Unlocking nonlinear dynamics and multistability from intensive longitudinal data: A novel method

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Abstract

The availability of smart devices has made it possible to collect intensive longitudinal data (ILD) from individuals, providing a unique opportunity to study the complex dynamics of psychological systems. Existing time-series methods often have limitations, such as assuming linear interactions or having restricted forms, leading to difficulties in capturing the complex nature of these systems. To address this issue, we introduce fitlandr, a method with implementation as an R package that integrates nonparametric estimation of the drift-diffusion function and stability landscape. The drift-diffusion function is estimated using the Multivariate Kernel Estimator (MVKE, Bandi & Moloche, 2018), and the stability landscape is estimated through Monte-Carlo estimation of the steady-state distribution (Cui et al., 2021, 2022). Using a simulated emotional system, we demonstrate that fitlandr can effectively recover bistable dynamics from data, even in the presence of moderate noise, and that it primarily relies on dynamic information from the system instead of distributional information. We then apply the method to two empirical single-participant ESM datasets and compared the results with the simulation datasets. Whereas both datasets show a bimodal distribution, fitlandr only revealed bistability in one of them, indicating that bimodality in ILD does not necessarily imply the existence of bistability in the underlying system. These results demonstrate the potential of fitlandr as a tool for uncovering the rich, nonlinear dynamics of psychological systems from ILD.

Translational Abstract

The availability of smart devices has made it possible to collect intensive longitudinal data (ILD) from individuals, providing a unique opportunity to study the complex interactions among variables over time. Existing time-series methods often have limitations, such as assuming linear interactions or having restricted forms, leading to difficulties in capturing more
complex features from the data. To address this issue, we introduce fitlandr, a method with an R package that conducts flexible estimation of the dynamic interactions among variables. The output of the model can be shown in a vector field, which depicts the changing tendency of the mental state starting from a given value, and a stability landscape which shows the stability of the system with the ball-and-landscape metaphor. Using a simulated emotional system, we demonstrate that fitlandr can effectively recover bistable dynamics from data, even in the presence of moderate noise, and that it primarily relies on the time dependencies of the data points instead of the distributional information. We then apply the method to two empirical single-participant ESM datasets and compared the results with the simulation datasets. Whereas both datasets show a bimodal distribution, fitlandr only revealed bistability in one of them, indicating that bimodality in ILD does not necessarily imply the existence of bistability in the underlying system. These results demonstrate the potential of fitlandr as a tool for uncovering the rich, nonlinear interactions of psychological variables from ILD.

Keywords: Time series, nonlinear dynamics, intensive longitudinal data, stability landscape.
Introduction

Psychologists have a long-standing interest in understanding changes in mental states over time. With the rise of smart devices, intensive longitudinal data (ILD) from individuals are now readily available, providing new opportunities to study the evolution of psychological systems (Hamaker et al., 2015; Myin-Germeys & Kuppens, 2021; Trull & Ebner-Priemer, 2009). The new type of data not only brings about new information but also bears challenges that call for innovative analytical methods. Psychological systems, like other complex systems in nature, consist of a large number of elements interacting continuously and nonlinearly (Olthof et al., 2020, 2023). As a result, bimodality and multimodality, which means that the data distribution has two or more modes instead of only one mode in a Gaussian distribution, are the rule rather than the exception (Delignières et al., 2004; Haslbeck et al., 2023; Haslbeck & Ryan, 2022). Also, change processes are characterized by sudden changes or regime shifts, indicating the nonstationarity of the data (Helmich et al., 2020; Olthof et al., 2020). Most existing models assume constant linear interactions of variables discretely and are therefore less suited to handle data that show multimodality and nonstationarity.

Additionally, previous methods primarily focus on the dynamics of a complex psychological system (i.e., indicating the most probable direction of the system at the next time point based on its current state.) But next to these dynamics, it is also essential to gain a deeper understanding of the system's stability. The stability of a system describes the number of qualitatively different phases in the system, the range of these phases, and in which phase the system is most likely to reside. Understanding the stability of psychological systems has important theoretical and practical implications. For example, there is a growing interest in sudden changes in experienced severity of symptoms (Cramer et al., 2016; Cui et al., 2022;
Haslbeck et al., 2022; Olthof et al., 2023; Wichers et al., 2019). This line of work conceptualizes mental disorders as a stable emergent phase\(^1\) of a person-specific mental system embedded in a specific context. The ball-and-landscape metaphor is often used to illustrate this concept, where the state of a person's mental system is like a ball on a landscape that fluctuates around a local minimum but may transition to another minimum under certain conditions, resulting in a change to a different phase (e.g., from the healthy phase to the depressive phase or the anxious phase). In this metaphor, the stability of the system is represented by the altitude of the landscape. A higher position indicates less stability and a tendency to fall, and a lower position indicates more stability and a tendency to remain in the same area. If multiple basins exist on the landscape, indicating there are multiple phases that the system can reside in, the system can be characterized as exhibiting multistability. It is important to distinguish multistability from multimodality: multimodality is only a feature of the data distribution, whereas multistability signifies the existence of distinct phases that the system tends to stay in a short time, but may occasionally transition between them. Although the multimodality of data may suggest the presence of multistability in the system, it is not always the case, as we will elaborate in subsequent sections. Whereas the ball-and-landscape metaphor is conceptually well recognized and accepted, quantifying the potential landscape function remains a challenge. Although some recent studies have proposed methods to estimate the stability landscapes for psychological formal models (Cui

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\(^1\) The term “attractor” is sometimes employed to convey the same idea as the “phase” we used here. However, “attractor” can be used to refer to a particular point at which the system is most stable, akin to a local minimum on the landscape rather than a basin of attraction encompassing the local minimum (Sayama, 2015). To prevent any potential confusion, we adhere to the term “phase” in this article, in line with our previous work (Cui et al., 2022).
et al., 2022), there are currently no methods available to estimate stability landscapes from multivariate continuous data, limiting further progress in this field.

