Strategic interplay between store brand introduction and online direct channel introduction

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ABSTRACT

This paper investigates the strategic interplay between a national brand manufacturer and a retailer in introducing an online direct channel and a store brand by constructing a game-theoretic model that incorporates the firms’ channel and brand strategies. We show that at equilibrium, the store brand is introduced but the online direct channel may or may not be introduced. Interestingly, the firms may be trapped in a prisoner’s dilemma when they choose to introduce the online direct channel and store brand. The online direct channel may be introduced if the store brand has been introduced; otherwise it may not be introduced.

1. Introduction

With the rapid development of the internet and logistics, consumers have increasingly accepted shopping through online channel. In this trend, manufacturers in various industries (e.g., Unilever, Procter & Gamble, Estée Lauder, L’Oréal, Nike, etc.) establish online direct channels to access customer segments they cannot reach through traditional retail channels. By introducing an online direct channel, a manufacturer becomes less reliant on the retail channel and competes with downstream retailers directly.

Aside from manufacturer encroachment in the downstream market through the online direct channel, the introduction of a store brand (SB) by retailers also frequently occurs in practice; it is reported that the sales of store brands reached up to $150 billion in the US in 2016 (Skrovan, 2017). For example, Sephora, one of the world’s most popular luxury cosmetics retailers, has launched a series of beauty brands that compete with the products of its national brand (NB) manufacturers, like Estée Lauder and L’Oréal (Safdar and Terlep, 2017). The retail giant Wal-Mart not only sells various consumer goods (e.g., food, chemical products) supplied by Unilever and Procter & Gamble but also offers substitutable products to customers through its private-label brand Great Value. A comparable situation exists for GOME Electrical Appliances (a leading Chinese retailer), which owns a number of store brands (e.g., cell phone brands) and sells substitutable NB products at the same time.

All the aforementioned examples involve both SB and online direct channel and therefore involve the market competition between the retailer and the NB manufacturer with respect to different products and channels. Intuitively, from a competitive perspective, the introduction of an SB by a retailer and the introduction of an online direct channel by an NB manufacturer most likely to be actions that are related to each other. However, such a relationship is generally overlooked by the existing literature, although the introduction of an SB or the introduction of an online direct channel alone has attracted considerable research interest before.

Departing from the previous literature, we study the interplay between the introduction of an SB and the introduction of an online
direct channel in a supply chain with one retailer and one NB manufacturer. We model four different scenarios depending on the retailer’s SB introduction strategy (i.e., whether the SB is introduced) and the NB manufacturer’s channel strategy (i.e., whether the online direct channel is introduced) based on utility-demand functions. The equilibrium outcomes under these scenarios are derived, and the optimal SB introduction and channel strategies are analysed.

Our study contributes to the literature in the following aspects. First, our study captures the interaction between a retailer’s SB introduction strategy and an NB manufacturer’s channel strategy, which has not been studied in previous research to the best of our knowledge. Meanwhile, our model considers two salient features related to consumer utility analysis: the quality differential between an NB product and an SB product and the perceived value differential between a retail channel and an online direct channel. Incorporating both features in the model has seldom been studied in the extant literature, and doing so provides a novel and important dimension to this paper.

Second, by constructing and analysing the game models under different scenarios, we derive the manufacturer’s and the retailer’s equilibrium decisions regarding channel strategy and brand strategy. In sum, we obtain two kinds of equilibrium results in the interaction between the manufacturer and the retailer: the online direct channel is not introduced, and the SB is introduced; and both the online direct channel and the SB are introduced.

Third, we obtain some interesting results by characterising the game’s equilibrium outcomes. For example, we find that the retailer and the NB manufacturer may get caught up in a prisoner’s dilemma in making their optimal decisions. That is, at equilibrium, the retailer and the NB manufacturer may choose to introduce the SB and the online direct channel, respectively, although they may be better off by not doing so. When the acceptance degree of the online direct channel is relatively low, the NB manufacturer will not launch the online direct channel if the retailer chooses not to introduce the SB, but the manufacturer can launch the online direct channel otherwise. Therefore, the SB introduction strategy and the channel strategy are correlated.

We also find that if the SB is not introduced, the introduction of the online direct channel by the NB manufacturer may benefit both the manufacturer and the retailer. Furthermore, when the perceived value of the NB product sold through the online direct channel is higher than that of the SB product sold through the retail channel, the price of the SB product is only influenced by the perceived value of the SB product and is unaffected by the acceptance degree of the online direct channel.

The remainder of this paper is organised as follows. Section 2 reviews the relevant literature. Section 3 specifies the model and derives the inverse demand functions for different scenarios. Then, we analyse the firms’ strategy combinations under different scenarios in Section 4 and characterise the equilibrium outcomes in Section 5. Section 6 extends this analysis to include price competition. Finally, we conclude the paper in Section 7. All the equilibrium solutions are included in Appendix A, and all the proofs of propositions are included in Appendix B.

2. Literature review

Our paper relates closely to two streams of literature: the literature on direct channel introduction and the literature on SB introduction. These two literature streams often concern an upstream or a downstream firm’s motivation to introduce a direct channel or an SB and how supply chain members’ optimal decisions and profits are affected by such an action. However, these studies generally investigate the direct channel introduction and the store brand introduction in isolation and therefore are quite different from our paper.

In the first literature stream, there are a number of studies that focus on the impacts of introducing a direct channel on the supply chain members. Generally, adding a new channel is likely to cause channel conflicts between supply chain members (Cai, 2010; Hsiao and Chen, 2013), and the effect of introducing a direct channel might be ambiguous when considering different situations (McGuire and Staelin, 1983). The majority of the extant literature finds that the introduction of direct channels by manufacturers does not always hurt supply chain members’ profits. Chiang et al. (2003) suggested that the introduction of an online direct channel can benefit both the manufacturer and the retailer by mitigating double marginalisation even when the manufacturer is less efficient than the retailer. Cattani et al. (2006) analysed the effect of introducing an online direct channel by a manufacturer on existing channel relationships under different pricing strategies. They found that the retailer might not necessarily view the manufacturer’s introduction of an online direct channel as a great threat.

