Chapter 4

Medium term capacity coordination

This chapter discusses the capacity coordination in the medium term. Lot sizing is at the heart of this. The Economic Lot Scheduling Problem (ELSP)– the problem of scheduling several products on a single facility– provides the bulk of the background knowledge. After a brief overview of the published ELSP approaches, we discuss the possible modifications and extensions to them that need to be adapted for application in food processing industry. Special attention has been given to specific characteristics of limited shelf life and a combination of MTO and MTS.

4.1 Introduction

Given the partitioning of the product range into MTO and MTS categories, the production and inventory (order acceptance rule, restocking and inventory control rules, etc.) policy needs to be determined. As discussed in chapter 2, the literature on the combined MTO-MTS has addressed these issues in a limited way. In some papers, in order to make solutions analytically tractable it is assumed that setups do not exist (e.g. Carr et al. 1993, Carr and Duenyas 2000). Some others advocate (cyclic) base-stock policies for the MTS items (e.g. Federgruen and Katalan 1994, 1999).

The literature does not pay any attention to two important food processing characteristics: limited shelf lives and sequence dependent setups. The perishable nature of products limits the possibilities for stocking. Sequence dependent setups (especially as there are families of related products) may make it relatively easier and economical to produce MTO items if a combination with MTS items from the same family can be made. Due to relatively large and sequence dependent setups, there is a tendency to produce in a recurring, cyclic pattern. Such cyclic patterns, although quite common in food processing companies, have been addressed by only Federgruen and Katalan (1994, 1999) in the MTO-MTS literature. Establishing such patterns has been the object of study of the literature on ELSP– Economic Lot Scheduling Problem (e.g. Elmaghraby 1978). In this text, we chose to adapt ELSP based approaches. The main reason is that the food processing industries are traditionally MTS companies and are in transition toward more MTO. The presence of regular demand products, high setup time makes cyclic schedules like those generated by ELSP more attractive than acyclic schedule approaches that might come from pure MTO based approaches.

The aim of this chapter is to develop an approach for finding a production cycle for the combined MTO-MTS situation in food processing industries. The objective is to make adaptations to ELSP approaches to be able to incorporate various food processing industry characteristics in the development of a production cycle. First, a brief description of ELSP is presented in section 4.2. Then, some extensions needed for food industries are discussed in section 4.3. Sections 4.4 and 4.5 address two of these extensions in more details– limited shelf life issues and the incorporation of MTO in ELSP respectively. Concluding remarks are provided in section 4.6.

4.2 Economic Lot Scheduling Problem

Chapter 2 described the main types of decisions in combined MTO-MTS situations. One of the decision areas is the determination of the production and inventory policy. In the context of make-to-stock only, a large number of publications deals the problem of producing a number of different products to stock on the same facility. We address the basic results from this research area (usually labelled as the Economic Lot Scheduling Problem (ELSP)) in order to use and adapt these for the combined MTO-MTS. The ELSP problem deals with scheduling the production of several products on a single facility. The objective is to minimise the sum of holding cost and setup cost in the long run, under the restriction that all demand is satisfied. This problem has been studied in the literature for around 45 years (see e.g. Rogers 1958). The basic assumptions of the problem are that demand is constant and continuous for each product; all the demand should be fulfilled immediately i.e. no backordering; production rate is constant and only one product can be produced at a time; usually a setup (sequence independent) is required for producing a product, incurring a setup cost; constant holding cost rate for carrying inventory of each product. Most methods are essentially built on the classical EOQ with the adaptation for the multi-product case. In principal, the decisions are made at an intermediate level between strategic and operational, comparable with the level— capacity coordination in medium term— of the hierarchy in section 2.5.

The ELSP problem is known to be NP-hard (Hsu 1983) and most solution approaches are heuristic algorithms (e.g. Elmaghraby 1978, Lopez and Kingsman 1991). Two approaches are widely used in the literature: the basic period approach (introduced by Bomberger 1966) and the common cycle approach). The basic period approach tries to find a fixed, relatively short period of time: the basic period. For each product a factor is determined that indicates whether it will be produced each period, every second, third or fourth, etc. period. Thus, the individual items have different cycle times, which are integer multiples of basic period. The aim is to find the optimal basic period and these factors for each product, while a feasible schedule is still possible. A feasible schedule means that the total time for production and setups does not exceed available capacity. Doll and Whybark (1973) and Haessler (1979) provide well-known heuristics in the basic period approach. The common cycle approach tries to find a cycle which accommodates production of every product exactly once during each cycle and is a special case of the basic Period approach. This common cycle should be long enough to produce every product. Dobson (1987) introduced a third approach called 'time varying lot size approach'. In this variant of a cyclic schedule, items are allowed to be produced more than once in a cycle like in the basic period approach. In addition, different lot sizes are allowed for any given item during a cyclic schedule.

In the next section, the applicability of ELSP approaches is discussed and extensions for the food processing industries are provided.

4.3 ELSP in food processing industries

As discussed in the previous section the emphasis in ELSP approaches has been on finding cost effective cyclic schedules. In a cyclic schedule the facility is setup for products in a sequence that repeats forever. The inventory positions repeat after each cycle. Key to this repeating property is the assumption of deterministic demand. In practice, however, demands are random. Further, assumption of no stockouts is too strict in the presence of randomness.

If we look at the food processing industries, cyclic schedules are common. Sequence dependent setup times are incurred before the production starts. The products do have limited shelf life also, which limits the amount of inventory that can be carried without spoilage. In a number of food processing companies, there are many product families as well. The setup structure is family setup. A major setup is incurred when product family changes and a minor setup within products of the same family. As discussed earlier in chapter 2, the companies are also moving from a pure make-to-stock (MTS) strategy and are producing more and more products to make-to-order (MTO). How to take care of MTO products within the cyclic schedules? In the following paragraphs, we review and discuss ELSP literature that can be adapted for various food industry characteristics mentioned above.

1. Case of random demand: Stochastic ELSP (SELSP)
   SELSP methods that are found in the literature have employed variable lots, idle times and safety stocks to take care of the random demands. Leachman and Gascon (1988) developed a heuristic procedure based on ELSP using target cycles combined with a continuous adjustment of production cycles. Their goal was to adequately space inventory runout times in order to balance the effect of random demand changes. Gallego (1990) uses optimal control theory to find an optimal target cyclic schedule and a linear recovery policy for the case of disruption to the cyclic schedule. Bourland and Yano (1994) examine the use of capacity slack and safety stock in SELSP. They use a static continuous review control policy and present a mathematical program that determines the cycle length, the allocation of idle times among the items, and the cycle safety stocks. Kelle et al. (1994) provide a model to determine target schedule and required safety stocks so as to minimise the sum of setup and holding cost subject to a service level constraint. They use a procedure similar to Doll and Whybark (1973) but also include safety stocks that, in turn, depend on cycle length. Sox et al. (1999) provide a detailed overview of SELSP literature.

