Synergies and end-effector kinematics in upper limb movements

Tuitert, Inge

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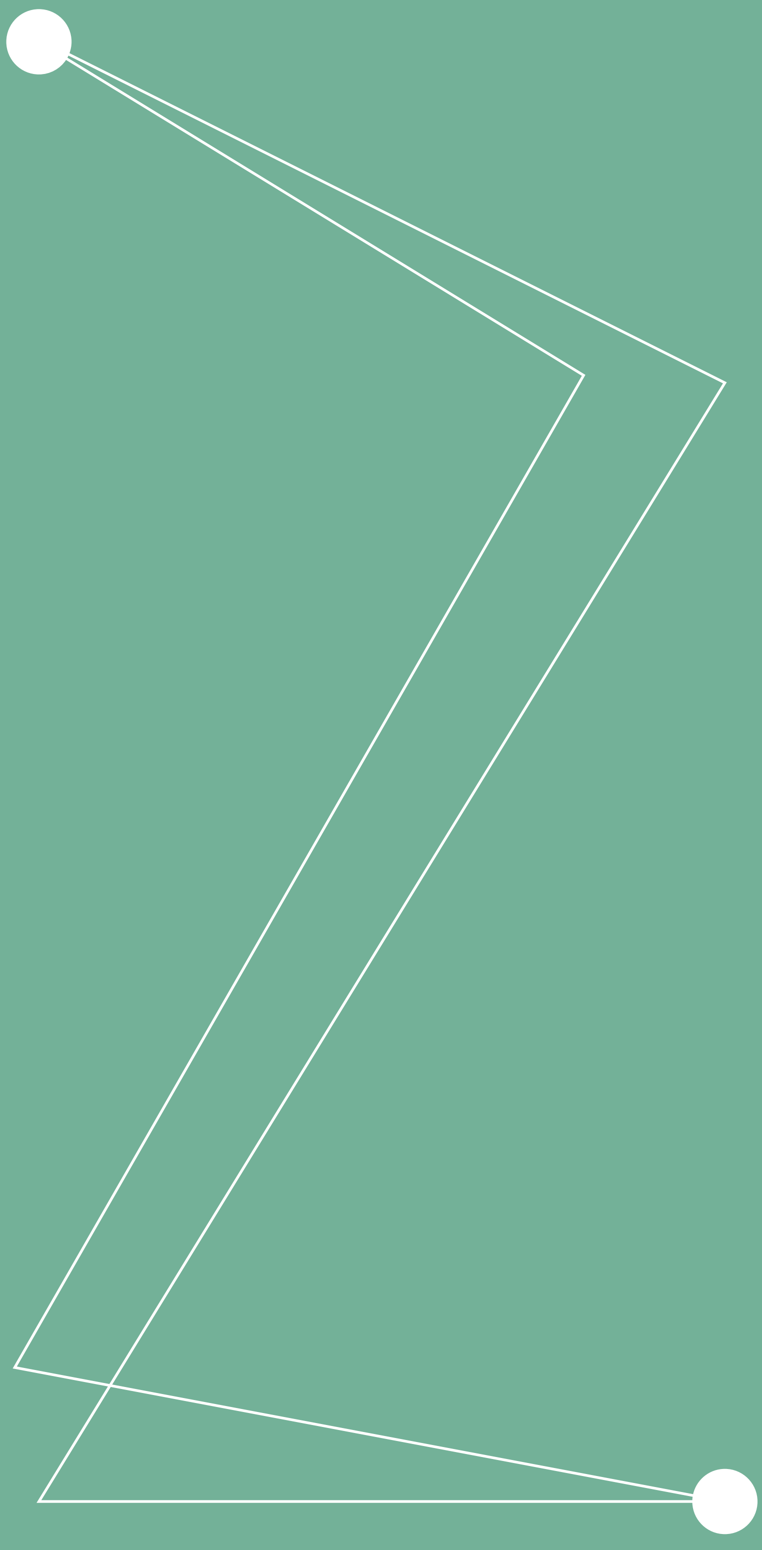

COMPARING DIFFERENT METHODS TO CREATE A LINEAR MODEL FOR THE UNCONTROLLED MANIFOLD ANALYSIS

Inge Tuitert, Tim A. Valk, Egbert Otten,
Laura Golenia, Raoul M. Bongers

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Abstract

An essential step in the uncontrolled manifold analysis is creating a linear model that relates changes in elemental variables to changes in performance variables. Such linear models are usually created by means of an analytical method. However, a multiple regression analysis is also suggested. Whereas the analytical method includes only averages of joint angles, the regression method uses the distribution of all joint angles. We examined whether the latter model is more suitable to describe manual reaching movements. The relation between estimated and measured fingertip-position deviations from the mean of individual trials, the relation between fingertip variability and V_{ort}, V_{ucm}, and V_{ort} indicated that the linear model created with the regression method gives a more accurate description of the reaching data. We therefore suggest the usage of the regression method to create the linear model for the uncontrolled manifold analysis in tasks that require the approximation of the linear model.

