for firm B we obtain $p_1^{A*} = \frac{6599}{11\,258} + \frac{3169}{5629}\delta$ and $p_1^{B*} = \frac{29\,115}{45\,032} + \frac{3169}{22\,516}\delta$. We require $p_1^{A*} > \widetilde{p}_+$ for the type III equilibrium. This condition can be written as $\delta < -\frac{2719}{3502}$, which is never satisfied. Thus, p_1^{A*} and p_1^{B*} are *not* the correct type III equilibrium prices. Instead, for any value of the parameter δ we should focus on the corner solution $p_1^{A*} = p_1^{B*} = \widetilde{p}_+$. Firm A's expected total profits are given by $\frac{979}{12\,150} - \frac{1831}{12\,150}\delta + \frac{1673}{6075}\delta^2$. For this corner solution, the condition $p_1^{A*} > \widehat{p}_-^A$ implies that we require $\delta > \frac{1}{17}$.

4. In order to find a type IV equilibrium we maximize

$$\Pi_1^A\left(p_1^A, p_1^B\right) + \frac{1}{2}\Pi_2^A\left(\widetilde{p}_+, \widetilde{p}_+, c_+\right) + \frac{1}{2}\left[\Pi_2^A\left(\widehat{p}_-^A, p_2^B\left(\widehat{p}_-^A\right), c_-\right) - \left(p_1^A - \widehat{p}_-^A\right) q_1^A\left(p_1^A, p_1^B\right)\right]$$

with respect to p_1^A, with $\widehat{p}_-^A$ as derived above. Combining the resulting FOC with that for firm B we obtain $p_1^{A*} = \frac{1162}{2479} + \frac{1144}{2479}\delta$ and $p_1^{B*} = \frac{1530}{2479} + \frac{286}{2479}\delta$. Expected total profits for firm A are given by $\frac{30\,575\,491}{331\,853\,814} - \frac{11\,795\,612}{165\,926\,907}\delta + \frac{43\,331\,234}{165\,926\,907}\delta^2$. We require $\widetilde{p}_{+\delta} > p_1^{A*}$, that is $\delta > -\frac{736}{763}$ which is always satisfied, and $\widehat{p}_-^A < p_1^{A*}$, which can be written as $\delta > \frac{47}{150}$.

5. Comparing the expressions for expected total profits to firm A in the candidate equilibria derived in steps 1-4, clearly the type IV equilibrium yields the highest profits. However, this equilibrium is feasible only for $\delta > \frac{47}{150}$. For $\delta \leq \frac{47}{150}$ the type II equilibrium yields highest expected total profits to firm A for all values of δ for which it is feasible ($\delta > \frac{1}{230}$). For $\delta \leq \frac{1}{230}$, only the type I equilibrium is feasible.

6. For $0 < \delta \leq \frac{1}{230}$ firm A's price does not rise if the cost increases, and does not fall if the cost decreases. Note that firm B's price adjusts to the cost change according to the standard reaction function (8.2), which implies that the adjustment is symmetric. For $\frac{1}{230} < \delta \leq \frac{47}{150}$, the price of firm A does increase in case of a cost increase, but remains at the first-period level in case of a cost decrease. Clearly, this implies asymmetric price adjustments for both firms, where firm A's price is completely rigid downward, and firm B's price is relatively more rigid downward. Finally, consider the following question. If $\frac{47}{150} < \delta < \frac{1}{2}$, firm A's price adjusts in either direction, but are these price adjustments symmetric? That is, consider the relative response of price, Δp, to the cost change Δc: $\frac{\Delta p}{\Delta c}$. When $c_2 = c_+$, we have

$$\left.\frac{\Delta p}{\Delta c}\right|_{c_2=c_+} = \frac{\widetilde{p}_+ - p_1^{A*}}{\delta} = \frac{2}{7437}\frac{763\delta + 736}{\delta},$$

whereas for $c_2 = c_-$,

$$\left.\frac{\Delta p}{\Delta c}\right|_{c_2=c_-} = \frac{\widehat{p}_-^A - p_1^{A*}}{-\delta} = \frac{20}{2479}\frac{150\delta - 47}{\delta}.$$

