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A risk-averse competitive newsvendor problem under the CVaR criterion

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\textbf{A B S T R A C T}

We study a risk-averse newsvendor problem with quantity competition and price competition. Under the Conditional Value-at-Risk (CVaR) criterion, we characterize the optimal quantity and pricing decisions under both quantity and price competition. For quantity competition, we consider two demand splitting rules, namely proportional demand allocation and demand reallocation. Although competition always leads to overstocking, interestingly it does not necessarily lead to a profit loss in certain competitive environments, such as demand reallocation, by avoiding/reducing overstocking that results from competition under the risk-neutral criterion. For price competition, we consider both additive and multiplicative demand. We find that the order quantity, sale price, and the expected profit decrease in the degree of risk aversion. Further, both high price sensitivity and competition intensity force decision makers to lower their prices. However, high price sensitivity always reduces the order quantity while competition can have the opposite effect.

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1. Introduction

The newsvendor problem, as one of the fundamental problems in supply chain management, is an one-time business decision in which a monopolist vendor orders inventory before a one period selling season with stochastic demand. The standard newsvendor problem has been widely studied and extended to more complex models. We refer interested readers to Khouja (1999), Petruzzi and Dada (1999) and Qin et al. (2011) for detailed reviews of the newsvendor problem and its extensions.

To reflect the situation in real markets, one major extension of the newsvendor problem is the replacement of the monopolist vendor by a set of competing vendors. A key element of the competitive newsvendor problem relates to how competition affects inventories and pricing. The two main research streams are quantity competition and price competition. In recent years, a number of papers have been devoted to competition analysis in the standard (risk neutral) newsvendor problem. They find that the quantity competition always leads to inventory overstocking (e.g., Lippman and McCardle, 1997; Cachon, 2003) and that price competition leads to lower equilibrium prices and higher stock levels in all firms (e.g., Chen et al., 2004). An overview of the key results is provided in Section 2.1.

Ordering and pricing decisions based on quantitative models in industrial practice and academics are often based on minimizing the expected cost or maximizing the expected profit, which implies the concept of risk neutrality of the decision maker. Nowadays, supply chains become more vulnerable to uncertainties. Besides the expected profit, decision makers focus more on risk or potential loss. Hence, the assumption of risk neutrality seems to be inadequate for contemporary supply chain management. In view of this, many researchers have called for models that deviate from the risk neutral assumption. For example, Tsay (2002) suggests that various players should be allowed to have different attitudes towards risk sensitivity. Recent empirical findings provide further support for the importance of incorporating risk preferences in business practices. After a survey of 1500 executives from 90 countries, a McKinsey research report (Koller et al., 2012) points out that the decision makers demonstrate extreme levels of risk aversion regardless of the size of the investment, even when the expected value of a proposed project is strongly positive. Through an experimental study, Schweitzer and Cachon (2000) show that the ordering decisions reflect risk aversion for high profit products. Motivated by these arguments, research on risk-averse models with different objective functions to reflect risk preferences has become an important stream.

Expected utility, mean-variance, and VaR/CVaR are the three main research streams of modeling risk averseness in inventory and pricing problems. For the framework of expected utility, although any concave increasing utility function (e.g., logarithmic...
utility function and loss-aversion utility function) could reflect risk-averse behaviour, the main challenge is how to explicitly specify the utility function for a decision maker. Note that a utility function represents both the degree of the diminishing utility in wealth and the decision makers’ attitude toward risk. However, these two aspects are inseparable from the utility function (e.g., Levy, 2006). Therefore, expected utility is not a dedicated risk measure and cannot be implemented in practice. The framework of mean-variance introduced by Markowitz (1952) is to address the trade-off between the expected return (mean) and the variation of return (variance). However, the mean-variance criterion suffers an inherent theoretical flaw in which both upside and downside variations from the mean are seen as risk. Although variance is still suitable for the case where the outcome distribution is close to a symmetric distribution, profit distributions of inventory models are asymmetric in general (e.g., Ismail and Louderback, 1979), which implies that variance may not be a reasonable risk measure. The above argument leads to the use of a downside risk measure to replace the variance. CVaR, as one important downside risk measure, is the conditional expected profit below the amount VaR, where VaR is defined as the maximum profit at a specified confidence level. The CVaR criterion is also a coherent risk measure that has some desirable structure and computational characteristics compared to the VaR criterion. Further, as Choi and Ruszczynski (2008) point out, CVaR represents a trade-off between the expected profit and a certain risk measure, which means that CVaR takes into account both the expected profit and the risk. Because of its numerous advantages, the CVaR criterion has been widely applied both in theoretical study and in practice. We review the key results of the newsvendor problem under the CVaR criterion in Section 2.2.

This paper differs from the previous literature on risk-averse newsvendor models in three ways. First, although several researchers (e.g., Wang, 2010) have recently considered risk-averse competitive newsvendor problem under a utility criterion, the aforementioned drawbacks of the expected-utility framework hinder practical implementation. Therefore, we replace the utility criterion by the CVaR criterion that has not been studied in the literature yet. Second, the literature has only explored the price competition in risk-neutral newsvendor games (see Chen et al., 2004), and studied the pricing and quantity decisions of a risk-averse newsvendor without competition (see Chen et al., 2009). The effect of risk averseness in pricing competition has not been investigated yet. Third, the literature on the competitive-newservendor problem assumes that the newsvendors are identical (e.g., Wang, 2010; Chen et al., 2004). The effect of heterogeneity in the newsvendors on the competitive problem is still unknown.

To fill these research gaps, we study the risk-averse competitive newsvendor problem, using CVaR as the risk measure and considering both quantity competition and price competition. For quantity competition, two types of demand splitting rules (proportional allocation and reallocation) are proposed. We show that there exists a unique Nash equilibrium for both competition types. We find that risk averseness decreases the newsvendor’s order quantity and may lead to inventory under stocking. Further, although competition leads to a higher total order quantity than that of integrated newsvendors (single newsvendor problem), the total profit of competing newsvendors may not always decrease. Specifically, the total profit of risk averse competitive newsvendors may be higher than that of integrated newsvendors in the demand reallocation problem. For price competition, the risk averse competition equilibrium has a lower retail price and a lower order quantity. Meanwhile, the total profit of competing newsvendors is always lower than that of integrated newsvendors. Moreover, we show that the equilibrium selling price decreases with both the elasticity of demand and the proportion of a newsvendor’s unsatisfied customers that switch to a competitor (intensity of price competition). The equilibrium order quantity decreases with the elasticity of demand, but first rises and then drops with the competition intensity. For the heterogeneous newsvendors, the newsvendor with a higher degree of risk aversion intends to adopt a more conservative strategy (e.g., a higher price and a less order quantity), which results in a smaller market share.

The remainder of this paper is organized as follows. In the next section, we review the literature on the competitive newsvendor problem and on the risk-averse newsvendor problem. Section 3 introduces the model setting. Section 4 considers the quantity competitive newsvendor model. In Section 5, we consider the pricing competitive newsvendor problem. Section 6 discusses the effects of risk aversion and competition. Section 7 discusses the heterogeneous competitive newsvendor problem and presents numerical results. Finally, Section 8 concludes the paper.

2. Literature review

The newsvendor literature is surveyed from two aspects: competition and risk aversion.