Introduction

The uncontrolled manifold (UCM) method is a well-established approach to assess the coordination of multiple degrees of freedom (DOF) in synergies that stabilize performance in human actions. The method has been applied to a variety of actions, such as sit-to-stance, finger-force production, and goal-directed reaching [31,34–40,43,60]. The current paper focuses on computational aspects of the UCM method in goal-directed manual reaching movements to illustrate the argument. When performing a reaching movement, the DOF, i.e., the joint angles of the arm, have to be coordinated to stabilize the index-finger position. To assess how the joint angles are coordinated, the UCM method is applied to evaluate the variability in the joint angles across trials. Joint-angle variability is partitioned into variability that does not influence the index-finger position (V_{ucm}) and variability that does (V_{ort}). If there is more V_{ucm} than V_{ort}, it is assumed that the joint angles of the arm are coordinated into a synergy that stabilizes the index-finger position.

The computation of V_{ucm} and V_{ort} with the UCM method requires four steps [4]. The first two steps consist of selecting the elemental variables and the performance variable, respectively. In goal-directed manual reaching, elemental variables are usually the nine joint angles of the arm (shoulder, elbow, wrist, and finger-joint angles) while the 3D position of the tip of the index finger is the performance variable. Subsequently, small changes in the joint angles are related to small changes in the index-finger position by means of a linear model (third step). These relations have to be approximated in goal-directed manual reaching movements and are represented in a Jacobian matrix. Lastly, this matrix is used to partition the joint-angle variability across trials; variability within the null space of the Jacobian corresponds to V_{ucm} and variability orthogonal to the null space corresponds to V_{ort}. The current paper focuses on the creation of a linear model, which can either be done by means of an analytical method or by means of multiple regression (see below). Although the analytical method [32] is the most often used of the two [61], the regression method uses more information of the data to create the linear model, which can influence the accuracy with which the model describes the data. In this paper we compare the accuracy of the two methods in a manual reaching task.

In reaching movements, the analytical method to create the linear model [32,61] employs the computation of the fingertip position with respect to the trunk, using segment origins and rotation matrices of joint angles (i.e., the computation of forward kinematics). These calculations express the position of the end effector (e.g., the tip of the index finger) in the coordinate frame of the segment origin (e.g., the sternum). The resulting expression is a function of the joint angles and the segment lengths, i.e., the geometry of the kinematic chain. We refer to these calculations as geometric transformations, which are typically obtained from motion capture data. The rotation matrices used for geometrical transformations are computed from average joint-angle configurations across repeated trials. To generate the linear model using the analytical method, the model is composed of the partial derivatives of the geometric transformations; this approach is often used for UCM analysis in the literature [32,61].

When multiple regression analysis [61–63] is applied, the linear model is created by entering the joint angles as independent variables and the index-finger position as the dependent variable. Their relative relationships are described in the regression equations (see equation 1; a separate equation is used for each direction of the fingertip position). In equation 1, $\hat{y}$ is the y value on the best-fit plane corresponding to x_k, where b_k are the coefficients, c is the constant, and k the number of joints [64,65]. To estimate the coefficients of the multiple regression equation, a least squares error solution is used (see equation 2). In equation 2, $\boldsymbol{y}$ is a vector with the y values of all trials (i.e., the position of the index finger), $\boldsymbol{x}$ is a vector with the x values of all trials (i.e., joint angles), resulting in a solvable equation with k unknowns ($\boldsymbol{b}_m$ is a vector including b_1-b_k) in j equations (maximum of j is k; independent counter; for a mathematical description of all steps to get from equation 1 to equation 2, see Zaiontz [64]). Note that to compute the coefficients, the covariance among all the joint angles and the covariance among all joint angles and the fingertip position are used. The coefficients (b_1-b_k) of the multiple regression analysis, representing partial derivatives, compose the linear model. The constant of the multiple regression equation (c) is not included in the Jacobian because this was the average of the end-effector position ($\boldsymbol{c} = \bar{y}$, see [64]). Until now, in UCM analysis the regression method to create the linear model has only been used when geometric transformations of the relations between elemental and performance variables were not available (e.g., when using EMG; [63]). However, we propose that the regression method to create the linear model should be considered, even when geometric transformations are available.