It can easily be verified that for any $\frac{47}{150} < \delta < \frac{1}{2}$, the former expression exceeds the latter, and therefore firm A's price rises more than it falls for these parameter values as well. Firm B follows the standard reaction function, so $p_2^B = \frac{1}{4}\left(1 + p_2^A + 2c_2\right)$, and the direct response of p_2^B to the cost change (via the term $\frac{1}{2}c_2$) is symmetric. If A has asymmetric price adjustments, then so does B (because of the term $+\frac{1}{4}p_2^A$). ■

The intuition behind this result is straightforward. In general, the mfc policy allows a firm to commit to a relatively high price in period 1. For that reason, a cost increase will not necessarily lead to a price increase in period 2 (since price was high already). Also, the rebate involved makes cost reductions very costly. Therefore, for very small cost changes, the price does not respond at all. For intermediate cost changes, the price does respond to a cost increase but not to a decrease. For very large cost changes, price also responds to a cost decrease, however, despite the costs of a rebate. Still, it will respond less to a cost decrease than to a cost increase, in order to limit the size of the rebate involved. Note that with an mfc policy, the rebate involved in period 2 when reducing price affects the first-period equilibrium price. In general (without a cost change in period 2) the mfc clause allows a firm to commit to a price above the Bertrand price $\widetilde{p}$ (see Cooper, 1986). In our model, however, a firm offering the policy may set a price *below* this level in the first period (it can be verified that for some values of δ, the first-period equilibrium price of firm A falls below $\widetilde{p}$). There is still a commitment effect of such an action, however, since the cost may fall to the low level c_- in period 2, which corresponds to the lower Bertrand price $\widetilde{p}_- < \widetilde{p}$.

8.4.3 Both firms offer the mfc clause: $M = (1, 1)$

Now we turn to the situation where both firms offer the clause. Now, both firms have a flat segment in their reaction curve. We focus on symmetric equilibria, and again use the defintion of the P2E given above. Again four possible cases can be distinguished with respect to the asymmetry in price adjustments; the only difference is that now the two firms always set the same price in equilibrium. Below we define the equilibrium concept for the competition subgame of periods 1 and 2 for $M = (1, 1)$, i.e. assuming that both firms have chosen to offer the policy.

Definition 8.3 *For $M = (1, 1)$, a symmetric SPNE of the competition subgame (periods 1 and 2) consists of a strategy $\left(p_1^*, p_+^*, p_-^*\right)$ for firm A, and a strategy $\left(p_1^*, p_+^*, p_-^*\right)$ for firm B, such that one of the four sets of conditions below is satisfied.*

- *Type I equilibrium:* (p_1^*, p_1^*, p_1^*) *is a type I equilibrium if*

    1. (p_1^*, p_1^*) *is a P2E if* $c_2 = c_+$;
    2. (p_1^*, p_1^*) *is a P2E if* $c_2 = c_-$;
    3. $p_1^* = \arg\max_{p_1^i} \Pi_1^i \left(p_1^i, p_1^*\right) + \mathrm{E}\widehat{\Pi}_2^i \left(p_1^i, p_1^*\right)$, $i = A, B$.

- *Type II equilibrium:* $(p_1^*, \widetilde{p}_+, p_1^*)$ *is a type II equilibrium if*

    1. $(\widetilde{p}_+, \widetilde{p}_+)$ *is a P2E if* $c_2 = c_+$;
    2. (p_1^*, p_1^*) *is a P2E if* $c_2 = c_-$;
    3. $p_1^* = \arg\max_{p_1^i} \Pi_1^i \left(p_1^i, p_1^*\right) + \mathrm{E}\widehat{\Pi}_2^i \left(p_1^i, p_1^*\right)$, $i = A, B$.

- *Type III equilibrium:* $(p_1^*, p_1^*, \widehat{p}_-)$ *is a type I equilibrium if*

    1. (p_1^*, p_1^*) *is a P2E if* $c_2 = c_+$;
    2. $(\widehat{p}_-, \widehat{p}_-)$ *is a P2E if* $c_2 = c_-$;
    3. $p_1^* = \arg\max_{p_1^i} \Pi_1^i \left(p_1^i, p_1^*\right) + \mathrm{E}\widehat{\Pi}_2^i \left(p_1^i, p_1^*\right)$, $i = A, B$.