In the current article, we propose a novel nonparametrically method to estimate the dynamic functions and the generalized potential function of psychological systems from ILD. The dynamics can be represented with vector fields, and the generalized potential function can be visualized by potential landscapes. By this method, we aim to provide a quantitative and rather assumption-free description of the dynamics and the stability of psychological systems. An R package, fitlandr, was developed to provide an implementation of the method and can be accessed at the Comprehensive R Archive Network (https://CRAN.R-project.org/package=fitlandr). The name fitlandr will be used throughout the current article for the whole method that we are proposing. In the following sections, we will first introduce the currently available methods for psychological ILD, and describe the similarities and differences with our method. After that, we describe our method and briefly introduce related algorithms. We will then demonstrate the method using simulation data and empirical data. With the simulation data, we examine if our method can recover known multistability from short time series, how much it is robust under noise, and how much it relies on the dynamical information (i.e., the information regarding how the current state of the system influences its state at the subsequent time point) from the data rather than the distributional information. With the empirical data, we examine if our method can recover multistability suggested by psychological theories and whether it provides insights regarding the nature of psychological systems. Finally, we discuss the advantages, disadvantages, and potential applications of this method. The necessary code for replicating the findings presented in this article can be obtained from the following URL: https://osf.io/s7dq4/.
A Survey of Current Methods and Models

In this section, we present a survey of commonly used methods and models for analyzing psychological ILD. We summarize their limitations in capturing complex dynamics and stability, and highlight the added value of the method we are proposing.

The vector autoregression (VAR) model is the most widely used approach for multivariate time series analysis and serves as a foundation for longitudinal network analysis (Bringmann et al., 2013). In a VAR model, the state of the system from the previous time point, represented as a vector, is used to predict the state of the system at the next time point through a regression model. Despite its simplicity, VAR provides a powerful way to gain insights from ILD. Yet, VAR models assume constant linear interactions of the variables discretely, thus fail to capture complex features such as nonstationarity and multistability (Haslbeck et al., 2022; Haslbeck & Ryan, 2022; Olthof et al., 2020).

To overcome this issue, many variants of the original VAR model have been developed, seeking to capture more complex dynamics in the system. For example, the time-varying VAR (TV-VAR) model allows for smooth change in the parameters of the VAR model, accounting for nonstationary behavior in the data (Bringmann et al., 2017; Haslbeck et al., 2021). Threshold-VAR (De Haan-Rietdijk et al., 2016; Tong & Lim, 1980) and Markov-Switching-VAR (Hamilton, 1989; Haslbeck & Ryan, 2022) are models that apply different regression equations based on a threshold variable or a latent variable that indicates which phase the system is in. Although these variants improve upon the VAR model, they do not explicitly model nonlinear dynamics. More concretely, even though these models allow the linear interaction coefficient to change nonlinearly over time, they still do not permit the function form to be nonlinear (i.e., the influence of any variable on itself or other variables is always linear). As a result, the system at
each time point is a linear system, which means that at any given moment, when a higher $x$ corresponds to a higher $y$, it follows that an even greater $x$ will consistently result in a proportionally higher $y$, without any exceptions. This limits the ability to capture nonlinear interactions among variables. Moreover, they represent nonstationary and multistable behaviors with one or more variables (such as the time-varying coefficients in the TV-VAR model, one or more assessed variables in the Threshold-VAR model, and latent states in the Markov-Switching-VAR model). These variables can undergo discrete or smooth changes, making the model undergo transformations, but they are not explanans per se. As a result, the model does not inherently account for the reasons behind their changes. Last, if the transition between different phases of the system is the primary focus rather than the specific dynamics, a hidden Markov model (HMM) can also be used, which assumes that the data come from multiple probability distributions (Haslbeck & Ryan, 2022; Visser, 2011). However, in HMM, only the specific transition probabilities between hidden states are defined, whereas within each hidden state, the time points are considered independent from each other, lacking any temporal dependencies. Consequently, HMM primarily focuses on capturing the transition possibilities between hidden states rather than providing a detailed description of the specific dynamics.

Another branch of methods for analyzing ILD is based on the drift-diffusion process, in which the moving trajectory of a system is governed by two types of forces: the drift part, which represents the deterministic forces, and the diffusion part, which represents the stochastic forces. The drift-diffusion process is modeled using stochastic differential equations instead of linear regressions, thus the models based on the drift-diffusion process are inherently continuous. The Fokker-Planck Equation Model (FPEM) by Tschacher and Haken (2020) is an example of a simple drift-diffusion model that uses the binned average of the differences between sample
points as the drift term and the variance as the diffusion term. The method calculates a potential landscape function by integrating the drift term. An advantage of this model is that the local dynamics and the global multistability are uniformly represented in one dynamical equation. The multimodality is the property of the system itself, not the consequence of switching between different systems. This method, however, requires enough data points in each bin to estimate the mean and the variance reliably, and the integral can only be calculated in a single dimension, thus the method is only applicable for one-dimensional systems. Another method based on the drift-diffusion process is the Ornstein-Uhlenbeck Model (OUM, also known as the DynAffect model, Kuppens et al., 2010; Oravecz et al., 2011). This model assumes that the drift is proportional to the distance between the current state of the system and a single point (the “homebase” of the system). The direction of the drift term always points towards this point. The diffusion term is set as a multidimensional Gaussian noise. This method can be applied to two or more dimensional systems, but it does not allow multistability due to its linear form in the drift part. Another related method is the Continuous-Time VAR (CT-VAR) model (Ryan et al., 2018; Ryan & Hamaker, 2022), which is a continuous-time extension to VAR models. It assumes linear continuous interactions among variables, thus the dynamics can also be described by a drift-diffusion process similar to the OUM, whereas the CT-VAR model is intended to be used for datasets with more variables and can be represented in a network. However, like the OUM, it assumes linear dynamics, thus does not show multistability. To explain the typical skewed and bimodal distribution in emotional ILD, Loossens et al. (2020) proposed the Affective Ising Model (AIM), which assumes a specific type of the drift function inspired by the Ising model in physics. This model assumes two subsystems: the positive emotion system and the negative emotion system. Each element strengthens other elements in the same system, and inhibits the
elements in the other system. The AIM can well represent a specific type of data distribution, namely the V-shaped distribution in the state space of positive and negative emotions. The model, however, is limited to a specific set of assumptions about positive and negative emotion systems and does not generalize to the ILD of other psychological processes.

