On competition and banking

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Chapter 4

The effects of an industry-wide cost change in banking

4.1 Introduction

This chapter investigates an extension of the well-known Klein-Monti model of a profit-maximizing bank, originally introduced by Klein (1971) and Monti (1972). For a discussion of this model, see chapter 2. There we argued that the original model, which analyzes the behavior of a single, monopolistic bank, can easily be extended to the symmetric Cournot case. Freixas and Rochet (1997, pp. 59-61) discuss this extension, assuming that all banks face the same linear management cost function. They also examine some comparative static properties of both the original model and the symmetric Cournot version with respect to changes in the exogenous interbank market interest rate or policy rate. Such changes can be made in the context of monetary policy by the central bank, in order to influence the volumes of loans and deposits of banks and the corresponding interest rates. Freixas and Rochet (1997, pp. 59-61) show that an increase in the policy rate decreases a bank's volume of loans, increases its volume of deposits, and increases the interest rates on both loans and deposits.

Here, we are interested in other forms of market structure, in partic-

[0]This chapter is a revised version of Toolsema and Schoonbeek (2000). Section 4.4 is based on Toolsema and Schoonbeek (1999).

ular in asymmetric extensions of the Klein-Monti model. Such asymmetries may affect the comparative static effects of policy rate adjustments. Two types of asymmetry can be considered. First, in the Cournot version of the model there could be differences across banks in the costs of managing loans and deposits. For an analysis of this case, see Toolsema and Schoonbeek (1999). It is shown there that for the case of two banks, the comparative static effects of a policy rate change on the individual volumes of loans and deposits of the bank with the lowest management cost may change direction. The second type of asymmetry refers to conduct. In this chapter, we focus on a Stackelberg-version of the Klein-Monti model, in which one bank is the leader, who sets its loan and deposit volumes first, and the other bank is a follower. We show that this asymmetric conduct may imply counterintuitive comparative static effects of a change in the policy rate on the leader's volumes of loans and deposits.

However, our results on the comparative static effects of a change in the policy rate in the context of the Stackelberg-version of the Klein-Monti model are more general. The results do not only hold for the banking case, but can be interpreted in terms of the general Stackelberg duopoly model. In that interpretation, the focus is on the comparative statics of a change in an industry-wide marginal cost. Therefore, the model is first presented in the broader context of a general Stackelberg duopoly. After presenting the comparative static effects of an industry-wide cost change, we turn to the case of banks. We briefly describe the Stackelberg-version of the Klein-Monti model of bank behavior, and reinterpret our results in terms of banks and policy rate adjustments. For a more extensive discussion of the banking case, see Toolsema and Schoonbeek (1999).

Thus, we first focus on a Stackelberg leader-follower duopoly with quantity competition on a market for a homogeneous good (see e.g. Shapiro, 1989). Suppose that both firms have the same constant marginal costs. For instance, both firms purchase their input in the same upstream market and use it to produce an output which they sell on the duopolistic downstream market. Alternatively, one can also interpret a change in the common marginal costs in terms of a change in an output tax faced by both firms. We want to investigate the comparative static effects of a change in these industry-wide costs in the Stackelberg equilibrium. We demonstrate that as a result of a cost increase the total market output will decrease. This is a standard result. However, it turns out that the individual output of the leader firm might *increase*. The intuition for this result runs as follows. A cost increase leads to a decrease in the follower's output, which implies a price increase. Furhter, an increase in the leader's quantity decreases price, but also decreases the follower's quantity which in turn increases price. We may have that the upward effects on the price dominate. In that case, the optimal response for the leader firm to a cost increase is to increase its output. We present and discuss in detail conditions under which the counterintuitive result for the leader firm occurs.

The observation that comparative static effects in oligopolistic markets might be counterintuitive is also made in some related studies. Kimmel (1992) investigates the effects of common cost changes in a Cournot oligopoly. Dixit (1986) investigates a general conjectural variations oligopoly with quantity competition. The conjectural variations may differ across firms. The Stackelberg case can be obtained as a special case by choosing them in an appropriate way. However, Dixit focuses mainly on the general methodology of comparative statics and only mentions the Stackelberg case in passing without further analyzing it. Katz and Rosen (1985) consider a similar kind of oligopoly in which the conjectural variations are identical for all firms. As a result, the Stackelberg case does not fit within their framework. Finally, Caputo (1996, 1998) discusses comparative static properties of, respectively, Nash equilibria and Stackelberg equilibria using a dual methodology which is based on the Envelope Theorem. However, Caputo (1996, 1998) focuses mainly on the general methodology of deriving comparative static effects, and our results do not readily follow from that analysis.