$$\hat{y} = c + b_1 x_1 + b_2 x_2 + \cdots + b_k x_k \qquad \text{equation (1)}$$

$$cov(\boldsymbol{y}, \boldsymbol{x_j}) = \sum_{m=1}^{k} \boldsymbol{b_m} \cdot cov(\boldsymbol{x_m}, \boldsymbol{x_j}) \qquad \text{equation (2)}$$

To understand why the accuracy of the two linear models described above might differ, the dissimilarities between the two need closer examination, especially since the two methods intuitively seem to be similar. The essence is that the regression method uses different information of the movement data than the analytical method does. The latter method only uses the averages of all joint angles and the averages of the 3D origins of the segments. The regression method, on the other hand, uses the (co)variance of the joint angles and the fingertip positions of all trials to estimate each of the coefficients (b_1-b_k), that make up the Jacobian. This implies that the regression method takes into account the distribution of the data, whereas the analytical method does not. To examine this we compared the two methods to describe goal-directed reaching movements, expecting that the linear model based on the regression method would be more accurate than that based on the analytical method.

Methods

Participants

The dataset used in the current paper is a subset of data presented in Valk et al. [66] and consisted of the data on the simple reaching condition obtained in 15 participants of whom seven were men (mean age: 21.3 years; standard deviation (SD): 1.4 years) and eight women (mean age: 20.5 years; SD 1.8 years). The study had ethical approval and all participants gave their informed consent.

Procedure

Participants were seated on a chair in front of a table. The backrest of the chair was extended with a plate to which the trunk of the participant was gently strapped to prevent movements of the origin (i.e., the sternum) while keeping the shoulder at approximately the same position in space without restricting shoulder motions. At the start of each trial, participants placed their index finger of the right dominant hand on the start location, a 1-cm diameter circle on the table, while resting the elbow on an arm rest to standardize the starting posture as much as possible across trials. Following a 'go' signal presented verbally by the experimenter, participants performed a forward movement in the sagittal plane to reach the 1-cm diameter target circle located 30 cm anterior of the start position. The experiment comprised a total of 50 trials. Participants were instructed to perform the movement as fast and as accurately as possible but were free to initiate the movement at their own convenience following the 'go' signal.

Materials and data collection

Movements were recorded using the Optotrak 3020 system (Northern Digital, Waterloo, Ontario, Canada). Using skin-friendly tape, six rigid PVC plates, each with three IREDs (infrared light-emitting diodes), were attached to the participant's sternum, the acromion, on the left side of the right upper arm below the insertion of the deltoid, proximal to the ulnar and radial styloids, to the dorsal surface of the hand [67], and to the index finger [43]. Following the procedure described by Van Andel et al. [67], for each individual participant, the 19 anatomical positions were recorded together with the rigid bodies using a standard pointer device. A small aluminum plate was taped under the index finger to prevent flexion-extension in the interphalangeal joints while allowing for flexion-extension and adduction-abduction in the metacarpophalangeal joint [43].

Preprocessing

The position data of the rigid bodies and their relations to the 19 anatomical positions in the calibration trials were used to compute the positions of the 19 anatomical positions in the global reference frame in measurement trials. X-Y-Z velocities were derived using the three-point central difference method. Tangential velocity was calculated at each point in time as the square root of the sum of the three squared velocities. For each trial, movement termination was determined by searching forward from the moment at which peak tangential velocity was reached. The end of the movement was identified as the first data point where the tangential velocity fell below a speed of 2.5 cm/s and the position of the pointer tip fell within a radius of 1 cm around the target. The instant of movement termination was used in the analyses. Averages of variables were calculated across trials at movement termination. For more information about the data collection and analysis, see Valk and colleagues [66].

Uncontrolled Manifold Method

The UCM was calculated at movement termination, using the four steps introduced earlier. These four steps are described in more detail below. At step three, we explain both the analytical and regression method to create the linear model.

Selection of the elemental variables

The elemental variables selected were the nine joint angles of the arm (θ_{1-9}): shoulder plane of elevation (θ_1), shoulder angle elevation (θ_2), shoulder endorotation-exorotation (θ_3), elbow flexion-extension (θ_4), forearm pronation-supination (θ_5), wrist abduction-adduction (θ_6), wrist flexion-extension (θ_7), finger abduction-adduction (θ_8), and finger flexion-extension (θ_9). These joint angles were computed following International Society of Biomechanics (ISB) guidelines for the upper extremity [68].