We summarize the key features of the above-mentioned methods in Table 1. Inspecting the table, we can see a clear trade-off between the flexibility of the model and the feasibility of the model estimation. Taking the representation of multistability as an example: models that specify the number of phases beforehand (such as Threshold-VAR, Markov-Switching-VAR, HMM, and AIM) can handle more variables and require fewer data points, whereas models that let the number of phases emerge from the estimation process (such as FPEM) require more data points and can handle fewer variables. This mostly comes from the methodological challenges posed by psychological ILD (van de Leemput et al., 2014): Psychological ILD often include a few dozens to one hundred data points from self-reported questionnaires. Although this length is large in the field of psychology, it falls short in comparison to other fields that typically have thousands of data points, and data collected through questionnaires are often more prone to measurement noise. For example, the animal movement data in ecology used by Brillinger (2007) has 1,571 data points measured by GPS with a precision in the meter level and range of kilometers, and the long-term geographical data used by Livina et al. (2010) has 3,000 data points of isotope concentration which could also be measured precisely comparing with the range of fluctuation. To compare with, ESM datasets often contains around a hundred data points, with the measurement noise comprising about a quarter of the total variation (Dejonckheere et al., 2022).
Nevertheless, in the middle of this trade-off, there is a gap in the existing range of methods: none of them can effectively represent all types of multistability in a multidimensional space for psychological ILD. This is where fitlandr comes in. The specifics of fitlandr will be discussed in depth in the subsequent sections.

The fitlandr Method

The fitlandr method draws inspiration from the dynamo project (Qiu et al., 2022), which aims to describe single cell RNA dynamics and stability with vector fields and potential landscapes. Whereas fitlandr uses similar steps as dynamo, different algorithms and procedures were implemented to accommodate the characteristics of psychological ILD, which are often shorter, more prone to noise, have fewer dimensions, and display more homeostatic behavior than single cell RNA data. The workflow of fitlandr is illustrated in Figure 1, with further explanations provided in the following subsections. The main characteristics of fitlandr, in comparison with previous methods, are shown in Table 1.

Step 1: Estimating System Dynamics

The first step of the method is to estimate the dynamics of the system from raw data. In fitlandr, we employ the general drift-diffusion form of stochastic differential equations to describe the dynamics of the system, written as follows:

$$dX = \mu(X)dt + \sigma(X)dW,$$

(1)

in which the first term in the right-hand side, $\mu(X)$, represent the deterministic force, and the

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2 Note that, although sharing the same mathematical origin, the model used in this context is not directly related to the drift-diffusion model (DDM) used in cognitive decision modeling (Ratcliff et al., 2016; Ratcliff & McKoon, 2008). DDM assumes a very specific process of cognitive decision making. In contrast, the method proposed in the current article is not intended to model any decision-making behavior, nor does it have strong assumptions regarding a specific mental process.
second term represents the noise of the system (the diffusion matrix, \( a = \sigma \sigma^T \), is more commonly used). In fitlandr, we use a flexible kernel algorithm, namely the Multivariate Kernel Estimator (MVKE, Bandi & Moloche, 2018) to estimate the drift-diffusion equation. Assuming \( x_i \ (i = 1, 2, 3, \ldots, n) \) are \( n \) observations of \( X_t \) in \( t = 1, 2, 3, \ldots, n \), the kernel estimators for \( \mu(x) \) and \( a(x) \) are given by (Bandi & Moloche, 2018):

\[
\hat{\mu}(x) = \frac{\sum_{i=1}^{n-1} K(x_i - x)(x_{i+1} - x_i)}{\sum_{i=1}^{n} K(x_i - x)},
\]

(2)

\[
\hat{a}(x) = \frac{\sum_{i=1}^{n-1} K(x_i - x)(x_{i+1} - x_i)(x_{i+1} - x_i)^T}{\sum_{i=1}^{n} K(x_i - x)},
\]

(3)

in which \( K(x) \) is a product kernel function, as follows in our implementation:

\[
K(x) = \frac{1}{h^D} \prod_{j=1}^{D} e^{-\frac{x_j}{h}},
\]

(4)

where \( x_j \) is the \( j \)-th element of \( x \), and \( h \) is a user-specified parameter controlling the width of the kernel estimator. Intuitively, MVKE is a weighted average of the dynamics from the data. The weight of a data point depends on its distance with the evaluation point, \( K(x_i - x) \). The closer the evaluation point is to a data point, the more similar the dynamics is.

Due to the use of a kernel algorithm, the output cannot be represented as parameter estimates. Instead, the force of the system in different directions is shown through a vector field. A vector field plot displays multiple arrows in a grid, representing the direction and tendency of how the system evolves from a given point. However, it is not as straightforward to see how variables influence one another. In a VAR model, a positive coefficient of variable \( X_1 \) on variable \( X_2 \), \( c_{1,2} \) indicates that a higher value of \( X_1 \) at time \( t \) predicts a higher value of \( X_2 \) at time \( t + 1 \). In a vector field, however, the influence of \( X_1 \) to \( X_2 \) may not be monotonous. A higher value of \( X_1 \) may predict a higher value of \( X_2 \) at the next time point in a certain range,
but the relationship may be reversed in another range, and the relationship may also depend on the value of $X_2$. It is exactly this flexibility that we see as the key advantage of the current method. Whereas previous time-series methods such as VAR and OUM can also be depicted with a vector field, they limit the shape of the vector field (see Appendix A). In the proposed method, we do not impose a specific form on the drift part. Instead, we use kernel methods to estimate it in a flexible manner.