2.1. The competitive newsvendor problem

For quantity competition, Lippman and McCardle (1997) consider a competitive newsvendor problem in which the total demand is allocated among competing newsvendors under certain demand splitting rules. Cachon (2003) considers the same newsvendor problem with a proportional demand allocation rule, i.e., the supplier allocates demand among the newsvendors proportional to their orders. Both studies find that quantity competition leads to over-stocking. Wang (2010) extends the newsvendor problem with a proportional demand allocation rule to a game setting where multiple newsvendors are loss averse under a utility criterion. They show that loss aversion leads to a decrease in the newsvendors’ total order quantity and may lead to a lower total inventory level of the decentralized supply chain than that of an integrated supply chain. By using the same utility criterion as in Wang (2010) and under a demand reallocation rule, Liu et al. (2013) consider the competitive loss averse newsvendor problem in which two substitutable products are sold to two identical retailers. They show that the order quantity of each retailer is increasing in the loss aversion coefficient and increasing in the substitution rate.

For price competition, Chen et al. (2004) investigate the price-dependent competitive newsvendor problem in which the firm uses the price to compete for demand. They find that competition leads to lower equilibrium prices and higher stock levels for all firms. Zhao and Atkins (2008) study a similar pricing competitive newsvendor problem. They also find that the competitive equilibrium has higher safety stocks and lower retail prices. Further, they show that retail prices and safety stocks strictly increase with the proportion of a newsvendor’s unsatisfied customers that switch to a competitor, but strictly decrease with the intensity of price competition.

2.2. The risk-averse newsvendor problem

Modelling risk-averse newsvendor problems has received considerable attention in recent years. Utility functions (e.g., Lau, 1980; Agrawal and Seshadri, 2000; Chen et al., 2007; Choi and Ruszczyński, 2011), mean-variance (e.g., Chen and Federgruen, 2000; Wu et al., 2009) and VaR/CVaR are the three main research streams. We refer interested readers to Jammernegg and Kischka (2012) for summaries of the ordering policies of newsvendors with various risk preferences. Here, we review the key contributions on the CVaR approach, which is one of the most important financial risk measures. CVaR is a coherent risk measure with attractive computational characteristics and consequently is widely used in the financial fields. For more
details of CVar, we refer to Rockafellar and Uryasev (2000, 2002) and the references therein.

Ahmed et al. (2007) consider a coherent risk measure for a multi-period inventory control problem. They show that the structure of the optimal solution of the risk-averse model is similar to that under the risk-neutral criterion. Gotoh and Takano (2007) investigate the risk-averse newsvendor problem under the CVar criterion. They show that downside risk measures including CVar are tractable due to their convexity. Jammermegg and Kischka (2007) analyze the ordering policy and its corresponding performance measures, such as the cycle service level. They consider the trade-off between CVar and the expected profit and conclude that a risk-averse (risk-seeking) newsvendor orders less (more) than a risk-neutral newsvendor. Chen et al. (2009) examine a price-dependent newsvendor model under the CVar criterion, and derive the optimal ordering and pricing decisions for both the additive and multiplicative demand model. Choi et al. (2011) study a multi-product risk-averse newsvendor under the law-invariant coherent measures of risk and show that increased risk aversion leads to decreased orders. Wu et al. (2013) explore the risk-averse newsvendor model with random capacity. The authors find that capacity uncertainty decreases the order quantity under the CVar criterion. Under the VaR constraint, capacity uncertainty leads to an order decrease for low confidence levels, but to an order increase for high confidence levels.

To the best of our knowledge, all literature on the CVar approach is based on the standard newsvendor problem in which there only is a monopoly vendor in the market. Competition under risk averoseness has not been studied yet in the literature. An interesting and unexplored question is what happens in a competitive environment with risk aversion. This is the main focus of our research, where we will consider the CVar approach in order to gain insights into the effects of both risk averseness and competition on the newsvendors’ optimal strategies and profit performance.

3. Model description

We consider two competing newsvendors selling a perishable product to the same market. For newsvendor i, the purchasing cost of the product is $c_i$ per unit, and the selling price is $r_i$ per unit. At the end of the selling season, we assume that the unsatisfied demand is lost and any leftover inventory can be salvaged at a unit value $s_i$. To avoid unrealistic and trivial cases, we assume that $r_i > c_i > s_i > 0$. We denote newsvendor i’s order quantity as $Q_i$ and assume that newsvendor i faces random demand $D_i$ with a cumulative distribution function (CDF) $F_i(\cdot)$ and probability density function (PDF) $f_i(\cdot)$. We also let $F(\cdot)$ and $F^*(\cdot)$ denote the PDF and CDF of the total random demand $D$ of all newsvendors, respectively. Without loss of generality, we assume that $F^*(0) = F_i(0) = 0$. Throughout this paper, we drop the subscript for identical newsvendors and suppose that information on each newsvendor’s demand distribution and cost structure is common knowledge to all newsvendors.

We consider the risk-averse newsvendor under the CVar criterion. CVar maximizes the average profit of the profit falling below a certain quantile level which is defined as the maximum profit at a specified confidence level. More formally, CVar is defined as

$$\text{CVar}_\eta(\pi(Q)) = \max_{\varphi \in \mathbb{R}} \left\{ \varphi(Q) \varphi = \varphi - \frac{1}{\eta} \left\{ \varphi(\varphi - \pi(Q)) \right\}^+ \right\},$$

where $\varphi$ reflects the target profit of the newsvendor and $\eta \in (0, 1]$ reflects the degree of risk aversion, i.e., a lower value implies a higher degree of risk aversion.

The aim of the risk averse decision maker is to minimize the downside risk of his profit. In other words, the object of decision maker is to maximize the CVar, i.e.,

$$\max_{Q} \text{CVar}_\eta(\pi(Q))$$

Note that, for $\eta = 1$, the CVar is equal to the expected profit.

4. Quantity competition

In this section, we consider two competition models with two risk averse competing newsvendors. Both models have a fixed retail price and competition occurs by allocating demand among the newsvendors proportional to their inventory, or by reallocating their excess demands among the newsvendors, respectively.

4.1. A proportional demand allocation problem

We assume that the total demand $D^*$ is divided among the newsvendors proportional to their ordering quantities. This gives

$$D_i = \frac{Q_i}{Q_i + Q_j}D^*$$

and

$$D_j = \frac{Q_j}{Q_i + Q_j}D^*.$$ 

Such a demand allocation rule is known as the proportional demand allocation rule in the literature. As pointed out by Cachon (2003), this rule is a reasonable model when customers have a relatively low search cost (e.g., online shopping) and the qualitative insights from this rule are consistent with other demand allocation rules considered in the literature (e.g., Lippman and McCardle, 1997).

The newsvendor $i$’s profit is

$$\pi(Q_i, Q_j) = (r_i - c_i)Q_i - (r_i - s_i)\left( Q_i - \frac{Q_i}{Q_i + Q_j}D^* \right)^+$$

and its expectation is

$$E(\pi(Q_i, Q_j)) = (r_i - c_i)Q_i - (r_i - s_i)\int_0^{Q_i + Q_j} F^*(x) \, dx.$$ 

Proposition 4.1 characterizes newsvendor $i$’s best response to the other newsvendor’s order quantity under the risk-averse criterion. The proofs of this result and of further results that will be presented are given in the appendix.

Proposition 4.1. Under the CVar criterion, for any given $Q_j \geq 0$, there exists a unique optimal order quantity $Q^*_i(Q_j)$ for a risk-averse newsvendor that satisfies the following first-order conditions.

