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Numerical methods for studying transition probabilities in stochastic ocean-climate models

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Sven Baars

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