**Step 2: Estimating Stability Landscape**

After obtaining the dynamic function of the system, we can go one step further to analyze the stability by constructing the system’s potential landscape. The formal representation of stability is based on the concept of a potential function, which assumes that the deterministic force of the system can be represented as the gradient of the potential. In this framework, a lower potential indicates greater stability, as the system naturally moves towards states with lower potential. The steeper the slope of the potential, the stronger the system's tendency to move towards lower potential states. However, because of the mathematical properties of multivariate functions (as explained in Cui et al., 2021, 2022), constructing a well-defined potential function from an arbitrary vector field is often impossible. This is because not all the forces in the vector field can be represented as the gradient of a stability function. Therefore, we use a generalized potential function defined by Wang et al. (2008). The generalized potential landscape provides an informative summary of the stability of the states, although it may not encompass all details of the system’s dynamics. This function is calculated as the negative logarithm of the steady-state distribution of the system,

$$U(\mathbf{x}) = -\ln p_{ss}(\mathbf{x}),$$

in which $U$ is the generalized potential function, the higher the value of $U$, the more unstable
the state is; and $P_{SS}$ is the steady-state distribution of the system and estimated through Monte Carlo simulation in the fitlandr package. The drift-diffusion function obtained from the previous step, is used to simulate the system from multiple starting points until the distribution converges. The simlandr package (Cui et al., 2021, 2022) is then used to calculate and visualize the potential landscape function and to find the position of the local minima and the saddle point on the potential landscape.

**Recover Nonlinear Dynamics and Stability from Simulation Data**

In this section, we evaluate the performance of fitlandr using model simulations. We use the mood system model by van de Leemput et al., (2014, also see Haslbeck & Ryan, 2021) for illustration. This model is based on a Generalized Lotka-Volterra model with four variables: $x_1$ and $x_2$ represent positive emotions and $x_3$ and $x_4$ represent negative emotions. Interactions between these variables are captured by the quadric terms $C_{ij}x_i x_j$ ($i,j = 1, 2, 3, 4$; corresponding to the four types of emotions), where positive emotions strengthen each other and weaken negative emotions, and vice versa. These variables also have self-reinforcing effects represented by $r_i x_i$ ($i = 1, 2, 3, 4$). The model also includes a white noise term, $\frac{\sigma_i}{d}dW$ ($i = 1, 2, 3, 4$), for each variable. Written together:

$$dx_i = \left(1.6 + r_i x_i + \sum_{j=1}^{4} C_{ij} x_i x_j\right) dt + \sigma_i dW \quad (i = 1, 2, 3, 4).$$  \hspace{1cm} (6)

We simulate the dataset from this model using the same parameter settings as in Haslbeck and Ryan (2021), namely $C = \begin{bmatrix} C_{ij} \end{bmatrix} = \begin{bmatrix} -0.2 & 0.04 & -0.2 & -0.2 \\ 0.04 & -0.2 & -0.2 & -0.2 \\ -0.2 & -0.2 & -0.2 & 0.04 \\ -0.2 & -0.2 & 0.04 & -0.2 \end{bmatrix}$, and $\sigma_i = 4.5$ ($i = 1, 2, 3, 4$). This is a four-variable model, for which the full vector field and the potential
landscape in the original state space can only be shown in a four-dimensional state space. However, with the aforementioned parameter settings, the dynamic equations for \( x_1 \) and \( x_2 \) are exactly symmetric, and the same holds for \( x_3 \) and \( x_4 \). As a result, the majority of the behavior of the system can be understood by examining \( x_1 \) and \( x_3 \). Therefore, we will only investigate \( x_1 \) and \( x_3 \) in the remaining part of this section. The true vector field and potential landscape of the system are shown in Figure 2.

The simulation was performed under five conditions to evaluate whether and when the bistability of the system can be recovered. The five conditions are: the baseline condition, the noisy condition, the permutation condition, the long interval condition, and the polarized interpretation condition. We first focus on explaining the first two conditions. In the baseline condition, the output time series was down-sampled to every 20 time units, and only the first 200 time points after down-sampling were retained (i.e., the variable values at the \( t = 0, 20, 40, \ldots, 4000 \) were used.) This was to ensure that the sampling frequency and the length of the data are comparable to psychological ILD. The resulting time series is shown in Figure 3a. In the noisy condition, we added white noise drawn from \( N(0,1) \) to the baseline data to evaluate the robustness of the method against noise. We assume this noise comes from the measurement but not the system itself. Therefore, we first run the simulation as in the baseline condition, and then add a white noise to the simulation result. The resulting time series is shown in Figure 3d.

We estimated the vector fields and the potential landscapes for both conditions. The results are shown in Figure 3b-c and Figure 3e-f, respectively. In the baseline condition, the estimated vector field (Figure 3b) was largely consistent with the true vector field (Figure 2a) in terms of direction, but not in magnitude. All the arrows in the vector fields point towards one of the two regions that the system is stable in. This can be attributed to the limited information
present in the short time series, which did not provide enough detail to perfectly estimate the movement tendency of the system in the unstable regions. Despite this, the potential landscape (Figure 3c) accurately captured the bistable nature of the system. There are clearly two basins on the potential landscape, and the positions of the local minima (denoted by the white dots) were close to the true positions (Figure 2b). The phases have similar stability, as indicated by the similar depth of the basins, which aligns with the characteristics of the true potential landscape as well. In the noisy condition, the addition of measurement noise disrupted the clear separation between the two phases of the system (Figure 3d). But the estimated vector field and potential (Figure 3e-f) still retained the general trends and stability features, despite becoming fuzzier. This indicates fitlandr is robust against moderate levels of noise. The results for smaller and larger noise levels can be found in Appendix B. The results demonstrate when the noise level is too large, fitlandr is not able to recover the bistability of the system anymore.