If $F^*(Q^*_i(Q_j) + Q_j) < \eta$, then

$$F^1(Q^*_i(Q_j) + Q_j) - \frac{Q_i}{Q^*_i(Q_j) + Q_j} \int_0^{Q^*_i(Q_j) + Q_j} x \, dF^*(x) = \frac{\eta(r_i - c_i)}{r_i - s_i},$$

otherwise,

$$\frac{Q_i}{Q^*_i(Q_j) + Q_j} \int_0^{Q^*_i(Q_j) + Q_j} x \, dF^*(x) = \frac{\eta(c_i - s_i)}{r_i - s_i}.$$ 

Due to the complexity of (2) and (3), we are unable to derive closed form expressions for the optimal ordering quantities in general. Next, we therefore consider firms that are identical.

Under the risk-neutral criterion, Cachon (2003) shows that, for identical newsvendors, there exists a unique symmetric Nash equilibrium in the risk-neutral demand allocation game. Specifically, the unique equilibrium order quantity $Q^*_i$ satisfies

$$F^1(2Q^*_i) - \frac{1}{4Q^*_i} \int_0^{2Q^*_i} x \, dF^*(x) = \frac{r_i - c_i}{r_i - s_i}.$$ 

Under the CVar criterion, for identical risk averse newsvendors, we have the following result.
Theorem 1. Under the CVaR criterion, the unique equilibrium order quantity of identical risk-averse newsvendors $Q_{CVaR}^*$ satisfies

$$F^2(Q_{CVaR}^*) = \frac{1}{4Q_{CVaR}^*} \int_0^{Q_{CVaR}^*} x \cdot dF(x) = \frac{{\eta(r - c)}}{{r - s}}. \quad (5)$$

4.2. A demand reallocation problem

In this model, there are two aspects of demand: the initial allocation and the reallocation. Demand is initially independent of the inventory levels chosen by the firms. However, if applicable, a portion of the excess demand is reallocated to other firms. This is considered to be the most general demand allocation scheme for competitive newsvendors (e.g., Lippman and McCardle, 1997).

Let $D_i$ denote the initial demand for firm $i$. If there is an unsatisfied demand at firm $j$, i.e., $D_j > Q_j$, then some portion of the unsatisfied customers attempts to make a purchase at firm $i$. This transfer of excess demand represents the reallocation. Further, we assume that $D_i$ and $D_j$ are independent and let $R_i$ denote the effective demand at firm $i$, including its reallocation, so that $R_i = D_i + \gamma_j(D_j - Q_j)^+ + \gamma_r$. This transfer of demand risk neutral newsvendor's satisfaction.

$$\gamma_r \quad 0 \leq \gamma_r \leq 1,$$ equals the proportion of firm $j$'s excess demand that is allocated to firm $i$.

We can express the newsvendor $i$'s profit $\pi(Q_i, R_i)$ as follows:

$$\pi(Q_i, R_i) = \pi(Q_i, Q_j) = \int (Q_i - D_i)\cdot dF_i(y) - \int (Q_j - D_j)\cdot dF_j(y)$$

and its expectation is

$$E(\pi(Q_i, Q_j)) = \int (Q_i - D_i)\cdot dF_i(y) - \int (Q_j - D_j)\cdot dF_j(y).$$

Proposition 4.2 characterizes the newsvendor $i$'s best response to the other newsvendor's order quantity under the risk-averse criterion.

Proposition 4.2. Under the CVaR criterion, for any given $Q_j > 0$, there exists a unique optimal order quantity $Q_{CVaR}^*$ for a risk-averse newsvendor that satisfies the first-order conditions:

$$\int_0^{Q_j} \int_0^{Q_j} dF_i(x)dF_j(y) = \int_0^{Q_j} \int_0^{Q_j} dF_i(x)dF_j(y) = \frac{\eta(r - c)}{r - s}. \quad (6)$$

As in Section 4.1, we consider the special case with two identical newsvendors, specifically in their abilities to attract customers in both the initial allocation and the reallocation.

Under the risk neutral criterion, by applying Theorem 6 in Lippman and McCardle (1997), the equilibrium order quantities $Q_{CVaR}^*$ of identical risk neutral newsvendors satisfies

$$F^2(Q_{CVaR}^*) + \int_{Q_j}^{(1 + \gamma_j)Q_j} F_i(1 + \gamma_j)Q_j = \frac{\eta(r - c)}{r - s}. \quad (7)$$

Under the CVaR criterion, by applying Theorem 6 in Lippman and McCardle (1997), the equilibrium order quantities of identical risk-averse newsvendors satisfies the following result.

Theorem 2. With identical firms and continuous distributions, for risk-averse newsvendors under the CVaR criterion, there exists a unique equilibrium order quantity $Q_{CVaR}^*$ that satisfies the following equation:

$$F^2(Q_{CVaR}^*) + \int_{Q_j}^{(1 + \gamma_j)Q_j} F_i(1 + \gamma_j)Q_j = \frac{\eta(r - c)}{r - s}. \quad (8)$$

5. Price competition

For tractability, we assume that there is no direct competition regarding inventory decisions, i.e., firms make combined price and inventory decisions and compete for market demand by using price as the strategy variable. For a general demand model, pricing behaviour is difficult to analyse even for a risk neutral newsvendor. In the literature of the newsvendor problem with pricing, two kinds of demand are extensively used: additive and multiplicative. Thus, two types of demand faced by the newsvendor $i$ are defined as follows:

$$D_i(r_i, r_j) = d(r_i, r_j) + \epsilon_i$$

where $d(r_i, r_j)$ is the mean of newsvendor $i$'s demand, i.e., $d(r_i, r_j) = E[D_i(r_i, r_j)]$ which is a continuous, strictly decreasing, non-negative, twice-differential function and has an increasing price elasticity (IPE), i.e., $\partial^2 d(r_i, r_j)/\partial r_i/\partial r_j \geq 0$. Here, $\epsilon_i$ is a random price-independent component of demand defined on a certain range, which is called demand risk. Further, $d(r_i, r_j)$ is defined on a closed interval $[c_i, \bar{r}]$ where $\bar{r}$ is the maximum admissible price for newsvendor $i$. We assume that $\epsilon_i$ has a continuous distribution $\Phi(\cdot)$ with density $\phi(\cdot)$. For additive demand, $\epsilon_i$ is defined on $[L, U]$ ($L < 0, U > 0$), and $E(\epsilon_i) = 0, \text{VAR}(\epsilon_i) < \infty$. For multiplicative demand, $\epsilon_i$ is defined on $[0, U]$ ($U > 0$) and $E(\epsilon_i) = 1, \text{VAR}(\epsilon_i) < \infty$.

To keep the exposition simple, we focus on the linear form of mean demand. Thus, mean demand faced by newsvendor $i$ is defined as follows:

$$d(r_i, r_j) = d - \alpha r_i + \beta (r_j - r_i),$$

where $d$ is the average demand for each newsvendor; $\alpha$ is the price-sensitivity factor and $\beta$ is the cross-price sensitivity factor which represents the portion of newsvendor $j$'s lost sales switching to newsvendor $i$. By definition, it is clear that $d(r_i, r_j)$ is an IPE and the demand faced by each newsvendor is dependent on their own prices as well as the price of the competing newsvendor. This linear form has been widely used in the literature on operations/marketing interfaces (e.g., Tsay and Agrawal, 2000). Furthermore, the demand risk $\epsilon_i$ is often assumed to exhibit an increasing failure rate (IFR). A random variable $\xi$ has an IFR if $\phi(\xi)(1 - \Phi(\xi)) \geq 0$ for all $\xi$ in its domain. Most widely used distributions (e.g., the uniform, normal, negative exponential, Weibull and gamma distributions) in the operations management literatures have an IFR (see Lariviere, 2006).