- *Type IV equilibrium:* $(p_1^*, \widetilde{p}_+, \widehat{p}_-)$ *is a type II equilibrium if*

    1. $(\widetilde{p}_+, \widetilde{p}_+)$ *is a P2E if* $c_2 = c_+$;
    2. $(\widehat{p}_-, \widehat{p}_-)$ *is a P2E if* $c_2 = c_-$;
    3. $p_1^* = \arg\max_{p_1^i} \Pi_1^i \left(p_1^i, p_1^*\right) + \mathrm{E}\widehat{\Pi}_2^i \left(p_1^i, p_1^*\right)$, $i = A, B$.

Note that if the firms find it optimal to raise their price, they raise it to the level $\widetilde{p}_+$; if they find it optimal to lower their price the equilibrium prices $\widehat{p}_-$ are determined by the intersection of the firms' reaction functions with the rebate. Using this equilibrium concept, we can derive the following proposition.

Proposition 8.2 *If* $M = (1, 1)$, *that is both firms offer the mfc clause, equilibrium price adjustments may be asymmetric. In particular, in our model, we have the following. For small cost changes* δ *(for* $0 < \delta \leq \frac{43}{154}$*) prices rise after a cost increase, but do not fall after a cost decrease, so prices are more (completely) rigid downward. For intermediate values of* δ *(for* $\frac{43}{154} < \delta < \frac{3}{10}$*), two equilibria may arise. The first is qualitatively similar to that for small* δ*; in the second, prices adjust in either direction but they rise more than they fall. For high values of* δ *(*$\frac{3}{10} \leq \delta < \frac{1}{2}$*) prices* *also adjust in either direction. In this case, prices rise more than they fall if δ is sufficiently small, but they fall more than they rise in this situation whenever $\delta > \frac{199}{434}$.*

Proof The proof of this proposition consists of six steps. In the first four steps, we derive strategies that satisfy the necessary (first-order) conditions for each of the four types of equilibrium. The corresponding SOCs for a maximum can easily be checked for each of the equilibrium prices derived below, and are not presented. Then, in the fifth step, we turn to the sufficient conditions and check whether a firm has an incentive to deviate from a given strategy to see which equilibrium types constitutes an SPNE of the competition subgame. That is, in the fifth step we consider deviating in a way that results in a different kind of price adjustment (say, for the type I equilibrium in which prices do not adjust, when a firm deviates it may consider adjusting its price upward if costs rise). Finally, we examine the relative price changes in period 2 for cost increases and decreases, respectively, and show that prices rise more than they fall if δ is sufficiently small, and fall more than rise if δ is large. Taken together, this proves the proposition.

1. We start again by considering the maximization problem of firm A in period 1, substituting the second-period behavior as described in conditions 1 and 2 for a type I equilibrium. Firm A's expected total profits in a type I equilibrium are given by

$$\Pi_1^A\left(p_1^A, p_1^B\right) + \frac{1}{2}\Pi_2^A\left(p_1^A, p_2^B, c_+\right) + \frac{1}{2}\Pi_2^A\left(p_1^A, p_2^B, c_-\right).$$

Note that now we should not substitute the standard reaction function for firm B because of symmetry - firm B now also offers the mfc policy. Taking the FOC with respect to p_1^A and imposing symmetry we obtain $p_1^{A*} = p_1^{B*} = p_1^* = \frac{2}{3}$. In a type I equilibrium prices never increase, nor decrease. So, we require $p_1^* > \widetilde{p}_+$ and $p_1^* < \widehat{p}_-$. Clearly, the first condition can never be satisfied (for any $0 < \delta < \frac{1}{2}$). Therefore, the only possibility for a type I equilibrium is the corner solution $p_1^* = \widetilde{p}_+ = \frac{2}{3}(1+\delta)$. Expected total profits are given by $\frac{2}{27} + \frac{4}{27}\delta - \frac{16}{27}\delta^2$. For this price to satisfy $p_1^* < \widehat{p}_-$, we have the following. First, we derive the price $\widehat{p}_-$, defined by the intersection of the two reaction curves with the rebates. In the previous subsection, we derived this reaction function for firm A. Combining it with a similar reaction function for firm B and plugging in first-period prices we obtain $\widehat{p}_- = \frac{7}{9} - \frac{8}{9}\delta$. We have $p_1^* < \widehat{p}_-$ if $\delta < \frac{1}{14}$.