At this point, it is tempting to suspect that fitlandr simply reflects the density distribution of the time series, given that the landscape results are similar to the density distribution of the raw data, and the density distribution of a system is known to be more robust than its dynamic features (Haslbeck & Ryan, 2022). We tested this with three other simulation conditions. In the permutation condition, the data from the baseline condition was randomly shuffled. This manipulation retains the distribution of the data points but removed the dynamical information from the data, and we use this condition to test if the method extracts the dynamical information from data or relies solely on the distributional information. The resulting time series is shown in Figure 3g. In the long interval condition, the time series was down-sampled to every 2,000 time points instead of 20 time points in the baseline condition, and the length of the data remained the same (i.e., the variable values at the \( t = 0, 2000, 4000, \ldots, 400000 \) were used.) This condition
was used to assess the impact of having a much lower sampling frequency compared to the time scale of the changing process of interest. The resulting time series is shown in Figure 3j. In addition, the polarized interpretation condition was not simulated using the bistable Generalized Lotka-Volterra model, but a monostable VAR model specified by:

$$
\begin{bmatrix}
    x'_{1,t+1} \\
    x'_{3,t+1}
\end{bmatrix} =
\begin{bmatrix}
    0.2 & 0.2 \\
    0.2 & 0.2
\end{bmatrix}
\begin{bmatrix}
    x'_{1,t} \\
    x'_{3,t}
\end{bmatrix} + \varepsilon, \quad \varepsilon \sim N\left(\mathbf{0}, \begin{bmatrix} 0.5 & 0.4 \\ 0.4 & 0.5 \end{bmatrix}\right),
$$

and the results were transformed with a hyperbolic tangent function,

$$
\begin{align*}
    x_{1,t} &= \tanh(x'_{1,t}), \\
    x_{3,t} &= \tanh(x'_{3,t}),
\end{align*}
$$

(8)

to create a bimodal sample distribution. This transformation makes $x_1$ and $x_3$ have a bimodal distribution (Figure 3m), so that it mimics a scenario where the psychological process is not inherently bistable, but the participants tend to use both extremes of the scale instead of the middle part, which creates bimodality in the data. In empirical studies, this can be induced, for example, when a 0-100 slider scale is used and the slider is initialized at 50. In that case, participants may be more likely to move the slider away from 50, thereby inducing bimodality in the data (Haslbeck et al., 2023).

For the permutation condition, the estimated vector field and potential function are shown in Figure 3h-i. It is clear that bistability cannot be recovered in this case. Both the vector field and the landscape showed a clear tendency for the system to move to the middle of the two distribution modes. With the time order randomized, it is equally likely for the system to remain in a stable phase or transition to another stable phase, causing the algorithm to identify the middle as the only stable phase of the system. We can also observe similar behaviors in the long interval condition and the polarized interpretation condition, in which the system also quickly transitioned between two distribution modes and the middle was recognized by the algorithm as
the only stable phase of the system (Figure 3k-l, Figure 3n-o). These similar behaviors have
different origins. In the permutation condition, the time order was shuffled, removing the time
dependency from the data. In the long interval condition, the system is actually bistable, but due
to the large measurement interval relative to the dynamics of interest, the dataset hardly retains
dynamic information about the bistability of the system. In the polarized interpretation condition,
the system is actually monostable, but the hyperbolic tangent transformation made the data
points bimodal. In this case, the monostable results from fitlandr correctly recovered the
monostable nature of the system.

To summarize, our results with simulated data demonstrate that fitlandr can effectively
capture the bistable nature of a system, displaying robustness in the presence of moderate noise.
The results indicate that the method primarily extracts dynamic information from the data rather
than simply reflecting its distributional characteristics. If the dataset lacks sufficient dynamic
information about the bistability of the system, fitlandr is unlikely to produce bistable outputs,
even if the data have a bimodal distribution.

Describe Nonlinear Dynamics and Stability from Empirical Data

After investigations with simulation models in the previous section, we now apply the
fitlandr method to two empirical datasets. It is important to note that these empirical applications
serve only as illustrations of the method. The selection of participants and variables was arbitrary
and not pre-registered. Hence, results shown in this section should not be regarded as empirical
claims about the behavior of these clinical groups.

The first dataset is from the Leuven BPD study (Houben et al., 2016) and was obtained
from the EMOTE database (Kalokerinos et al., in preparation) with the data request ID
32YD13R54M. This dataset involves participants with a diagnosis of bipolar personality disorder
(BPD) and a healthy control group reporting their emotional states 10 times a day over eight consecutive days. The details of the data collection procedure are described in Houben et al. (2016). We chose this dataset because patients with BPD have “unstable emotional experiences and frequent mood changes” (DSM-5TM, American Psychiatric Association, 2022). Therefore, we expect that their mood system may be multistable. In the current study, the time series data from the participant with a BPD diagnosis and with the longest record was used to estimate the vector field and potential landscape. The time series of this participant (P1) contained 4 missing values and 75 valid points. Two variables, the self-rated emotional arousal and valence, were used in this study. Both variables were measured together from an emotional grid scale and took integer values between 0 and 91. The mean values of emotional arousal and variance of this participant were 25.09 (SD = 23.05) and 44.81 (SD = 33.86), respectively. Both variables are bimodally distributed, as shown in Figure 4a.

The second dataset used in the current study is from Delignières et al. (2004) and is freely available online. In this study, four participants filled in the Physical-Self Inventory (PSI-6, Ninot et al., 2001) twice a day for 512 consecutive days, yielding 1024 time points without missing values. We used the data from the second participant (P2) due to its clear bistable patterns (as shown in Figure 4d). Two items from the scale, perceived fitness and physical self-worth, were used in this study. The mean values of perceived fitness and physical self-worth for this participant were 5.88 (SD = 0.94) and 6.28 (SD = 0.79), respectively.

We estimated the vector fields and the potential landscapes for both participants, and the results are presented in Figure 4. Although both variables of P1 have a bimodal distribution

