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Practice-inspired contributions to inventory theory

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Document Version

Publisher's PDF, also known as Version of record

Publication date:

2019

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Prak, D. R. J. (2019). *Practice-inspired contributions to inventory theory*. University of Groningen, SOM research school.

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CHAPTER 1

Introduction

A substantial share of firms' investments is tied up in inventories. Although EU national accounts do not report inventory investment values, accounting data from the U.S. reveal that in November 2018, inventories held by U.S. businesses alone totaled to approximately 2,000 billion U.S. Dollars (€1,700 billion), which is around 10% of that country's Gross Domestic Product and a 5% increase compared to a year earlier (United States Census Bureau, 2019). For every €1 of monthly sales, approximately €1.35 worth of inventory is held. Good inventory management naturally aids a firm's profitability, but also helps to reduce waste and pollution. Inventories relate naturally to the selling of commercial goods, but are also key for services. Expensive spare parts of e.g. aircraft, wind mills, or production machines are demanded infrequently, but if requested, are needed fast to minimize downtime, requiring careful planning. Also for health care operations, public services like libraries, and military purposes, to name a few examples, inventory management plays a key role in enabling a streamlined operation.

Inventory control is at the core of management science. It is dedicated to developing decision models that yield policies with which an optimal trade-off can be achieved between customer service or product availability on the one hand, and costs on the other hand. An inventory policy dictates how many items should be ordered when. Since the Economic Order Quantity model by Harris (1913), and especially since the pioneering of cost-minimizing inventory control with stochastic demand in the 1950's (Arrow et al., 1951, 1958; Dvoretzky et al., 1952a,b; Scarf, 1959), hundreds if not thousands of models have been and continue to be developed in order to cope with new developments such as inventory pooling, vendor-managed inventories, product returns, and sustainability goals. Commercial software relies

on inventory control models from the literature to devise solutions that companies can use to define their replenishment strategies in practice. The trade-off that inventory control models address is that between product availability and customer service on the one hand, and inventory-related costs (such as holding and ordering costs) on the other hand. In order to correctly make that trade-off, it is important that the inventory model describes the setting to which it is applied as accurately as possible. It should capture the future demand, inventory level reviews, ordering possibilities, replenishment lead times, service measures, and all costs. Only if the model accurately reflects these circumstances, then an order strategy can be derived that performs well in practice. However, the majority of the theoretical inventory control literature approaches this question from the opposite angle. In their search for optimal order strategies they define model assumptions that are defensible, but - more importantly - facilitate or simplify the analysis. This causes the inventory control literature to alienate from its practical purpose, which is to control real inventories.

The main purpose of holding inventories is to account for the uncertainty in future demand, and a forecast of the latter is key in every inventory control model. However, the research fields of demand forecasting and inventory control have developed almost in complete isolation of each other. Inventory control literature and textbooks typically assume that demand follows a certain probability distribution with known parameters, and start their analysis from there. This totally ignores the fact that in practice such information is not available. Demand parameter estimates have to be obtained from historical data, as also happens in case studies (e.g. Lengu et al., 2014; Turrini and Meissner, 2019) and in commercial forecasting and inventory control software packages. However, it is typically ignored that these parameter estimates have errors, which are particularly large if not too many data points are available. Several leading textbooks, such as Zipkin (2000) and Waters (2012), do not discuss at all how these parameters can be obtained from real data. Other textbooks, such as Axsäter (2015) and Silver et al. (2017), do briefly mention a parameter estimator for specific demand distributions. However, they subsequently fail to correctly integrate those estimates and their errors in the decision model, thereby ignoring part of the uncertainty around future demand.