Hsiao and Chen (2014) investigated the problems of when and why a manufacturer or a retailer should introduce an online channel. Arya et al. (2007) found that the manufacturer, retailer and consumers can benefit from the introduction of a direct channel by the manufacturer. In contrast to Arya et al. (2007), Yoon (2016) showed that when the investment spillover effect is considered, the retailer could still benefit from the introduction of a direct channel by the manufacturer even if the manufacturer is as efficient as the retailer in selling products. In addition, Pu et al. (2017) studied the problem of cross-channel free-riding when an online direct channel is introduced by the manufacturer. Qing et al. (2017) analysed the impact of direct channels by applying Nash bargaining solution in the context of partial forward integration. Wang et al. (2018) investigated manufacturers’ e-channel decisions; they mainly compared the introduction of an online direct channel and the introduction of a consignment e-channel for a manufacturer.

Though many extant papers discuss how introducing a direct channel affects supply chain members from various aspects, these papers ignore the fact that the introduction of an SB by the retailer can be a countermeasure to cope with the challenge posed by direct channels. If we consider that the retailer has the option to introduce an SB, some more complicated problems emerge. For example, if the manufacturer anticipates that the retailer will introduce an SB, would they still decide to introduce a direct channel? If so, considering the possibility of an SB introduction, what impact does the introduction of a direct channel have on the participants’ profits? In this regard, our paper contributes to the channel management literature by revealing how a retailer’s SB decision affects the manufacturer’s direct channel choice and the profits of chain members.
The second stream of literature is related to SB introduction. The introduction of an SB alters the interaction between the manufacturer and retailer and brings new challenges to their channel relationships (Nasser et al., 2013; Cui et al., 2016; Hara and Matsubayashi, 2017). In general, the majority of studies show that introducing SBS may benefit retailers and hurt the manufacturers under the Manufacturer Stackelberg game (Raju et al., 1995; Choi and Coughlan, 2006). For example, Raju et al. (1995) developed a game-theoretic model to investigate how the introduction of an SB influences the retailer’s profit. They found that introducing the SB may be profitable for the retailer in certain product categories. However, there is some research reporting that SB introduction can benefit the NB manufacturer. Ru et al. (2015) found that an SB may benefit the manufacturer under the Retailer Stackelberg game. Hence, introducing an SB may lead to a win-win situation for both the NB manufacturer and the retailer. In addition, some other papers take the SB introduction as a product-line problem for the retailer and focus on the problem of how to position the SBS (Morton and Zettelmeyer, 2004; Kuo and Yang, 2013; Chung and Lee, 2017). Sayman et al. (2002) addressed the SB positioning problem and identified the conditions under which the SB should be positioned closer to the stronger NB. Groznik and Heese (2010) examined the problem of SB introduction in a supply chain under retailer competition. They found that retailers are more likely to adopt randomised SB introduction strategies under retailer competition. Though many extant papers discuss SB introduction strategies, none of them studies a retailer’s SB introduction strategy by taking into account the introduction of the manufacturer’s direct channel.

In sum, there has been significant research on manufacturers’ channel strategies and retailers’ SB introduction strategies separately, but few studies have focussed on the interaction of manufacturers’ channel strategies and retailers’ SB introduction strategies except for Jin et al. (2017) and Amrouche and Yan (2012). Jin et al. (2017) mainly investigated manufacturer’s channel strategies to cope with the entry of an SB. They considered whether manufacturers should adopt a single-channel strategy or a dual-channel strategy (i.e., choosing one or two retailers to distribute their products). But they did not take into account the introduction of a direct channel by the manufacturer. The study closest to our work was conducted by Amrouche and Yan (2012). Amrouche and Yan (2012) considered the introduction of an online direct channel as the manufacturer’s countermeasure to compete against the retailer’s SB introduction strategy. They only compared three contexts and analysed the comparative results from the standpoint of the manufacturer’s implementation of an online store. The retailer’s decisions on whether to introduce an SB to compete against the manufacturer was not considered in their paper. Thus, the dynamic between an online direct channel introduction strategy and an SB introduction strategy is ignored in their paper. Differing from Amrouche and Yan (2012), we focussed on the interaction between the introduction of an SB and the introduction of an online direct channel.

3. The models

3.1. Model description and assumptions

Our model encompasses a supply chain that consists of a NB manufacturer and a retailer. Besides selling through the retailer, the manufacturer has the option to launch an online direct channel and sell the NB product on its own. Meanwhile, the retailer can choose to introduce an SB. The subscripts i(=1, 2) denote the NB product and the SB product, respectively, and the subscripts j(=1, 2) denote the traditional retail channel and the online direct channel, respectively. Let w denote the manufacturer’s wholesale price for the NB product sold to the retailer.

Depending on whether the online direct channel and the SB are introduced, we considered four different scenarios, which are shown as in Fig. 1. First, the manufacturer sells the NB product through the retail channel exclusively, and the retailer does not introduce the store brand. This scenario is denoted by the abbreviation nn. Second, the online direct channel is introduced, and the SB is not introduced (denoted by dn). Third, the online direct channel is not introduced, and the SB is introduced (denoted by ns). Fourth, both the online direct channel and the SB are introduced (denoted by ds). Furthermore, we use K = nn, dn, ns, ds to denote these four scenarios, respectively.