2. For a type II equilibrium we maximize

$$\Pi_1^A\left(p_1^A, p_1^B\right) + \frac{1}{2}\Pi_2^A\left(\widetilde{p}_+, \widetilde{p}_+, c_+\right) + \frac{1}{2}\Pi_2^A\left(p_1^A, p_2^B, c_-\right).$$

Taking the FOC with respect to p_1^A and imposing symmetry we obtain $p_1^* = \frac{2}{3} - \frac{2}{9}\delta$. Corresponding expected total profits are given by $\frac{2}{27} + \frac{8}{81}\delta^2$. For this type II equilibrium we require $p_1^* < \widetilde{p}_+$, which can easily be verified. We also require $p_1^* < \widehat{p}_-$, which can be rewritten as $\delta < \frac{3}{10}$. For $\delta \geq \frac{3}{10}$ for a type II equilibrium we may only have the corner solution $p_1^* = \widehat{p}_-$, $i = A, B$. This can be solved to give $p_1^{i*} = \frac{3}{4} - \frac{1}{2}\delta$. Corresponding profits are given by $\frac{35}{432} + \frac{1}{108}\delta - \frac{1}{108}\delta^2$. For $p_1^* < \widetilde{p}_+$ we now require $\delta > \frac{1}{14}$ which is satisfied for any feasible $\delta \geq \frac{3}{10}$.

3. In order to find a type III equilibrium, we maximize

$$\begin{aligned}&\Pi_1^A\left(p_1^A, p_1^B\right) + \frac{1}{2}\Pi_2^A\left(p_1^A, p_2^B, c_+\right)\\ &+\frac{1}{2}\left[\Pi_2^A\left(\widehat{p}_-^A, \widehat{p}_-^B, c_-\right) - \left(p_1^A - \widehat{p}_-^A\right) q_1^A\left(p_1^A, p_1^B\right)\right]\end{aligned}$$

with respect to p_1^A. Taking the FOC, and imposing symmetry we obtain $p_1^* = \frac{76}{121}\delta + \frac{69}{121}$. It can be verified that the condition $p_1^* > \widetilde{p}_+$ is never satisfied. Therefore the only candidate for a type III equilibrium is the corner solution $p_1^* = \widetilde{p}_+ = \frac{2}{3}(1+\delta)$. Corresponding expected total profits are given by $-\frac{26}{243}\delta + \frac{38}{243}\delta^2 + \frac{43}{486}$. The condition $p_1^* > \widehat{p}_-$ now implies $\delta > \frac{1}{14}$.

4. For a type IV equilibrium we maximize

$$\begin{aligned}&\Pi_1^A\left(p_1^A, p_1^B\right) + \frac{1}{2}\Pi_2^A\left(\widetilde{p}_+, \widetilde{p}_+, c_+\right)\\ &+\frac{1}{2}\left[\Pi_2^A\left(\widehat{p}_-^A, \widehat{p}_-^B, c_-\right) - \left(p_1^A - \widehat{p}_-^A\right) q_1^A\left(p_1^A, p_1^B\right)\right]\end{aligned}$$

with respect to p_1^A. Taking the FOC and imposing symmetry, this yields $p_1^* = \frac{62}{107}\delta + \frac{48}{107}$. This yields expected total profits equal to $-\frac{23\,474}{309\,123}\delta + \frac{52\,058}{309\,123}\delta^2 + \frac{56\,233}{618\,246}$. For a type IV equilibrium we require $p_1^* < \widetilde{p}_+$, which can easily be verified, and $p_1^* > \widehat{p}_-$, which implies $\delta > \frac{43}{154} \simeq 0.27922$.