5.1. Additive demand

Newsvendor $i$'s profit $\pi(r_i, r_j, Q_i)$ is given by

$$\pi(r_i, r_j, Q_i) = r_i(Q_i - D(r_i, r_j)) + \epsilon_i(Q_i - D(r_i, r_j)) + \gamma_r\pi(Q_i)$$

$$= (r_i - c_i)Q_i - (r_i - s_i)(Q_i - D(r_i, r_j))^+, \quad (8)$$

and its expectation is

$$E(\pi(r_i, r_j, Q_i)) = (r_i - c_i)\int_{Q_i}^{(1 + \gamma_j)Q_i} F_i(1 + \gamma_j)Q_j = \frac{\eta(r - c)}{r - s}. \quad (8)$$

For newsvendor $i$, the relevant decision is to find a price $r_i^* \in \overline{[c_i, \bar{r}]}$ that maximizes its expected profit, and the corresponding optimal order quantity is $Q_i^*(r_i^*)$.

Proposition 5.1 characterizes the newsvendor $i$'s best response to the other newsvendor's pricing strategy under the CVaR criterion.
Proposition 5.1. For any given \( r_i \), if each demand risk distribution \( \Phi_i \) has an IFR, then there exists a unique optimal price \( r^*_i(r_j) \) for a risk-averse newsvendor that satisfies the following first-order conditions:

\[
\frac{c - s}{r^*_i(r_j) - s} - \Phi_i^{-1}\left( \frac{r_i^*(r_j) - c}{r^*_i(r_j) - s} \right) + d(r^*_i(r_j), r_j) - (c + \beta)(r^*_i(r_j) - s) = 0,
\]

and the optimal order quantity \( Q^*_i(r^*_i) \) is

\[
Q^*_i(r^*_i) = \Phi_i^{-1}\left( \frac{r_i^*(r_j) - c}{r^*_i(r_j) - s} \right) + d(r^*_i(r_j), r_j).
\]

Next, we consider the special case with two identical newsvendors in order to obtain closed-form results for ordering quantities.

Under the CVaR criterion, the equilibrium order quantities and selling prices of identical risk-averse newsvendors satisfy the following result.

Theorem 3. With identical firms and continuous demand risk distributions, for risk-averse newsvendors with additive demand under the CVaR criterion, if each demand risk distribution \( \Phi_i \) has an IFR, then there exists a unique equilibrium pricing strategy \( r^*_{i\text{cvar}}(\cdot) \) that satisfies

\[
\frac{c - s}{r^*_{i\text{cvar}}(\cdot) - s} - \Phi_{i\text{cvar}}^{-1}\left( \frac{r_i^*(\cdot) - c}{r^*_{i\text{cvar}}(\cdot) - s} \right) + d(r^*_{i\text{cvar}}(\cdot), r(\cdot)) - (c + \beta)(r^*_{i\text{cvar}}(\cdot) - s) = 0,
\]

and the equilibrium order quantity \( Q^*_{i\text{cvar}}(\cdot) \) is

\[
Q^*_{i\text{cvar}}(\cdot) = \Phi_{i\text{cvar}}^{-1}\left( \frac{r_i^*(\cdot) - c}{r^*_{i\text{cvar}}(\cdot) - s} \right) + d(r^*_{i\text{cvar}}(\cdot), r(\cdot)).
\]

Under the risk neutral criterion, the equilibrium order quantities and selling prices of identical risk neutral newsvendors satisfy the following result.

Corollary 1. With identical firms and continuous demand risk distributions, for risk-neutral newsvendors with additive demand, if each demand risk distribution \( \Phi_i \) has an IFR, then there exists a unique equilibrium pricing strategy \( r^*_i(\cdot) \) that satisfies

\[
\frac{c - s}{r^*_i(\cdot) - s} - \Phi_i^{-1}\left( \frac{r_i^*(\cdot) - c}{r^*_i(\cdot) - s} \right) + d(r^*_i(\cdot), r(\cdot)) - (c + \beta)(r^*_i(\cdot) - s) = 0,
\]

and the equilibrium order quantity \( Q^*_i(\cdot) \) is

\[
Q^*_i(\cdot) = \Phi_i^{-1}\left( \frac{r_i^*(\cdot) - c}{r^*_i(\cdot) - s} \right) + d(r^*_i(\cdot), r(\cdot)).
\]

5.2. Multiplicative demand

Newsvendor \( i \)'s expected profit is

\[
E(\pi(Q_i, D_i, r_i)) = (r_i - c) Q_i - (r_i - s) \int_{0}^{Q_i/d(r_i, r_j)} (Q_j - d(r_i, r_j)x) d\Phi_i(x).
\]

Proposition 5.2 characterizes newsvendor \( i \)'s best response to the other newsvendor's pricing strategy under the CVaR criterion.

Proposition 5.2. For any given \( r_i > 0 \), if each demand risk distribution \( \Phi_j \) has an IFR, then there exists a unique optimal price \( r^*_j(r_i) \) for risk-averse newsvendor that satisfies the following first-order conditions:

\[
\frac{c - s}{r^*_j(r_i) - s} - \Phi_j^{-1}\left( \frac{r_j^*(r_i) - c}{r^*_j(r_i) - s} \right) + d(r^*_j(r_i), r_i) - (c + \beta)(r^*_j(r_i) - s) = 0,
\]

and the optimal order quantity \( Q^*_j(r^*_j) \) is

\[
Q^*_j(r^*_j) = \Phi_j^{-1}\left( \frac{r_j^*(r_i) - c}{r^*_j(r_i) - s} \right) + d(r^*_j(r_i), r_i).
\]

Next, we consider the special case with two identical newsvendors in order to obtain closed form results for the ordering quantities.

Under the CVaR criterion, the equilibrium order quantities and selling prices of identical risk-averse newsvendors satisfies the following result.

Theorem 4. With identical firms and continuous distributions, for risk-averse newsvendors with multiplicative demand under the CVaR criterion, if each demand risk distribution \( \Phi_i \) has an IFR, then there exists a unique equilibrium pricing strategy \( r^*_{i\text{cvar}}(\cdot) \) that satisfies

\[
\frac{c - s}{r^*_{i\text{cvar}}(\cdot) - s} - \Phi_{i\text{cvar}}^{-1}\left( \frac{r_i^*(\cdot) - c}{r^*_{i\text{cvar}}(\cdot) - s} \right) + d(r^*_{i\text{cvar}}(\cdot), r(\cdot)) - (c + \beta)(r^*_{i\text{cvar}}(\cdot) - s) = 0,
\]

and the equilibrium order quantity \( Q^*_{i\text{cvar}}(\cdot) \) is

\[
Q^*_{i\text{cvar}}(\cdot) = \Phi_{i\text{cvar}}^{-1}\left( \frac{r_i^*(\cdot) - c}{r^*_{i\text{cvar}}(\cdot) - s} \right) + d(r^*_{i\text{cvar}}(\cdot), r(\cdot)).
\]

Under the risk neutral criterion, the equilibrium order quantities and selling prices of identical risk neutral newsvendors satisfies the following result.

Corollary 2. With identical firms and continuous distributions, for risk-neutral newsvendors with multiplicative demand, if each demand risk distribution \( \Phi_i \) has an IFR, then there exists a unique equilibrium pricing strategy \( r^*_i(\cdot) \) that satisfies

\[
\frac{c - s}{r^*_i(\cdot) - s} - \Phi_i^{-1}\left( \frac{r_i^*(\cdot) - c}{r^*_i(\cdot) - s} \right) + d(r^*_i(\cdot), r(\cdot)) - (c + \beta)(r^*_i(\cdot) - s) = 0,
\]

and the equilibrium order quantity \( Q^*_i(\cdot) \) is

\[
Q^*_i(\cdot) = \Phi_i^{-1}\left( \frac{r_i^*(\cdot) - c}{r^*_i(\cdot) - s} \right) + d(r^*_i(\cdot), r(\cdot)).
\]

6. The effect of risk aversion and competition

Having characterized the optimal price and quantity decisions for risk averse newsvendors in various competitive settings, we discuss the effects of both risk averseness and competition on the newsvendors' optimal strategies.