Next, we provide the assumptions in our model as follows:

1) We assume that the NB product is preferred by consumers when both the NB product and the SB product are sold through the same channel. This kind of assumption regarding quality beliefs has been widely adopted in previous analytical models (Mills 1995, 1999; Chen et al. 2011; Choi and Coughlan, 2006; Fang et al., 2013; Ru et al., 2015). Normalising the perceived value of the NB product sold through retail channel to 1, we use \( \delta \) to represent the perceived value of the NB product when it is sold through the online direct channel owned by the NB manufacturer. Parameter \( \delta \) is also defined as the acceptance degree of the online direct channel in much of the extant research (Kacen et al., 2002; Chiang et al., 2003; Yan et al., 2016). Following Kacen et al. (2002)’s empirical study on the acceptance degree of the online direct channel regarding various product categories, we assume that \( 0 < \delta < 1 \). We use \( \theta \) to represent the quality or the perceived value of the SB product sold by the retailer. We assume that \( 0 < \theta < 1 \) because NB products are usually considered to have a higher quality than SB products for customers (Chen et al., 2011; Li et al., 2018b). Therefore, the NB product sold through the retail channel has the highest value perceived by consumers, and the perceived value of the NB product sold through the online direct channel may be higher or lower than that of the SB product.

2) We assume that consumers are heterogeneous in their willingness to pay for the product and buy at most one unit of product. Let \( v_{ij} \) and \( p_{ij} \) denote the perceived value and the retail price of product \( i (=1, 2) \) sold through channel \( j (=1, 2) \), respectively. We let \( \gamma \) represent the consumer’s willingness to pay for the perceived value of product \( i \) sold through channel \( j \), which follows a uniform distribution over \([0, 1]\). Therefore, a consumer obtains a utility \( U_{ij} = v_{ij} - p_{ij} \) when buying one unit of product \( i \) through channel \( j \), where \( v_{11} = 1, v_{12} = \delta \) and \( v_{21} = \theta \), respectively.
3) Finally, we follow the literature (e.g., Raju et al., 1995; Narasimhan and Wilcox, 1998; Sayman et al., 2002; Choi and Coughlan, 2006; Choi and Fredj, 2013; Liang et al., 2013) to normalise the production and selling costs of the NB products and the SB products to zero.

3.2. Demand functions

Here, we discuss the retailer’s and the NB manufacturer’s demand functions. From Section 3.1, a customer’s utility is \( r - p_{11} \) when purchasing a unit of NB product through the retail channel, \( \delta r - p_{12} \) when purchasing a unit of NB product from the retailer and \( \delta r - p_{21} \) when purchasing a unit of SB product from the retailer.

In the former three scenarios, the demand functions are straightforward to obtain based on the consumer’s utilities. In the last scenario, in which both the online direct channel and the SB are introduced, we first need to determine the rank order in terms of perceived value. To achieve this, we consider both the cases where \( \delta < \theta < 1 \) and \( \theta < \delta < 1 \), in which the perceived value of the NB product sold through the online direct channel is higher or lower than that of the SB product sold through the retailer, respectively. Fig. 2 illustrates the utility functions in the case where \( \theta < \delta < 1 \) and Fig. 3 illustrates the utility functions in the case where \( \delta < \theta < 1 \).

In the case where \( \theta < \delta < 1 \), the market has four segments: (1) consumers with the highest willingness to pay buy the NB product through the traditional retail channel, (2) consumers with the second highest willingness to pay buy the SB product through the traditional retail channel, (3) consumers with the third highest willingness to pay purchase the NB product through the online direct

![Fig. 1. Four possible scenarios.](image1)

![Fig. 2. Consumer utility in the case where \( \theta < \delta < 1 \).](image2)
channel and (4) consumers with the lowest willingness to pay choose to buy nothing. The consumer who is indifferent between buying NB product through retail channel and buying SB product through retail channel is located at \( p_{21} - p_{11} \), the consumer who is indifferent between buying NB product through retail channel and nothing to buy is located at \( p_{21} - p_{12} - \frac{\delta}{1 - \delta} \). Thus, the demand functions are

\[
q_{11} = 1 - \frac{p_{11} - p_{12}}{1 - \delta}, \quad q_{12} = \frac{p_{11} - p_{21} - p_{22} - \delta}{\delta - 1}, \quad \text{and} \quad q_{21} = \frac{p_{21} - p_{22} - p_{22}}{\delta - 1}.
\]

In the case where \( \delta < \theta < 1 \), the market also has four segments: (1) consumers with the highest willingness to pay buy the NB product through the traditional retail channel, (2) consumers with the second highest willingness to pay buy the NB product through the online direct channel, (3) consumers with the third highest willingness to pay buy the SB product through traditional retail channel and (4) consumers with the lowest willingness to pay choose to buy nothing. The consumer who is indifferent between buying NB product through retail channel and online direct channel is located at \( p_{11} - p_{p12} \), the consumer who is indifferent between buying NB product through online direct channel and nothing to buy is located at \( p_{21} - p_{22} - \frac{\delta}{1 - \delta} \). Thus, the demand functions are

\[
q_{11} = 1 - \frac{p_{11} - p_{12}}{1 - \delta}, \quad q_{12} = \frac{p_{11} - p_{21} - p_{22} - \delta}{\delta - 1}, \quad \text{and} \quad q_{21} = \frac{p_{21} - p_{22} - p_{22}}{\delta - 1}.
\]

### 4. Model analysis

In this section, we model the game and analyse the equilibrium outcomes under the four different scenarios (as described in Section 3.1) when the retailer and the manufacturer conduct Cournot competition, where firms choose quantities rather than prices (Arya et al., 2007; Xiong et al., 2012; Yoon, 2016; Li et al., 2016; Li et al., 2018a).\(^1\)

We use the superscript “ \( K \) ” (recall that \( K = nn, dn, ns, ds \)) to denote the equilibrium outcome under different scenarios. Please see the game solving process and all the equilibrium solutions of the models in Appendix A.

