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Relationship between Granger non-causality and network graph of state-space representations

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Chapter 8

Conclusions

Complex dynamical systems, arising as networks of several subsystems, appear in fields ranging from cyber physical systems to systems biology and neuroscience. Often, the way the subsystems are interconnected is not known. It is then of interest to find these interconnections from the observed behavior. More precisely, it is of interest to find the so-called network graph of the system: the network graph is a directed graph whose nodes correspond to the subsystems generating components of the output process, and whose edges correspond to interactions among the subsystems.

This thesis addressed the problem of relating the network of subsystems of a dynamical system to statistical properties of its output process. The dynamical system under consideration is from one of the following families: linear time-invariant state-space (LTI-SS) representation, LTI transfer matrix or general bilinear state-space (GB-SS) representation. For LTI-SS representations and LTI transfer matrices, the relevant statistical property of the output process is the so-called conditional and unconditional Granger causality. Conditional Granger causality of the output process could be related to the existence of an LTI-SS representation or LTI transfer matrix with a certain network graph. For a GB-SS representation, the relevant statistical property of the output is the so-called GB-Granger causality. The contribution of this thesis is to provide formal relationship between the network graph of dynamical systems and statistical properties of their output processes.

Contribution

As a first step, in Chapter 2 we dealt with LTI-SS representations. We related the lack of Granger causality between two components of an output process generated by an LTI-SS representation to the network graph of that representation. We have shown that if one output component does not Granger cause the other then it is equivalent to the existence of an LTI-SS representation in the so-called block triangular form whose network graph has two nodes and one directed edge. That is, LTI-SS representations of this type are compositions of two subsystems, where each subsystem

generates a component of the output process, and the subsystem representing the component that does not cause the other component, does not send information to the other subsystem, information only can flow in the other way.

As a next step, in Chapter 3 we have studied the relationship between a collection of conditional and unconditional Granger causalities of an output process of LTI-SS representations and the network graph of that representation. It has been shown that if a collection of conditional and unconditional Granger causalities hold then there exists an LTI-SS representation in the so-called coordinated form whose network graph is a star graph. The latter LTI-SS representations are compositions of $n \geq 2$ subsystems, among which there is one subsystem, called coordinator who sends information to the other subsystems but there is no any other information exchange between the subsystems.

The most general case for LTI-SS representations that we studied in Chapter 4 is when a collection of conditional and unconditional Granger causalities of an output process of an LTI-SS representation are related to a transitive acyclic directed network graph of that representation. It has been shown that if a collection of conditional and unconditional Granger causalities hold then there exists an LTI-SS representation whose network graph is a transitive acyclic graph. The latter LTI-SS representations are compositions of subsystems which subsystems correspond to the nodes of the network graph and who send information to each other according to the directed edges of the network graph.

The above-mentioned LTI-SS representations with different network graphs can be calculated algorithmically. In the corresponding chapters, we proposed algorithms for these calculations whose inputs are either arbitrary LTI-SS representations or the second order statistics of the output processes and whose outputs are the desired LTI-SS representations. To illustrate the practical value of the algorithms, we also presented variations of the algorithms to calculate the LTI-SS representations with different network graphs from data. Note that all the algorithms allow for distributed state and parameter estimation which, compared to other algorithms on calculating LTI-SS representations, potentially decreases the estimation error. This has been illustrated in Chapter 7.

In Chapter 6 we have extended the results in Chapter 2 on Granger non-causality and LTI-SS representations in block triangular form to GB-SS representations. It has been shown that the components of an output process satisfy a certain statistical conditions, called GB-Granger causality, if and only if the output can be represented by a GB-SS representation whose network graph is the two node graph with one directed edge. The statistical property of GB-Granger causality is a generalization of Granger causality. Furthermore, LTI-SS representations form a subclass of GB-SS representations.

Perspectives

The primary goal of this thesis was to formalize the relationship of the network graph of certain dynamical systems and statistical properties of their output processes. However, there are several applications that can be developed in the future, using the results of this thesis, such as distributed system identification/parameter estimation/control, reverse engineering the network structure and structure preserving model reduction. Below, we discuss these potential applications.

Consider an LTI-SS representation that can be transformed to an LTI-SS representation with a certain network graph that is the subject of the thesis. Then, using the results in Chapters 2-5, we can estimate its state and its system matrices in a distributed manner. The estimation of interconnected systems in a distributed manner is the first necessary step for their distributed control. Note that distributed estimation of the state and system matrices can also be applied in the case of innovation GB-SS representations whose network graph is the two node graph with one directed edge.

By reverse engineering of the network graph we mean finding out the network graph of a system based on the observed output of that system. The understanding of when the observed behavior can be realized by a system with a specific network graph is essential for solving such problem. In fact, the representations studied in this thesis can be constructed algorithmically from the observed behaviour. Moreover, the causal properties of the observed process that are necessary to realize the process with representations discussed in this thesis are statistical properties that can be tested. Therefore, the results of this thesis open the possibility for reverse engineering of the network graph of the LTI-SS representations and GB-SS representations that are subject of this thesis.

By the methods of this thesis, one can estimate systems with interconnected subsystems in a distributed way. That is, the subsystems can be identified by only using certain components of the observed process.

The research problems that have been presented in this thesis can be continued in several directions. First, note that in GB-SS representations we only dealt with network graphs that have two nodes and one directed edge. We believe that the results can be generalized to GB-SS representations with transitive acyclic network graphs in a similar way that it has been accomplished for LTI-SS representations. Second, LTI-SS representations with non-transitive and non-acyclic network graphs could be studied, however, we believe that it requires major additional assumptions on the processes and the matrices of the representations, or needs to be related to a new definition of causality. Third, the dynamical systems that are studied here have several limitations, therefore, it would be desirable to solve similar problems

in more complex systems. Perhaps the most straightforward step would be to study LTI-SS representations with additive inputs. However, when the input is not a stationary process, Granger causality cannot be directly defined between the output components. Another approach would be to find different statistical properties than Granger causality of the observed processes that are related to structural properties of the system. For this, the well established system identification of LTI-SS systems with non-stationary input processes (see ([Lindquist and Picci, 2015](#), Chapter 17) and ([Caines, 1988](#), Chapter 9)) can provide a great help.

The extension of the results to the class of bilinear state-space representations with additive inputs can also be interesting. However, if the input is non-stationary, the same problem arises with GB-Granger causality as with Granger causality in LTI-SS representations with additive input.

At last, the algorithms presented in Chapter 7 on the calculation of LTI-SS representations with certain network graphs showed appealing results on simulated data. Namely, that they can reduce the estimation error of the LTI-SS representations at hand and that they provide models that guarantee a collection of Granger causalities to hold among the components of the observed process. Such properties can be of interest in application in e.g., economics or neuroscience.