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3 The dataset is available at https://didierdelignieresblog.wordpress.com/recherche/databank/.
(Figure 4a), the vector field and the potential landscape (Figure 4b-c) both show that the system has the tendency to move to the only local minimum (denoted by the white dot on the landscape) from any starting point. Thus, only one phase was identified for the system, with a moderate level of emotional valance and arousal, which means that the dataset does not contain enough bistability information about the system. As shown in the simulation study, it is possible that the system is monostable but has a bimodal data distribution due to the reaction tendency of the participant or that the system is bistable but the measurement method did not capture enough information to reveal the bistability (e.g., due to inadequate sampling frequency or measurement precision). Notably, the vector field of P1 showed a rotation tendency. The arrows from the low arousal, low valance region, and the high arousal, high valance region both tend to first move to the low arousal, high valance region before moving to the local minimum. This paves the way for future studies to investigate whether this holds true for the emotion regulation of this particular participant. From the time series of P2, both the vector field and the potential landscape indicate that there are two phases in the system (Figure 4e-f), with positions close to the two modes of the data distribution. This means that the participant’s self-esteem regarding physical strength is likely to have bistability. The centers of the phases, which are the local minima, are denoted by the white dots on the landscape. One phase is more stable, characterized by a low level of physical self-worth and perceived fitness, and the other phase is more unstable, with a higher level of physical self-worth and perceived fitness. Because the length of the two time series differs notably, we also estimated vector fields and landscapes with a small section of the time series from P2 as well as a down-sampled time series which is more sparse than the original one. The resulting vector field and potential landscape (Appendix C) showed similar features as for the original time series (Figure 4e-f).
Discussion

In the current article, we introduce fitlandr, a novel method for estimating the dynamic function and the generalized potential function of psychological systems using ILD. The method comprises two steps. The first step involves estimating the drift-diffusion using the multivariate kernel estimation algorithm (MVKE, Bandi & Moloche, 2018). Compared with previous methods, MVKE is a non-parametric, rather assumption-free algorithm, which offers adequate flexibility for recovering complex dynamics from ILD. The second step of the method entails estimating the generalized potential function from the drift-diffusion function, based on the definition by Wang et al. (2008). This definition is grounded on the steady-state distribution of the system, which can be estimated through Monte-Carlo simulation of the drift-diffusion function. We evaluated the performance of fitlandr using both simulation and empirical data. Results from simulation data showed that fitlandr is capable of recovering bistability of the system from relatively short time series (200 sample points) and is robust against moderate levels of noise. We also showed three conditions in which the datasets had a bimodal distribution but did not contain information of bistable dynamics. In those cases, fitlandr produces monostable results. Our analysis of two empirical datasets demonstrated that fitlandr generates meaningful results about the stability of the system. Although the two datasets displayed bimodality in their variable distributions, only one of the outputs from fitlandr showed bistability.

The results of fitlandr highlight the distinction between bimodality and bistability. Whereas bimodality describes the data distribution, bistability refers to the stability of the system: if a system is bistable, and if we know the system is currently in its first phase, then it is likely that the system will still stay around the same phase in the near future. In the simulation study, we showed three cases where the bimodality in the data did not correspond to the
bistability of the system. In the permutation condition and the long interval condition, although the data points were generated from a bistable system, there was almost no dynamical information in the data that showed the bistability. As a result, the algorithm was unable to determine whether the data are from a genuine bistable system, or randomly drawn from a bimodal distribution. When the data points are shuffled (in the permutation condition) or sampling frequency is much slower than the average time that the system transitions between two stable phases, it is equally likely that the system stays around its previous phase or moves to a new phase. Therefore, it is not possible to tell if bistability exists in the system. This can also be found in the last condition, the polarized interpretation condition. In this condition, the system is actually monostable, and the bimodal data distribution was a result of a transformation. When the underlying data generation process is unknown, those three conditions are indistinguishable to the algorithm, and hence, the algorithm produces similar monostable results. Here we also note an important difference between the characteristics of the underlying system and the information contained in the data. If the data quality is not good enough, in the sense that it does not contain enough information about the true characteristics of the system, no matter how good an algorithm is, it will always lead to faulty outputs.

The same also applies to the empirical analysis, in which we used two N = 1 datasets to estimate the dynamical function and the potential landscape. The formal theories about which psychological processes involve bistability is sparse (Fried, 2020; Robinaugh et al., 2021). Based on verbal theories, one may expect the emotional system of a patient with BPD (P1) to be bistable, whereas there may not be an explicit theory suggesting that the physical self-concept of a person (P2) is bistable. However, we found the opposite in the results by fitlandr. There is only one phase for P1, but two phases for P2. The dataset of P1 did show a bimodal distribution, but
we did not recover bistability of the system. As shown in the simulation study, multiple reasons can lead to this result. First, the person may have two qualitatively different phases, one with high arousal and low valence emotion and one with low arousal and high valence emotion, but the sample intervals were much larger than the average time of switching between these two phases (e.g., the average time of switching between the two affective phases may be twenty minutes, but the assessment is conducted every two hours). As a result, the data obtained were too coarse-grained to capture the switches between these phases effectively. Second, it may be that the person actually has only one psychological phase, but the person’s responses are influenced by a polarized tendency, which means that the person tends to use the extremes of the scale instead of the middle of the scale. Without additional information, it is impossible to distinguish between these two explanations. Moreover, for other participants in the same group of those studies, we did not find similar results as P1 or P2 (Appendix D), which suggests that the number and position of stable phases are highly idiographic.

The kernel method used in fitlandr enables the description of various nonlinear dynamics of psychological systems. Although the fitting procedure is rather assumption-free, there are underlying assumptions in the drift-diffusion function. The drift part and the diffusion part are only the functions of the state of the system, \( x \). This implies that during the data collection process, only the state of the system changes, but the dynamic function of the system remains unchanged. This assumption may not be correct when for instance a clinical intervention is administrated during data collection, as this could alter the way that the variables interact with each other (e.g., a cognitive restructuring may change how physical arousal influence perceived threat, Robinaugh et al., 2019). Future development of this method may involve a moving window approach to allow the dynamics of the system to slowly change over time, as in the TV-
VAR model (Bringmann et al., 2017; Haslbeck et al., 2021). Another assumption of the drift-diffusion function is the Markov property, which means that the evolution of the system only depends on the current state of the system and not on historical states. As a result, it cannot directly predict periodic changes, such as those in cyclothymic disorder (Akiskal & Pinto, 1999; American Psychiatric Association, 2022a). Including previous time points in the dynamic function may help to remedy this issue, as in phase space reconstruction methods (Marwan et al., 2007; Takens, 1981). MVKE also assumes equal distance between observations, which is often not true. Future statistical development of the method extends the method to handle different time intervals, as in the CT-VAR model. However, it is unclear if the physical time interval should be used in the modeling. Physically, 8 hours from 11 PM to 7 AM and 8 hours from 9 AM to 5 PM may be equal, but the psychological changes during those two time slots are not likely to be the same. Further research is needed to determine the appropriate time scale for measuring psychological variables.

