Model reduction for controller design for infinite-dimensional systems

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Chapter 1

Introduction

In this thesis we study systems that can formally be described by the equations

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ x(0) &= w \\ y(t) &= Cx(t) + Du(t), \end{aligned} \tag{1.1}$$

or by the equations

$$\begin{aligned} x_{n+1} &= Ax_n + Bu_n \\ x_0 &= w \\ y_n &= Cx_n + Du_n. \end{aligned} \tag{1.2}$$

Here A, B, C and D are linear operators on Hilbert spaces.

The main problem in the field of linear systems and control theory is, for a given system (1.1), designing a system

$$\begin{aligned} \dot{x}_c(t) &= A_c x_c(t) + B_c u_c(t) \\ x_c(0) &= w_c \\ y_c(t) &= C_c x_c(t) + D_c u_c(t), \end{aligned} \tag{1.3}$$

such that if we put $u_c = y$ and $u = y_c$, then the resulting closed-loop system has some prespecified properties. The system (1.1) is usually referred to as the plant and the system (1.3) as the controller. This type of control is called feedback control since the output y of the system (1.1) is, after being processed by the system (1.3), fed back into the system (1.1) (see figure 1.1).

Which properties the closed-loop system is required to have depends very much on the particular application. We will focus on two properties that are almost always required: stabilization and low complexity of the controller. To

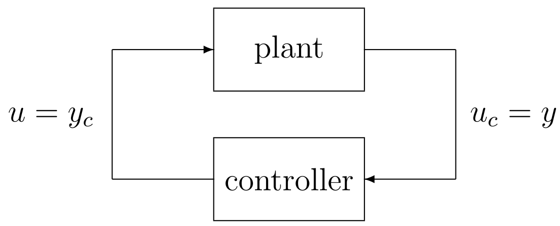

Figure 1.1: Feedback interconnection of plant and controller.

start with the second property, the measure of 'complexity' of the controller (1.3) we will use is the dimension of the Hilbert space in which the state x_c of the controller takes its values. In engineering applications it is often paramount that this dimension is small. Most standard controller design methods lead to a controller with the same state space as the plant. In the examples we are interested in the plant has an infinite-dimensional state space, so applying these standard controller design procedures to the plant is not an attractive option. There are two possible solutions to this problem. The first is to develop completely new controller design methods that do not have this disadvantage of producing in a controller with the same state space as the plant. The second is to use standard controller design procedures, but not for the plant, but for a low-dimensional approximation of it. One then has to prove that the controller designed based on the approximation, when interconnected with the plant, results in a closed-loop system that has the prespecified properties (other than having low complexity of the controller). We will focus on this second alternative. The other prespecified property we will look at is, as mentioned earlier, stabilization. There are several types of stability that one can demand the closed-loop system to have. We concentrate on obtaining so-called input-output stability of the closed-loop system, since under appropriate stabilizability conditions this is equivalent to the other (a priori stronger) types of stability.

Infinite-dimensional continuous-time systems

Examples we are interested in are systems described by partial differential equations. Continuous-time systems like (1.1) are an abstract representation of such systems described by partial differential equations. The operators A, B, C and D that result from writing the partial differential equation examples in this form are usually unbounded, which introduces severe technical difficulties. These difficulties already start with a correct notion of 'solution' of the equations (1.1). For an arbitrary quadruple of unbounded operators

A, B, C, D it is impossible to do even this, let alone developing a theory for control design. During the last decades much effort has gone into the question under which conditions on the quadruple of operators one can develop a theory of control design for stabilization. The present state-of-the-art class of systems for which there is a reasonably complete theory (well-posed linear systems, see Staffans [89]) does not include all systems described by partial differential equations that one would like to study. In part II of this thesis we present a new class of systems, larger than the class of well-posed linear systems, which also includes many interesting partial differential equations that are not well-posed. We were able to complete the program outlined above at this level of generality, showing that a reasonably complete control theory for this class of systems is certainly feasible. We note that the completion of the program outlined above is new even for well-posed linear systems (in fact, even for systems where all operators are bounded it is new). We were able to complete the program outlined above for our general class of systems by making the connection with discrete-time systems using a Cayley transform approach.

Infinite-dimensional discrete-time systems

In contrast to the infinite-dimensional continuous-time case, the infinite-dimensional discrete-time case does not provide problems due to unbounded operators. For all purposes one may assume that A, B, C and D are bounded. As mentioned before, our approach to carrying out the program outlined above is to first obtain the results in discrete-time and then to translate these results, using the Cayley transform, to continuous-time. It turns out that to be able to translate the results using the Cayley transform the results have to be 'optimal', i.e. one has to prove the desired theorems under the weakest possible assumptions. This meant that we had to rewrite large parts of infinite-dimensional discrete-time systems theory, since the existing results were proven under conditions that were too strong and therefore do not translate well under the Cayley transform. In particular, we had to prove theorems under weaker stability/stabilizability conditions than the standard power stability/stabilizability.

Outline of this thesis

This thesis consists of two parts. Part I is the longest and deals with discrete-time systems. Part II deals with continuous-time systems.

We now briefly outline the contents of Part I. In Chapter 2 some basic notions are defined. Chapter 3 deals with stability and Chapter 4 with stabilizability. The aspects of energy preserving systems that are needed in this thesis are collected in Chapter 5. The very important linear quadratic regulator problem is the subject of Chapter 6. In Chapter 7 coprime factorizations are studied. The existence result for (strongly) coprime factorizations proven in this chapter is probably one of the most important results presented in this thesis. Robustly stabilizing controllers are the topic of Chapter 8. The metric in which we measure the distance between systems, the gap metric, is the topic of Chapter 9. Part I concludes with Chapter 10 on balanced realizations. These balanced realizations are used to define the desired approximations of the plant. In this chapter we show that, under certain conditions, a robustly stabilizing controller based on an approximation of the plant stabilizes the original infinite-dimensional system.

Part II starts with Chapter 11 in which we introduce our new class of systems. In Chapter 12 we illustrate how systems described by partial differential equations fit into this abstract framework. The Cayley transform, which we use to translate results from discrete-time to continuous-time, is studied in Chapter 13. In Chapter 14 the continuous-time counterparts of the most important results obtained in discrete-time are presented. Chapter 15 illustrates the model reduction for controller design approach outlined in this thesis using an example of a system described by a partial differential equation (a beam).

In Chapter 16 the most important results obtained in the preceding chapters are collected.

There are two appendices; in the first one some basic results in Hardy space theory are recalled and in the second one some rather tedious algebraic calculations with algebraic Riccati equations are performed. At the end of the thesis one can find a short summary (both in English and in Dutch), a list of notations, a bibliography and an index.