2. Sequence dependent setup time
   The common cycle method results in a solution where the cycle times for each product are the same and each product is produced exactly once during the cycle. No constraint on the order in which products are produced is imposed. We can thus first attempt to produce a sequence of products that minimises the total setup time required in the cycle. These values can then be used in the calculation of the optimal cycle time. To put it simply, we use a fixed sequence strategy. This product sequencing is essentially a variant of the well-known travelling salesman problem (TSP) and

is known to be NP-hard. In basic period ELSP approaches, the only possible option to incorporate sequence dependent setups is to use an average value for each product in the beginning to decide on the production frequencies. These production frequencies are then used to create an initial production schedule which is then modified using the sequence dependent setup time data. This problem is a Travelling Salesman Problem with sub-tours. Dobson (1992), Lopez and Kingsman (1991) use this approach.

3. Family structure setup
   Inman and Jones (1993), McGee and Pyke (1996) provide methods for disaggregation of family schedules in part schedules. McGee and Pyke first, use an iterative goal seeking program (in Microsoft Excel) for allocating the major setup cost and setup time to the products in the family. Then they use a procedure similar to that of Doll and Whybark (1973). Ham et al. (1985) provide a common cycle approach that considers a family structure setup.

4. Shelf life
   The additional constraints are to be added to ELSP problem to take care of limited shelf life for products. These constraints require that cycle time for a product should be less than its shelf life to avoid any spoilage. The existing literature that considers ELSP and shelf life for products, e.g. Silver (1995), Viswanathan and Goyal (1997), has assumed a common cycle approach and an unrealistic assumption of possibility of deliberately reducing the production rate. In many food processing industries, where limited shelf life for products is common, changing the production rates is not allowed at all because it may result in products with poor quality.

5. MTO-MTS
   No research has been done so far on the determination of production cycle and lot-sizes under combined MTO-MTS using ELSP approach with the notable exception of Federgruen and Katalan (1994, 1999). They use a cyclic base stock policy to address lot sizing in combined MTO-MTS production situation. Their method is essentially a variant of common cycle policy i.e. every product is produced exactly once in a cycle. It has been widely acknowledged that the performance of such a policy deteriorates as the product variety increases. We provide other possibilities of incorporating MTO in ELSP in section 4.5.

While all the characteristics discussed above are relevant and important, we look at only the last two extensions in more details in section 4.4 and 4.5 respectively. The other characteristics are open for further research.

In the next section, we allow products to be produced more than once in a cycle and do not allow reducing production rates. A comparative review of various ELSP approaches, Lopez and Kingsman (1991), report that Haessler's algorithm (1979) is superior in performance, in terms of 'achieving economical and realistic schedules', to all other basic period approaches because of its convenient blend of analytical calculation, and limited enumeration. It may also be noted that this algorithm has an explicit built-in procedure for feasible schedule generation. A modification to Haessler's basic period procedure to account for the shelf life is presented. The proposed 'branch-and-bound like' procedure exploits the extra shelf life constraints to efficiently achieve a feasible solution. Numerical examples are presented to show that our approach outperforms the common cycle approach with shelf life considerations.

4.4 ELSP with shelf life considerations[1]

In food processing industries, products do have limited shelf life that restricts the amount of inventory that can be carried without spoilage. This adds another dimension to ELSP. There are a limited number of ELSP research papers that deal with shelf life considerations. We now briefly review these.

4.4.1 Literature overview

Silver (1989) studies an instance of the ELSP problem with shelf life considerations where an unconstrained common cycle solution violates the shelf life constraint of exactly one item. He considers options of either (a) deliberately slowing down the production rate for that item, or (b) reducing the common cycle time. He proves that it is effective to reduce the production rate. Sarker and Babu (1993) consider the same model as Silver while incorporating the production time costs. Their computational results indicate that the choice between 'reducing production rate' and 'reducing the production cycle time' is not as straightforward as Silver has suggested. It depends on the problem parameters such as shelf life, machine setup times (and costs), the production times and unit inventory costs. Goyal (1994) and Viswanathan (1995) postulate that it may be economical to produce some of the items more than once in a cycle but raise concerns about obtaining feasible schedules in such cases. Silver (1995)

[1]The content of this section has been published in: Soman, C. A., Van Donk, D. P. and Gaalman, G. (2004b), A basic period approach to the economic lot scheduling problem with shelf life considerations, *International Journal of Production Research*, **42**(8), 1677–1689.

and Viswanathan and Goyal (1997) provide heuristics for simultaneous optimization of cycle times and production rates in the cases where exactly one, and more than one item has binding shelf life constraint respectively. Viswanathan and Goyal (2000) show that allowing backordering can reduce the costs substantially while respecting the shelf life constraint. Chowdhury and Sarker (2001) and Goyal and Viswanathan (2002) discuss the problem of determining the optimum production schedules and raw material ordering policy under three options – adjusted production rate, adjusted cycle time, simultaneous adjustment of production rate and cycle time.

Most of these papers assume that reduction of a production rate does not incur additional costs. Also, in all papers, it is assumed that idle capacity cannot be used for other (more profitable) purposes. We think that these are not realistic assumptions in many situations. In the cases of high system utilisation, there are obviously limits on reducing the production rates. Also in many cases, like in food processing industry where limited shelf life for products is common, changing the production rates is not allowed at all because it may result in products with poor quality. We will, hence, stick to fixed production rates in the discussion. All the papers discussed above use the common cycle policy. This has some practical implications. Consider an example using the common cycle approach– if the shelf life of some product is very small, it will significantly reduce the cycle length for other products as well, which may have higher 'natural' cycle lengths and hence will lead to a solution with higher costs ('Natural' cycle lengths are the single product cycle lengths, as if they are produced independently). A motivation for allowing multiple production runs for some products stems from this. In other words, the products differ from each other in terms of production rates, demand rates, inventory costs, setup time and costs etc. and hence do have different natural cycle lengths. If the diversity in these natural cycle lengths is large, the common cycle policy may not be the most economical option. Hence, we will allow products to be produced more than once in a cycle.

The main contribution of this section is the development of a basic period solution procedure for the ELSP problem with shelf life considerations. In contrast to the previous work, it does not allow reducing production rate and does not restrict the solution to a common cycle solution. The heuristic presented here builds on Haessler's procedure (1979) for 'solving ELSP and feasible schedule generation'.

The organization of the section is as follows. Section 4.4.2 presents the problem formulation. The heuristic for finding a solution to the ELSP problem with shelf life is described in section 4.4.3. Section 4.4.4 presents computational results. A numerical example illustrates the heuristic procedure. Then the comparison of the suggested procedure and common cycle solution using the well-known problem data set is presented. The conclusions and future research directions are outlined in section 4.4.5.