#### 4.1. Online direct channel not introduced and SB not introduced (\( nn \))

Here, we discuss the scenario when both the direct channel and the store brand are not introduced (\( K = nn \)). According to Section 3.2, the demand function is \( q_{11} = 1 - p_{11} \). Hence, the reverse demand function is \( p_{11} = 1 - q_{11} \). Under this scenario, the manufacturer and the retailer interact in two stages. In stage 1, the manufacturer determines the wholesale price of the NB product sold through the retail channel \( w \). In stage 2, the retailer decides the sale quantity of the NB product \( q_{11} \). Table 1 summarises the equilibrium outcomes.

#### 4.2. Online direct channel introduced and SB not introduced (\( dn \))

When \( K = dn \), the retailer’s and the manufacturer’s demand functions are \( q_{11} = 1 - \frac{p_{11} - p_{12}}{1 - \delta} \) and \( q_{12} = \frac{p_{11} - p_{21} - p_{22} - \delta}{\delta - 1} \), respectively (refer to Section 3.2). It follows that the reverse demand functions are \( p_{11} = 1 - q_{11} - \theta q_{12} \) and \( p_{12} = \theta (1 - q_{11} - q_{12}) \).

The manufacturer and the retailer interact in three stages. In stage 1, the manufacturer determines the NB product’s optimal wholesale price \( w \). In stage 2, the retailer chooses the NB product’s sale quantity \( q_{11} \). In stage 3, the manufacturer decides the sale quantity \( q_{12} \) of the NB product sold through the online direct channel. Here, as in Arya et al. (2007) and Xiong et al. (2012), we assume that the manufacturer makes the sale quantity decision in the last stage. This may reflect the practice that manufacturer may supply the products to the retailer first and then consider the quantity supplied for the online direct channel since retailer takes the advantage in the sale process. That is to say, the online direct channel is treated as a complementary marketing channel. Table 2 summarises the equilibrium outcomes.

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\(^1\) Our main results are robust under Bertrand competition under which firms compete in price. The related discussions are available in Section 6 and Appendix C.
Table 1
Equilibrium outcomes in the scenario where $K = \text{nn}$.

<table>
<thead>
<tr>
<th>$q_{11}^{\text{nn}}$</th>
<th>1/4</th>
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<tbody>
<tr>
<td>$p_{11}^{\text{nn}}$</td>
<td>1/7</td>
</tr>
<tr>
<td>$w_{11}^{\text{nn}}$</td>
<td>1/7</td>
</tr>
<tr>
<td>$\pi_{m}^{\text{nn}}$</td>
<td>1/5</td>
</tr>
<tr>
<td>$\pi_{r}^{\text{nn}}$</td>
<td>1/16</td>
</tr>
</tbody>
</table>

Table 2
Equilibrium outcomes in the scenario where $K = \text{dn}$.

<table>
<thead>
<tr>
<th>$q_{11}^{\text{dn}}$</th>
<th>$2-\theta_1$ $4-3\theta_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{12}^{\text{dn}}$</td>
<td>2 - 6 $3-2\theta_1$</td>
</tr>
<tr>
<td>$p_{11}^{\text{dn}}$</td>
<td>$2(0-\theta)^2$ $6-6\theta$</td>
</tr>
<tr>
<td>$p_{12}^{\text{dn}}$</td>
<td>$(2-\theta)^2$ $4-4\theta$</td>
</tr>
<tr>
<td>$w_{1\theta}^{\text{dn}}$</td>
<td>$\theta(2-\theta)$ $4-4\theta$</td>
</tr>
<tr>
<td>$\pi_{m}^{\text{dn}}$</td>
<td>$(2-\theta)(1-\theta)^2$ $(4-3\theta)^2$</td>
</tr>
<tr>
<td>$\pi_{r}^{\text{dn}}$</td>
<td>$2(2-\theta)(1-\theta)^2$ $(4-3\theta)^2$</td>
</tr>
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</table>

When the manufacturer introduces the online direct channel, the retail price in the online direct channel always equals the wholesale price set in the traditional retail channel. Though the consumers prefer the traditional retail channel to the online direct channel, the manufacturer can set a high wholesale price in the retail channel to compete against the retailer in the end market as long as the wholesale price is not higher than the retail price in the online direct channel. In this situation, the manufacturer has the cost advantage in the online direct channel to seize some market share from the retailer.

**Proposition 1.** Suppose that $K = \text{dn}$. As the acceptance degree of the online direct channel $\theta$ increases, the optimal sale quantities in both channels decrease, the optimal wholesale price of the NB product and the optimal retail price in the online direct channel increase, and the retailer’s optimal selling price decreases first and then increases.

Proposition 1 shows that an increased acceptance degree of the online direct channel $\theta$ enhances the manufacturer’s wholesale price and direct retail price and thereby lowers the sale quantities in both channels. This result stems from the manufacturer’s incentive to offset the retailer’s competitive advantage in the retail channel. Furthermore, due to the increased wholesale price, the retailer’s selling price also increases in the acceptance degree of the online direct channel $\theta$.

**Proposition 2.** Suppose that $K = \text{dn}$. Compared with the scenario where $K = \text{nn}$, both the optimal wholesale price of the NB product and the retailer’s optimal selling price of the NB product decrease, and the retail margin decreases if $\theta > 0.6$ and otherwise increases. The retailer’s optimal sale quantity lowers if $\theta > 0.8$ and is otherwise increased.

Proposition 2 indicates that a high acceptance degree of the online direct channel $\theta$ lowers the retail margin (i.e., the retailer’s selling price of the NB product $p_1$ minus the wholesale price of the NB product $w$) and sale quantity in the retail channel because it intensifies the market competition between the manufacturer and the retailer. From Proposition 2, we can also infer that the introduction of an online direct channel will hurt the retailer’s profit if $\theta$ is higher than 0.8.