We used around 200 time points in the current article, which is feasible for an ESM study, but is not always easy to conduct. Here, we provide three general principles to guide the data collection for using the fitlandr method. First, the period of data collection should be long enough to observe different phases of the system. For a patient with a bipolar disorder, for example, data points from both the depressive phase and the manic phase should be collected to describe the possible bistability of the system. Second, as fitlandr assumes that the dynamics and stability of the system does not change over time, the data collection period should not be too long that it contains significant developmental changes or treatment changes. Alternatively, if the time series does contain significant changes of the system dynamics, it may be better to use the fitlandr method in moving windows instead of applying it to the entire time series. Third, the
measurement frequency should match the change rate of the phenomena, so that there is sufficient information about the stability of the system in the dataset. It is difficult to provide guidance about exactly how many data points are sufficient to produce stable results, because the results shown in the current study may not be applicable to all real-life scenarios, and the dynamic properties of different people and different variables can differ in so many ways. Future work may investigate different specific scenarios and provide estimations of the number of data points required for this method.

Currently, fitlandr can only be applied to a single participant dataset. It might be appealing to develop multilevel extensions to fitlandr, as in the Dynamic Structural Equation Models (DSEM, Hamaker et al., 2018). It is technically possible to build a multilevel model for nonparametric methods (e.g., Li et al., 2006). However, as the dynamic functions and potential landscapes can be very different within the same group of people (Appendix D), we doubt if aggregating the landscapes will yield meaningful results. It is good to have a method that helps to generalize the results to a larger group, but this is only meaningful to the extent that the result is generalizable. Therefore, we highly suggest the current method to be used as an N = 1 method unless there is sufficient evidence suggesting the dynamic functions or stability landscapes at the group level are similar to the landscapes of the individuals that make up the group (Fisher et al., 2018; Hamaker, 2023; Hekler et al., 2019; Molenaar, 2004; Olthof et al., 2023).

Conclusion

We introduce the fitlandr method in the current article to flexibly estimate the dynamical drift-diffusion function and generalized potential landscape function for psychological systems. This approach uses MVKE, a nonlinear kernel method, to estimate the drift-diffusion function, employs Monte Carlo simulation to determine the steady-state distribution, and uses the simlandr
package to calculate the generalized potential landscape function. Our method is effective in detecting bistability from both simulation and empirical data, is robust against noise, and relies on the dynamic information from the data instead of the distributional information.

Reference


Wichers, M., Schreuder, M. J., Goekoop, R., & Groen, R. N. (2019). Can we predict the
direction of sudden shifts in symptoms? Transdiagnostic implications from a complex
https://doi.org/10.1017/S0033291718002064
Table 1. A comparison of the commonly used ILD methods and the fitlandr method presented in the current article.

<table>
<thead>
<tr>
<th>Method</th>
<th>Multidimensional¹</th>
<th>Nonlinear dynamics</th>
<th>Continuous dynamics</th>
<th>Number of stable states</th>
<th>Transitions</th>
<th>Selected references</th>
</tr>
</thead>
<tbody>
<tr>
<td>VAR-based</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VAR</td>
<td>Yes (high)</td>
<td>No</td>
<td>No</td>
<td>1</td>
<td>N/A</td>
<td>(Bringmann et al., 2013; Hamilton, 1994)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(Bringmann et al., 2017; Haslbeck et al., 2021)</td>
</tr>
<tr>
<td>TV-VAR</td>
<td>Yes (high)</td>
<td>No</td>
<td>No</td>
<td>1 (smooth changes allowed)</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>Threshold-VAR</td>
<td>Yes (high)</td>
<td>No</td>
<td>No</td>
<td>≥ 2 (prespecified)</td>
<td>Change models</td>
<td></td>
</tr>
<tr>
<td>Markov-Switching-VAR</td>
<td>Yes (high)</td>
<td>No</td>
<td>No</td>
<td>≥ 2 (prespecified)</td>
<td>Change models</td>
<td>(De Haan-Rietdijk et al., 2016; Tong &amp; Lim, 1980)</td>
</tr>
<tr>
<td>Distribution-based</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HMM</td>
<td>Yes (high)</td>
<td>N/A</td>
<td>N/A</td>
<td>≥ 2 (prespecified)</td>
<td>Change models</td>
<td>(Haslbeck &amp; Ryan, 2022; Visser, 2011)</td>
</tr>
<tr>
<td>VAR- and drift-diffusion-based</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CT-VAR</td>
<td>Yes (high)</td>
<td>No</td>
<td>Yes</td>
<td>1</td>
<td>N/A</td>
<td>(Ryan et al., 2018; Ryan &amp; Hamaker, 2022)</td>
</tr>
<tr>
<td>Drift-diffusion-based</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FPEM</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Any (emerged)</td>
<td>Within model</td>
<td>(Tschacher &amp; Haken, 2020)</td>
</tr>
<tr>
<td>OUM</td>
<td>Yes (low)</td>
<td>No</td>
<td>Yes</td>
<td>1</td>
<td>N/A</td>
<td>(Kuppens et al., 2010; Oravecz et al., 2011)</td>
</tr>
<tr>
<td>AIM</td>
<td>Yes (2)</td>
<td>Yes (specific type)</td>
<td>Yes</td>
<td>2 (specific type)</td>
<td>Within model</td>
<td>(Loossens et al., 2020)</td>
</tr>
</tbody>
</table>
Notes: VAR-based models can be easily fitted for dozens of variables, and the resulted model can be meaningfully represented in a network structure, thus practically they more often used for rather high-dimensional data. Distribution- and drift-diffusion-based models sometimes, in principle, can also be applied for high dimensional data, but the required amount of data is often larger, and the resulted model is often presented as distribution plots, vector fields, or potential functions that can only be represented in a lower dimensional space. Therefore, they are often used for up to three-dimensional data.
Figure 1. The workflow of fitlandr.