Selection of the performance variable

The performance variable selected was the 3D fingertip position (r_X , r_Y , r_Z). According to the ISB guidelines, the coordinate system was defined as follows: positive X was the forward position, positive Y the upward position, and positive Z the rightward position.

Creating a linear model of the system

The deviation from the mean of the performance variable relates to the deviation from the mean of the joint configuration $\Delta\theta_j$ as:

$$\Delta r_j = J * \Delta\theta_j \qquad \text{equation (3)}$$

where J is a Jacobian matrix (see equation 4) and *j* represents the trial. The Jacobian was computed as follows:

$$J = \begin{matrix} \frac{\delta r_1}{\delta\theta_1} & \cdots & \frac{\delta r_1}{\delta\theta_9} \\ \vdots & \ddots & \vdots \\ \frac{\delta r_3}{\delta\theta_1} & \cdots & \frac{\delta r_3}{\delta\theta_9} \end{matrix} \qquad \text{equation (4)}$$

The elements of this matrix were the partial derivatives of the coordinates of the performance variable with respect to each joint angle.

Analytical method

The analytical partial derivative was calculated using geometric transformations of joint-angle means and segment-origin means. These transformations are shown in equation 5, where $R_{\theta 1-\theta 9}$ are the rotation matrices of each angle (number for each angle, see selection of elemental variables), D_{1-5} the positions of the segments' origins with respect to the sternum (i.e., the origin of the segment chain): D_1: glenohumeral, D_2: ulnar styloid, D_3: metacarpal 3, D_4: metacarpophalangeal 2, D_5: fingertip, and *r* the position of the fingertip in three directions. The Jacobian is obtained by differentiating equation 5 with respect to the independent variables (i.e., joint angles). The results of these computations are united the Jacobian matrix to create the linear model [32,61].

$$r = D_1 + R_{\theta 1}R_{\theta 2}R_{\theta 3}D_2 + R_{\theta 1}R_{\theta 2}R_{\theta 3}R_{\theta 4}R_{\theta 5}D_3 + R_{\theta 1}R_{\theta 2}R_{\theta 3}R_{\theta 4}R_{\theta 5}R_{\theta 6}R_{\theta 7}D_4 + R_{\theta 1}R_{\theta 2}R_{\theta 3}R_{\theta 4}R_{\theta 5}R_{\theta 6}R_{\theta 7}R_{\theta 8}R_{\theta 9}D_5 \qquad \text{equation (5)}$$

Regression method

Contrary to the previous method where the mean across trials was used to create the linear model, in the multiple regression method [61–63] the (co)variance across trials was included. In the multiple regression analysis, the dependent variable was the fingertip position and the independent variables were the joint angles. The multiple regression equation and the least square error solution equation are shown in equations 1 and 2. Three separate multiple linear regression analyses were run for each dimension of the fingertip position. The constants of the regressions were excluded from the model because these were the averages of the end-effector positions ($c = \bar{y}$, see [64]); note that this was the case because the regressions were not run 'mean-free' as done by de Freitas and colleagues [61,62]. The coefficients of the regression analysis composed the linear model, which were equal to $\frac{\delta r_n}{\delta \theta_m}$, i.e., the partial derivative of the regression formula to a certain joint angle, which makes these coefficients suitable as a linearized model (where *n* is the dimension of the fingertip position and *m* is the number of the joint angle).

Partitioning of variance into V_{ucm} and V_{ort}

The variance per DOF was partitioned into two components: V_{ucm} and V_{ort} (see [32]). The nullspace of J represents those changes in the joint-angle configurations that do not cause any changes in the performance variable: $\Delta\theta^{Vucm}$. Variance that does not affect the performance variable (V_{ucm}) and corresponds to the variance per DOF, which lies within the nullspace of J, was defined as:

$$V_{ucm} = \frac{{\Delta\theta^{V_{ucm}}}^2}{DF - DV} \qquad \text{equation (6)}$$

Here, DF is the number of involved DOF; in our reaching example, DF was 9 and DV, the dimension of the performance variable, was 3. The variance affecting the performance variable ($\Delta\theta^{Vucm}$; V_{ort}) and corresponding to the variance per DOF of the orthogonal component was defined as:

$$V_{ort} = \frac{{\Delta\theta^{V_{ort}}}^2}{DV} \qquad \text{equation (7)}$$

Testing the Linearized Models

To test these linearized models we used three measures: 1) the estimated fingertip-position deviations from the mean of individual trials, 2) the relation between the fingertip variability and V_{ort}, and 3) V_{ucm} and V_{ort}.

