On competition and banking

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Chapter 5

Reserve requirements and double Bertrand competition in banking

5.1 Introduction

In chapter 4 we applied the well-known and widely used Klein-Monti model of banking (Klein, 1971; Monti, 1972). In this model and its extensions quantities are the strategic variables. This may not be the case in the real world for banks in particular, since banks generally set interest rates instead of quantities of loans and deposits. Therefore, a model in which prices (interest rates) are the strategic variables may be more relevant. We consider such a model in this chapter. Also, in the remainder of the thesis, we focus on interest rates as being the strategic variables.

In this chapter we analyze the effects of imposing a reserve requirement on competition in banking. For this purpose we apply Stahl's (1988) model of competition between intermediaries and apply it to the case of banks in order to model financial intermediation. In this model, banks compete for deposits as well as for loans. We analyze in this context the implications of a reserve requirement for equilibrium interest rates and the (constrained) efficiency of the equilibrium.

As Stahl (1988) observes, price competition between intermediaries constitutes a special case of competition, which can be modelled as a

[0]This chapter is a revised version of Toolsema (2001b).

two-stage game. In the first stage the intermediary or merchant buys stock from producers, which he sells to consumers in the second stage. Alternatively, the order of the stages can be reversed using forward contracts. This model has double Bertrand competition in the sense that there is price competition in both stages. In the model, a type of capacity is introduced. However, the capacity is not exogenously given and fixed, but is determined by the outcome of the first stage of the game. Yanelle (1987, 1989) shows that the model can be applied to financial intermediaries, i.e. banks (see also Freixas and Rochet, 1997, Chapter 3). In this chapter we consider the application of Stahl's (1988) model of double Bertrand competition to the case of banks, interpreting deposits as inputs and loans as outputs. In particular, we investigate in the context of this model what happens if the banks are obliged to hold a fraction of their deposits as a non-interest bearing reserve at the central bank. In that case, a fraction of the endogenous capacity cannot be used to issue loans. This setup allows us to analyze the effects of a reserve requirement on competition between financial intermediaries.

Stahl (1988) shows that if (loan) demand is elastic the unique subgame-perfect Nash equilibrium (SPNE) of the game is the standard perfect competition or Walrasian outcome. In this Walrasian outcome, the demand for loans equals the supply of deposits, and banks earn zero profits. This equilibrium is efficient. However, if (loan) demand is inelastic then, depending on the tie-breaking rule used if the banks offer the same first-stage deposit rate, either no SPNE exists, or an inefficient SPNE will arise. In this inefficient equilibrium, prices are different from Walrasian prices, there is a monopolist in the second stage, and the monopolist has excess stock. Fingleton (1997) shows that even if direct trade between agents is possible (next to trade via an intermediary), this result still holds. Since with forward contracts the equilibrium of the game always exists and is efficient, see Stahl, 1988, we will concentrate on the case in which deposits are acquired in the first stage, and loans are granted in the second stage.

In this chapter we show that the introduction of a reserve requirement can have an effect on the existence and efficiency properties of the SPNE of the model. More precisely, it makes the non-existence result and the inefficient outcome less likely to occur, in the sense that the range of parameters for which this outcome obtains shrinks. We show that if the original equilibrium is inefficient, i.e. the second-stage monopolist bank has excess funds (more deposits than loans) and interest rates are above the Walrasian level, then imposing a (fine-tuned) reserve requirement may lead to an equilibrium with no excess funds, without increasing (changing) the lending rate. We refer to this new equilibrium as constrained efficient, because of the absence of excess funds and because interest rates are at the constrained Walrasian level (that is, constrained by the size of the reserve requirement). That is, if the banks are obliged by the central bank to hold a fraction of deposits as a reserve in order to influence the appeal of banks to the money market or to limit liquidity risk, the resulting equilibrium may be constrained efficient even if the corresponding equilibrium without the reserve requirement is not efficient. So in addition to protecting depositors by reducing liquidity risk, a reserve requirement may also help to achieve a (constrained efficient) equilibrium. This provides an additional rational for or at least an interesting side effect of imposing reserve requirements.