4.4.2 Problem Formulation

The basic assumptions of the ELSP problem with shelf life considerations are –

- There is a constant, continuous demand rate for each product. All the demand should be fulfilled immediately i.e. no backordering is allowed.
- The production rate for each product is constant. The facility can produce only one product at a time.
- A sequence independent setup time (and cost) may be incurred before the start of the production of a product.
- There is a constant holding cost rate for each product for carrying inventory.
- The products have limited shelf life and no spoilage is allowed.
- Stock is used on a first-in-first-out basis.

The following notation is used in this section. For each item *i=1,...,N* ; the following parameters are assumed to be given:

d_i	:	constant demand rate (units per unit time)
p_i	:	constant production rate (units per unit time)
c_i	:	setup cost per production lot of product i ($)
u_i	:	setup time per production lot of product i (unit time)
h_i	:	inventory holding cost ($ per unit per unit time)
s_i	:	shelf life (unit time)

We define T_i as the cycle time for product i, i.e. the time that elapses between the commencement of two successive production runs of product i. The ELSP problem with shelf life constraints is to find feasible cycle times $T_1, \ldots, T_N$ for which, the average cost per unit time (setup costs plus inventory costs), V, of producing all N products is minimized.

The total cost V is equal to $\sum V_i$ where V_i is the cost per unit for producing product i and is given by

$$V_i = c_i/T_i + h_i T_i d_i (1 - d_i/p_i)/2 \tag{4.1}$$

We will use the basic period approach where each T_i is an integer multiple k_i of a time interval T_{BP} that we refer to as a basic period. Thus for each product:

$$\begin{aligned} T_i &= k_i T_{BP} && \text{: the cycle time for product } i \\ PT_i &= T_i di/p_i && \text{: the processing time per lot for product } i \\ TPT_i &= u_i + T_i d_i/p_i && \text{: the total production time per lot for product } i \end{aligned}$$

Using above definitions and (4.1), the total cost V can now be written as:

$$V = \sum V_i = \sum c_i/k_i T_{BP} + h_i k_i T_{BP} d_i (1 - d_i/p_i)/2 \tag{4.2}$$

Each item has a shelf life of s_i time units. A too large value of cycle time T_i could lead to spoilage of some units of item i, which would violate our assumption. Given the assumption that stock is used on first-in-first-out basis, and production and demand rate of products are constant, the maximum duration an item is held in stock is $T_i(1 - d_i/p_i)$. Thus, to avoid spoilage of any item we have the condition [as in Silver 1989]

$$T_i(1 - d_i/p_i) \leq s_i$$

Since we are using the basic period approach where $T_i = k_i T_{BP}$, the above shelf life constraint can be rewritten as

$$T_{BP} \leq \left\{ \frac{s_i}{k_i(1 - d_i/p_i)} \right\} \tag{4.3}$$

It is clear from (4.3) that for a given set K, which consists of k_i values for each product, T_{BP} has an upper bound and is given by

$$T_{BP} \leq \min_i \left\{ \frac{s_i}{k_i(1 - d_i/p_i)} \right\} \tag{4.4}$$

Keeping in mind that there must be sufficient time to handle the average number of setups required for all products, a lower bound for T_{BP} can be specified as

$$T_{BP} \geq \frac{\sum u_i/k_i}{(1 - \sum d_i/p_i)} \tag{4.5}$$

Here, it may be noted that this is a necessary condition but is alone not sufficient to ensure a feasible schedule generation. Given that total time required on the facility for a production of item i is TPT_i, and production runs of product i must begin at k_iT_{BP} time units apart, a feasible schedule is the schedule of length that is equal to the least common multiple (LCM) of k_i values times T_{BP} and which does not have interference between the times that each product has to occupy on the facility.

To get a more clear idea of what a 'feasible schedule' is, an illustration is given in figure 4.1. It shows a production schedule that satisfies the problem described in previous paragraphs. The manufacturer produces items 1,2,3,4 with basic period of T_{BP} and k_i values of 1,2,2,4 respectively. Item 1 is produced in every basic period while items 2 and 3 are both produced in the first and third basic periods. Item 4 is produced in the second basic period only. The total cycle length is of $LCM(k_i) * T_{BP}$ time units. The inventory versus time profiles for only items 1 and 3 are drawn. The cycle time and the maximum duration for which a unit of product 3 can be held in the stock are also shown. This maximum duration should be less than s_i, the shelf life, in order to avoid spoilage.

Obtaining the solution (T_{BP}, K), which comprises of the basic period length T_{BP}, the vector K of item multiplier values k_i, that minimizes (4.2) while satisfying (4.3) and (4.5) and is capable of generating a feasible schedule is the focus of this section.

4.4.3 The basic period approach solution

In this section, we present a modification of Haessler's procedure (1979) to find out the basic period length and production frequencies that minimize total setup and inventory costs while satisfying shelf life constraints. Before presenting the solution approach, we would like to state some preliminary remarks that will help us in achieving feasible and better cost solutions.

Preliminary remarks

The method described in the section is essentially a search procedure within a solution space (T_{BP}, K), where K is a vector of integer multiples k_i. The method iteratively reduces the search space and then performs a quite exhaustive search to get improvement in the solution. It may be noted that the solution space (T_{BP}, K) is bounded because of the following reasons–