Estimated Fingertip-Position Deviations from the Mean of Individual Trials

We computed the difference between the estimated fingertip positions and the mean fingertip positions for the two methods, after which we compared the differences to the measured fingertip-position deviations from the mean for all individual trials. We computed the relations between these two dependent variables for the two methods separately (i.e., based on the analytical and the regression method, respectively). The method for which this relation was strongest describes the data better.

The estimated fingertip-position deviations from the mean of the linearized models were calculated using equation 3 (following [32]). For each of the two linearized models, the joint-angle deviations from the mean $(\theta_j - \bar{\theta})$ were computed for each trial and were subsequently multiplied with the Jacobian matrix. Two sets (each based on one of the linear models) of three vectors (one vector for each dimension of the index finger) were obtained, representing the estimated deviation of individual trials from the mean fingertip position $(\Delta\hat{r}_j)$.

To compare these estimated fingertip-position deviations from the mean $(\Delta\hat{r}_j)$ the measured position deviations from the mean (Δr_j), we calculated the Pearson correlation coefficient (PCC) between $\Delta\hat{r}_j$ and Δr_j for each individual participant and dimension. A correlation was valued as high if PCC was greater than 0.6 and as medium if PCC was greater than 0.4 and less than 0.6 [69]. A MANOVA on PCC with the three directions as dependent variables and Method (Analytical Method and Regression Method) as within-subject variable was conducted to compare the PCCs of the linear model created with the analytical method and the linear model created with the regression method for all directions. Furthermore, we fitted a regression line through the data of two participants, one participant with a low end-effector variability and one with a high end-effector variability, to visualize the relation between $\Delta\hat{r}_j$ and Δr_j for each method in different situations.

Relation between Fingertip Variability and V_{ort}

We examined the relation between the SD of the measured fingertip position at movement termination (we refer to this as the fingertip variability) and V_{ort} for the two methods. If the data is described appropriately by the linear model, then there should be a relation between the fingertip variability and V_{ort}.

Fingertip variability was computed as the SD of the tangential fingertip positions at movement termination. The tangential position was calculated as the square root of the sum of the 3D position. To examine the relation between fingertip variability and V_{ort}, we calculated the PCC between these two variables for each method. A regression line was fitted through the data to illustrate this relation for each method.

V_{ucm} and V_{ort}

We compared the V_{ucm} and V_{ort} of the linear models created using the two methods. The manual reaching task is a simple task of which it has been repeatedly shown that the position of the index finger is stabilized, showing that V_{ucm} is larger than V_{ort} [35,43,70]. Moreover, the data used in the current study showed a low variability of the index finger at the end of the movement [66]. This underscores the notion that, if the linear model is a good description of the data, then the stabilization of the index finger, as reflected by a high V_{ucm} and a low V_{ort}, is stronger.

To compare V_{ucm} and V_{ort} of the two linear models, we conducted a MANOVA with V_{ucm} and V_{ort} as dependent variables and Method (Analytical Method and Regression Method) as within-subject variable. To correct for non-normal data distributions, V_{ucm} and V_{ort} were log-transformed prior to statistical analysis [71], as indicated by the subscript log.

Additionally, we quantified the difference between the two Jacobians by comparing the nullspace and the orthogonal space of the Jacobians of the two methods through the cosine of principal angles [72]. This analysis reveals the shared dimensions by the subspaces. The threshold for similarity was set at 0.9 [73].

For all statistical analyses the level of significance was set at $\alpha = 0.05$. All variables that were subjected to statistical analyses were normally distributed according to the Kolmogorov-Smirnov test (p's < 0.05).