For quantity competition, we have the following results.

Theorem 5. For both proportional demand allocation and demand reallocation problems, the equilibrium order quantity \( Q^*_{i\text{cvar}}(\cdot) \) of two identical risk averse newsvendors decreases with respect to the degree of risk aversion. In other words, risk aversion leads to a decrease in the optimal order quantities of both firms.

Theorem 5 indicates that risk averse newsvendors are less inclined to increase the order quantity. Therefore, the negative
impact of overstock on the profit caused by quantity competition (Lippman and McCardle, 1997) can be reduced.

For price competition, we have the following results.

**Theorem 6.** Consider two identical risk averse newsvendors and suppose that each demand risk distribution $\Phi$ has an IFR.

(a) Additive demand: both the equilibrium selling prices $r_{\text{Newsv}}^*$ and the equilibrium order quantities $Q_{\text{Newsv}}^*$ decrease with respect to the degree of risk aversion.

(b) Multiplicative demand: if $\Phi$ satisfies $(\Phi(x)/x\Phi(x)) \geq 0$, then both the equilibrium selling prices $r_{\text{Newsv}}^*$ and the equilibrium order quantities $Q_{\text{Newsv}}^*$ decrease with the degree of risk aversion.

**Remark 1.** The condition $(\Phi(x)/x\Phi(x)) \geq 0$ is satisfied by most common distributions, such as the uniform, negative exponential, Weibull and gamma distributions, all of which also have IFR (Chen et al., 2009). However, this condition is not satisfied by the normal distribution although that does have an IFR. We perform a numerical investigation in Section 7.4 to discuss the effect of risk averseness for normal distributed demand risk.

Similar to quantity competition, risk averseness leads to reduced order quantities for price competition. However, different from quantity competition, it is not necessary beneficial for the newsvendors. Whereas risk neutral newsvendors always overstock under quantity competition (Lippman and McCardle, 1997), this is not always true under price competition, as stated in Theorem 7.

**Theorem 7.** Consider two identical risk neutral or risk averse newsvendors and suppose that each demand risk distribution $\Phi$ has an IFR. For both additive demand and multiplicative demand, the following holds:

(a) The equilibrium selling price $r_{\text{Newsv}}^*$ (or $r_\beta^*$) is decreasing in the intensity of price competition $\beta$. However, the equilibrium order quantity $Q_{\text{Newsv}}^*$ (or $Q_\beta^*$) may increase or decrease with respect to the intensity of competition $\beta$. Specifically, there exist a unique $\beta^* \in [0, 1]$ so that if $\beta \leq \beta^*$, then the equilibrium order quantity increases with respect to $\beta$, otherwise the equilibrium order quantity is decreasing.

(b) Both the equilibrium selling price $r_{\text{Newsv}}^*$ (or $r_\beta^*$) and the equilibrium order quantity $Q_{\text{Newsv}}^*$ (or $Q_\beta^*$) are decreasing in the price sensitivity $\alpha$.

From Theorem 7, we find that both intensive price competition and high price sensitivity force decision makers to reduce their order quantities and lower their prices. In such situations, further risk aversion incentives to reduce order quantities may lead to a high possibility of out-of-stock compared with an integrated newsvendor perspective. Furthermore, it has also been shown (see e.g., Chen et al., 2004; Zhao and Atkins, 2008) that competition leads to a lower (than integrally optimal) equilibrium price for risk neutral newsvendors, and Theorem 6 shows that the price drops even further for risk averse newsvendors.

### 7. Nonidentical newsvendors: a numerical investigation

In this section, we consider the heterogeneous case with one risk neutral and one risk averse vendor. In order to obtain insights into the effects of risk perspective on the competitive outcome for newsvendors, in Sections 7.1-7.4, we will consider quantity competition with proportional demand allocation and demand reallocation, price competition with additive demand and multiplicative demand, respectively. For all four models, we first derive a pair of optimality conditions. Then, we perform a numerical study on equilibrium quantities and prices. We remark that parameter settings other than those considered lead to the same qualitative findings.

#### 7.1. A proportional demand allocation problem

For heterogeneous newsvendors, based on a similar analysis as for Section 4.1, we can prove that the optimal equilibrium solutions are given by the first-order conditions. More specifically, for a risk-neutral newsvendor and a risk-averse newsvendor, under the CVaR criterion, the optimal pair of order quantity ($Q_{\text{Newsv}}^*, Q_{\text{CVaR}}^*$) satisfies the following first-order conditions.

If $F^\prime(Q_{\text{Newsv}}^* + Q_{\text{CVaR}}^*) < \eta$, then

$$
F(Q_{\text{Newsv}}^* + Q_{\text{CVaR}}^*) - \left(\frac{Q_{\text{CVaR}}^*}{Q_{\text{Newsv}}^* + Q_{\text{CVaR}}^*}\right) \int_0^{Q_{\text{CVaR}}^*} Q_{\text{CVaR}}^* + Q_{\text{Newsv}}^* \cdot x \cdot dF(x) = \frac{\eta(\epsilon - \epsilon^*)}{r - s},
$$

otherwise,

$$
F(Q_{\text{Newsv}}^* + Q_{\text{CVaR}}^*) - \left(\frac{Q_{\text{CVaR}}^*}{Q_{\text{Newsv}}^* + Q_{\text{CVaR}}^*}\right) \int_0^{Q_{\text{CVaR}}^*} Q_{\text{CVaR}}^* + Q_{\text{Newsv}}^* \cdot x \cdot dF(x) = \frac{\eta(\epsilon - \epsilon^*)}{r - s}. 
$$

Let us consider the following example: $r = 3$, $c = 1$, $s = 0.5$ and $D^T \sim N(4, 2)$. Tables 1 and 2 give the order quantities for the heterogeneous and the homogeneous case with both two risk neutral or two risk averse firms, for different levels of risk averseness $\eta$.

Keeping in mind that a lower value for $\eta$ implies a higher degree of risk averseness, we observe that, for the heterogeneous case, the market share of the risk neutral firm grows with the level of risk averse for the other firm. In fact, for smaller values of $\eta$ in the considered range, the risk averse newsvendor is almost completely pushed out of the market.

It further appears from Tables 1 and 2 that the total order quantity of heterogeneous newsvendors is lower than that of identical risk neutral newsvendors. However, the total profit of heterogeneous newsvendors may be higher than that of identical risk neutral newsvendors. This relates to the observation in Section 5 that risk neutral newsvendors overstock in this type of competitive environment, and that risk averseness can help counter this effect. We remark that the optimal order quantity of the integrated newsvendor (the single newsvendor problem) is 5.19 for this example and the total order quantity of the risk neutral competitive newsvendors is 7.043.

Comparing these two values, we obtain that the competition between risk neutral newsvendors leads to a 92.7% overstock and 29% profit reduction. However, if either both newsvendors are mildly risk averse ($0.6 < \eta \leq 0.8$) or one of them is very risk averse ($\eta \leq 0.5$), then the profit reduction is less than 3%.