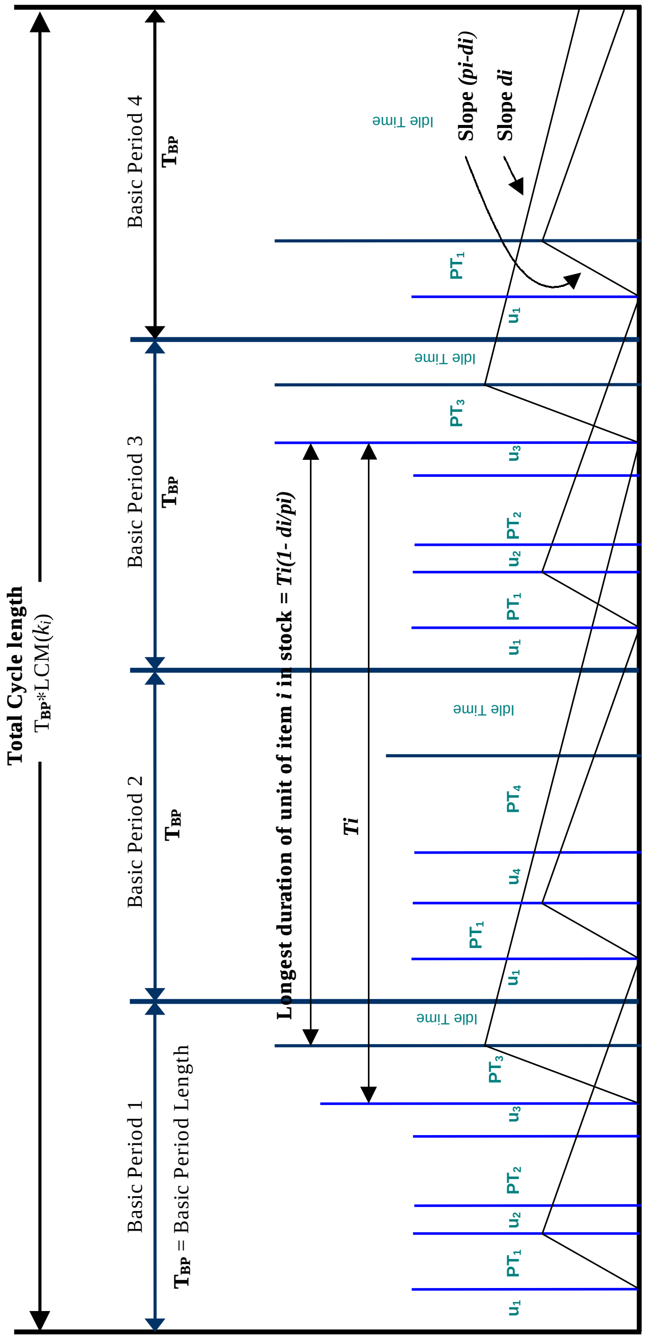


Figure 4.1: Example of a feasible solution for ELSP with shelf life considerations

1. Because of the shelf life constraints, for a given set K, T_{BP} has an upper bound given by (4.4). There is also an overall upper bound on T_{BP} irrespective of K and is given by– $\min\{s_i/(1-d_i/p_i)\}$

2. T_{BP} has a lower bound greater than zero, given by (4.5).

3. Since k_i can take only positive integer values, it has a lower bound of 1.

4. From (4.5), it is clear that if the denominator is greater than zero (i.e. utilisation level less than 100%), the T_{BP} has a lower bound greater than zero in order to get a feasible solution. This prevents each k_i from getting infinitely large since $T_i = k_i T_{BP}$ and can take only finite values. Thus, there are bounds on k_i as well.

It is obvious that, the number of values that k_i can take within these bounds may still be very large. We will, however, restrict these values to power-of-two i.e. $\{1, 2, 4, 8, 16, \ldots\}$. This will reduce the search space drastically without too much sacrifice of optimality. Maxwell and Singh (1983) provide an economic rationale and justification of such power-of-two restriction that is widely used in ELSP literature. They prove that using such policy results in a 6% costlier solution in the worst case. The construction of repetitive feasible schedule also becomes easier in case of power-of-two, since $LCM(k_i)$ takes the value of $max(k_i)$.

In the basic period ELSP approaches without shelf life (e.g., Haessler, 1979), for a fixed K, if a feasible schedule can be generated for a certain basic period length of T'_{BP} then a feasible schedule can be generated for all basic period lengths which are greater than T'_{BP}. The converse is also true – if a feasible schedule cannot be generated for a basic period length of T'_{BP} then a feasible schedule cannot be generated for the basic period lengths smaller than T'_{BP}. During our search procedure, we will make use of this property effectively after ensuring that the shelf life constraints are not violated.

Since shelf life constraints put an upper bound on T_{BP} values for a fix set K, we will check if a feasible schedule can be generated for T_{BP} equal to this upper bound, which is given by (4.4). If a feasible schedule exists, then only it makes sense to search for T_{BP} value less than this upper bound in order to get a feasible and least cost solution for the current values of k_i.

If a feasible schedule cannot be generated using the T_{BP} upper bound, then the only way of achieving feasibility is to increase the production frequencies (i.e. reducing k_i) of some item(s). This will allow T_{BP} to take higher values than before since new k_i values will be providing higher upper bound. This in turn may yield a feasible schedule. Once the feasibility is achieved, an analysis of sensitivity of k_i with respect to current T_{BP} value is done in pursuit of a lower cost solution.

In some cases, it may happen that the successive iterations do not yield any feasible solution and thus all k_i values will be reduced to 1. In such cases, we get the common cycle solution, which is the same as the option of reducing the common cycle time so as to meet shelf life constraint (as in Silver 1989, and Sarker and Babu 1993).

The basic period approach for ELSP with shelf life

Now we present a modified basic period algorithm for ELSP with shelf life considerations. The solution procedure has the following main steps – 1) Finding production frequencies for each product, 2) Checking for shelf life constraint violation, 3) Forming a production schedule 4) Achieving feasibility and lower cost solutions by increasing T_{BP} or reducing k_i through sensitivity analysis.

Step 1: Use Doll and Whybark (1973) procedure with the power-of-two policy to get good starting values of k_i and T_{BP}. This procedure is described in appendix A.1 at the end of this chapter.

Step 2: Ensure that T_{BP} satisfies the constraint given by (4.5).

$$T_{BP} = \max\left\{T_{BP}, \frac{\sum_i u_i/k_i}{(1-\sum_i d_i/p_i)}\right\}$$

Step 3: Ensure that T_{BP} satisfies the shelf life constraint given by (4.4).

$$T_{BP} = \min\left\{T_{BP}, \min_i\left[\frac{s_i}{k_i(1-d_i/p_i)}\right]\right\}$$

A check is done if a feasible schedule can be generated for the basic period length of $T_{BP} = \min_i\{s_i\ /k_i(1-d_i/p_i)\}$. If yes go to step 4, otherwise go to step 5. The feasibility check is done using the procedure described in appendix A.2 at the end of this chapter.

Step 4: The basic period T_{BP} as obtained in step 2 is systematically increased till feasibility is achieved or till it reaches $\min_i\{si\ /ki(1-di/pi)\}$. If this feasible solution is the lowest cost feasible solution so far, save it as the 'current best solution' and continue to step 5.

Step 5: This step involves following substeps

- **(a)** If $max(k_i) = 1$, go to step 7.
- **(b)** For each product with $k_i > 1$, halve the value of k_i and calculate the lower bound of the cost per unit time, V, using (4.2), where T_{BP}= {2[$\sum c_i/k_i$]/[$\sum h_i d_i k_i(1 - d_i/p_i)$]}$^{1/2}$.
- **(c)** Sort the products in ascending order of their cost V and store in a list.
- **(d)** If there is any 'current best solution' stored (in step 4), ignore those products which give higher minimum costs than the 'current best solution' and update the list. If the list is empty the procedure terminates and the 'current best solution' is the final solution.
- **(e)** Choose the first product in the list.
- **(f)** Using equation (4.4), calculate upper bound with new k_i values. i.e. k_i value for this product is halved while others retain their k_i values from step 2.
- **(g)** Check if it is possible to generate a feasible schedule for new K vector and with T_{BP} equal to new upper bound as in (f). If a feasible schedule can be generated, go to step 6.
- **(h)** Choose the next product in the list and go to (f). If end of the list is reached, choose the first product in the list and go to step 6.