Results

Estimated Fingertip-Position Deviations from the Mean of Individual Trials

The MANOVA of the correlation between the estimated and measured fingertip-position deviations from the mean revealed a significant effect of Method ($F(3,12) = 53.93$, $p < 0.001$). The separate univariate ANOVAs on the X ($F(1,14) = 53.34$, $p < 0.001$), Y ($F(1,14) = 112.57$, $p < 0.001$), and Z ($F(1,14) = 47.18$, $p < 0.001$) directions indicated that PCCs were higher in the regression method compared with the analytical method in all directions (see Figure 1). Figure 2, which depicts the relations between and for a participant with low and a participant with high fingertip variability, elucidates that the linearized model created with the regression method revealed a higher correlation between and in all movement directions (PCCs > 0.42; see Figure 2, lower panels) than that of the analytical method (PCCs < 0.17; see Figure 2, top panels) for participants with low and high fingertip variability. To check whether the regression method had higher correlations than the analytical method for the complete movement, we also calculated the correlations between and at each instant of the (time-normalized) movement trajectory. Visual inspection of these correlations also revealed higher correlations for the linearized model created with the regression method compared to the analytical method in 100% of the instances in all movement directions. Taken together, these results support our expectation that the estimated position deviations from the mean of the index finger computed using the regression-based linear model describe the data better than those computed using the analytical linear model.

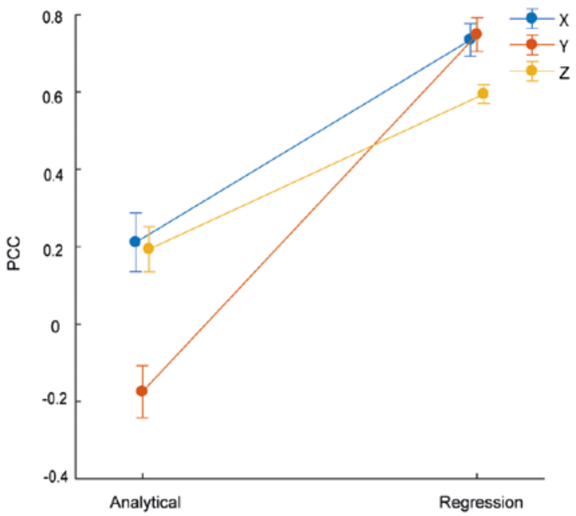

Figure 1. Means and standard errors of the correlations between $\Delta\hat{r}_j$ and Δr_j for the analytical and the regression method to compute the linear model and each dimension of the position of the index finger (X, Y, and Z).