One would expect that the research fields of demand forecasting and inventory

control are closely linked. The forecasting of future demand should not be seen as a goal in itself, but rather as an input for making decisions, in this case inventory control decisions. However, this is generally not the case. The demand forecasting literature is typically solely concerned with producing the (point) forecast of demand per period, and forecasting techniques are ranked based on their accuracy with respect to symmetrical loss measures (examples are Hyndman and Koehler, 2006; Kim and Kim, 2016; Petropoulos et al., 2018). In many cases, the link with inventory control and demand parameter estimation is missing completely. If a connection is made with inventory control, then a normal approximation of demand is used to set a safety stock based on the one-period ahead forecast error (e.g. Chatfield et al., 2004; Wang et al., 2010; van Wingerden et al., 2014). However, even if the normality assumption of demand is justified, then (contrarily to what textbooks such as Axsäter (2015) propose) the auto-correlation of future forecast errors cannot be ignored. An even more striking problem occurs when individual customer demand parameters need to be obtained. Period demand forecasts are not suitable for this purpose, yet several combined demand forecasting and inventory control software packages do mis-use them as such, leading to biased estimates and flawed inventory calculations.

It is peculiar and undesirable that the demand forecasting literature and the inventory control literature seem to operate in complete isolation from each other, even though several authors remark that the interactions between the two are not studied extensively enough (e.g. Goodwin, 2009; Babai et al., 2013; Silver et al., 2017). Three chapters of this thesis are devoted to exactly this mismatch. These are further introduced in Sections 1.1 and 1.2. Next, however, we first put the remaining chapters in a general perspective. These deal with other modeling aspects on which the inventory control literature departs from business practice.

A peculiarity in the inventory control literature is that stock review moments and ordering possibilities are almost always merged, whereas this is in practice not needed and may be undesirable. The inventory control literature (including all major textbooks) is roughly divided into continuous and periodic review models, meaning that the stock level is either always known or only updated after a fixed number of periods. In the latter case, it is implicitly assumed that replenishment orders can also only be placed and received at those stock review moments. Section 1.3 elaborates further on this topic and introduces our suggestion to allow order placements

in-between stock reviews, albeit without knowledge of the precise stock level.

We finally move to the repair services industry, where another unrealistic ordering-related assumption is commonly used to simplify the analysis. A stream of literature has developed around the so-called Repair Kit Problem, which deals with the question how many units of various items an engineer needs to store in his kit of spare parts when performing on-site service jobs. An assumption has always been that engineers can replenish their kit at the end of each tour, without a lead time and without any costs. In practice, parts often need to be shipped from central warehouses, which takes time during which an unknown number of repair jobs may occur, each requiring an also unknown combination of different parts. In Section 1.4 we discuss the Repair Kit Problem as studied to date and introduce - motivated by a real-life case - positive lead times and fixed order costs.

This thesis' overall research question is "How can theoretical inventory control research be extended to enhance its practical applicability?" We subdivide this question into two main topics. Chapters 2, 3, and 4 focus on enhancing the interface with demand forecasting, whereas chapters 5 and 6 discuss specific improvements on the inventory control modelling side. Below we elaborate on the separate contributions.

1.1 Demand parameter uncertainty

Almost all inventory models, and certainly all standard methods that are included in software and applied in practice, are based on the assumption that the probability distribution of future demand and all its parameters are completely known. Such information is obviously never available in practice, where a demand distribution has to be fitted based on available data. There exists literature on forecasting demand from a set of historical observations, and this literature has developed in various ways. There are simple forecasting techniques such as the moving average, but also (auto-correlated) regression models (possibly with seasonality effects), bootstrapping methods, and recently also machine learning approaches. However, these papers are focused solely on producing that demand forecast, and not on using it as an input in inventory control models. That is, the forecasting literature produces a point forecast of the demand per period or during the lead time, and possibly an estimate of its accuracy such as the Mean Square Error (MSE), Mean Absolute Devi-

ation (MAD), or Mean Absolute Percentage Error (MAPE), but stops there instead of yielding the probability distribution that is required in all the inventory control literature. This implies that there is a missing link between these two research strands.