This chapter is organized as follows. Section 4.2 gives the Stackelberg duopoly model. The results are presented in section 4.3. Section 4.4 discusses the banking case. Section 4.5 concludes.

4.2 The Stackelberg duopoly model

Consider a duopolistic market for which the inverse market demand function is $p(q)$, with derivative $p'(q) < 0$. Here $q \equiv q_1 + q_2$ denotes total output, and q_j is the output of firm j, $j = 1, 2$. Firm 1 is the leader (i.e., it determines its output first, in period 1), and firm 2 the follower (who determines its output in period 2). The cost function of firm j is assumed to be linear and given by $c(q_j) = cq_j + d_j$, with $c > 0$ and $d_j \geq 0$, $j = 1, 2$. Thus, both firms have common constant marginal costs c, and possibly different fixed costs d_j. For analytical tractability, we have not included firm specific marginal costs.

The two-stage model is solved using backward induction. Starting in the second stage, the profit maximization problem of the follower is

$$\max_{q_2} \quad \pi_2(q_2) = p(q_1 + q_2)q_2 - cq_2 - d_2,$$

where we assume that the profit function $\pi_2(\cdot)$ is strictly concave in q_2. The corresponding first-order condition (FOC) for profit maximization reads

$$p'(q_1 + q_2)q_2 + p(q_1 + q_2) - c = 0. \tag{4.1}$$

This condition implicitly defines the reaction function $q_2 = f(q_1, c)$ of firm 2. The partial derivatives of $f(q_1, c)$ with respect to q_1 and c are denoted as $f'_q(q_1, c)$ and $f'_c(q_1, c)$, respectively. Differentiating (4.1) with respect to q_1 we have

$$f'_q(q_1, c) = -\frac{p'(\cdot) + p''(\cdot)f(q_1, c)}{2p'(\cdot) + p''(\cdot)f(q_1, c)}, \tag{4.2}$$

where the first-order and second-order derivatives of $p(\cdot)$ are evaluated in $q_1 + f(q_1, c)$. The denominator equals the second-order derivative of $\pi_2(\cdot)$ and therefore is negative. Hence $f'_q(q_1, c) > -1$. We restrict ourselves to the case $f'_q(q_1, c) < 0$, that is, we focus on decreasing reaction functions. This is the normal case with quantity competition (see Shapiro, 1989, p. 337). It is equivalent to assuming that $p''(\cdot) < -p'(\cdot)/f(q_1, c)$, i.e. that the inverse demand function is not too convex. Analogous to (4.2), differentiating (4.1) with respect to c shows

$$f'_c(q_1, c) = \frac{1}{2p'(\cdot) + p''(\cdot)f(q_1, c)}.$$

As a result, $f'_c(q_1, c) < 0$.

Turning to the first stage, the leader firm takes into account how the follower will respond to its choice of q_1. The profit-maximization problem for the leader is

$$\max_{q_1} \quad \pi_1(q_1) = p(q_1 + f(q_1, c))q_1 - cq_1 - d_1,$$

where we assume that the profit function $\pi_1(\cdot)$ is strictly concave. The FOC for the leader is

$$p'(q_1 + f(q_1, c))[1 + f'_q(q_1, c)]q_1 + p(q_1 + f(q_1, c)) - c = 0. \tag{4.3}$$

We assume that a unique (positive) Stackelberg equilibrium $(\widetilde{q}_1, \widetilde{q}_2)$ exists, characterized by (4.1) and (4.3). That is, we concentrate on the case where both $\widetilde{q}_1$ and $\widetilde{q}_2$ are strictly positive. The total equilibrium output is $\widetilde{q} \equiv \widetilde{q}_1 + \widetilde{q}_2$, and the equilibrium price is $\tilde{p} \equiv p(\widetilde{q})$. It follows from (4.1) and (4.3) that

$$\widetilde{q}_2 = f(\widetilde{q}_1, c) = [1 + f'_q(\widetilde{q}_1, c)]\widetilde{q}_1. \tag{4.4}$$

As a result $\widetilde{q}_1 > \widetilde{q}_2$, i.e. the equilibrium output of the leader firm is largest.