The chapter is organized as follows. Section 5.2 describes a model of double Bertrand competition for the special case of banks, including a reserve requirement. Section 5.3 discusses the equilibrium of the model without the reserve requirement for expositional convenience. Section 5.4 analyzes the implications of the reserve requirement for the equilibrium of the model. Section 5.5 concludes.

5.2 The model

Consider commercial banks that obtain deposits from agents and grant loans to (the same or other) agents. Such banks are financial intermediaries, who facilitate the redistribution of assets between agents. Competition between this type of banks can be modelled in a two-stage setting. In the first stage banks obtain funds from depositors, which they can lend to agents in the second stage. We assume the strategic variables of the banks to be prices, i.e. rates of interest on loans and deposits. The model thus has price competition both in the first stage and in the second stage, that is, double Bertrand competition (see also Stahl, 1988; and Yanelle, 1987, 1989, and Freixas and Rochet, 1997, chapter 3, for applications to banking). For simplicity we concentrate on the case of two banks. Analogous to Stahl's (1988) arguments, the results of the model can be generalized to the case of $N \geq 2$ banks.

Assume constant marginal management costs, normalized to zero for simplicity (see also Freixas and Rochet, 1997, p. 65). Assume that there is no infinitely elastic source of funds. Let $L(r_L)$ denote loan demand as a function of the rate of interest on loans r_L, which is continuously differentiable with $L'(r_L) < 0$. Similarly, let $D(r_D)$ denote deposit supply, with $D'(r_D) > 0$. The reserve requirement is determined as an exogenous fraction α of deposits, with $0 \leq \alpha < 1$.

In the first stage, the bank offering the highest deposit rate 'wins' and obtains the total supply of deposits. If the deposit rates of the banks are equal, a tie occurs. Two tie-breaking rules can be used in this situation: the equal-sharing rule, and the random tie-breaking rule. With the equal-sharing rule, the banks share the supply of deposits equally. With the random rule, one of the banks is chosen randomly to obtain the total supply. The former rule leads to a more realistic outcome in which both banks are active in the second stage, but this SPNE does not always exist. Stahl (1988) argues that this non-existence problem results from the continuous strategy space and that the problem can be solved by using a discrete approximation of the game. In that case, banks can only choose from a finite interest rate grid and a mixed-strategy SPNE does exist under the equal-sharing rule. As the grid becomes finer, the probability of ties vanishes and it can be shown that the bank's strategies become concentrated at the deposit rate a monopolist bank would charge. Therefore the outcome in the limit approximates the random tie-breaking outcome (see Stahl, 1988, p. 195). Thus, we focus on the random tie-breaking rule in the analysis below. Evidently, if $\alpha > 0$ the stock of a bank available for lending will not be equal to the volume of deposits acquired in the first stage, but to $1 - \alpha$ times this volume. The amount of loans granted in the second stage is thus constrained by the stock of deposits obtained in the first stage, minus the obligatory reserve.

The Walrasian, constrained efficient equilibrium of this game is given by the interest rates r_L^W and r_D^W that equate demand for loans and supply of deposits, exclusive of the reserve requirement, i.e.

$$L\left(r_L^W\right) = (1-\alpha)\, D\left(r_D^W\right), \tag{5.1}$$

and for which the banks' profits are zero, i.e.

$$r_L^W L\left(r_L^W\right) = r_D^W D\left(r_D^W\right). \tag{5.2}$$

This implies

$$r_L^W = r_D^W / (1-\alpha). \tag{5.3}$$

We assume that the Walrasian equilibrium exists and is unique. If $\alpha = 0$, $r_L^W = r_D^W$, but the Walrasian equilibrium clearly cannot have $r_L^W = r_D^W$ if $\alpha > 0$. Observe that any two of the three conditions (5.1), (5.2), and (5.3) describing the Walrasian equilibrium imply the third.