5. For any feasible δ, it can easily be verified that the corner solutions derived in steps 1 and 3 cannot be the SPNE. A firm is better off by deviating, charging a different price and increasing it in period 2 if the cost level turns out to be high. Similarly, the corner solution derived in step 2 for $\delta \geq \frac{3}{10}$ is not an equilibrium. This can be seen as follows. Suppose firm B charges $p_1^{B*} = \widehat{p}_- = \frac{3}{4} - \frac{1}{2}\delta$ in period 1, and firm A considers deviating and lowering price in period 2 if costs are low. Suppose for now that B does not lower its price in period 2 if costs are low, and check later whether this is indeed true in the resulting equilibrium. Firm A now maximizes

$$\Pi_1^A\left(p_1^A, p_1^{B*}\right) + \frac{1}{2}\Pi_2^A\left(\widetilde{p}_+, \widetilde{p}_+, c_+\right)$$
$$+\frac{1}{2}\left[\Pi_2^A\left(\widehat{p}_-^A, p_1^{B*}, c_-\right) - \left(p_1^A - \widehat{p}_-^A\right) q_1^A\left(p_1^A, p_1^{B*}\right)\right],$$

and we find $p_1^{A*} = \frac{1353}{2704} + \frac{449}{1352}\delta$. Expected total profits to firm A of this strategy are given by $\frac{65\,545}{584\,064} - \frac{29\,023}{146\,016}\delta + \frac{49\,273}{146\,016}\delta^2$, which exceeds the profits computed in step 2. It can be verified that $p_1^{A*} < \widetilde{p}_+$ and $p_1^{B*} < \widetilde{p}_+$. Further, both for $p_1^{A*} > \widehat{p}_-^A$ and for $p_1^{B*} < \widehat{p}_-^B$ we require $\delta > \frac{3}{10}$, which is satisfied as well. So, for any $\delta > \frac{3}{10}$ this strategy is feasible and yields strictly higher profits to firm A. That is, we have shown that for $\delta > \frac{3}{10}$, the corner solution calculated in step 2 is not the SPNE. It can be verified that the firms have no incentive to deviate from the type II equilibrium for $\delta < \frac{3}{10}$, nor from the type IV equilibrium for $\delta > \frac{43}{154}$. Thus, for $0 < \delta < \frac{3}{10}$ the type II equilibrium derived above is an SPNE; for $\frac{43}{154} < \delta < \frac{1}{2}$ the type IV equilibrium from step 4 is an SPNE. Note that for $\frac{43}{154} < \delta < \frac{3}{10}$ we have two SPNEs. To see what is going on, note that for $\delta = \frac{43}{154}$ the price in the type IV equilibrium is slightly higher than the price in the type II equilibrium. Also, the first equilibrium price is increasing in δ, whereas the latter is decreasing in δ. So, for $\frac{43}{154} < \delta < \frac{3}{10}$, either the firms set a relatively low price and do not lower it if costs are low, or they set a relatively high price and lower it later if costs turn out to be low. Although the type IV equilibrium yields higher profits, the type II equilibrium is also an SPNE of the competition subgame.

6. For δ small (for $0 < \delta \leq \frac{43}{154}$, and possibly for $\frac{43}{154} < \delta < \frac{3}{10}$), the equilibrium price does go up in case of a cost increase, but remains

at the first-period level in case of a cost decrease, so price adjustments are asymmetric (this is the type III equilibrium). Finally, we consider the case of large δ ($\delta \geq \frac{3}{10}$, and possibly $\frac{43}{154} < \delta < \frac{3}{10}$), the type IV equilibrium, where the equilibrium price adjusts in either direction, and ask whether prices rise more than they fall in this case. For $c_2 = c_+$ we have

$$\left.\frac{\Delta p}{\Delta c}\right|_{c_2=c_+} = \frac{\widetilde{p}_+ - p_1^{A*}}{\delta} = \frac{14}{321}\frac{5+2\delta}{\delta},$$

whereas for $c_2 = c_-$,

$$\left.\frac{\Delta p}{\Delta c}\right|_{c_2=c_-} = \frac{\widehat{p}_-^A - p_1^{A*}}{-\delta} = \frac{1}{107}\frac{154\delta - 43}{\delta}.$$