Step 6: Halve the k_i value of the product obtained in step 5. If $max(k_i) > 1$ go to step 2, otherwise go to step 7.

Step 7: Stop the procedure. If there is a 'current best solution', it is the final solution otherwise use the common cycle approach (the option of reducing the production cycle time as in Silver 1989).

The method differs from Haessler's procedure mainly in steps 3–6. In Haessler's procedure T_{BP} can be increased till a feasible schedule is achieved for a fixed values of k_i whereas we can't do it always because of the upper bound on T_{BP} imposed by shelf life constraints. We, thus, have to resort to reducing k_i values instead. We perform sensitivity analysis on k_i values not only for searching better cost solutions but also to achieve feasibility.

The method presented above has concepts similar to those in *branch & bound* methods. In step 3 and/or 4, we start with the root node. Steps 5(b), 5(c) provide a list of next nodes to be examined, while step 5(d) helps us in reducing search space by discarding non-promising nodes. Steps 5(e), 5(f), 5(g) and 6

Table 4.1: Bomberger problem with shelf life of products – 88% utilisation

Product	Cost per item*	Setup cost	Production rate Items/day	Demand rate Items/Day	Setup time Hours	Shelf life Days
1	0.0065	15	30000	400	1	100
2	0.1775	20	8000	400	1	150
3	0.1275	30	9500	800	2	100
4	0.1000	10	7500	1600	1	30
5	2.7850	110	2000	80	4	100
6	0.2675	50	6000	80	2	100
7	1.5000	310	2400	24	8	200
8	5.9000	130	1300	340	4	150
9	0.9000	200	2000	340	6	150
10	0.4000	5	15000	400	1	150
Common cycle solution				Suggested procedure		
Common cycle length (days)		38.136		Basic period length (days)		23.630
Daily Cost ($)		41.374		Daily cost ($)		31.951

* Annual inventory cost = 10% of item cost and one year = 240-8 hour days

deploy *depth-first-search* technique on remaining nodes to arrive at feasible and better cost solution.

We would also like to point out that there is no feasible solution for even the common cycle approach if the shelf life of some product is too short to accommodate the setups for different products. i.e.

If $\min_i \left\{ \frac{s_i}{(1-d_i/p_i)} \right\} \leq \frac{\sum u_i}{(1-\sum d_i/p_i)}$, then there is no feasible 'common cycle solution'.

The details of Doll and Whybark (1973) procedure with the power-of-two policy used in step 1 and the procedure for generating production schedule used in Steps 3–5 are given in the appendix at the end of this chapter.

4.4.4 Computational results

In this section, we report our computational results using the proposed procedure. First, a numerical example illustrating the procedure is presented. Later, results from different experiments are also presented.

We will use the well-known Bomberger problem data with additional shelf life factor for each product (see table 4.1). The step by step illustration of the procedure is provided in table 4.2.

The feasible schedule (in terms of allocation of products to basic periods) for the saved best-cost solution is given in table 4.3. The total of setup and inventory costs for this schedule is $31.951 per day.

Table 4.2: Illustration of solution procedure

Step	Iteration 1	Iteration 2	Iteration 3
Step 1	$K = \{8, 2, 2, 1, 2, 4, 8, 1, 4, 2\}$ $T_{BP} = 20.379$, Cost $= 31.956$		
Step 2	$T_{BP}^{LB} = 12.889$ $T_{BP} = 20.379$	$K = \{8, 2, 2, 1, 2, 4, 8, 1, 2, 2\}$ $T_{BP} = 23.534$, Cost $= 31.922$ $T_{BP}^{LB} = 14.484$, $T_{BP} = 23.534$	$K = \{4, 2, 2, 1, 2, 4, 8, 1, 2, 2\}$ $T_{BP} = 23.630$, Cost $= 31.951$ $T_{BP}^{LB} = 12.889$, $T_{BP} = 23.630$
Step 3	$T_{BP}^{UB} = 12.669$, No feasibility Go to Step 5	$T_{BP}^{UB} = 12.669$, No feasibility. Go to Step 5	$T_{BP}^{UB} = 25.253$, Feasible Solution. Go to Step 4
Step 4			Solution Saved. $T_{BP} = 23.630$ 'Current best solution' $= 31.951$
Step 5	Product, T_{BP}^{UB}, V 9, 12.669, 31.922 1, 25.253, 32.005 10, 12.669, 32.012 2, 12.669, 32.250 3, 12.669, 32.276 6, 12.669, 32.381 7, 12.669, 33.158 5, 12.669, 33.554 No feasible Solution using new k_i values and T_{BP}^{UB}. First product, product 9 is selected	Product, T_{BP}^{UB}, V 1, 25.253, 31.951 10, 12.669, 31.951 3, 12.669, 32.083 2, 12.669, 32.122 6, 12.669, 32.238 7, 12.669, 32.786 5, 12.669, 33.037 9, 12.669, 34.491 Feasible Solution using $k_1 = 4$ and new T_{BP}^{UB} Product 1 is selected	Product, T_{BP}^{UB}, V 10, 25.253, 31.980 1, 25.253, 32.084 3, 25.253, 32.107 2, 25.253, 32.148 6, 25.253, 32.264 7, 25.338, 32.806 5, 25.253, 33.053 9, 25.253, 34.500 Step 5 (d) Gives empty list. Procedure Terminates
Step 6	$k_9 = 2$, Go to Step 2	$k_1 = 4$, Go to Step 2	
Step 7			

Table 4.3: Allocation of products to basic periods – 88% utilisation case

Product	k_i	TPT_i	1	2	3	4	5	6	7	8
8	1	6.680	6.680	6.680	6.680	6.680	6.680	6.680	6.680	6.680
4	1	5.166	5.166	5.166	5.166	5.166	5.166	5.166	5.166	5.166
9	2	8.784	8.784		8.784		8.784		8.784	
3	2	4.230		4.230		4.230		4.230		4.230
2	2	2.488	2.488		2.488		2.488		2.488	
5	2	2.390		2.390		2.390		2.390		2.390
10	2	1.385		1.385		1.385		1.385		1.385
6	4	1.510		1.510				1.510		
1	4	1.385		1.385				1.385		
7	8	2.890				2.890				
Total time used			23.118	22.747	23.118	22.742	23.118	22.747	23.118	19.852

It can be seen that the procedure results in lower costs than the common cycle solution approach (Option of reducing cycle times as in Silver 1989, Sarker and Babu 1993) which gives a solution with common cycle of 38.136 days and cost of $41.374 per day.