**Proposition 3.** The manufacturer benefits from the introduction of the online direct channel if $10\theta^3 + 480\theta^3-41\theta^3-16 > 0$ and is otherwise hurt from the introduction of an online channel. The retailer benefits from the introduction of the online direct channel if $119\theta^3 + 48-136\theta^3-328\theta^3 > 0$ and is otherwise hurt from the introduction of an online channel.

Due to the complexity of the optimal solutions, we show the results of a numerical analysis based on Proposition 3 here. When the acceptance of the online direct channel is sufficiently high (e.g., $\theta \in [0.7, 0.9]$), the manufacturer’s profit is enhanced and the retailer’s profit is reduced by its introduction. However, if the acceptance of the online direct channel is sufficiently low (e.g., $\theta \in [0.3, 0.6]$), to prevent the wholesale price from exceeding the retail price in the online direct channel, the NB manufacturer has to lower the wholesale price or else the retailer will source the NB product from the online direct channel. As a result, the manufacturer will eventually be hurt from the introduction of an online direct channel. In this case, the retailer could be better off by its introduction. In addition, with medium acceptance of the online direct channel (e.g., $\theta \in [0.6, 0.7]$), both the NB manufacturer and the retailer can be better off by its introduction.
4.3. Online direct channel not introduced and SB introduced (ns)

When $K = ns$, the retailer’s and the manufacturer’s demand functions are $q_{11} = 1 - \frac{p_{11} - p_{s1}}{1 - \delta}$ and $q_{21} = \frac{2p_{11} - p_{s1}}{(1 - \delta)\delta}$ (refer to Section 3.1). It follows that the reverse demand functions are $p_{11} = 1 - q_{11} - \delta q_{21}$ and $p_{21} = \delta (1 - q_{11} - q_{21})$. The firms interact in two stages. In stage 1, the manufacturer determines the wholesale price of NB product $w$. In stage 2, the retailer decides the sale quantities of the NB product $q_{11}$ and the SB product $q_{21}$. Table 3 summarises the equilibrium outcomes.

Proposition 4. Suppose that $K = ns$. Compared with the scenario where $K = nn$, both the optimal wholesale price and the retailer’s optimal selling price of the NB product decrease, the retail margin increases and the retailer’s optimal sale quantity of the NB product remains unchanged.

Under Proposition 4, by introducing the SB product, the retailer can realise a lower wholesale price of the NB product and a higher margin from selling the NB product. Meanwhile, the sale quantity of the NB product is unaffected by the introduction of the SB product. Hence, the retailer can benefit from the NB product if an SB product is introduced. We can conclude that the retailer not only earns extra profit from the introduction of an SB product but also gets more profit from the NB product by forcing the manufacturer to reduce the wholesale price. Due to the competition from the SB product, the wholesale price of the NB product lowers, but the sale quantity of the NB product remains unchanged, which decreases the manufacturer’s profit.

Proposition 5. The manufacturer’s profit is lower and the retailer’s profit is higher when $K = ns$ than when $K = nn$.

Unlike the introduction of the online direct channel, the introduction of the SB product does not benefit the supply chain partner (i.e., the manufacturer) because it causes market competition between different kinds of products. The retailer, on the other hand, can attain a higher profit because they benefit from both the SB product and the NB product.

4.4. Online direct channel introduced and SB introduced (ds)

Once the online direct channel is introduced by the manufacturer and the SB product is introduced by the retailer, there are two different cases: the case where $\delta < \delta < 1$ and the case where $\delta < \delta < 1$. In the first case, the perceived value of the NB product sold through the online direct channel is lower than that of the SB product sold through the traditional retail channel, while in the second case, the perceived value of the NB product sold through the online direct channel is higher than that of the SB product sold through the traditional retail channel.

The manufacturer and the retailer interact in three stages. In stage 1, the manufacturer chooses the wholesale price of the NB product $w$. In stage 2, the retailer decides the sale quantity of the NB product $q_{11}$ and the sale quantity of the SB product $q_{21}$. In stage 3, the manufacturer decides the sale quantity of the NB product $q_{11}$. Here, following the same logic as in Section 3.2, we assume that the manufacturer decides on the quantity of NB product in the last stage. The online direct channel is treated as a complementary marketing channel.

4.4.1. The case where $\delta < \delta < 1$

According to Section 3.2, the reverse demand functions are given by $p_{11} = 1 - \delta q_{21} - \delta q_{21} - q_{21}$, $p_{12} = \delta (1 - q_{11} - q_{21})$ and $p_{21} = \delta (1 - q_{11} - q_{21})$. If $\delta < \delta < 1$, we next characterise the equilibrium results under different conditions.

Suppose that $\delta > \delta \geq 2(1 - \delta)$. Then, we have an interior solution of the optimal wholesale price $w_{11}^d = \frac{1 - \delta}{2}$. By substitution, other equilibrium results are as follows: $q_{11}^d = \frac{1}{7}$, $q_{21}^d = \frac{1}{7}$, $q_{12}^d = \frac{1}{7}$, $P_{11}^d = \frac{3 - \delta}{4}$, $P_{21}^d = \frac{3 - \delta}{4}$, and $P_{21}^d = \frac{\delta}{4}$ and $P_{11}^d = \frac{\delta}{4}$. It follows that the optimal profits are $\pi_{m}^d = \frac{1 - \delta}{8} + \frac{\delta}{16}$ and $\pi_{r}^d = \frac{1}{16} + \frac{\delta}{16} - \frac{\delta}{8}$ (see Appendix A).