To see whether this constrained efficient Walrasian equilibrium actually arises, the model can be solved using backward induction. But first we require some additional definitions. Define the 'zero-monopoly-rent' rate of interest on deposits $\widetilde{r}_D$ as the highest deposit rate yielding nonnegative profits in case the bank is a monopolist in the second stage. Let the function $M(x)$ describe the maximum second-stage revenues attainable with given stock x, that is

$$M(x) = \max\{r_L L(r_L) | L(r_L) \leq x\}.$$

The function $M(x)$ is itself independent of α, but $\alpha > 0$ does affect the available stock x. Clearly, $M(x)$ is nondecreasing in x. Therefore, the zero-monopoly-rent deposit rate $\widetilde{r}_D$, defined by setting the profits of the second-stage monopolist equal to zero, satisfies

$$M((1-\alpha) D(\widetilde{r}_D)) - \widetilde{r}_D D(\widetilde{r}_D) = 0.$$

This shows that $\widetilde{r}_D$ depends on α for $\alpha > 0$. Now define $\widehat{r}_L$ as the loan rate that maximizes the strictly concave second-stage revenues $r_L L(r_L)$ (without taking capacity into account). This loan-revenue-maximizing rate $\widehat{r}_L$ is left unaffected by the reserve requirement α. Observe that these definitions imply that if $\widehat{r}_L$ is smaller than or equal to r_L^W, the bank can do no better than to set the Walrasian lending rate in the second stage and $\widetilde{r}_D$ will equal r_D^W in the first stage. So, $\widehat{r}_L \leq r_L^W$ implies $\widetilde{r}_D = r_D^W$. Similarly, $\widehat{r}_L > r_L^W$ means that the winner can earn second-stage profits, which implies $\widetilde{r}_D > r_D^W$ in the first stage. The prospect of future profits leads banks to compete more aggressively for deposits in the first stage. Each of these two cases gives rise to a specific type of equilibrium.

Below, we first discuss the (two types of) equilibrium of the model, focusing on the case $\alpha = 0$ for expositional convenience. Then, we turn to the case $\alpha > 0$ and analyze the effects of a reserve requirement. Clearly, for the case $\alpha = 0$, the lemma's and propositions derived by Stahl (1988) carry over. In this case, the only difference between his model and ours is a change of notation. Anticipating on the discussion of the effects of a reserve requirement ($\alpha > 0$), it can be verified that Stahl's results with respect to (the qualitative properties of) the equilibrium of the model carry over to the case $\alpha > 0$ as well. That is, the value of α does not affect the equilibrium analysis of the model in general. The formal derivation of the equilibrium is analogous to that in Stahl (1988) and is not repeated here. Note, however, that α does affect the *levels* of the equilibrium rates r_L^W, r_D^W, and $\widetilde{r}_D$ (but not $\widehat{r}_L$), as we argued above, and thereby it may affect the precise type of equilibrium resulting for particular parameter values. This issue will be discussed in more detail in section 5.4. Applying Stahl's (1988) analysis, the results presented in the next section can thus be stated for the general case $\alpha \geq 0$, although we focus on the case $\alpha = 0$ for expositional convenience there.

5.3 Discussion of equilibrium: The case $\alpha = 0$

To analyze the equilibria, assume for a moment that $\alpha = 0$ and let $r_L = g(r_D)$ describe the combinations of r_L and r_D for which demand for loans equals supply of deposits (equation (5.1) with $\alpha = 0$). This function is graphed both in figure 5.1, where $\widetilde{r}_D = r^W$, and in figure 5.2, where $\widetilde{r}_D > r^W$. It is downward sloping, because loan demand is a decreasing function, whereas deposit supply is increasing. Since $g(r_D)$ describes equality of demand and supply of funds, to the left of $g(r_D)$, there is excess demand, and to the right of $g(r_D)$, there is excess stock. The zero-profit ($\pi = 0$) curve consists of two parts. Below $g(r_D)$, where there is excess demand, zero profits imply $r_L = r_D$. On $g(r_D)$ itself, demand equals supply, and therefore $r_L > r_D$ implies strictly positive profits. Therefore, the second part of the zero-profit curve must lie above and to the right of $g(r_D)$.