This missing link has led to the use of two main approaches in practice, both of which are wrong. The first approach is that companies and software make a lead time demand forecast, derive the MSE, and calculate order levels based on ad-hoc approximative methods, thereby ignoring the abundance of models available in the inventory control literature to deal with all kinds of demand probability distributions and inventory scenarios. The second approach is that inputs are sought for the models from the inventory control literature, but that the produced point forecasts are directly substituted as the parameters for those probability distributions. This ignores the so-called estimation uncertainty.

It is obvious that a demand mean estimate based on 5 observations is more volatile than one based on 100 observations. In statistics, inference can be done on the true parameter in the form of e.g. confidence bounds. After 5 observations these bounds are still relatively wide and the true parameter can be within a large range of values. After 100 observations, under some degree of stationarity, one can be much more certain of the true parameter value. If this distinction between the estimated demand parameter and the true demand parameter (the estimation uncertainty) is ignored, as is done in practice, then inventories will as a result be set too low, and the targeted customer service will not be attained. However, it is not trivial to apply these concepts from statistics to inventory control. There, decisions are derived assuming knowledge of the true parameters, and knowing a range in which a true parameter lies by itself is not enough.

Chapters 2 and 3 of this thesis deal with the problem of estimation uncertainty in inventory models. The approach in both chapters is to derive a so-called predictive demand distribution, which results from the distributional assumption of the inventory model and the parameter estimates, and takes their uncertainty into account. In chapter 2 we show that future forecast errors (over e.g. a lead time) are correlated, which is ignored in the literature and in textbooks. For example, an underestimation of mean demand persists throughout all future periods. This leads to too low order levels and hence to underachieving the target service level. We derive

corrected expressions for the lead time demand forecast error variance for any stationary demand process and for two popular forecasting methods (Simple Moving Average and Single Exponential Smoothing). Thereafter, we show how these corrected expressions lead under normally distributed demand to a closed-form predictive lead time demand distribution that takes the estimation uncertainty of both the demand mean and variance into account. Finally, we present adjusted order levels that achieve the target service levels.

In Chapter 3 we generalize the idea of creating a predictive demand distribution to deal with estimated parameters and their estimation uncertainty. We use concepts from Bayesian probability theory to create a framework that transforms any demand distribution and its parameter estimators into a predictive distribution. Other than in the Bayesian literature, any parameter estimator is allowed, as long as the distribution of its estimation error can be derived or approximated. We discuss a mean-stationary demand model, a model where demand is auto-correlated, a random walk model, and a trend model. The latter allows for especially large cost savings, as the mis-estimation of a trend parameter has a cumulative effect throughout the lead time. We finally show that an approximate error distribution is generally easy to derive, and relatively robust under misspecification of the demand model.

The concept of combining parameter estimates and the corresponding parameter uncertainty into a predictive demand distribution yields a simple adaptation that keeps standard inventory control models applicable also under the assumption of unknown demand parameters. An inventory mark-up is needed to account for the uncertain parameters, which leads to marginally higher holding costs in case demand was overestimated, but to large (backorder) cost savings if demand was underestimated. The latter effect is clearly dominant. Cost savings (or service level differences) are particularly large if the parameter uncertainty is large, i.e. if estimates are based on limited data. After substitution of the original demand distribution with the newly derived predictive distribution, the inventory analysis remains unchanged. In conclusion, when applying theoretical inventory control models to real data, one should take parameter uncertainty into account, and the proposed framework allows to do so exactly or approximately, without dramatic changes to the analysis.

1.2 Demand parameter estimation from periodic data

In several scenarios, the point forecasts that are produced in the forecasting literature actually do not match with the unknown parameters of the demand distribution assumed by the inventory control model. Chapter 4 deals with one of the most common problems in that respect: obtaining demand parameters at the individual customer level from periodically stored data. Croston (1972) was the first to separately estimate the average period demand size (per e.g. day, week, or month) and the number of periods between two periods with a positive demand. Forecasting methods in this spirit are called Size-Interval methods. Companies typically store demand data on a periodic basis and several commercial software packages use Size-Interval methods to forecast period demand. On the other hand, many of these companies use continuous inventory control policies, for which individual demand arrivals and sizes must be modeled. A common distributional assumption is that of the compound Poisson distribution. However, this distribution requires demand size and arrival rate parameters at the individual customer level. By using period estimates as individual customer parameters, one cannot distinguish between e.g. few large or many small demands in a period, leading to flawed inventory calculations.