#### 7.2. A demand reallocation problem

Similar to Section 7.1, for heterogeneous newsvendors, we can prove that the optimal equilibrium solutions are given by the first-order conditions and the optimal pair of order quantity ($Q_{\text{Newsv}}^*, Q_{\text{CVaR}}^*$).
The optimal pair of order quantity

\[ Q^*_{\text{En}}, Q^*_{\text{CVaRn}} \]

satisfies the following first-order conditions:

\[ F(Q^*_{\text{En}}, P^*_{\text{En}}) + I(\alpha + \beta) - F(Q^*_{\text{CVaRn}}, P^*_{\text{CVaRn}}) \]

\[ F(Q^*_{\text{En}}, P^*_{\text{En}}) + I(\alpha + \beta) - F(Q^*_{\text{CVaRn}}, P^*_{\text{CVaRn}}) \]

Suppose that each demand is normally distributed with mean 2 and standard deviation 1, i.e., \( \eta = 2 \) and \( \sigma = 1 \), that if \( \gamma = 1 \), then the market can be regarded as a fully competitive market (i.e., a newsvendor's lost sale fully switches to a competitive). The equilibrium order quantity and profit on equilibrium order quantity with different \( \eta \) are shown in Tables 3 and 4.

Different from the proportional demand model, the gain in the total profit from having one or two risk averse newsvendors is much bigger than that in the proportional demand model. This is explained by the fact that only part of the demand can now be reallocated.

As for the example in Section 7.1, the optimal order quantity and corresponding profit of the risk-neutral integrated newsvendor are 5.19 and 7.043, respectively. Note from Tables 3 and 4 that two risk-neutral competitive newsvendors actually achieve a higher profit (7.38) by ordering more (5.908), which is counterintuitive. The explanation is that under the demand reallocation, the market size is enlarged due to the potential customer switching. For this reason, there is also less benefit of reduced order quantities through risk aversion, although having one risk averse newsvendor can still lead to a small gain (less than 1%) compared to having two competing risk-neutral newsvendors.

### 7.3. Price competition with additive demand

For heterogeneous newsvendors, based on a similar analysis as for Section 5.1, we can prove that the optimal equilibrium solutions are given by the first-order conditions. The optimal pair of selling price \( (\ell^*, r^*) \) satisfies the following first-order conditions:

\[ \frac{c - s}{r^*_{\text{En}}} \Phi^{-1} \left( \frac{r^*_{\text{En}}}{r^*_{\text{En}} - s} \right) + d - \alpha r^*_{\text{En}} + \beta (r^*_{\text{En}} - r^*_{\text{CVaRn}}) \]

\[ - \left( r^*_{\text{CVaRn}} - c \right) \left( \alpha + \beta \right) \left( \frac{1}{\eta} \Phi^{-1} \left( \frac{r^*_{\text{En}}}{r^*_{\text{En}} - s} \right) \right) \]

\[ \frac{c - s}{r^*_{\text{CVaRn}}} \Phi^{-1} \left( \frac{r^*_{\text{CVaRn}}}{r^*_{\text{CVaRn}} - s} \right) + d - \alpha r^*_{\text{CVaRn}} + \beta (r^*_{\text{En}} - r^*_{\text{CVaRn}}) \]

\[ - \left( r^*_{\text{En}} - c \right) \left( \alpha + \beta \right) \left( \frac{1}{\eta} \Phi^{-1} \left( \frac{r^*_{\text{CVaRn}}}{r^*_{\text{CVaRn}} - s} \right) \right) \]

The optimal pair of order quantity \( (Q^*_{\text{En}}, Q^*_{\text{CVaRn}}) \) is

\[ Q^*_{\text{En}} = \frac{c - s}{r^*_{\text{En}}} \Phi^{-1} \left( \frac{r^*_{\text{En}}}{r^*_{\text{En}} - s} \right) + d - \alpha r^*_{\text{En}} + \beta (r^*_{\text{En}} - r^*_{\text{CVaRn}}) \]

\[ Q^*_{\text{CVaRn}} = \frac{c - s}{r^*_{\text{CVaRn}}} \Phi^{-1} \left( \frac{r^*_{\text{CVaRn}}}{r^*_{\text{CVaRn}} - s} \right) + d - \alpha r^*_{\text{CVaRn}} + \beta (r^*_{\text{En}} - r^*_{\text{CVaRn}}) \]

Suppose that each demand risk is normally distributed with mean 2 and standard deviation 1, i.e., \( \epsilon_i \sim N(0, 1) \). Let \( d = 5, c = 2, s = 1, \alpha = 0.2, \) and \( \beta = 0.4 \). The equilibrium selling price, order quantity and profit on equilibrium order quantity with different \( \eta \) are shown in Tables 5–7.

For the considered example, the optimal selling price and order quantity of the integrated newsvendor are 13.44 and 6.61, respectively. The profit of the integrated newsvendor is 50.46. Tables 5–7 show that risk neutral competitive newsvendors again overstocking (36%). They also lower their price by 43%, which leads to a 26% profit reduction. As for quantity competition, having one risk averse newsvendor helps us to reduce the overstocking. However, the risk averse newsvendor (further) reduces its selling price to hedge against the demand
uncertainty, leading to an even lower overall profit. Indeed, the profit loss, compared to having two competing risk neutral newsvendors, can be up to 29%. This implies that, although risk averseness can avoid overstocking, competition eventually leads to profit loss in the pricing competition environment.

Next, we present another example with $\eta=0.1$ to illustrate the effect of competition intensity. The equilibrium selling price, order quantity and profit on equilibrium order quantity with different values of $\beta$ are shown in Tables 8–10.

Tables 8–10 show that both homogeneous newsvendors offer a lower price and obtain a lower profit when competition becomes more fierce. However, the order quantity first rises and then falls. Since the selling price is decreasing with respect to the intensity of competition, the increase of safety stock $\Phi^{-1}((\eta r_{EN}^{CVaR} - c)/ (r_{EN}^{CVaR} - s))$ exceeds the decrease of average demand $d(r_{EN}^{CVaR} - r_{EN})$ in a less competitive environment. However, if competition becomes more fierce, the decrease of average demand exceeds the decrease of safety stock which leads to a lower order quantity.

For heterogeneous newsvendors, most of the observations for homogeneous newsvendors still apply. It further appears from Tables 8 to 10 that differences in decisions between heterogeneous newsvendors decrease with the intensity of price competition. This implies that, in a cut-throat, heavily competitive market (i.e., $\beta$ is sufficient large), heterogeneous newsvendors apply the same pricing and ordering policies, even if they have different risk attitudes. In other words, fierce competition moderates the effect of risk averseness.

One more example with $\eta=0.1$ illustrates the effect of price sensitivity. The equilibrium selling price, order quantity and profit on equilibrium order quantity with different $\alpha$ are shown in Tables 11–13. These results show that both homogeneous newsvendors offer a lower price and get a lower profit when demand becomes more sensitivity to price changes. However, again the order quantity first rises and then falls. For heterogeneous newsvendors, most of the observation for homogeneous newsvendors still apply. Also, the differences in decisions between heterogeneous newsvendors and the total profit of both newsvendors decrease with price sensitivity. This implies that, in a market which has numerous substitutable goods (i.e., $\alpha$ is sufficient large), heterogeneous newsvendors select almost the same price. Moreover, none of them is able to maintain a high profit.