We also conducted different experiments to compare the suggested procedure to the common cycle procedure with shelf life considerations (see table 4.4). They provide an evaluation of two factors– utilisation and product diversity. High levels of utilisation make it more difficult to develop feasible schedules where multiple runs are evenly spread over time. The product diversity factor determines the range of values for production rate p_i and demand rate d_i. It is well known from ELSP literature that high product diversity leads to some products being produced more frequently than others. We use the Bomberger data set for different utilisation levels with addition of shelf life for each product. The impact of shelf life can be easily seen through the changes in production frequencies of the products. The products are required to be produced more often to avoid spoilage. The comparison with common cycle solution is also presented in table 4.4. It is evident that the suggested procedure results in lower cost than the common cycle solution approach at all utilisation levels. The cost savings are up to 40% in case of low utilisation.

In the data set of Table 4.1, the highest demand item (product 4) has the lower shelf life. More frequent production of this product ($k_i = 1$) could take care of the situation. We conducted another set of experiments with a low demand item (product 7) having a low shelf life. It is interesting to note that in the case of a shelf life of 30 days for product 7 (all other parameters being same as in Table 4.1), there is no feasible solution. This shelf life for product 7 is too short even to accommodate the setups for different products. If the shelf life of this product is 40 days, we get a common cycle solution (which of course is quite costlier as compared to the situation if shelf life for this item was relatively higher). This example definitely provides a motivation for designing products with higher shelf life in order to get lower cost production schedules. These experiments also support a generally accepted fact that the low demand items with low shelf lives are candidates for make-to-order (see Soman et al. 2004a). Allowing backordering for such items as in Viswanathan and Goyal (2000) is an another possibility.

4.4.5 Concluding remarks

The researchers working on ELSP with shelf life considerations have focussed too much on the option of deliberately reducing production rates. In many in-

Table 4.4: Experiments at various utilisation levels

Product	Utilisation level								
	22%			44%			66%		
	Step1 k_i values	Final k_i values	Production Periods	Step1 k_i values	Final k_i values	Production Periods	Step1 k_i values	Final k_i values	Production Periods
1	8	4	1,5	8	4	1,5	8	4	2,6
2	4	2	1,3,5,7	2	2	1,3,5,7	2	2	1,3,5,7
3	2	2	1,3,5,7	2	2	1,3,5,7	2	2	1,3,5,7
4	1	1	ALL	1	1	ALL	1	1	ALL
5	4	2	1,3,5,7	4	2	1,3,5,7	2	2	1,3,5,7
6	8	4	1,5	8	4	1,5	4	4	2,6
7	16	8	1	16	8	1	8	8	2
8	1	1	ALL	1	1	ALL	1	1	ALL
9	4	4	1,5	4	2	1,3,5,7	4	2	1,3,5,7
10	2	2	1,3,5,7	2	2	1,3,5,7	2	2	2,4,6,8
CC solution (days)		31.690			33.582			35.714	
Cost per day ($)		32.357			35.373			38.379	
Basic period solution (days)		25.063			25.126			25.189	
Cost per day ($)		18.765			24.449			28.459	

dustries and especially, in food processing industry this option is not at all applicable since it will result in products with quality and yield that is different than expected. Also, the previous research has considered only the common cycle approach. This is in spite of the fact that a large body of ELSP literature has shown that the basic period approach outperforms the common cycle approach.

Haessler's procedure has been adapted for determining the cycle times for the lot-scheduling problem to account for constraints imposed by shelf life of products. Unlike other ELSP literature with shelf life considerations, the proposed algorithm allows products to be produced more than once in a cycle. The procedure presented here can never result in higher cost solutions than the common cycle approach. In the worst case, the procedure yields the common cycle solution. If the shelf life of some product is quite different than others, then the cost benefits that are achieved through the use of our procedure are quite significant (up to 40% lower costs in the experiments carried out).

The products having limited shelf life are very common in food processing industries. However, these industries are also characterized by sequence dependent setups times (and costs). The procedure presented in this section cannot be directly used in such situation. The economic lot scheduling problem that considers both shelf life constraints and sequence dependent setups is a challenging problem for future research. In this section, for each item the production lots are of equal size and are equally spaced and the solution procedure can leave idle times in the schedule. In view of this, it may be a logical extension to modify other ELSP approaches like time-varying lot sizes (Dobson 1987) to take care of shelf life constraints. As pointed out in chapter 2 and Soman et al. (2004a), combined make-to-stock and make-to-order food production system are becoming more common. In this context, it will be interesting to study the ELSP procedures so as to incorporate make-to-order and is explored in section 4.5. Our numerical results with low-demand, short shelf life products show that further research on this is needed. We would also like to mention, as pointed out by Van Donk (2001), that in practice many products have a reasonably long technical shelf life but retailers do not accept successive deliveries with identical 'best-before-dates'. The result is that from a technical point of view products are fresh but are commercially obsolete. Thus, schedulers at food manufacturers may not only want to reduce storage time of products in their warehouses and provide longer storage possibilities for retailers but also need to ensure that successive deliveries do not have identical best-before-dates. This, however, may lead to more frequent production. The existing literature on lot scheduling problem with shelf life considerations has not dealt with the effects of such commercial compulsions on the food manufacturers. We think that developing models for reducing storage life is an interesting area for further research.

4.5 Incorporating MTO in ELSP[2]

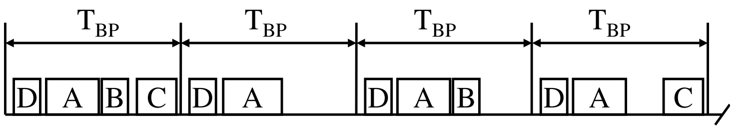
The logic of ELSP approaches is that a product is manufactured during a cycle and that inventory will be sufficient to cover demand until the next lot will be produced. As discussed in section 4.3, the normal ELSP (as some other EOQ-based policies) is not directly applicable for real-life situations. But within the ELSP approach hardly any attempts have been made to incorporate make-to-order production, so far. The objective of this section is to explore how we can adapt ELSP procedures for the combined MTO-MTS in an abstract sense. The second objective is to provide some directions for determining a production cycle in real-life situations.