If $\delta < \min(2(1 - \delta), \delta)$, then we have a corner solution of the optimal wholesale price $w_{11}^d = \frac{\delta}{2}$. By substitution, we have other equilibrium results as follows: $p_{11}^d = \frac{1 - \delta}{2}$, $p_{21}^d = \frac{\delta}{2}$, $q_{11}^d = \frac{1}{2}$, $q_{21}^d = \frac{1}{2}$, $q_{12}^d = \frac{1}{2}$, $q_{22}^d = \frac{1}{2}$, $\pi_{m}^d = \frac{\delta}{16} - \frac{\delta^2}{32(1 - \delta)}$ and $\pi_{r}^d = \frac{1 - \delta}{4} + \frac{\delta^2}{56(1 - \delta)}$ (see Appendix A).
We find that in the case where \( \theta < \delta < 1 \), if the acceptance degree of the online direct channel is relatively high, the manufacturer will lower the wholesale price to induce the retailer to sell more NB product as the quality of the SB product increases. However, if the acceptance degree of the online direct channel is very low, the manufacturer has to set a sufficiently low wholesale price that is less than the retail price in the online direct channel (otherwise, the retailer would source the NB product from the online direct channel). Thus, the wholesale price is affected by the acceptance degree of the online direct channel.

When \( \theta < \delta < 1 \), regardless of whether the optimal solution is an interior solution or a corner solution, the optimal price of the SB product sold through the retail channel is always \( p_{ds}^{nt} = \frac{\delta}{2} \), and the optimal price of the NB product sold through the online direct channel is always \( p_{ds}^{nt} = \frac{\delta}{2} \). That means the price of the SB product is not only influenced by the quality of the SB product but also by the acceptance degree of the online direct channel. The price of the NB product in the online direct channel is only influenced by the acceptance degree of the online direct channel. Interestingly, the quality of the SB product does not impact the optimal price of the NB product in the online direct channel \( p_{ds}^{nt} \), although market competition exists between the SB product and the NB product. This result relates to the fact that when \( \theta < \delta < 1 \), the reverse demand function \( p_{ds} = \theta(1-q_{nt}^{2}-q_{nt}^{0}) \) does not include the term on the quality of the SB product \( \delta \). Notice that when \( \delta < \theta < 1 \), the optimal price \( p_{ds}^{nt} \) will be affected by the quality of the SB product.

**Proposition 6.** Suppose that \( K = ds \) and \( \delta > \theta \geq 2(1-\delta) \). Compared with the scenario when \( K = ns \), the optimal wholesale price and the retailer’s optimal sale quantities of the NB product and the SB product remain unchanged, and the retailer’s optimal selling prices of the NB product and the SB product decrease.

Compared with the scenario in which an online direct channel is not introduced and an SB is introduced, the introduction of an online direct channel will not influence the manufacturer’s profit in the traditional retail channel but can bring additional profit for the manufacturer by attracting consumers through the online direct channel. On the other hand, with an unchanged wholesale price and sale quantity, the decrease in the retail price causes the retailer’s profit to be reduced. Hence, the profit of manufacturer is enhanced, and the profit of the retailer is reduced.

**Proposition 7.** Suppose that \( \delta > \theta \geq 2(1-\delta) \). Compared with the scenarios where \( K = ns \) and \( K = nn \), the retailer’s optimal sale quantity of the NB product remains unchanged when \( K = ds \). Among the three scenarios where \( K = ds \), \( K = ns \) and \( K = nn \), the manufacturer’s profit is the highest when \( K = nn \) and is the lowest when \( K = ns \); the retailer’s profit is the highest when \( K = ns \) and is the lowest when \( K = nn \). The scenario where \( K = ds \) is the second-best scenario for both the manufacturer and retailer.

Proposition 7 implies that if \( \delta > \theta \geq 2(1-\delta) \), introducing the online direct channel does not impact the retailer’s optimal sale quantity of the NB product when store brand is introduced. Meanwhile, the manufacturer earns the highest profit when both the online direct channel and the SB are not introduced and the retailer earns the highest profit when the online direct channel is not introduced and the SB is introduced. Therefore, if \( \delta > \theta \geq 2(1-\delta) \), the co-introduction of the online direct channel and the SB is not optimal for both the manufacturer and the retailer.

**Proposition 8.** Suppose that \( K = ds \) and \( \theta < \min[2(1-\delta), \delta] \). Compared with the scenario where \( K = ns \), the optimal wholesale price decreases, while the retailer’s optimal sale quantity of the NB product increases; the optimal sale quantity of the SB product decreases; and the manufacturer’s profit increases, while the retailer’s profit change situation is vague.

Proposition 8 indicates that when the SB is introduced and \( \theta \) is sufficiently small, the introduction of the online direct channel increases the retailer’s sale quantity of the NB product, but lowers the sale quantity of the SB product. Furthermore, given that the SB is introduced, the manufacturer may still have the motivation to introduce the online direct channel even when \( \theta \) is sufficiently small.

### 4.4.2. The case where \( \theta < \delta < 1 \)

When \( \delta < \theta < 1 \), the reverse demand functions are given by \( p_{11} = 1-\delta q_{11}-\theta q_{12}-q_{11} \), \( p_{12} = \theta(1-q_{nt}^{2}-q_{nt}^{0})-\delta q_{12} \) and \( p_{21} = \delta(1-q_{nt}^{2}-q_{nt}^{0}) \) (refer to Section 3.2). In this case, regardless of whether the optimal solution is an interior solution or a corner solution, the optimal price of the SB product is always \( p_{ds}^{nt} = \frac{\delta}{2} \). Unlike the price of the SB product when \( \delta < \theta < 1 \), it is counter-intuitive that the price of the SB product is unaffected by the acceptance degree of the online direct channel.

It is also interesting to find that the price of the SB product may equal half of what it was before the introduction of online direct channel. That means that the introduction of the online direct channel forces the retailer to reduce the retail price of SB products by half under the condition where \( \theta < \delta < 1 \). The equilibrium results in this case are very complicated, and we resorted to numerical analysis for managerial insights in the following section.

5. Equilibrium decisions of channel strategy and brand strategy

This section investigates the manufacturer’s and the retailer’s equilibrium decisions regarding channel strategy and brand strategy based on the equilibrium outcomes under different scenarios studied in Section 4. In order to facilitate our analysis, we assume that the acceptance degree of the online direct channel was \( \theta \in [0.3, 0.9] \), which follows Kacen et al. (2002)’s empirical result that \( \theta \) is less than 1 for a number of product categories.