4.3 The results

Now consider the effects of a change in the common marginal cost c. For the follower, we have

$$\frac{\mathrm{d}\widetilde{q}_2}{\mathrm{d}c} = f'_q(\widetilde{q}_1, c)\frac{\mathrm{d}\widetilde{q}_1}{\mathrm{d}c} + f'_c(\widetilde{q}_1, c),$$

which implies

$$\frac{\mathrm{d}\widetilde{q}}{\mathrm{d}c} = \frac{\mathrm{d}\widetilde{q}_1}{\mathrm{d}c} + \frac{\mathrm{d}\widetilde{q}_2}{\mathrm{d}c} = [1 + f'_q(\widetilde{q}_1, c)]\frac{\mathrm{d}\widetilde{q}_1}{\mathrm{d}c} + f'_c(\widetilde{q}_1, c). \tag{4.5}$$

Using this, we can derive the following lemma.

Lemma 4.1 *In the Stackelberg model where firm 1 is the leader and firm 2 the follower, we have* $d\widetilde{q}_1/dc = A_1/A_3$ *and* $d\widetilde{q}/dc = A_2/A_3$*, with* $A_3 < 0$ *the second-order derivative of the (strictly concave) profit function of firm 1 with respect to* q_1*, and*

$$\begin{aligned} A_1 &= -6(f'_q(\cdot))^2 - 6f'_q(\cdot) - 1 + B \\ A_2 &= -(f'_q(\cdot))^2 + 1 > 0, \end{aligned}$$

where

$$B = \frac{[1 + f'_q(\cdot)]p'''(\cdot)(f(\cdot))^2}{2p'(\cdot) + p''(\cdot)f(\cdot)},$$

and all expressions are evaluated in the Stackelberg equilibrium.

PROOF In order to provide the proof, the following equations are helpful:

$$f'_q(\cdot) = -\frac{p'(\cdot) + p''(\cdot)f(\cdot)}{2p'(\cdot) + p''(\cdot)f(\cdot)} \tag{4.6}$$

$$1 + f'_q(\cdot) = \frac{p'(\cdot)}{2p'(\cdot) + p''(\cdot)f(\cdot)} \tag{4.7}$$

$$f'_c(\cdot) = \frac{1}{2p'(\cdot) + p''(\cdot)f(\cdot)} \tag{4.8}$$

$$\frac{\partial^2 f(\cdot)}{\partial q_1 \partial c} = \frac{-[p'''(\cdot)f(\cdot) + 2p''(\cdot)][1 + f'_q(\cdot)]f'_c(\cdot) - p''(\cdot)f'_q(\cdot)f'_c(\cdot)}{2p'(\cdot) + p''(\cdot)f(\cdot)} \tag{4.9}$$

$$\frac{\partial^2 f(\cdot)}{\partial q_1^2} = \frac{-[p'''(\cdot)f(\cdot) + p''(\cdot)][1 + f'_q(\cdot)]^2 - 2p''(\cdot)f'_q(\cdot)[1 + f'_q(\cdot)]}{2p'(\cdot) + p''(\cdot)f(\cdot)}, \tag{4.10}$$

where the derivatives of $f(\cdot)$ have been computed by differentiating the FOC (4.1) of the follower.

First concentrate on the leader. Differentiating the FOC (4.3) with respect to c, and solving for $\mathrm{d}q_1/\mathrm{d}c$ gives the result that in the Stackelberg equilibrium we have $\mathrm{d}\widetilde{q}_1/\mathrm{d}c = A_1/A_3$, where

$$A_1 = 1 - p'(\cdot)f'_c(\cdot) - p''(\cdot)[1 + f'_q(\cdot)]\widetilde{q}_1 f'_c(\cdot) - p'(\cdot)\widetilde{q}_1 \frac{\partial^2 f(\cdot)}{\partial q_1 \partial c} \tag{4.11}$$

and

$$A_3 = 2p'(\cdot)[1 + f'_q(\cdot)] + p''(\cdot)[1 + f'_q(\cdot)]^2 \widetilde{q}_1 + p'(\cdot)\widetilde{q}_1 \frac{\partial^2 f(\cdot)}{\partial q_1^2} < 0.$$

Here, all derivatives of $p(\cdot)$ are evaluated in the point $\widetilde{q}_1 + f(\widetilde{q}_1, c)$, and $f(\cdot)$ and its derivatives are evaluated in $(\widetilde{q}_1, c)$. Using (4.6), (4.7), (4.8), and (4.9), it can be verified that (4.11) can be rewritten as

$$A_1 = -f'_q(\cdot) - \frac{p'(\cdot)p''(\cdot)[1 + f'_q(\cdot)]\widetilde{q}_1}{[2p'(\cdot) + p''(\cdot)f(\cdot)]^2} - \frac{2\,(p''(\cdot))^2\, f(\cdot)[1 + f'_q(\cdot)]\widetilde{q}_1}{[2p'(\cdot) + p''(\cdot)f(\cdot)]^2} + B, \tag{4.12}$$

where

$$B \equiv \frac{[1 + f_q'(\cdot)]p'''(\cdot)(f(\cdot))^2}{2p'(\cdot) + p''(\cdot)f(\cdot)}.$$

