Relationship between Granger non-causality and network graph of state-space representations

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Summary

In this thesis we study dynamical systems that consist of interconnected subsystems. We address the problem of relating the network of subsystems to statistical properties of the output process of the dynamical system. The considered systems are: linear time invariant state-space (LTI–SS) representation, LTI transfer matrix and general bilinear state-space (GB–SS) representation.

The network of subsystems of the dynamical system is represented by a directed graph that we call network graph whose nodes correspond to the subsystems and whose edges correspond to the directed communication between the subsystems. The statistical property of the output process is, in LTI systems, the so-called conditional and unconditional Granger causality and, in GB–SS representation, the so-called GB–Granger causality.

As a first step, we relate the lack of Granger causality between two components of an output process generated by an LTI–SS representation to the network graph of that representation. We show that if one output component does not Granger cause the other then it is equivalent to the existence of an LTI–SS representation whose network graph has two nodes and one directed edge. That is, LTI–SS representations of this type are compositions of two subsystems, where each subsystem generates a component of the output process. Furthermore, the subsystem representing the component that does not cause the other component, does not send information to the other subsystem; information can only flow in the other direction.

The results above are extended to a collection of conditional and unconditional Granger causalities of an output process of LTI–SS representations. It is shown that the existence of an LTI–SS representation whose network graph is a transitive acyclic graph can be characterized by a collection of conditional and unconditional Granger causalities. The latter LTI–SS representations are compositions of subsystems which correspond to the nodes of the network graph and which send information to each

other according to the directed edges of the transitive and acyclic network graph.

Besides extending the results on LTI–SS representations whose network graph has two nodes and one directed edge to more complex network graphs, we also extended these results to more complex systems, namely to GB–SS representations. LTI–SS representations form a specific subclass of GB–SS representations for which GB–Granger causality reduces to Granger causality. It is then shown that if one output component of a GB–SS representation does not Granger cause the other, then it is equivalent to the existence of a GB–SS representation whose network graph has two nodes and one directed edge.

All dynamical systems considered in this thesis can be calculated algorithmically. After presenting the results on each representation with a given network graph, we propose realization algorithms whose outputs are the desired representations with the predefined network graph, if they exist. The input of these algorithms are either arbitrary representations of the output process or second order statistics of the output process. Since the second order statistics can be estimated from data (of the output process), this opens up the possibility of using the above-mentioned realization algorithms to define identification algorithms. In fact, for LTI–SS representations, to illustrate the practical value of the algorithms, we present identification algorithms to calculate the LTI–SS representations with given network graphs from data. The identification algorithms allow for distributed state and parameter estimation which potentially decreases the estimation error compared to other algorithms for calculating LTI–SS representations. This feature of the algorithms is illustrated by simulated examples.

In summary, this thesis provides a formal relationship between the network of subsystems of a dynamical system and statistical properties of its output process. In addition, for constructing the dynamical systems under consideration we suggest realization and identification algorithms. The results can be of interest in application in e.g., systems biology, neuroscience, and economics.