Figure 2. The true (a) vector field and (b) potential landscape for the model.
Figure 3. The simulated time series, their vector field estimation results, and their landscape construction results. The first row is the baseline condition; the second row is the noisy condition; the third row is the permutation condition; the fourth row is the long interval condition; the fifth row is the polarized interpretation condition. The three columns are the time series with density plot aside, the vector field estimation result, and the landscape construction result. In the vector field plots, gray arrows represent the estimated vector field; the black arrows represent the vector samples from the datasets used for estimation. In the potential landscape plots, the color at the purple side represents higher stability; the color at the yellow side represents lower stability; the white dots represent local minimums of the phases; when there are two phases on the landscapes, the red dots represent the saddle point on the path connecting the two local minimums.

Figure 4. The empirical time series, their vector field estimation results, and their landscape construction results. The first row corresponds to the data from P1 (Houben et al.,
2016); the second row corresponds to the data from P2 (Delignières et al., 2004). The three columns are the time series with density plot aside, the vector field estimation result, and the landscape construction result. The meaning of the symbols in the vector fields and the potential landscapes is the same as in Figure 3.
Appendix A. Equivalent Vector Fields and Potential Functions for VAR and OUM

In the main text, we present fitlandr, a method to estimate vector fields and potential landscapes for psychological systems. Many previous methods, such as the VAR model and the OUM can also be represented in vector fields and potential landscapes, but in their specific, restricted forms.

In a two-dimensional VAR(1) model, the state of the system is a linear function of the previous state plus a noise term:

\[ x(t + 1) = c + Ax(t) + e(t), \]

in which \( c \) is a constant term and \( A \) is the parameter matrix. We can rewrite this equation in terms of the differences between time points:

\[ \Delta x = x(t + 1) - x(t) = (A - I)x(t) + c + e(t). \]

Only the first two terms are deterministic. Therefore, \( f(x) = (A - I)x(t) + c \) is the vector field for the VAR(1) model. As in the difference equation, the vector field function is also restricted to be linear. We draw the vector field of an arbitrary example, where \( A = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix} \), \( c = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} \), in Figure A1a. Note that because the equation is linear, in usual cases there is only one point where \( f(x) = 0 \), which means there is only one stable point.\(^4\) Therefore, it is not possible to show bistability with a VAR(1) model.

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\(^4\) In some special cases there may be infinite number of solutions to \( f(x) = 0 \). For example, if \( A = I \) and \( c = 0 \), then every \( x \) satisfies \( f(x) = 0 \). However, this situation is not likely to happen in a VAR(1) model estimated from empirical data.
To construct a potential landscape for a VAR(1) model, we estimate the steady-state distribution from simulation, with the noise term drawn from an arbitrary bivariate normal distribution with \( \Sigma = \begin{bmatrix} 0.1 & 0.05 \\ 0.05 & 0.1 \end{bmatrix} \). The resulted landscape is shown in Figure A1b.

The OUM assumes the system always tends to go back to a balance point (Kuppens et al., 2010; Oravecz et al., 2011):

\[
dx = -\beta^T(x - \mu)dt + \sigma dB, \tag{A3}
\]

where \( \mu \) is the local minimum (referred to as the home base by Kuppens et al., 2010) of the system, \( \beta \) is a vector specifying the stability of the phase (referred to as the strength that the system is attracted to the home base by Kuppens et al., 2010), and \( \sigma dB \) is a noise term. The vector field representation of OUM is \( f(x) = -\beta^T(x - \mu) \), which is again, a linear function.

We draw an arbitrary example where \( \beta = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix} \), \( \mu = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix} \) in Figure A1c.

The Jacobian of \( f(x) \) is a diagonal matrix \( \text{diag}(\beta) = \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix} \), which is symmetric. Therefore, it is a gradient model, and its potential function is in a quadratic form:

\[
U(x) = \int f(x)dx = -\frac{1}{2}(x - \mu)^T\text{diag}(\beta)(x - \mu) + U_0. \tag{A4}
\]

We draw the potential landscape with the noise term drawn from an arbitrary bivariate normal distribution with \( \Sigma = \begin{bmatrix} 0.1 & 0.05 \\ 0.05 & 0.1 \end{bmatrix} \). The landscape plot is shown in Figure A1d.
Figure A1. The vector fields and potential landscapes for a two-dimensional VAR(1) model and an OUM.
Appendix B. Performance of fitlandr under Different Levels of noise

In the main text, we demonstrated that fitlandr is robust under moderate noise (\(SD = 1\)). When the level of noise is lower (\(SD = 0.5\)), the results tend to be closer to the baseline condition (Figure B1a-c), but when the noise is too high (\(SD = 2\)), the bistability cannot be accurately recovered (Figure B1d-f).

Figure B1. The vector fields and potential landscapes for the model used in the simulation study, with lower and higher levels of noise.
Appendix C. Results from the Shortened Data of P2

The length of the data from P2 is much longer than that of P1. To further demonstrate the robustness of our method, we also present results from shortened data from P2. The data was shortened in two ways: (1) by using only the 700th to 799th data points, and (2) by using every 10th data point from the first data point to the 991st data point. The results are shown in Figure C1a-c and Figure C1d-f, respectively. Despite the reduction in data length, the vector field and the potential landscape generated by fitlandr showed similar features as those based on the original time series.

Figure C1. The vector fields and potential landscapes estimated from shortened data from P2.
Appendix D. Vector Fields and Potential Landscapes for Other Participants

In the main text, we showed the results from two participants from two studies as examples. We also applied the same procedure for four additional participants from the same studies. We selected two participants from the Leuven BPD study (Houben et al., 2016), one without a BPD diagnosis (P3) and one with a BPD diagnosis (P4), both with relatively long records. We also selected two participants from the study by Delignières et al. (2004), referred as P5 and P6. The resulted vector fields and potential landscapes are shown in Figure D1.

Comparing with the results of P1 and P2, it is clear that each participant has unique dynamics and stability features.
Figure D1. The vector fields and potential landscapes estimated from the data from four additional participants. Each row presents the results for a different participant, P3-P6.