Recalling (4.4), we substitute $f(\cdot) = [1 + f_q'(\cdot)]\widetilde{q}_1$ into (4.12). Rewriting the resulting expression using (4.6) and (4.7) shows that A_1 can be written as

$$A_1 = -6(f_q'(\cdot))^2 - 6f_q'(\cdot) - 1 + B, \tag{4.13}$$

which proves the part concerning A_1 of lemma 4.1.

Next, in order to demonstrate the part concerning A_2, we observe that it follows from (4.5) that $\mathrm{d}\widetilde{q}/\mathrm{d}c = A_2/A_3$, where

$$A_2 = (1 + f_q'(\cdot))A_1 + f_c'(\cdot)A_3. \tag{4.14}$$

By making use of (4.10) and applying the same methods as above, $f_c'(\cdot)A_3$ can be shown to satisfy

$$f_c'(\cdot)A_3 = 6(f_q'(\cdot))^3 + 11(f_q'(\cdot))^2 + 7f_q'(\cdot) + 2 - [1 + f_q'(\cdot)]B. \tag{4.15}$$

Substituting (4.13) and (4.15) into (4.14) gives

$$A_2 = -(f_q'(\cdot))^2 + 1,$$

which completes the proof. ■

Note that the sign of B is opposite to the sign of $p'''(\cdot)$, the third-order derivative of $p(\cdot)$ in $\widetilde{q}$.

Using $-1 < f_q'(\widetilde{q}_1, c) < 0$ and lemma 4.1, we easily obtain the following proposition on the effects of a change in the marginal costs c.

Proposition 4.1 *In the Stackelberg model where firm 1 is the leader and firm 2 is the follower, the following holds:*

(a) $\frac{d\widetilde{q}}{dc} < 0$ *and* $\frac{d\widetilde{p}}{dc} > 0$*;*

(b) Let $p'''(\widetilde{q}_1+\widetilde{q}_2) = 0$*. Then* $\frac{d\widetilde{q}_1}{dc} < 0$ *if and only if* $s_1 < f_q'(\widetilde{q}_1, c) < s_2$*;*

(c) Let $p'''(\widetilde{q}_1 + \widetilde{q}_2) < 0$*. Then* $f_q'(\widetilde{q}_1, c) \in (s_1, s_2)$ *implies* $\frac{d\widetilde{q}_1}{dc} < 0$*;*

(d) Let $p'''(\widetilde{q}_1 + \widetilde{q}_2) > 0$*. Then* $f_q'(\widetilde{q}_1, c) \in (-1, s_1]$ *or* $f_q'(\widetilde{q}_1, c) \in [s_2, 0)$ *implies* $\frac{d\widetilde{q}_1}{dc} > 0$*;*

where $s_1 = -\frac{1}{2} - \frac{1}{6}\sqrt{3} \approx -0.79$ *and* $s_2 = -\frac{1}{2} + \frac{1}{6}\sqrt{3} \approx -0.21$.

We make a number of remarks with respect to proposition 4.1. First, part (a) shows that the comparative static effect on the total output $\widetilde{q}$ has the 'normal' negative sign. As a result, the comparative static effect on the price $\tilde{p}$ has the 'normal' positive sign. Second, parts (b) and (d) point out that the effect on the output of the leader firm can be counterintuitive, i.e. $\mathrm{d}\widetilde{q}_1/\mathrm{d}c > 0$. Third, the critical values s_1 and s_2 are located symmetrically around $-\frac{1}{2}$. If the inverse demand function is linear, we have $f'_q(\widetilde{q}_1, c) = -\frac{1}{2}$. Thus, in the situations in which $\mathrm{d}\widetilde{q}_1/\mathrm{d}c > 0$, the slope $f'_q(\widetilde{q}_1, c)$ of the follower's reaction function is sufficiently different from its value in the linear case. Loosely speaking, counterintuitive effects for the leader can occur if we are sufficiently far away from the linear case. Notice that if $p(\cdot)$ is a quadratic function, then a priori part (b) of the proposition is relevant. Hence, in that case the counterintuitive effect for the leader occurs if and only if the slope $f'_q(\widetilde{q}_1, c)$ of the follower's reaction function lies in either $(-1, s_1]$ or $[s_2, 0)$. Fourth, the conditions in proposition 4.1 are related to the curvature of the inverse demand function $p(\cdot)$ in the equilibrium $\widetilde{q}$. In particular, the value of $p''(\widetilde{q})$ determines whether $p(\widetilde{q})$ is (strictly) concave or (strictly) convex in $\widetilde{q}$. In turn, the value of $p'''(\widetilde{q})$ determines the change in the concavity or convexity of $p(\cdot)$ in $\widetilde{q}$. For instance, if $p''(\widetilde{q}) < 0$ and $p'''(\widetilde{q}) < 0$, then $p(\cdot)$ is concave in $\widetilde{q}$ and the extent of concavity is increasing in $\widetilde{q}$ as well.