5.1. The case where \( \theta < \delta < 1 \)

In the case where \( \theta < \delta < 1 \), we discuss the manufacturer and the retailer’s equilibrium decisions regarding channel strategy and
brand strategy both when $\delta \geq 2(1-\delta)$ and when $\delta < 2(1-\delta)$.

$\theta \geq 2(1-\delta)$

A game matrix is used to compare the profits of the manufacturer and the retailer under different scenarios and characterised the equilibrium decisions of the firms. Fig. 4 plots the game matrix when $\delta > 2(1-\delta)$. There is only one equilibrium result: both the online direct channel and the SB product are introduced (i.e., $K = ds$). By comparing the retailer’s profits, we find $\pi_m^{ns} < \pi_m^{ds}$ and $\pi_r^{ds} = \frac{(2-\theta)(6-5\theta)}{4(4-3\theta)^2} < \frac{1}{16} + \frac{\theta^2}{16} = \pi_r^{ds}$; hence, the retailer is always better off by introducing the SB product. Meanwhile, we have $\pi_m^{ns} < \pi_m^{ds}$. At equilibrium, the manufacturer’s profit decreases and the retailer’s profit increases compared with when both the online direct channel and the SB product are not introduced (i.e., $K = nn$).

$\theta < 2(1-\delta)$

Fig. 5 shows the game matrix when $\theta < \min\{2(1-\delta), \delta\}$. By comparison, we have $\pi_m^{ns} = \frac{2(2-\theta)(1-\theta)^2}{(4-3\theta)^2} - \frac{1}{4} + \frac{\theta^2}{64(1-\delta)} = \pi_r^{ds}$. There are two possible equilibrium results depending on the comparison of the manufacturer’s profits, which are $K = ns$ and $K = ds$. First, the online direct channel is not introduced and the SB product is introduced (i.e., $K = ns$), which occurs if the acceptance degree of the online direct channel $\theta$ is low and the perceived value of the SB product $\delta$ is relatively low, and therefore $\pi_m^{ns} = \frac{1}{8} + \frac{\theta^2}{16} - \frac{3\theta^3}{32(1-\delta)} = \pi_m^{ns}$. Second, both the online direct channel and the SB product are introduced ($K = ds$), which occurs if $\theta$ is high and $\delta$ is relatively high, and therefore $\pi_m^{ns} = \frac{1}{8} + \frac{\theta^2}{16} < \frac{5\theta^2}{32(1-\delta)} = \pi_m^{ds}$.

Notice that if $\delta$ is sufficiently high, $K = ds$ arises as the only equilibrium result because with a sufficiently high $\delta$, the manufacturer’s profit decreases significantly when the retailer introduces the SB product (in the extreme case, the manufacturer’s profit would drop to zero if $\delta = 1$). To deal with the intensified competition from the SB product, the manufacturer has to launch an online direct channel.

We also find that in both the equilibrium cases, the manufacturer’s profit decreases and the retailer’s profit increases compared with when both the online direct channel and the SB product are not introduced (i.e., $K = nn$). Therefore, when the perceived value of the NB product sold through the online direct channel is lower than that of the SB product, the retailer is better off, but the
manufacturer is worse off in the interaction between SB introduction and online direct channel introduction.

5.2. The case where $\delta < \theta < 1$

In the case where $\delta < \theta < 1$, we discuss the manufacturer and the retailer’s equilibrium decisions regarding channel strategy and brand strategy both when $\delta \geq \frac{40 - 4\theta^2}{2 - \theta}$ and when $\delta < \frac{40 - 4\theta^2}{2 - \theta}$.

(1) $\delta \geq \frac{40 - 4\theta^2}{2 - \theta}$

Fig. 6 shows the game matrix when $\delta < \theta < 1$ and $\delta \geq \frac{40 - 4\theta^2}{2 - \theta}$ (note that the expressions of $\pi_m^{ds}$ and $\pi_r^{ds}$ in Fig. 6 are very complicated and are provided in Appendix A). By numerical analysis on the complicated profit functions, we have $\pi_m^{nn} < \pi_m^{ns}, \pi_r^{dn} < \pi_r^{ds}$ and $\pi_m^{ns} < \pi_m^{ns}$. Hence, there is only one equilibrium result: both the online direct channel and the SB product are introduced (i.e., $K = ds$). Furthermore, we find that at equilibrium, the manufacturer’s profit decreases, and the retailer’s profit increases as compared with the profits in the scenario where $K = nn$.

$$\delta < \frac{40 - 4\theta^2}{2 - \theta}$$

(3)

Fig. 7 shows the game matrix when $\delta < \theta < 1$ and $\delta < \frac{40 - 4\theta^2}{2 - \theta}$ (as before, the expressions of $\pi_m^{ds}$ and $\pi_r^{ds}$ in Fig. 7 are very complicated and are provided in the Appendix A). By a profit comparison, we have $\pi_r^{nn} < \pi_r^{nn}, \pi_r^{ds} < \pi_r^{ds}$ and $\pi_m^{nn} < \pi_m^{nn}$; hence, the manufacturer introduces the online direct channel and the retailer introduces the SB product at equilibrium (i.e., $K = ds$). At equilibrium, both the manufacturer’s profit and the retailer’s profit decreases in comparison with the scenario where $K = nn$.