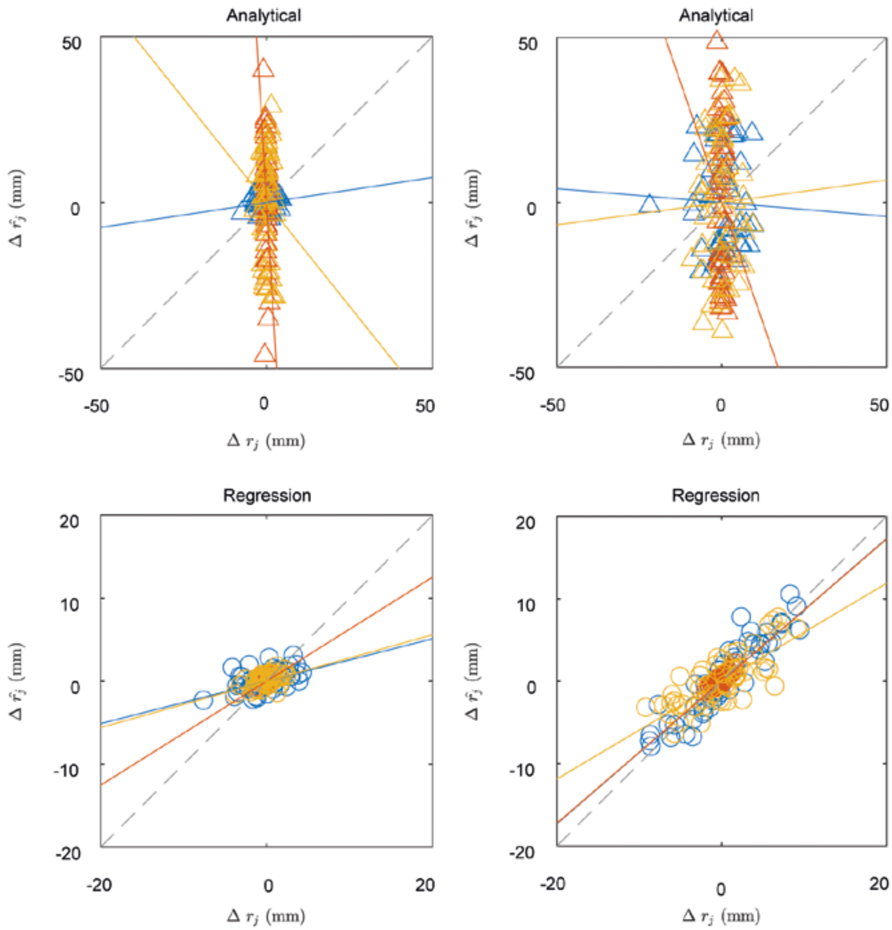

Figure 2. Relations between $\Delta\hat{r}_j$ of the linearized model and Δr_j for each method in a separate plot and each direction in a different color (top row: analytical method; bottom row: regression method). The dashed grey line represents the hypothesis $\Delta\hat{r}_j$ where and Δr_j show a PCC of 1. The left panels show the least variable participant and the right panels the most variable participant (end-effector variability). The shades of grey for the different directions are as follows: X direction blue, Y direction orange, and Z direction yellow. Note that the axes of the upper panels are different from the axes of the lower panels.

Relation Between Fingertip Variability and V_{ort}

The PCC of the relation between the fingertip variability and V_{ort} (see Figure 3) was medium to high in the linear model created with the regression method (r = 0.59), whereas it was very low in the model created using the analytical method (r = -0.01). This result suggested that in the linear regression model joint-angle variability was partitioned into V_{ort} when appropriate whereas this was not the case in the analytical model.

V_{ucm} and V_{ort}

The MANOVA of V_{ucm}^{log} and V_{ort}^{log} comparing the models generated by the two methods revealed a significant Method effect (F(2,13) = 7.94, p = 0.006). Separate univariate ANOVAs on V_{ucm}^{log} and V_{ort}^{log} indicated that in the regression-based model more joint-angle variability was partitioned into V_{ucm}^{log} (F(1,14) = 14.02, p = 0.002) and less into V_{ort}^{log} (F(1,14) = 16.67, p =

0.001) than in the analytical-based model (see Figure 4). Given the small variability of the position of the index finger, these results suggested that the linear model created through regression described the data best.

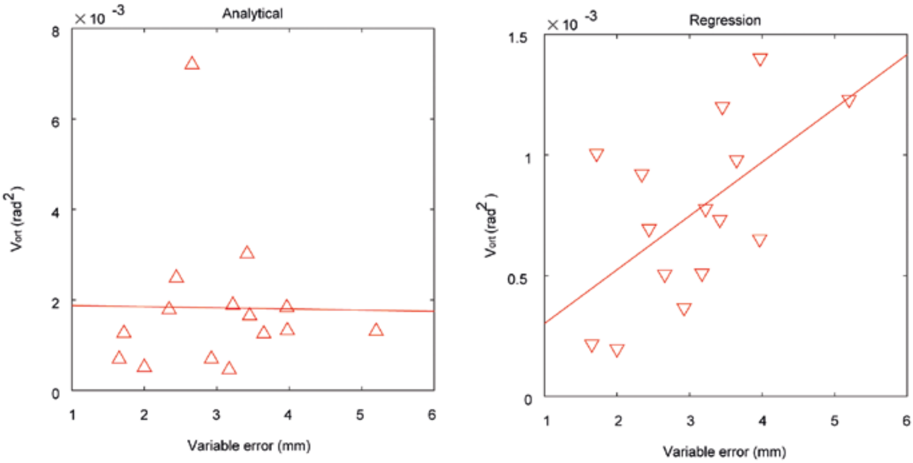

Figure 3. The relation between fingertip variability and V_{ort} for each method, where each triangle represents one participant. Note that the y axis of the upper panel is different from the y axis of the lower panel.

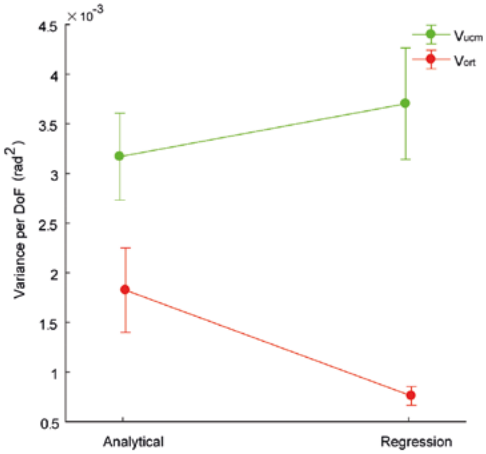

Figure 4. Means and standard errors of the means of V_{ucm} and V_{ort} for the analytical and the regression method.