In order to better understand the counterintuitive effect on the leader's output, consider part (b) of proposition 4.1 and suppose that c increases. If the outputs of both firms would remain constant, then the price and revenues of both firms would not change, the total cost of both firms would increase, and thus their profits would decrease. However, both firms will react to the increase in c. To begin with, we can identify a direct effect on the output of the follower; i.e. supposing that the output of the leader remains constant, the follower will decrease its own output (since $f'_c(\widetilde{q}_1, c) < 0$). In turn, this gives a downward effect on the total cost of the follower, and, more important for what follows, also has an upward impact on the price (which weakens the decrease in the follower's revenues caused by its lower output). We denote this price effect as the 'direct follower price-effect'. A similar (direct) negative effect on the output might be discussed with respect to the leader. However, the leader also takes into account the indirect effect of changes in its own output on the output of the follower. The latter effect depends on the size of $f'_q(\widetilde{q}_1, c)$. Recall that $f'_q(\widetilde{q}_1, c) < 0$, i.e. a change in the leader's output leads to an opposite change in the follower's output. The question is under which circumstances the leader will increase its output if c increases. Notice that in that case the total cost of the leader certainly will increase.

Summarizing this intuition, a cost increase leads to a decrease in the follower's output, which drives the price up (the 'direct follower price-effect'). If the leader chooses to increase its output, the direct effect of this action is to lower the price. However, it also leads to an additional reduction of the the follower's output, which drives the price up again (the indirect effect). We argue below that if either of the upward effects on price is strong enough, the leader finds it optimal to increase its output.

The relative magnitudes of the 'direct follower price-effect' and the indirect effect are related through the equality $p'(\widetilde{q})f'_c(\widetilde{q}_1, c) = 1 + f'_q(\widetilde{q}_1, c)$. Note that $0 < 1 + f'_q(\widetilde{q}_1, c) < 1$. Thus, if 'direct follower price-effect' $p'(\widetilde{q})f'_c(\widetilde{q}_1, c)$ is large, then $f'_q(\widetilde{q}_1, c)$ is small in absolute value (say $f'_q(\widetilde{q}_1, c) \in [s_2, 0))$. Similarly, if $p'(\widetilde{q})f'_c(\widetilde{q}_1, c)$ is small, then $f'_q(\widetilde{q}_1, c)$ is large in absolute value (say $f'_q(\widetilde{q}_1, c) \in (-1, s_1])$. This relationship, indicating that the direct effect is strong when the indirect effect is weak and vice versa, is the underlying idea behind the discussion below.

Suppose first that the 'direct follower price-effect' $p'(\widetilde{q})f'_c(\widetilde{q}_1, c)$ is large, i.e. through this channel there is a large upward effect on the price. In this situation, one can imagine that it is profitable for the leader to increase its output. Admittedly, such an increase in the leader's output has a (direct) downward effect on the price; however, it also has a (small) downward indirect effect on the follower's output, which in turn reinforces the large upward 'direct follower price-effect'. Summarizing, although the increase in the leader's output has a (direct) downward effect on the price, one can still expect that the upward effect dominates because of the large upward 'direct follower price-effect'. The price increase that ultimately results and the increase in the output of the leader both give a positive effect on the revenues of the leader. The occurrence of this situation is confirmed in part (b) of proposition 4.1 (for $f'_q(\widetilde{q}_1, c) \in [s_2, 0))$.

