

University of Groningen

Numerical methods for studying transition probabilities in stochastic ocean-climate models

Baars, Sven

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version

Publisher's PDF, also known as Version of record

Publication date:

2019

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Baars, S. (2019). *Numerical methods for studying transition probabilities in stochastic ocean-climate models*. Rijksuniversiteit Groningen.

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Numerical methods for studying transition probabilities in stochastic ocean-climate models

Sven Baars

The work in this thesis has been carried out at the Bernoulli Institute for Mathematics, Computer Science and Artificial Intelligence of the University of Groningen. It is part of the Mathematics of Planet Earth research program, which is financed by the Netherlands Organization for Scientific Research (NWO).

Copyright © 2019 Sven Baars

Printed by Gildeprint

ISBN 978-94-034-1710-3 (printed version)

ISBN 978-94-034-1709-7 (electronic version)



rijksuniversiteit
 groningen

Numerical methods for studying transition probabilities in stochastic ocean-climate models

Proefschrift

ter verkrijging van de graad van doctor aan de
 Rijksuniversiteit Groningen
 op gezag van de
 rector magnificus prof. dr. E. Sterken
 en volgens besluit van het College voor Promoties.

De openbare verdediging zal plaatsvinden op

vrijdag 21 juni 2019 om 14:30 uur

door

Sven Baars

geboren op 15 augustus 1990
 te Ulrum

Promotor

Prof. dr. ir. R.W.C.P. Verstappen

Copromotor

Dr. ir. F.W. Wubs

Beoordelingscommissie

Prof. dr. R.H. Bisseling

Prof. dr. D.T. Crommelin

Prof. dr. A.J. van der Schaft

CONTENTS

1	Introduction	1
2	Basic concepts	7
2.1	Newton's method	7
2.2	Iterative methods	8
2.3	Bifurcation analysis	11
2.3.1	Pseudo-arclength continuation	12
2.4	Stochastic differential equations	15
2.4.1	Brownian motion	15
2.4.2	Stochastic differential equations	16
2.4.3	The Euler–Maruyama method	17
2.4.4	The stochastic theta method	17
2.5	Governing equations	18
2.5.1	The Navier–Stokes equations	18
2.5.2	The ocean model	18
2.5.3	Stochastic freshwater forcing	20
3	Linear systems	21
3.1	The two-level ILU preconditioner	24
3.1.1	Initialization phase	25
3.1.2	Factorization phase	27
3.1.3	Solution phase	28
3.2	The multilevel ILU preconditioner	28
3.3	Skew partitioning in 2D and 3D	30
3.4	Numerical results	33

3.4.1	Weak scalability	35
3.4.2	Strong scalability	39
3.4.3	Lid-driven cavity	41
3.5	Summary and Discussion	44
4	Lyapunov equations	47
4.1	Methods	48
4.1.1	Formulation of the problem	48
4.1.2	A novel iterative generalized Lyapunov solver	50
4.1.3	Convergence analysis	52
4.1.4	Restart strategy	54
4.1.5	Extended generalized Lyapunov equations	55
4.2	Problem setting	57
4.2.1	Bifurcation diagram	57
4.2.2	Stochastic freshwater forcing	58
4.3	Results	58
4.3.1	Comparison with stochastically forced time forward simulation	59
4.3.2	Comparison with other Lyapunov solvers	60
4.3.3	Numerical scalability	66
4.3.4	Towards a 3D model	68
4.3.5	Continuation	70
4.3.6	Extended Lyapunov equations	71
4.4	Summary and Discussion	72
5	Transition probabilities	75
5.1	Definition	75
5.2	The Eyring–Kramers formula	76
5.2.1	Double well potential	77
5.2.2	Computing the transition probability	77
5.3	Covariance ellipsoids	78
5.3.1	Example	79
5.4	Most probable transition trajectories	80
5.5	Computing transition probabilities	81
5.5.1	Direct sampling	81
5.5.2	Direct sampling of the mean first passage time	82
5.5.3	Adaptive multilevel splitting	82
5.5.4	Trajectory-Adaptive Multilevel Sampling	91
5.5.5	Genealogical Particle Analysis	95
5.5.6	Comparison	96
5.6	Summary and Discussion	101
6	Transitions in the Meridional Overturning Circulation	103
6.1	Projected time-stepping in TAMS	104

6.2	Problem setting	105
6.2.1	Bifurcation diagram	105
6.2.2	Stochastic freshwater forcing	106
6.2.3	Reaction coordinate	106
6.3	Results	107
6.4	Summary and Discussion	108
7	Conclusion	111
	Publications and preprints	115
	Software	117
	Bibliography	119
	Summary	131
	Samenvatting	135
	Soamenvatting	139
	Acknowledgments	141