Applying Stahl's (1988) analysis, the following results can be stated. For an SPNE, it is necessary that each bank follows a pure strategy concentrated at the zero-monopoly-rent rate of interest on deposits, $\widetilde{r}_D$ (see Stahl, 1988, pp. 199-200). This result holds independent of the sharing rule used. Intuitively, maximum second-stage revenues are obtained at the rate $\widehat{r}_L$. First-stage competition for these revenues drives r_D up, up till the value $\widetilde{r}_D$ for which profits (setting $r_L = \widehat{r}_L$) will be zero. As figures 5.1 and 5.2 illustrate for the case $\alpha = 0$, the bank's expected profits will never be positive at the interest rate $\widetilde{r}_D$. Evidently, if expected profits are negative, no SPNE exists. Further, the precise SPNE depends on the sharing rule. (We remark here that these results carry through for the general case $\alpha \geq 0$. For a discussion, see the next section.)

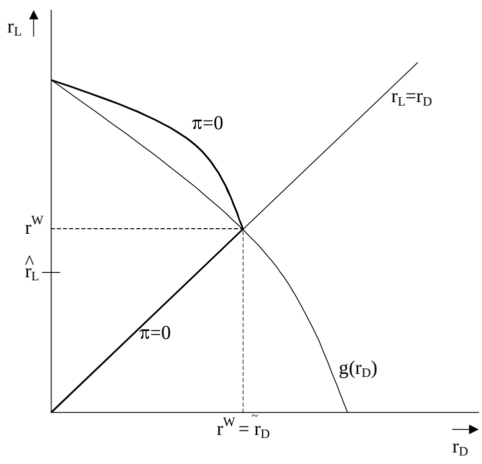

Figure 5.1: Double Bertrand competition among banks without the reserve requirement ($\alpha = 0$): the case $\widetilde{r}_D = r^W$.
Source: Stahl (1988).

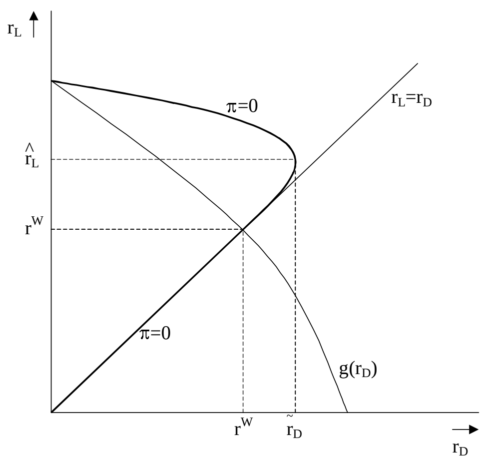

Figure 5.2: Double Bertrand competition among banks without the reserve requirement ($\alpha = 0$): the case $\widetilde{r}_D > r^W$.
Source: Stahl (1988).

First consider the equal-sharing rule. In the subcase where $\widehat{r}_L \leq r_L^W$, the Walrasian outcome is the unique SPNE, so each bank sets $r_D = \widetilde{r}_D = r_D^W$ and $r_L = r_L^W$. If $\widehat{r}_L > r_L^W$, no SPNE exists (see Stahl, 1988, pp. 194-195). Intuitively, in the latter case, if both banks offer $\widetilde{r}_D > r_D^W$ to depositors, they can only make nonnegative profits in the second stage by setting their loan rate equal to $\widehat{r}_L$. However, since banks have excess stock in this case (illustrated in figure 5.2), this cannot be an equilibrium, because by asking a slightly lower loan rate a bank could make positive profits. In the former case (see figure 5.1), a bank cannot raise profits by undercutting and the equilibrium does exist.