Now suppose that the upward 'direct follower price-effect' is small. Because in that situation $f'_q(\widetilde{q}_1, c)$ is large in absolute value, again one can imagine that it is profitable for firm 1 to increase its output. Clearly, this again has a (direct) downward effect on the price. However, now the increase in the leader's output is (almost) completely offset by a decrease in the follower's output, and thus has (almost) no effect on the price. Summarizing, we expect that in this case the upward 'direct follower price-effect' dominates. Thus, ultimately the price increases, which together with the increase in the output of firm 1 gives a positive effect on the revenues of the leader. The occurrence of this situation is confirmed in part (b) of proposition 4.1 (for $f'_q(\widetilde{q}_1, c) \in (-1, s_1])$.

Now consider the case in which $p'(\widetilde{q})f'_c(\widetilde{q}_1, c)$ is of intermediate size. In that case, $f'_q(\widetilde{q}_1, c)$ is in absolute value of intermediate size as well (say $f'_q(\widetilde{q}_1, c) \in (s_1, s_2)$). Suppose that in this case, the leader would increase its output. As always, this has a (direct) downward effect on the price. However, now this downward effect is neither dominated by a large upward 'direct follower price-effect', nor offset by an (almost) equal upward price effect caused by the decrease in the output of the follower (that results from the increase in firm 1's output). Hence, one might expect that ultimately the price would decrease, and that in this case an increase in its output would not be profitable for firm 1. This is indeed confirmed in part (b) of proposition 4.1 (for $f'_q(\widetilde{q}_1, c) \in (s_1, s_2)$).

Concluding this section, we make two final comments. First, using lemma 4.1 we see that there is a counterintuitive effect with respect to the output of the follower firm, i.e. $\mathrm{d}\widetilde{q}_2/\mathrm{d}c > 0$, if and only if $A_2 - A_1 < 0$. One can easily verify that this situation can only occur if both $p'''(\widetilde{q}) < 0$ and B is relatively large. Ceteris paribus, the latter means that the size of the third-order derivative $p'''(\widetilde{q})$ must be relatively large in absolute value. Second, we remark that analysis of the comparative static effects on the firms' profits is cumbersome, and does not lead to clear-cut interesting results.

4.4 The banking case

This section returns to the case of banks. We briefly present a Stackelberg-version of the well-known Klein-Monti model of a representative, profit-maximizing bank, originally introduced by Klein (1971) and Monti (1972). The Klein-Monti model considers banks as profit-maximizing firms that offer services to agents. These services are described by the securities that banks buy from agents (i.e. loans) and sell to agents (i.e. deposits). So, banks are operating on two markets at the same time. The difference between the volume of deposits and the volume of loans is the bank's (net) position on the interbank or money market.

In chapter 2 we briefly discussed the original Klein-Monti model of a monopolistic bank. There, we also presented the first-order conditions for profit maximization, from which the optimal volumes of loans and deposits (L^* and D^*) can be derived. Note that in this model of a profit-maximizing bank there are two markets, for loans and deposits, respectively. The results of the general model of sections 4.2 and 4.3 can be applied to both markets, as we show below. For details, see Toolsema and Schoonbeek (1999).

Intuitively, one would expect an increase in the interbank market rate to lead to a decrease in a bank's volume of loans, an increase in its volume of deposits, and increases in the interest rates on loans and deposits. This is exactly what occurs both in the original, monopolistic Klein-Monti model and in the symmetric Cournot version of Freixas and Rochet (1997, pp. 59-61). In this section we illustrate that this result does not necessarily hold in asymmetric oligopolistic generalizations of the model. We focus on the Stackelberg case here, where conduct is asymmetric.

We start by introducing some notation, and relating it to the notation of sections 4.2 and 4.3. In general, both the market for loans and the market for deposits can be interpreted as a Stackelberg duopoly, and the results derived above apply to each of these markets. The most notable difference is that some signs are reversed for the deposit market, since in that market consumers do not *demand* a product (as they do with loans) but instead *supply* it.

The volume of loans L and the volume of deposits D correspond to the quantity q in the general model above. Interest rates on the loan market and deposit market are denoted by r_L and r_D, respectively, and correspond to the price p. The inverse demand function for loans is given by $r_L(L)$, with derivative $r'_L(L) < 0$, and the inverse supply function of deposits is $r_D(D)$, with derivative $r'_D(D) > 0$. Both functions correspond to the function $p(q)$ of the general model, although $r_D(D)$ is described here as a supply function instead of a demand function. An alternative interpretation of this function is that of the demand for deposit services. For bank j, $j = 1, 2$, the cost of managing an amount L_j of loans and an amount D_j of deposits is given by the management-cost function $C_j(L_j, D_j)$. For simplicity, this function is assumed to be linear, i.e. $C_j(D_j, L_j) = \gamma_{L,j} L_j + \gamma_{D,j} D_j$. The functions $r_L(\cdot)$ and $r_D(\cdot)$ are continuously differentiable up to any order. Let i denote the exogenous interest rate on the interbank market, and α be the exogenous fraction of deposits that is required as a non-interest bearing reserve ($0 \leq \alpha < 1$). Both i and α are set by the central bank. The rate i can thus be interpreted as a policy rate, or alternatively as the industry-wide marginal cost c.