In Chapter 4 we study this problem. Size-Interval methods are especially popular for intermittent demand patterns, which contain many periods without demand. Several integrated forecasting and inventory control software packages use for example Croston's method for forecasting. Subsequently, they wrongly use the obtained period arrival rate and average period demand size as demand parameters for the compound Poisson distribution, and based on that perform inventory control analysis. We show how this leads to even asymptotically biased parameter estimates, dramatically overshoot service levels, and therefore also too high inventory costs. Case studies in the inventory control literature, on the other hand, typically fit compound Poisson distributions based on the standard method-of-moments estimator (e.g. Lengu et al., 2014; Turrini and Meissner, 2019), as advocated e.g. in the textbook by Axsäter (2015). However, this estimator, even though it is consistent, also has large biases in small samples, especially if the demand pattern is intermittent. We therefore present a new method-of-moments variant that retains the idea of separately estimating the average demand size and the arrival rate. We show that for

intermittent demand this estimator significantly outperforms the standard method-of-moments estimator. With this new estimator one can fit compound Poisson distributions simply and accurately to period demand data. This allows for continuous review inventory control, modeling individual customer arrivals and demand sizes, while minimizing efficiency loss due to the periodic storage of demand data.

1.3 Decoupling stock reviews and order moments

Next to the issue of periodic demand data storage, there are several practical situations in which complete stock level information is not continuously available. A reason may simply be that inventories are only counted periodically, but also counting inaccuracies, perishability, product quality issues, misplacement, and theft hinder continuous and complete stock overviews (see Raman et al., 2001; Yano and Lee, 1995; Nahmias, 1982; Fleisch and Tellkamp, 2005). There exists a large stream in the literature on so-called period review inventory systems, and also all major textbooks, such as Axsäter (2015), Silver et al. (2017), and Zipkin (2000) discuss periodic review models. However, periodic review inventory systems do not only limit the stock review moments to certain time intervals, but also the replenishment opportunities, an unnecessary restriction that forces inventory accumulations and thereby drives up inventory costs.

In Chapter 5 we therefore study the effects of decoupling inventory review moments and replenishment orders. Specifically, we assume that orders can be placed at any time and that inventory reviews are only allowed at fixed points in time. In-between reviews orders can be placed under knowledge of the last observed stock level and the distribution of the demand that happened since that last observation. This leads to a stream of orders that gradually build up safety stock during the period to account for the increasing demand uncertainty. Shortly before the review, this build-up stops and the review is awaited, after which inventory is typically first depleted again, because the degree of uncertainty has dropped.

We show that by allowing to place orders in-between reviews, large holding cost savings can be achieved compared to forcedly ordering only at the review moment. It is remarkable that in this model, typically no order is placed at or around the review moment, whereas the classical periodic review literature only allows orders

at those moments in time. This mixture situation thus leads to order policies that are fundamentally different from those advocated by the standard, restrictive periodic review models.

1.4 Lead times and order costs in repair services

The Repair Kit Problem deals with the question of how many units of each spare part should be kept in an engineer's kit to optimize the delivered service while minimizing the total investment value of the units carried. A major difficulty here is that service should be defined in terms of completed jobs, whereas job completion depends on the simultaneous availability of various spare parts. This makes this problem notoriously hard to study. Major textbooks such as Axsäter (2015), Silver et al. (2017), and Zipkin (2000) do not mention it at all, whereas studies in the literature make several simplifying assumptions.