### 7.4. Price competition with multiplicative demand

For heterogeneous newsvendors, based on a similar analysis as for Section 5.2, we can prove that the optimal equilibrium solutions are given by the first-order conditions. The optimal pair of selling price $(r_{EN}^{CVaR}, r_{EN})$ satisfies the following first-order conditions:

\[
\begin{align*}
\frac{c-s}{r_{EN}^{CVaR} - s} - \Phi^{-1}\left(\frac{r_{EN}^{CVaR} - c}{r_{EN}^{CVaR} - s}\right) & \left(d-\alpha r_{EN}^{CVaR} + \beta r_{EN}^{CVaR} - r_{EN}\right), \\
\left(d-\alpha r_{EN}^{CVaR} + \beta r_{EN}^{CVaR} - r_{EN}\right) & = 0,
\end{align*}
\]

The optimal pair of order quantity $(Q_{EN}^{CVaR}, Q_{EN})$ is

\[
\begin{align*}
Q_{EN}^{CVaR} = & \Phi^{-1}\left(\frac{r_{EN}^{CVaR} - c}{r_{EN}^{CVaR} - s}\right)\left(d-\alpha r_{EN}^{CVaR} + \beta r_{EN}^{CVaR} - r_{EN}\right), \\
Q_{EN} = & \Phi^{-1}\left(\frac{r_{EN}^{CVaR} - c}{r_{EN}^{CVaR} - s}\right)\left(d-\alpha r_{EN}^{CVaR} + \beta r_{EN}^{CVaR} - r_{EN}\right).
\end{align*}
\]

Since most of the properties for multiplicative demand are similar to that for additive demand, we consider a normal distributed demand risk to explore what happens if the condition of part (b) in Theorem 6 is not satisfied. Let us consider the following example: $c=2, s=1, d=3, \alpha=0.2, \beta=0.4$ and $\epsilon \sim N(1, 0.5^2)$. The equilibrium selling price, order

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$r_{EN}^{CVaR}$</th>
<th>$r_{EN}$</th>
<th>$Q_{EN}^{CVaR}$</th>
<th>$Q_{EN}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>7.023</td>
<td>5.235</td>
<td>(6.545, 5.706)</td>
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<tr>
<td>1</td>
<td>5.192</td>
<td>4.067</td>
<td>(4.854, 4.398)</td>
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</tr>
<tr>
<td>1.5</td>
<td>4.325</td>
<td>3.512</td>
<td>(4.070, 3.761)</td>
<td>3.5</td>
</tr>
<tr>
<td>2</td>
<td>3.820</td>
<td>3.188</td>
<td>(3.617, 3.186)</td>
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<table>
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<th>$Q_{EN}$</th>
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<td>4.673</td>
<td>2.691</td>
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<td>1.5</td>
<td>4.657</td>
<td>2.744</td>
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<td>4.609</td>
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<tr>
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</tr>
<tr>
<td>1</td>
<td>11.639</td>
<td>5.532</td>
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<tr>
<td>1.5</td>
<td>8.676</td>
<td>4.131</td>
</tr>
<tr>
<td>2</td>
<td>6.837</td>
<td>3.263</td>
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<th>$Q_{EN}$</th>
</tr>
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<tbody>
<tr>
<td>0.2</td>
<td>7.663</td>
<td>5.644</td>
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<tr>
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<td>5.397</td>
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<tr>
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<th>$Q_{EN}$</th>
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<tr>
<td>0.2</td>
<td>4.503</td>
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</tr>
<tr>
<td>0.4</td>
<td>3.599</td>
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<tr>
<td>0.6</td>
<td>2.950</td>
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<th>$Q_{EN}$</th>
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<tr>
<td>0.4</td>
<td>8.638</td>
<td>3.865</td>
</tr>
<tr>
<td>0.6</td>
<td>4.590</td>
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<th>$Q_{EN}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>2.534</td>
<td>0.842</td>
</tr>
<tr>
<td>0.4</td>
<td>1.370</td>
<td>0.312</td>
</tr>
<tr>
<td>0.6</td>
<td>0.678</td>
<td>0.025</td>
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<table>
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<tr>
<th>$\alpha$</th>
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<th>$Q_{EN}$</th>
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<tbody>
<tr>
<td>0.2</td>
<td>3.243</td>
<td>1.316</td>
</tr>
<tr>
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<td>1.370</td>
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</tr>
<tr>
<td>0.6</td>
<td>0.678</td>
<td>0.025</td>
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<th>$Q_{EN}$</th>
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<tbody>
<tr>
<td>0.2</td>
<td>2.534</td>
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</tr>
<tr>
<td>0.4</td>
<td>1.370</td>
<td>0.312</td>
</tr>
<tr>
<td>0.6</td>
<td>0.678</td>
<td>0.025</td>
</tr>
</tbody>
</table>
quantity and profit on equilibrium order quantity with different \( \eta \) are shown in Tables 14–16.

Tables 14–16 show that, for homogeneous newsvendors, moderate risk averseness still leads to a lower price and a lower order quantity, which is consistent with the competition under the additive demand. However, if newsvendors are very risk averse, then they offer a higher selling price, which is different from that in the additive demand model. The explanation for this result is that, although a higher price leads to a lower average demand, it also reduces the uncertainty. Note that the normal distribution does not satisfy the condition of part (b) in Theorem 6, which implies that newsvendors may offer a higher price rather than lower the price. Therefore, it is important to carefully select the distribution of demand risk.

Compared with the additive demand, it further appears from Tables 14 to 16 that the total profit of heterogeneous newsvendors may be higher than that of identical risk neutral newsvendors. This relates to the fact that risk averseness could avoid overstocking and raise selling prices. For the considered example, the optimal selling price and order quantity of the integrated newsvendor are 8.806 and 3.884, respectively. The profit of the integrated newsvendor is 15.61. However, the selling price of the risk neutral competitive newsvendors is 5.607, the total order quantity is 5.226 and the total profit is 11.63, which results in 34.6% overstock, a 36.3% price dropping and a 25.5% profit decrease.

For the heterogeneous newsvendors, compared to the integrally optimal solution, for part of the considered range of values for \( \eta \), having one rather than two risk averse newsvendors leads to a total ordering quantity close to the integrated optimum of 3.884. However, the asymmetry in order quantities can still lead to a considerable profit loss, compared to having two competing risk neutral newsvendors, of up to 6.3%. This implies that, although risk averseness can avoid overstocking, competition eventually leads to a lower average demand, it also reduces the uncertainty. We assume the same parameters in Tables 17–19 and in Tables 20–22, respectively. From Tables 17 to 22, we observe similar phenomena as for the additive model.

Now, we briefly summarize the effect of heterogeneity on equilibrium quantities and prices. For quantity competition, risk averseness can avoid overstocking under both demand splitting rules. The risk-neutral newsvendor gains a bigger market share and higher profit than the risk-averse newsvendor does. Under the proportional demand allocation, newsvendors begin to reduce the overstock through risk averseness. However, reducing the overstock may lead to a lower profit under demand reallocation. For price competition, risk averseness avoids overstocking and leads to a lower selling price in both the additive demand model and the multiplicative demand models. However, the impact of risk averseness on the optimal strategies becomes weak in an intensely competitive market.