Now consider the Stackelberg model of quantity leadership. Suppose that there are two banks. Bank 1 is the leader, who can set its quantities L_1 and D_1 first, and bank 2 is the follower. For simplicity we assume that the management-cost functions of the banks are equal. That is, the only asymmetry is in the way of conduct. This two-stage model is solved backwards. In the second stage, bank 2 maximizes its profits

$$\begin{aligned}\pi_2(L_2, D_2) &= [r_L(L_1 + L_2) - i]L_2 \\ &\quad + [i(1-\alpha) - r_D(D_1 + D_2)]D_2 - C(L_2, D_2),\end{aligned}$$

taking as given the output (L_1, D_1) of bank 1. This maximization problem is the same as that of a Cournot bank. The FOCs for the follower are given by

$$r'_L(L_1 + L_2)L_2 + r_L(L_1 + L_2) - i - \gamma_L = 0 \quad (4.16)$$

$$i(1-\alpha) - r'_D(D_1 + D_2)D_2 - r_D(D_1 + D_2) - \gamma_D = 0. \quad (4.17)$$

It is convenient to write the reaction function related to the loans of firm 2 as $L_2 = f(L_1, i)$, i.e. we include i explicitly as an argument and omit the subscript '2' of $f(\cdot)$. As a matter of notation, the first-order partial derivatives of $f(\cdot)$ with respect to L_1 and i will be abbreviated as $f'_L(\cdot)$ and $f'_i(\cdot)$, respectively. These derivatives correspond directly to the derivatives $f'_q(q_1, c)$ and $f'_c(q_1, c)$ in section 4.3. In a similar way, we write the reaction function for the deposit side of bank 2 as $g(D_1, i)$, with first-order partial derivatives $g'_D(\cdot)$ and $g'_i(\cdot)$. Clearly, these derivatives also correspond to $f'_q(q_1, c)$ and $f'_c(q_1, c)$. We assume again that reaction functions are downward sloping and obtain $-1 < f'_L(\cdot) < 0$ and $-1 < g'_D(\cdot) < 0$ (see Toolsema and Schoonbeek, 1999, for details).

Next, consider the first stage of the model. Bank 1 wants to choose the amounts L_1 and D_1 such that its profit is maximized, taking into account how bank 2 will respond to its choice. The problem for bank 1 is therefore

$$\begin{aligned}\max_{(L_1, D_1)} \pi_1(L_1, D_1) &= [r_L(L_1 + f(L_1, i)) - i]L_1 \\ &\quad + [i(1-\alpha) - r_D(D_1 + g(D_1, i))]D_1 - C(L_1, D_1),\end{aligned}$$

where we again assume that the profit function $\pi_1(\cdot)$ is strictly concave. The FOCs for the leader are

$$\begin{aligned} r_L'(L_1 + f(L_1, i))[1 + f_L'(\cdot)]L_1 & \\ +r_L(L_1 + f(L_1, i)) - i - \gamma_L &= 0 \qquad (4.18) \\ i(1-\alpha) - r_D'(D_1 + g(D_1, i))[1 + g_D'(\cdot)]D_1 & \\ -r_D(D_1 + g(D_1, i)) - \gamma_D &= 0. \qquad (4.19) \end{aligned}$$

We again assume that a unique (positive) Stackelberg equilibrium exists. It is characterized by the four FOCs (4.16), (4.17), (4.18), and (4.19), and denoted by $\widetilde{L}_1$, $\widetilde{L}_2$, $\widetilde{D}_1$ and $\widetilde{D}_2$. The corresponding total equilibrium volumes are $\widetilde{L} \equiv \widetilde{L}_1 + \widetilde{L}_2$ and $\widetilde{D} \equiv \widetilde{D}_1 + \widetilde{D}_2$, and the corresponding interest rates are $\widetilde{r}_L$ and $\widetilde{r}_D$. It follows from (4.16) and (4.18) and the assumption that the marginal management costs of loans of the two banks are equal that

$$f(\cdot) = \widetilde{L}_2(\cdot) = [1 + f_L'(\cdot)]\widetilde{L}_1. \qquad (4.20)$$

As a result, $\widetilde{L}_1 > \widetilde{L}_2$, i.e. the volume of loans of the leader bank is largest. Similarly, it can be shown that $\widetilde{D}_1 > \widetilde{D}_2$.