Second, consider the random tie-breaking rule. Since with this rule the first stage comes down to auctioning the right to become a monopolist in the second stage, expected profits will again be zero. With random tie breaking, a unique SPNE always exists in which $r_D = \widetilde{r}_D$ and there is a randomly chosen monopolist in the second stage (see Stahl, 1988, p. 196). The nature of the SPNE however depends on the subcase. In the case in which $\widehat{r}_L \leq r_L^W$ the Walrasian prices arise, so both banks set $r_D = \widetilde{r}_D = r_D^W$ and the second-stage monopolist sets $r_L = r_L^W$ (as illustrated in figure 5.1). The standard Walrasian outcome is obtained. In the subcase where $\widehat{r}_L > r_L^W$, however, $r_D = \widetilde{r}_D > r_D^W$ and the monopolist sets $r_L = \widehat{r}_L > r_L^W$. Thus, in the latter subcase the SPNE is inefficient as prices are above the Walrasian prices and there is excess stock in the second stage (see also figure 5.2; there is exess stock since the equilibrium is to the right of $g(r_D)$).

The two subcases can be reformulated in terms of demand elasticities, using the condition that $r_L L(r_L)$ is strictly concave. Under this assumption, the elasticity of loan demand $\eta \equiv -r_L L'(r_L)/L(r_L)$ is strictly increasing in r_L. For the loan-revenue-maximizing rate of interest, $\widehat{r}_L$, the first-order condition is

$$L(\widehat{r}_L) + \widehat{r}_L L'(\widehat{r}_L) = 0,$$

which implies that in this point, the elasticity of demand is $\eta(\widehat{r}_L) = 1$. Therefore, $\widehat{r}_L \leq r_L^W$ if and only if $\eta(r_L) \geq 1$. Similarly, $\widehat{r}_L > r_L^W$ if and only if $\eta(r_L) < 1$.

Summarizing, modelling financial intermediation as double Bertrand competition shows that the efficient Walrasian equilibrium does not necessarily occur. Only if loan demand is elastic, the outcome will be efficient. The intuition behind this result is that in the inelastic case the bank is able to exert market power because borrowers are not very responsive to interest rate changes.[1]

5.4 Effects of a reserve requirement

Now we consider the implications of the introduction of a reserve requirement α, where α is exogenously given and $0 < \alpha < 1$. We compare the resulting equilibrium to the equilibrium without a reserve requirement, i.e. with $\alpha = 0$. As we argued above, the value of α does not affect the equilibrium analysis (as presented in the previous section) of the model in general, but it does influence the precise levels of the equilibrium interest rates. In particular, the Walrasian rates r_D^W and r_L^W as well as the 'zero-monopoly-rent' rate of interest on deposits $\widetilde{r}_D$ depend on α. In contrast, the loan-revenue-maximizing rate $\widehat{r}_L$ is left unaffected by the reserve requirement α.

The Walrasian equilibrium can be characterized by equality of demand and supply, $L\left(r_L^W\right) = (1-\alpha)\, D\left(r_D^W\right)$, and the condition $r_L^W = r_D^W / (1-\alpha)$ (equations (5.1) and (5.3)). The comparative static properties of the Walrasian equilibrium can be derived from this system of two equations by taking the derivative with respect to α and solving for $\mathrm{d}r_L^W/\mathrm{d}\alpha$ and $\mathrm{d}r_D^W/\mathrm{d}\alpha$. This results in

$$\begin{aligned}
\frac{\mathrm{d}r_L^W}{\mathrm{d}\alpha} &= \frac{1}{\Delta}\left(\frac{\mathrm{d}D\left(r_D^W\right)}{\mathrm{d}r_D}\frac{r_D^W}{1-\alpha} + \frac{D\left(r_D^W\right)}{1-\alpha}\right) \\
\frac{\mathrm{d}r_D^W}{\mathrm{d}\alpha} &= \frac{1}{\Delta}\left(\frac{\mathrm{d}L\left(r_L^W\right)}{\mathrm{d}r_L}\frac{r_D^W}{1-\alpha} + D\left(r_D^W\right)\right),
\end{aligned}$$

where

$$\Delta = -\frac{1}{1-\alpha}\frac{\mathrm{d}L\left(r_L^W\right)}{\mathrm{d}r_L} + (1-\alpha)\frac{\mathrm{d}D\left(r_D^W\right)}{\mathrm{d}r_D} > 0.$$