The main problem in incorporating MTO items is that demand is not known in quantity and/or timing. This complicates the standard ELSP but the combination of MTO and MTS offers (as suggested by Bemelmans 1986) the possibility to buffer part of this uncertainty with additional stock of MTS. Some further complications arise because food processing industries usually have family setup structures that favour production in a (family-)cycle (Van Donk 2001). A cycle usually ends with a major cleaning of the equipment. In order to structure the discussion we distinguish some different situations with respect to demand and the nature of setups. In the normal ELSP approaches demand is supposed to be deterministic, so we start with situations that only in a limited way deviate from that basic situation. Next, in case of MTO items uncertainty with respect to demand can be uncertainty in quantity (and capacity needed) or in the specification of the product (but with a stable capacity requirement). Next to exploring the influence of demand of MTO, we investigate the setup structure by assuming a family structure.

4.5.1 Stable MTO demand, no family structure

Suppose the aggregate demand for MTO items is deterministically known and constant over time, but demand for each single MTO item is not known. A practical example might be that the colour or type of packaging is specified just

[2] Earlier version of this section has appeared in Van Donk, D. P., Soman, C. A. and Gaalman, G. (2003), ELSP with combined make-to-order and make-to-stock: Practical challenges in food processing industry, *1st joint POMS-EUROMA International Conference, Como Lake, Italy*, vol. II, pp. 769–778.



Items A, B, C are MTS while D denotes the MTO items

Figure 4.2: Capacity reservation for MTO products

before production. Now, we can treat all MTO items together as an additional product with known demand. Under these assumptions, the normal ELSP procedures can be followed to determine the production cycle, the amounts to be produced, based on the minimisation of costs. In fact, we treat the MTO items here as MTS items that have no stock. Depending on the parameters, an amount of capacity is reserved for the MTO items. This is illustrated in figure 4.2. The capacity planned for MTO can be used to produce the MTO items that are actually ordered during each cycle. If the due date of MTO items is known, the length of it can be used as an additional constraint in determining the cycle length. The due date can be used for the determination of a 'natural' cycle length for the MTO, as starting values in procedures like Doll and Whybark (1973). Due dates for MTO can also be determined as a result of making a feasible schedule. Then, a trade-off exists between the length of the due date (longer due dates can be associated with higher costs) and the setup costs. In general, the length of the cycle determines the maximum due date for MTO. From a more practical point of view, it is interesting to note that order acceptance and due date determination are quite simple. Given the cycle time, due dates are fixed and all MTO orders can be accepted and delivered.

4.5.2 Unstable MTO demand, no family structure

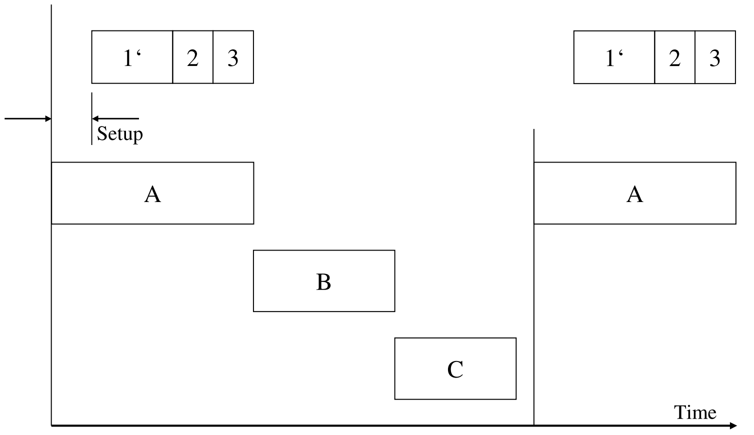
Here we assume that the aggregate demand needed for MTO is not deterministic and has a large variance over time. If demand is low compared to capacity, it is easy to reserve enough capacity to cope with even the largest variations in demand of MTO items. If capacity is more restricted, finding a cycle is not that easy. Making a reservation (e.g. based on the expected value) for the MTO items is rather risky, as either demand for MTO will be much higher or much lower. However, on average capacity for MTO will be needed and used. In order to cope with the variance in demand of MTO, two possibilities exist. Varying the due date of MTO: which is basically a buffer in time, or buffering uncertainty of MTO with an (additional) inventory of a MTS item (following the idea of Bemelmans 1986). The safety stock of the MTS is consumed less if demand for MTO is low and is consumed more if demand for MTO is high. It is interesting to note that we even need safety stock in case demand of MTS is totally deterministic and randomness only in MTO items. The cycle length can be determined in the same way as in the previous case, based on the average demand for MTS items. The remaining problem is to determine safety stock levels for the MTS item. This strategy needs a clear operational control in order to maintain the level of stock and accept MTO orders. It is very important to keep the cycle length constant: then the availability of production capacity is guaranteed. From a practical point of view, clear operational control will be needed and occasionally orders need to be refused due to lack of capacity or when safety stocks are too low to deliver MTS items.

4.5.3 Family structure setup

Here we assume that the products have sequence dependent setups and more specifically some kind of family structure: with large setups for a family and smaller setups for the family members. It is worth noticing, that in contrast to normal ELSP, setup control grows in importance. In general, a family consists of both MTO and MTS items and given the large family setup it is preferred to produce all products of a family after a family setup to minimise family setups. Given the family structure, again the two above situations (stable or unstable demand for MTO) can be assumed. Figure 4.3 illustrates one such situation. There are three product families each having a few products. Product 1 in family A carries extra inventory to account for unstable MTO demand.

If we assume that MTO items are fairly stable in demand on a family level, we might use the approach of Atkins and Iyogun (1988) that is based on deterministic demand. This method determines the frequency for each family first (based on the family setup and aggregate family demand) and then the production frequency for each item within a family, based on the allocation of the major family setup to the items. One major problem with two applications (McGee and Pyke 1996, Strijbosch et al. 2002) is, however, that 20-30% idle time (including maintenance and breakdowns) is allowed, while in food processing capacity utilisation is high. Note that the stable demand, no family structure situation discussed above now holds for each family.

If it is assumed that the variance in demand of MTO items is large, more problems arise. Now we might estimate total capacity needed for all MTO items during a cycle: assuming that the individual variance of MTO items is absorbed in the aggregate forecast. The reserved capacity is (as part of the execution of



A, B, C are product families. 1, 2, 3 are products within family A.
1‘ indicates that product 1 carries buffer inventory to account for MTO

Figure 4.3: Additional inventory for MTS items

production control) allocated to different families on the basis of actual orders for MTO items. An alternative is to estimate the demand on a family level in which case the second situation of above (unstable MTO demand, no family structure) holds. However, if variability in MTO demand differs across families or if it is assumed that some families contain only MTO or MTS items, it can be rather difficult to find a buffer in time or buffer in stock of a MTS item. In all cases, one has to decide how uncertainty in MTO can be buffered by MTS items and which items from what family are the best candidates for carrying buffer inventory. Under tight capacity utilisation keeping the run length of families constant as well as the cycle time for individual items might be problematic. All in all, production control: both in planning MTO, controlling capacity, checking inventory levels and determining due dates will need a lot of attention.