In the original Repair Kit Problem formulation (Smith et al., 1980) there was only a single job, and only one unit of each part could be picked. After the job, a free replenishment of the kit takes place. Later, the problem has been extended to multiple jobs per tour and multiple units per part by Teunter (2006), but he simplified the analysis by adopting a part fill rate service measure, that does not measure actual job completion, but the availability of different parts. Bijvank et al. (2010) introduced the job fill rate, making the model significantly more realistic and applicable in practice. However, all studies of the Repair Kit Problem assume free restocking at the end of a tour, with immediate delivery. This makes the analysis much easier, as only starting stock levels of each part at the start of a tour have to be derived. However, in practice spare parts come in various specific types which are not all locally available, and may have to be shipped from a central warehouse, which takes time and involves order costs.

Chapter 6 revisits the Repair Kit Problem, while introducing positive replenishment lead times and fixed ordering costs. This study is motivated by a real-life case of an Italian supplier of printing equipment, who has two warehouses, each with a different replenishment lead time. This leads to a completely different analysis from those performed in earlier Repair Kit Problem studies, as now the starting number of units per spare part at the beginning of a tour cannot be directly controlled any-

more. Contrarily, reorder levels and order-up-to levels have to be derived, that decide when an order is triggered and what the size of that order should be. The order then arrives after the lead time, during which a number of tours have occurred with stochastic numbers of jobs, each requiring stochastic numbers of each spare part.

We present an exact calculation of the job fill rate, and discuss heuristic approaches to optimize the reorder level and order-up-to level of an (s, S) policy for each spare part, so that a service threshold is met with minimal inventory holding and ordering costs. We study the model and its solution procedures first theoretically and on benchmark instances, and then apply them to the real-life case. By introducing lead times and fixed ordering costs, the practical applicability of the Repair Kit Problem is enhanced. This comes at the cost of a more complicated analysis and computationally demanding solution procedures, but we find that simpler order rules derived in previous literature do not perform well under these more realistic assumptions.

The topics addressed in this thesis each in their own way bring inventory models closer to their ultimate goal: being applicable to control inventories in real-life scenarios. The lacking bridge between demand forecasting and inventory control causes on the one hand an inability to correctly fit demand models that are assumed in the inventory literature, and on the other hand a partial neglect of the demand uncertainty against which inventories are supposed to guard. We present solutions to adapt the demand distribution so that it takes parameter uncertainty into account. Furthermore, we suggest a better estimator to fit an individual customer demand distribution based on periodic data. Except for these amended distribution-fitting procedures, no fundamentally different inventory control analysis is needed. This is not the case for the last two topics that we study. The coupling of stock reviews and order moments is a restriction that leads to unnecessarily high peak inventories around these review moments. A more general policy in which orders can also be placed and received in-between stock reviews is desirable. Finally, the assumption of free immediate replenishments of engineers at the end of their tour in the Repair Kit Problem can often not be met in practice. The five upcoming chapters of this thesis present the specific solutions to enhance the practical applicability of general and certain specific inventory control models.

1.5 Academic articles

This thesis is based on the following academic articles.

Chapter 2

Prak, D.R.J., R.H. Teunter, A.A. Syntetos. 2017b. On the calculation of safety stocks when demand is forecasted. *European Journal of Operational Research* **256**(2) 454–461.

Chapter 3

Prak, D.R.J., R.H. Teunter. 2019. A general method for addressing forecasting uncertainty in inventory models. *International Journal of Forecasting* **35**(1) 224–238.

Chapter 4

Prak, D.R.J., R.H. Teunter, M.Z. Babai, J.E. Boylan, A.A. Syntetos. 2019. Compound Poisson parameter estimation for inventory control - submitted for publication.

Chapter 5

Prak, D.R.J., R.H. Teunter, J. Riezebos. 2015. Periodic review and continuous ordering. *European Journal of Operational Research* **242**(3) 820–827.

Chapter 6

Prak, D.R.J., N. Saccani, A.A. Syntetos, R.H. Teunter, F. Visintin. 2017a. The Repair Kit Problem with positive replenishment lead times and fixed ordering costs. *European Journal of Operational Research* **261**(3) 893–902.