Clearly, $\mathrm{d}r_L^W/\mathrm{d}\alpha > 0$. The introduction of a reserve requirement will therefore increase the Walrasian loan rate. The first term between brackets in the expression for $\mathrm{d}r_D^W/\mathrm{d}\alpha$ is negative, whereas the second term is positive. Thus, the effect on the Walrasian deposit rate is ambiguous, and depends on the size of α. Depending on the precise specifications

[1] Fase (1995, p. 110) discusses estimates of long-run elasticities of demand for short-term credit from various studies for The Netherlands, France, and Belgium, which are all between 0.1 and 0.2. This suggests that the inefficient equilibrium may well be relevant in practice.

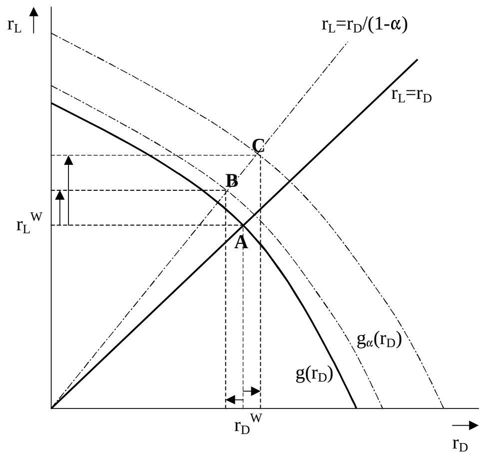

Figure 5.3: Double Bertrand competition among banks: the implications of a reserve requirement.

of the model, $\mathrm{d}r_D^W/\mathrm{d}\alpha$ might be positive if α is small, and it must be negative if α is large (close to 1). The effect on the loan rate can alternatively be observed from the function $r_L = g_\alpha(r_D)$, which describes equality of demand and supply of funds. Evidently, $g_\alpha(r_D)$ is affected by the reserve requirement. Equality of demand and supply is given by $L(r_L) = (1-\alpha) D(r_D)$ so that

$$r_L = L^{-1}((1-\alpha) D(r_D)) = g_\alpha(r_D).$$

This equation shows that an increase in α implies an increase in the value $g_\alpha(r_D)$, given r_D. Therefore, the introduction of a reserve requirement causes an upward (but not necessarily parallel) shift in the function $g_\alpha(r_D)$.

These results are illustrated in figure 5.3, which shows the implications of the introduction of a reserve requirement for the Walrasian equilibrium. The original Walrasian equilibrium with $\alpha = 0$ (see figures 5.1 and 5.2), in which $r_L^W = r_D^W = r^W$, is given by point A. The Walrasian equilibrium of the model with a reserve requirement $\alpha > 0$ is determined by the intersection of the new $g_\alpha(r_D)$ curve (which lies above the old $g(r_D)$ curve) and the line $r_L = r_D/(1-\alpha)$. As shown in figure 5.3, there are two possible cases, illustrated by points B and C.

The new Walrasian equilibrium B refers to the case in which the introduction of a reserve requirement increases r_L^W but decreases r_D^W. The Walrasian equilibrium C illustrates the case in which the introduction of the reserve requirement causes both r_L^W and r_D^W to rise.