It can be shown that the first expression exceeds the second, that is, prices rise more than they fall if $\delta < \frac{199}{434} \simeq 0.45853$. Recall that the type IV equilibrium price p_1^* is increasing in δ. That is, for very large δ ($\delta > \frac{199}{434}$) the firms commit to a relatively high price in period 1 and therefore price falls more than it rises in period 2. The two subcases are easy to understand if one observes that the expression $\left.\frac{\Delta p}{\Delta c}\right|_{c_2=c_+}$ is decreasing in δ, whereas $\left.\frac{\Delta p}{\Delta c}\right|_{c_2=c_-}$ is increasing in δ. For small δ, the former exceeds the latter, but as δ increases, the latter exceeds the former. ■

Intuitively, the proposition states that whenever both firms offer the mfc policy, the first-period equilibrium price is sufficiently low for it to be adjusted upward as the cost level rises for any value of δ. If the cost change δ is sufficiently high (in absolute value), the equilibrium price will also be adjusted downward if the cost level falls, despite the cost of the rebate involved. In the latter case, two subcases can be distinguished. In the first, if δ is relatively small, prices rise more than they fall. In the second subcase, if δ is relatively large, the first-period price is relatively high and therefore prices will fall more than rise in period 2. Thus, for $M = (1,1)$ we may have more downward rigidity (for small δ) as before, but in the extreme case of very large δ we find more upward rigidity.

8.4.4 Equilibrium policy choice

Now we turn to the policy choice at time zero. The equilibrium policy choice can be derived by comparing the firms' total profits for any value of δ. This completes the derivation of the SPNE of the three-stage game.

Proposition 8.3 *In the SPNE of our three-stage model, the policy will be offered in equilibrium by one firm if* $\frac{1}{230} < \delta \leq \frac{4245}{134\,366} + \frac{1188}{67\,183}\sqrt{41} \simeq 0.14482$*, and it will be offered by both firms if* $\frac{4245}{134\,366} + \frac{1188}{67\,183}\sqrt{41} < \delta < \frac{1}{2}$.

Proof In order to prove this proposition, we first compare the cases $M = (0,0)$ and $M = (1,0)$. A simple repetition of the Bertrand game (without the mfc) yields expected profits $\frac{2}{27}\left(1 + 2\delta^2\right)$. It can easily be verified that this exceeds the profits for the type I equilibrium of $M = (1,0)$, for any value of δ for which this case may occur ($\delta \leq \frac{1}{230}$). However, the profits corresponding to the type II and type IV equilibria for $M = (1,0)$ are always strictly larger than this expression. Thus, for $\delta \leq \frac{1}{230}$ no firm has an incentive to introduce the mfc clause in equilibrium, and for $\frac{1}{230} < \delta < \frac{1}{2}$, a firm does have an incentive to offer the policy.

Now, suppose that one firm, say firm A, is already offering the policy. Does the other firm, firm B have an incentive to offer it as well? That is, could both firms offer the policy in equilibrium? To analyze this question, compare the profits computed for $M = (1,1)$ to those of the firm *not* offering the policy (firm B) when $M = (1,0)$. Note that these profits have not yet been calculated above. We have to distinguish the three types of equilibrium that may occur for $M = (1,0)$: type I for $0 < \delta \leq \frac{1}{230}$; type II for $\frac{1}{230} < \delta \leq \frac{47}{150}$; and type IV for $\frac{47}{150} < \delta < \frac{1}{2}$. Firm B's profits can easily be calculated by substituting equilibrium prices in the expression for total expected profits (taking into account that if firm B lowers its price in period 2, it does not incur a rebate since it does not offer the mfc policy). In the type I equilibrium, expected total profits of firm B are given by $\frac{507}{6728}$. In the type II equilibrium we find $\frac{124\,963}{1672\,704} + \frac{1415}{418\,176}\delta + \frac{18\,907}{418\,176}\delta^2$, and in the type IV equilibrium $\frac{11\,081\,474}{165\,926\,907} + \frac{8362\,003}{165\,926\,907}\delta - \frac{1593\,742}{165\,926\,907}\delta^2$. Comparing these profits to the profits in the SPNE(s) for $M = (1,1)$ we find that for any $\delta < \frac{4245}{134\,366} + \frac{1188}{67\,183}\sqrt{41}$, profits of firm B with $M = (1,0)$ exceed the profits with $M = (1,1)$, and for any $\delta > \frac{4245}{134\,366} + \frac{1188}{67\,183}\sqrt{41}$, profits are higher with $M = (1,1)$. Thus, in equilibrium, no firm will offer the policy if δ is very small; one firm will offer it for intermediate values of δ; and both firms will offer the policy if δ is sufficiently large. ■