Thus, the discussion of both the loan and the deposit market in this Stackelberg-version of the Klein-Monti model is completely analogous to the general model analyzed in sections 4.2 and 4.3. Applying the results from section 4.3, we obtain the following corollary of proposition 4.1.

Corollary 4.1 *In the Stackelberg version of the model, where bank 1 is the leader and bank 2 is the follower, the following holds:*

(a) $\frac{d\widetilde{L}}{di} < 0$ *and* $\frac{d\widetilde{r}_L}{di} > 0$*;*

(b) Let $r_L'''(\widetilde{L}_1 + \widetilde{L}_2) = 0$*. Then* $\frac{d\widetilde{L}_1}{di} < 0$ *if and only if* $s_1 < f_L'(\widetilde{L}_1, i) < s_2$*;*

(c) Let $r_L'''(\widetilde{L}_1 + \widetilde{L}_2) < 0$*. Then* $f_L'(\widetilde{L}_1, i) \in (s_1, s_2)$ *implies* $\frac{d\widetilde{L}_1}{di} < 0$*;*

(d) Let $r_L'''(\widetilde{L}_1 + \widetilde{L}_2) > 0$*. Then* $f_L'(\widetilde{L}_1, i) \in (-1, s_1]$ *or* $f_L'(\widetilde{L}_1, i) \in [s_2, 0)$ *implies* $\frac{d\widetilde{L}_1}{di} > 0$*;*

(e) $\frac{d\widetilde{D}}{di} > 0$ *and* $\frac{d\widetilde{r}_D}{di} > 0$*;*

(f) *Let* $r'''_D(\widetilde{D}_1+\widetilde{D}_2) = 0$. *Then* $\frac{d\widetilde{D}_1}{di} > 0$ *if and only if* $s_1 < g'_D(\widetilde{D}_1, i) < s_2$;

(g) *Let* $r'''_D(\widetilde{D}_1 + \widetilde{D}_2) > 0$. *Then* $g'_D(\widetilde{D}_1, i) \in (s_1, s_2)$ *implies* $\frac{d\widetilde{D}_1}{di} > 0$;

(h) *Let* $r'''_D(\widetilde{D}_1 + \widetilde{D}_2) < 0$. *Then* $g'_D(\widetilde{D}_1, i) \in (-1, s_1]$ *or* $g'_D(\widetilde{D}_1, i) \in [s_2, 0)$ *implies* $\frac{d\widetilde{D}_1}{di} < 0$;

where $s_1 = -\frac{1}{2} - \frac{1}{6}\sqrt{3} \approx -0.79$ *and* $s_2 = -\frac{1}{2} + \frac{1}{6}\sqrt{3} \approx -0.21$.

This corollary directly corresponds to proposition 4.1; the main change is that it has now two 'components'. The first component (parts (a)-(d)) refers to the market for loans and the second (parts (e)-(h)) to that for deposits. We will not discuss the individual components of this corollary in detail; the interpretation is similar to that of proposition 4.1. The main result is that in a Stackelberg-version of the Klein-Monti model, i.e. with asymmetric conduct among banks, there may be counterintuitive comparative static effects of a change in the policy rate on the leader's volumes of loans and deposits. That is, policy rate changes may have adverse effects on the leader bank's volumes of loans and deposits.

4.5 Conclusion

In this chapter, we considered a homogeneous Stackelberg leader-follower duopoly with quantity competition, in which both firms face the same constant marginal costs. We investigated the comparative static effects of a cost change. We have shown that a cost increase will lead to a decrease of the total market output, which is a standard result. However, the output of the leader firm might increase. Intuitively, a cost increase leads to a decrease in the follower's output, and thus increases price. An increase in the leader's quantity decreases price, but also decreases the follower's quantity, which in turn leads to a price increase. The upward effects on price may dominate, in which case it is optimal for the leader firm to respond to a cost increase by increasing its output. We presented conditions under which this counterintuitive effect occurs. Also, we reinterpreted the results in the context of a Stackelberg version of the well-known Klein-Monti model of banking. There, the industry-wide marginal cost refers to the interbank or policy rate. We have shown that an increase in this rate might increase the leading bank's volume of loans, and decrease its volume of deposits.