The loan rate that maximizes second-stage revenues, $\widehat{r}_L$, is not affected by α. By concentrating on the loan side, it is clear now that the situation in which $\widehat{r}_L > r_L^W$ is less likely to occur after the introduction of the reserve requirement, in the sense that it obtains for a smaller set of parameter values if α is larger. That is, with the equal-sharing rule, the equilibrium will exist more often, and with the random tie-breaking rule, the inefficient equilibrium is less likely to occur. Moreover, with random tie breaking, even if the new equilibrium is again inefficient, the reserve requirement may reduce the level of inefficiency (since equilibrium lending rates will be closer to the Walrasian interest rates; note that this does not necessarily hold for deposit rates as well). It should be noted that the Walrasian equilibrium with a reserve requirement is only constrained efficient. Nevertheless, this analysis shows that imposing a fine-tuned reserve requirement may lead to an equilibrium with no excess funds without increasing the lending rate, and possibly without decreasing the deposit rate.

We illustrate this fine-tuning of the reserve requirement with a simple example below. Consider the case of linear loan demand and linear deposit supply, where

$$\begin{aligned} L(r_L) &= \lambda_1 - \lambda_2 r_L \\ D(r_D) &= \delta_1 + \delta_2 r_D \\ &\quad \lambda_1, \lambda_2, \delta_1, \delta_2 > 0. \end{aligned}$$

In order to determine the value of α for which the equilibrium becomes constrained efficient, the equation $\widehat{r}_L = r_L^W$ has to be solved for α (note that this condition implies that the lending rate charged by the banks is left unaffected by the introduction of a reserve requirement of this particular size). In this case, we have $\widehat{r}_L = \lambda_1 / 2\lambda_2$ and

$$r_L^W = \frac{\lambda_1 - (1-\alpha)\,\delta_1}{\lambda_2 + (1-\alpha)^2\,\delta_2}.$$

Solving for α gives

$$\alpha^* = 1 + \frac{1}{\lambda_1 \delta_2}\left(\lambda_2 \delta_1 - \sqrt{\lambda_2^2 \delta_1^2 + \lambda_1^2 \lambda_2 \delta_2}\right).$$

For the initial equilibrium to be inefficient, it must be the case that $\alpha^* > 0$. For the constrained efficient equilibrium to be feasible, it must be the case that $\alpha^* < 1$. If these conditions are satisfied, it is possible to obtain a constrained efficient outcome by introducing a reserve requirement that satisfies $\alpha^* \leq \alpha < 1$. As a numerical example with an interesting interpretation, consider the case of $\delta_1 = 0$, that is, deposit supply crosses the origin $(D(0) = 0)$. This implies $\alpha^* = 1 - \sqrt{\lambda_2/\delta_2}$. If the loan demand function is steeper than the deposit supply curve, the initial equilibrium will be efficient. If loan demand is less steep than deposit supply, the constrained efficient equilibrium can be reached by introducing a reasonable reserve requirement provided that the slopes of loan demand and deposit supply are (different but) similar.

Summarizing, the effect of the reserve requirement in a model of double Bertrand competition among banks is to increase the Walrasian loan rate and to either increase or decrease the Walrasian deposit rate. Focusing on the loan side and combining this with the result that the loan-revenue-maximizing rate $\widehat{r}_L$ is left unaffected gives the conclusion that a reserve requirement makes (constrained) efficiency of the SPNE more likely, in the sense that it obtains for a larger set of parameter values. If the initial SPNE is inefficient, the introduction of a reserve requirement may result in a constrained efficient equilibrium. This can be done without changing the loan rate (by setting α such that the new value of r_L^W equals the initial loan rate $\widehat{r}_L$). The effect on the deposit rate is ambiguous.

5.5 Conclusion

Reserve requirements may be used by the central bank to influence the appeal of banks to the money market, or to decrease the liquidity risk of banks, and through that increase safety and soundness of the banking sector. The above analysis has shown that reserve requirements can also be applied in order to achieve a constrained efficient equilibrium. If borrowers are not very responsive to loan rates, a bank can exert market power and set interest rates above the perfect-competition level, leading to an excess stock of funds at the bank. Since a reserve requirement decreases the excess amount of funds, its introduction may force the bank to set its lending rate at the Walrasian level, thus achieving constrained efficiency.