### Table 14
Equilibrium selling price for the quantity and pricing model with multiplicative demand.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \eta )</th>
<th>( Q^*_\beta )</th>
<th>( P^*_\beta )</th>
<th>(( Q^<em>_\beta ), ( P^</em>_\beta ))</th>
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<tr>
<td>0.5</td>
<td>0.1</td>
<td>5.607</td>
<td>5.353</td>
<td>(5.182, 7.871)</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>5.607</td>
<td>5.890</td>
<td>(6.076, 6.976)</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
<td>5.607</td>
<td>5.686</td>
<td>(5.923, 6.527)</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
<td>5.607</td>
<td>5.730</td>
<td>(5.787, 6.228)</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>5.607</td>
<td>5.541</td>
<td>(5.761, 6.056)</td>
</tr>
</tbody>
</table>

### Table 15
Equilibrium order quantities for the quantity and pricing model with multiplicative demand.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \eta )</th>
<th>( Q^*_\beta )</th>
<th>( P^*_\beta )</th>
<th>(( Q^<em>_\beta ), ( P^</em>_\beta ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>5.607</td>
<td>2.633</td>
<td>(3.355, 4.199)</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>5.607</td>
<td>2.317</td>
<td>(5.762, 2.986)</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
<td>5.607</td>
<td>2.193</td>
<td>(5.911, 0.953)</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
<td>5.607</td>
<td>2.123</td>
<td>(5.821, 1.220)</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>5.607</td>
<td>1.627</td>
<td>(2.758, 1.462)</td>
</tr>
</tbody>
</table>

### Table 16
Profits for the quantity and pricing model with multiplicative demand.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \eta )</th>
<th>( \pi^*_\beta )</th>
<th>( \pi^<em>_\beta,(Q^</em><em>\beta ,P^*</em>\beta ) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>5.607</td>
<td>1.237</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>5.607</td>
<td>1.204</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
<td>5.607</td>
<td>1.192</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
<td>5.607</td>
<td>1.201</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>5.607</td>
<td>1.213</td>
</tr>
</tbody>
</table>

### Table 17
Equilibrium selling price for the quantity and pricing model with multiplicative demand.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \eta )</th>
<th>( Q^*_\beta )</th>
<th>( P^*_\beta )</th>
<th>(( Q^<em>_\beta ), ( P^</em>_\beta ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>5.607</td>
<td>2.633</td>
<td>(3.355, 4.199)</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>5.607</td>
<td>2.317</td>
<td>(5.762, 2.986)</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
<td>5.607</td>
<td>2.193</td>
<td>(5.911, 0.953)</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
<td>5.607</td>
<td>2.123</td>
<td>(5.821, 1.220)</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>5.607</td>
<td>1.627</td>
<td>(2.758, 1.462)</td>
</tr>
</tbody>
</table>

### Table 18
Equilibrium order quantities for the quantity and pricing model with multiplicative demand.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \eta )</th>
<th>( Q^*_\beta )</th>
<th>( P^*_\beta )</th>
<th>(( Q^<em>_\beta ), ( P^</em>_\beta ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>5.607</td>
<td>2.633</td>
<td>(3.355, 4.199)</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>5.607</td>
<td>2.317</td>
<td>(5.762, 2.986)</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
<td>5.607</td>
<td>2.193</td>
<td>(5.911, 0.953)</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
<td>5.607</td>
<td>2.123</td>
<td>(5.821, 1.220)</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>5.607</td>
<td>1.627</td>
<td>(2.758, 1.462)</td>
</tr>
</tbody>
</table>

### Table 19
Profits for the quantity and pricing model with multiplicative demand.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \eta )</th>
<th>( \pi^*_\beta )</th>
<th>( \pi^<em>_\beta,(Q^</em><em>\beta ,P^*</em>\beta ) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>5.607</td>
<td>1.237</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>5.607</td>
<td>1.204</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
<td>5.607</td>
<td>1.192</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
<td>5.607</td>
<td>1.201</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>5.607</td>
<td>1.213</td>
</tr>
</tbody>
</table>

### Table 20
Equilibrium selling price for the quantity and pricing model with multiplicative demand.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \eta )</th>
<th>( Q^*_\beta )</th>
<th>( P^*_\beta )</th>
<th>(( Q^<em>_\beta ), ( P^</em>_\beta ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>5.607</td>
<td>2.633</td>
<td>(3.355, 4.199)</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>5.607</td>
<td>2.317</td>
<td>(5.762, 2.986)</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
<td>5.607</td>
<td>2.193</td>
<td>(5.911, 0.953)</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
<td>5.607</td>
<td>2.123</td>
<td>(5.821, 1.220)</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>5.607</td>
<td>1.627</td>
<td>(2.758, 1.462)</td>
</tr>
</tbody>
</table>
Affects the supplier's performance, an interesting direction for further research is to investigate the impact of risk aversion on the optimal strategy and profit performance in a decentralized supply chain. Furthermore, the supplier may not know the degree of risk aversion of the newsvendor in practice. Therefore, another direction is to study the issue of information asymmetry with respect to the risk attitude.

Acknowledgements

The author is very grateful to the referees and the editor for patiently giving critical and insightful comments and suggestions, which helped to significantly improve the models, results, and presentation. The research was supported in part by Netherlands Organisation for Scientific Research under Grants 040.03.017 and 040.02.009, the European Regional Development Fund for the ITRAC-T extension, National Natural Sciences Foundation of China under Grants 71271092, 71101099, 71171088, and 71171205, the Fundamental Research Funds for the Central Universities under Grants 2014SCU04A06, and Sichuan University under Grants PKQY201408.

Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.ijpe.2014.05.009.

References


8. Conclusion

In this paper, we study the effect of risk aversion for the competitive newsvendor problem, using CVaR as the risk measure and considering both quantity competition (proportional demand allocation and demand reallocation) and price competition (additive demand and multiplicative demand). For quantity competition, competition always leads to overstocking compared with the integrated newsvendor, and risk averseness can help counter this effect. However, whether reducing the overstock leads to increased profits depends on the demand splitting rule. Under proportional demand allocation, it does. Under demand reallocation, overstocks induce demand-expanding switching behaviour, and reduced stocks through risk averseness may hurt profits.

Price competition always leads to overstocking and profit loss compared with the integrated newsvendor. Since buying fewer products and offering a lower price increase the probability of selling all those products, risk averseness can avoid overstocking. In the additive demand model, this is indeed what always happens. However, in the multiplicative demand model with normal distributed demand risk, buying fewer products and offering higher price is another way to reduce risk, and so the effect of risk averseness is more complex. This shows that from a supply chain perspective, suppliers should carefully select retailers with different risk preferences in different competitive environments.

The results developed in this paper provide valuable insights for firms in retail industries. In particular, in a financial crisis situation, firms face more fierce competition than ever and managers need decisions tools to assist them in choosing the optimal strategies in order to hedge the potential risk. To meet this challenge, our paper provides a comprehensive understanding of different competition strategies under a risk-averse environment. Under quantity competition, a comparison of two demand splitting rules reveals that when retailers have their own original market, then they benefit from risk averseness by avoiding the overstock. Under price competition, the equilibrium price depends on how the price affects the potential market demand. For an additive demand function, retailers order fewer items and offer a lower price. For a multiplicative demand function, retailers may choose to order less and charge a higher price. Our findings also provide insights into how a supplier chooses its own retailers with different risk preferences to balance out the effect of risk aversion on the profit.

Since our results indicate that the risk attitude of the newsvendors affects the supplier's performance, an interesting direction for further research is to investigate the impact of risk aversion on the optimal strategy and profit performance in a decentralized supply chain. Furthermore, the supplier may not know the degree of risk aversion

<table>
<thead>
<tr>
<th>Table 21</th>
<th>Equilibrium order quantities for the quantity and pricing model with multiplicative demand.</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>Q*</td>
</tr>
<tr>
<td>0.2</td>
<td>2.613</td>
</tr>
<tr>
<td>0.4</td>
<td>1.680</td>
</tr>
<tr>
<td>0.6</td>
<td>1.103</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 22</th>
<th>Profits for the quantity and pricing model with multiplicative demand.</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>β</td>
</tr>
<tr>
<td>0.2</td>
<td>5.816</td>
</tr>
<tr>
<td>0.4</td>
<td>2.222</td>
</tr>
<tr>
<td>0.6</td>
<td>0.933</td>
</tr>
</tbody>
</table>