The practical implications for management are the same as in the two situation described. The determination of the whole cycle will be rather complex in this case.

4.5.4 High variance in MTS and family structure

If we assume that MTS items show a rather large variance in demand, the possibilities to buffer MTO will be more difficult. Now, an alternative for the normal ELSP-like approaches can be derived from the 'can-order' policies, also described as (s, c, S) policy. Here s is the reorder point, c is the can-order point and S is the order-up-to level. This policy is used to initiate production for a product that has reached its reorder point and to produce other items from the same family that have reached their can-order level (see Silver et al. 1998, Federgruen et al. 1984). This approach can be adapted for the combined MTO-MTS case.

We determine a family sequence, based on aggregate demand for a family. During a family run both MTO and MTS (that are on or below their reorder point) must be produced, supplemented with can-orders to fill family capacity. (Silver et al. 1998) suggest that optimising can-order policies (without MTO) is complicated and inferior to the method of Atkins and Iyogun (1988). However, as far as we know it is not tested explicitly for the combined MTO-MTS case. We think that this approach could be well implemented if the aggregate demand on a family level is rather stable. Silver et al. (1998) remark that this approach is well suited for situations where savings in setups are important as in the food processing industry. If the MTO orders are large compared to MTS, then the start of a family might be induced by the arrival of a MTO item order that is supplemented with MTS. However, a fixed cycle is then abandoned.

From a managerial point of view, it can be noted that this type of control needs a clear order acceptance policy and a good collaboration between scheduling and order acceptance.

4.5.5 High utilisation and controlling setups

The last situation we explore is the case of high capacity utilisation as is often found in food processing and other process industries. Now the main aim is to control the time for setups and/or the amount of setups. This approach is more top-down, starting with limiting the time for setups for a year and then determining the ŞavailableŤ setups for a month or week. A possible solution method is to allocate setup time to families based on the demanded amounts. Another approach is to add restrictions to the previously mentioned methods. Limiting the number of setups for a time period can also pose natural constraints on the number of MTO orders that can be accepted during that period. In any case the amount of production capacity or time available for manufacturing is fixed in advance. We think that further elaboration of the different options into more concrete decision tools is needed. We assume that some of the approaches are less usable if the percentage of MTO items is large, because buffering possibilities with MTS items is then limited. Next, actually determining the sizes of buffers is not directly straightforward. Introducing another food processing characteristic such as limited shelf life will give additional problems as discussed in the previous section.

4.6 Conclusion and discussion

This chapter investigated the capacity coordination in the medium term. The main problem that has to be addressed at this level is the allocation of capacity to different products and product families. ELSP literature has been widely used for these purposes in the case of pure MTS situations. This chapter builds on basic period ELSP approaches and provides solutions to problems arising from certain food processing characteristics. Shelf life constraints have been included in a formulation and a heuristic is presented which will never perform worse than the existing common cycle approaches. Incorporating MTO in ELSP has been addressed for the first time in the research literature. We specially looked at various demand patterns, presence of family structure setup. Several ideas and managerial insights are provided on various production situations. Although the discussion is conceptual in nature, it can be used to translate the ideas into analytical models. However, we do not do that so far.

In this chapter, we relied heavily on ELSP procedures developed for pure MTS situations. This seems logical given the fact that the food processing industries are traditionally make-to-stock companies and are only now producing a small proportion of their production on make-to-order basis. However, with the product portfolios of the companies expanding rapidly, the proportion of MTO products is bound to increase. In such cases, the ELSP approaches minimising the sum of inventory and setup costs may not be helpful as the inventory costs will be less relevant. The cyclic policies will, however, still be attractive given the environment consisting of product families and high setup time. The determination of cycle times (for families) is then essentially a trade-off between required customer order lead time and the setup times. Examples of such production planning and control rules based on cyclic production and pure make-to-order environment are suggested by Bertrand et al. (1990) in chapter 9, and Dellaert (1989). The control of cycle time, so as to avoid extra setups and ensure productive capacity, at the operational level can generally be achieved in two ways: (1) The standard customer order lead time for a product should be at least equal to the cycle time of its product family. This will ensure that each product will have at least one production opportunity during its lead time. (2) Order acceptance function– while accepting a order the workload level (already planned plus workload from the new order using the standard customer lead time) is checked against the productive capacity available. In case a capacity problem occurs, then either the order is rejected or the other possibilities are to be considered. These possibilities include a use of parallel equipment (if available), negotiable due-date and price.

Appendix

For the sake of completeness, we have chosen to spell out in details two important steps in the basic period procedure suggested in section 4.4.3. These are namely the Doll and Whybark (1973) procedure for finding the starting solution (step 1) and the procedure for generating schedules (used in steps 3–5).

A.1 Doll and Whybark procedure with the power-of-two policy

This is an iterative procedure to simultaneously determine product multipliers k_i and the basic period T_{BP}.

Step a Determine T_i independently for each product

$$T_i = (2c_i/[h_i d_i(1 - d_i/p_i)])^{1/2}$$

Step b Select the smallest T_i as the initial estimate of the basic period T_{BP}.

$$T_{BP} = \min(T_i)$$

Step c Determine the integer multiple k_i^- and k_i^+ for each product defined by

$$k_i^- \leq T_i/T_{BP} \leq k_i^+$$

Where k_i^- = {1,2,4,8,16,...} the next lowest power-of-two integer multiple, and k_i^+ = {1,2,4,8,16,...} the next higher power-of-two integer multiple.

Step d The k_i value is set to either k_i^- or k_i^+, the one incurring less costs using equation (4.1).

Step e Recompute the basic period time T_{BP} using the new estimates of k_i.

$$T_{BP} = \{2[\sum c_i/k_i]/[\sum h_i d_i k_i(1 - d_i/p_i)]\}^{1/2}$$

Step f Return to step c to determine new k_i^- and k_i^+, using T_{BP} from step e. The procedure terminates when consecutive iterations produce identical values of k_i at step d.

This method gives T_{BP} , k_i and hence the production times TPT_i can be calculated.

A.2 Creating a Production schedule

1. The complete rotation cycle is of length $T = max(k_i)T_{BP}$ and has $max(k_i)$ slots each having T_{BP} units of time available.

2. Sort the item in ascending order of their k_i. Products with the same k_i are sorted in descending order of total production times TPT_i.

3. Assign the item at the top of the list to the basic period slot that has sufficient time available for its production, and in a similar fashion to subsequent slots with intervening gaps of k_i - 1 slots. Until $max(k_i)/K_i$ assignments for that item have been made. If such assignments are not possible, a feasible schedule cannot be generated and the procedure stops.

4. Update time available in each slot and delete the item from the list. If the list is non-empty return to step 3 above; otherwise a feasible schedule has been generated and the procedure stops.