Thus, in the SPNE of the three-stage model presented in section 8.3, if δ is small, no firm offers the policy, prices are determined by the standard reaction functions and price adjustments are symmetric. However, for intermediate values of δ, a single firm will offer the mfc policy and there will be relatively more downward rigidity in price adjustments. In particular, the firm offering the policy will not adjust its price at all if costs go down. If δ is sufficiently large, both firms offer the policy. Again this results in relatively more downward rigidity, except for very large δ, which implies relatively more upward rigidity.

Note that we have chosen a very stylized model and fixed many parameter values (i.e. demand parameters, as well as the first-period cost level). For other parameters, the SPNE(s) might be different. However, using this simple model we established our main result that the mfc policy may result in asymmetric price adjustments. Also, our results hold in the more general model in which demand, the first-period cost level, and the cost change δ are multiplied by a given, positive constant (clearly, the resulting equilibrium prices would be multiplied by that same constant).

8.5 Conclusion

We argued that in the presence of an mfc clause there may well be asymmetric price adjustments following a cost change. We illustrated our argument using a simple linear model of a price setting duopoly with heterogeneous goods. In the equilibrium, if the cost change is very small, no firm offers the policy. For intermediate values of the cost change, one firm offers the policy, and this firm does not adjust its price at all in case of a cost decrease. For large cost changes, both firms offer the policy. In this case, if the cost change is sufficiently large, the firms do adjust their prices downward if the cost level falls. However, prices still rise more than they fall, except for very large cost changes, when there is relatively more upward rigidity. In general, our model thus predicts more downward rigidity. However, depending on the parameters of the model, there may be more upward price rigidity in some cases. This finding of upward rigidity with an mfc clause is surprising since one would expect firms to be reluctant to *decrease* price because of the cost of rebates.

The intuition behind these results is related to the fact that the equilibrium price changes we refer to above are cost-change induced. They are not unilateral responses to price changes by the competitor. Indeed, if costs do not change but a competitor reduces its price for some reason, a firm offering the clause will unilaterally choose to keep its price constant because of the cost of rebates otherwise incurred (if the price reduction by the competitor is not too large). This is reflected in the shape of the reaction curves. However, equilibrium prices are determined by the interaction of the two firms' price-setting behavior. That is, they are determined by the intersection of the reaction curves of the two firms. If costs change these reaction curves shift. So the response of equilibrium prices to a cost change is described by the difference between the intersections of the old reaction curves and the new reaction curves, respectively. Even if a firm is unilaterally reluctant to decrease its price when the other firm does so, this does not necessarily imply that prices never fall, or that they are more rigid downward after a cost change. Note that the firms anticipate these changes, however. Therefore, the first-period equilibrium price is also affected.

For the most likely case of relatively more downward rigidity, the intuition runs as follows. The mfc policy generally allows a firm to commit to a relatively high price in the first period. Therefore, a cost increase does not necessarily imply a price increase (since price was relatively high already). Further, cost reductions are costly because of the rebate involved. Thus, for very small cost changes, the price does not respond at all. For intermediate cost changes, the price does respond to a cost increase but not to a decrease. For even larger cost changes, however, price also responds to a cost decrease, despite the costs of the rebate. But, in order to limit the size of the rebate involved, it will respond less to a cost decrease than to a cost increase.