To examine the orientations of the Jacobians for the two methods, we compared the nullspace and orthogonal space of both Jacobians separately using the cosine of principle angles and found the averages across participants to be higher than the similarity criterion of 0.9 for the first four dimensions of the nullspace and for all three dimensions of the orthogonal space. This indicated that the differences between the Jacobians of the regression and analytical methods to create the linear model were in the fifth and the sixth dimension of the nullspace, implying that the subspaces of the two Jacobians are different, and, therefore, that the regression-based and analytical Jacobians were indeed different, showing that the differences between the two methods on the other measures were valid.

Discussion

When multi-joint coordination is studied with the UCM method, the linear model is usually created using the analytical method rather than a regression method. One major difference between the two approaches is that in the analytical method only the averages of joint angles are used, whereas in the regression method the distribution of angular values of the joints and positional values of the end-effector across repetitions are used. Comparing the linear models the two methods computed for the data obtained in a manual reaching task, we first found higher correlations between the estimated and the measured fingertip-position deviations from the mean in the regression-based linear model. Second, the relationship between fingertip variability and V_{ort} indicated that if the linear model is created with the regression method, an appropriate amount of joint-angle variability is partitioned into V_{ort} whereas this is not the case if the linear model is created analytically. Moreover, we showed that with the regression method more joint-angle variability was partitioned into V_{ucm} and less into V_{ort}. Taken together, these results demonstrate that the linear model created with the regression method provided a more accurate description of the data in our goal-directed reaching task, which is why we propose using the regression method to create the linear model for the UCM analysis when analyzing reaching data.

Although we only examined goal-directed reaching, we argue that this recommendation could be extended to other actions, such as in sit-to-stance or walking. We hypothesized that the regression method described the data better than the analytical method because it incorporated the distribution of the joint angles and fingertip positions across repetitions into the linear model, whereas the analytical method considered only the averages of the joint angles and origins of the segments. Confirming our assumption, we have shown the added value of including these changes across repetitions into the analysis. Given that the distribution of the data across repetitions of the performance variable and the elemental variables also plays a role in other motor tasks, including this distribution in the creation of the model may also improve the linear model for these tasks. Note that this recommendation only applies to tasks in which the linear model has to be approximated. In tasks that are by definition linear, the regression method should not be considered. This is, for instance, the case in a finger force-production task where a certain amount of force needs to be exerted with four fingers (DOF) at a certain point in time [74,75]. Here, the production of the total force is a linear combination of the four DOF and hence the exact linear model, which makes the use of the regression method superfluous. It is for tasks that require a linear approximation (i.e., tasks that are not exactly linear) that we suggest using the regression method to describe the data more accurately.

To do a UCM analysis in a reaching task, a linear approximation of the relations between changes in joint angles and changes in fingertip position is used, although these relations are actually non-linear [4,76]. This is exemplified by a 3D plot with a curved (non-linear) solution manifold where three joint angles (three axes) keep the fingertip at one specific location at an instant in time. If the joint-angle ranges across repetitions are small, only a small part of this curved manifold is exploited, allowing this part to be approximated by a linear model because in a small part of the manifold the deviations from linearity are small. Given that in our simple reaching task the ranges of the joint-angle rotations across repetitions are small and the estimated deviations from the mean of the fingertip do not differ much from the measured deviations from the mean, the non-linear behavior is suitable for approximation using a linear model [77–79], facilitating the UCM analysis.

While in the current task linearization is appropriate, in other tasks where the ranges of joint angles (or other elemental variables) across repetitions is larger and thus also the scattering of trials on the curved solution manifold, a linearized manifold to approximate the solution manifold is less suitable. In such cases, one might consider using a non-linear method to assess variability across trials. Müller and Sternad [80], for example, proposed a surrogate non-linear data analysis that was adapted and applied by Ambike and collegues ([78]; see also [81]) for an inverse piano finger-force task. Having created a surrogate data set by randomizing the original data set, they found the surrogate data set to show a much larger variance than the original one, implying that there was less co-variance in the surrogate data set and indicating that in the original data the variance was mainly co-variance along the non-linear solution manifold. In short, if the joint-angle ranges in a multi-joint task are small, the usage of a linearized model is appropriate, whereas if the joint-angle ranges are larger, a non-linear method would be the better option. Note that there are also other discussions regarding the analysis of variability in redundant tasks [5,82], but these are beyond the scope of this paper.

In conclusion, our results show that in goal-directed reaching the regression method to create the linear model is preferred to the analytical method. We argue that if the UCM analysis is applied to tasks that require the approximation of the linear model, the use of the regression method to create the linear model should be considered.