Interestingly, the retailer and the manufacturer may get caught up in a prisoner’s dilemma because both their equilibrium profits are lower than their profits when both the SB and the online direct channel are not introduced (i.e., $\pi_r^{ds} < \pi_r^{nn}$ and $\pi_m^{ds} < \pi_m^{nn}$). The reason for this result is twofold. First, the introduction of an SB is dominant for the retailer and adversely affects the manufacturer’s profit. By considering the manufacturer’s possible channel decision in the game, the retailer must introduce a low-quality SB product to compete against the manufacturer’s introduction of an online direct channel with low consumer acceptance. Second, to mitigate this negative effect, the manufacturer will launch the online direct channel, which adversely affects the retailer’s profit in turn.
Consequently, both firms’ profits decrease at equilibrium. This result implies that if collusion between the manufacturer and the retailer is permitted, both of them can be better off by giving up the introduction decisions and entering the scenario where $K = nn$. We find that when the perceived value of the SB product $\delta$ becomes bigger, $\delta \geq \frac{4\theta - \theta^2}{2 - \delta}$ is more likely to hold. When the perceived value of the SB product $\delta$ becomes smaller, $\delta < \frac{4\theta - \theta^2}{2 - \delta}$ is more likely to hold. This can be inferred as follows.

In the case where $\delta < \theta < 1$, the equilibrium result of the game is always that both the online direct channel and the SB product are introduced ($K = ds$). Compared with the scenario where $K = nn$, when the perceived value of the SB product is relatively great, the manufacturer’s profit decreases and the retailer’s profit increases. When the perceived value of the SB product is relatively small, the profits of the manufacturer and retailer will decrease, and they will fall into a prisoner’s dilemma.

In summary, if the perceived value of the NB product sold through the online direct channel is lower than that of the SB product ($\theta < \delta < 1$), then the equilibrium result is $K = ns$ if $\theta$ is relatively low. Otherwise, the equilibrium result is $K = ds$. In either case, the manufacturer’s profit decreases, and the retailer’s profit increases in comparison with the scenario where $K = nn$. If $\delta < \theta < 1$, then $K = ds$ would be the only equilibrium outcome. In this case, it is possible that both the manufacturer’s and the retailer’s equilibrium profits are lower than their profits in the scenario where $K = nn$, which causes a prisoner’s dilemma for them. This implies that collusion between the manufacturer and the retailer may benefit both firms, although such collusion may adversely affect customers.

Notice that our analysis also indicates that the introduction of an online direct channel and the introduction of an SB interact. For example, when the acceptance of the online direct channel $\theta$ is sufficiently low, the manufacturer does not have the incentive to introduce an online direct channel if an SB has not been introduced (as discussed in Section 4.2). However, given that introducing the SB is the retailer’s dominant strategy, the manufacturer may have to choose the online direct channel under the competitive pressure from the retailer even when $\theta$ is sufficiently low.

On the other hand, the retailer’s profit is enhanced in most situations but is hurt only when the perceived value of the NB product sold through the online direct channel is higher than the perceived value of the SB product sold through the traditional retail channel and the perceived value of SB product $\delta$ is very low. In other words, the introduction of the SB product is a proactive behaviour for the retailer in most situations.

6. Price competition

This section considers an extension of the analysis in which the manufacturer and the retailer engage in price competition in the market. When an online direct channel is not introduced, (i.e., $K = nn$ and $K = ns$), there is no difference between the equilibrium outcomes under price competition and under quantity competition. Hence, we only need to derive the equilibrium outcomes in the scenarios where $K = dn$ and $K = ds$, which are provided in Appendix C. The equilibrium decisions regarding channel strategy and brand strategy are also discussed in Appendix C.

We find that when the online direct channel is introduced and the SB is not introduced (i.e., $K = dn$), our main results under quantity competition (refer to Section 4.2) continue to hold under price competition. The only difference lies in the value of the parameters of some specific conditions. For example, Proposition 2 indicates that compared with the situation when $K = nn$, the retail margin decreases if $\theta > 0.6$ and otherwise increases. Here, we find that this result holds if we replace the condition $\theta > 0.6$ with the condition $\theta > \frac{1}{2}$ (see Proposition C1 in Appendix C). Similarly, Proposition 3 continues to hold if we replace the conditions $10\theta^3 + 48\theta^2 - 41\theta^2 - 16 > 0$ and $119\theta^2 + 48 - 136\theta - 32\theta^3 > 0$ with the conditions $-16 + 32\theta - 9\theta^2 > 0$ and $48 - 88\theta + 31\theta^2 > 0$, respectively (see Proposition C2).

In the scenario where $K = ds$, if $\theta < \delta < 1$, the interior solution of the optimal wholesale price remains unchanged, and the corner solution of the optimal wholesale price becomes more complicated compared to that under quantity competition (refer to Appendix C). Propositions C3–C5 in Appendix C parallel Propositions 6–8, respectively, and show that our main insights under quantity competition continue to hold under price competition when $\theta < \delta < 1$.

We find that there are also two kinds of equilibrium results under price competition: the online direct channel is not introduced, and the store brand is introduced; and both the online direct channel and the store brand are introduced. Similarly, we find that if $\theta < \delta < 1$, both equilibrium results may arise. Otherwise, the latter equilibrium result will arise. In particular, in the case where $\delta < \theta < 1$, the manufacturer and the retailer may be trapped in a prisoner’s dilemma when they choose their channel and brand strategies. Furthermore, the manufacturer’s channel strategy may be affected by the retailer’s brand strategy. That is, the manufacturer may introduce the online direct channel if the retailer introduces the store brand but otherwise may not introduce the online direct channel.

7. Conclusion

The introduction of an online direct channel and the introduction of an SB are very common in business practice. However, the existing literature generally investigates the channel strategy and the brand strategy in isolation. By capturing these two issues simultaneously, this paper investigates the interaction between a manufacturer with an option to introduce an online direct channel and a retailer with an option to introduce an SB. A game-theoretic model that incorporates four different scenarios depending on the firms’ channel strategy and brand strategy was built, and the equilibrium outcomes were derived. In our model, the competition between different channels and that between different brands enable us to capture two salient features related to consumer utility analysis: the perceived value differential when the NB product is sold through different channels; and the quality differential between the NB product and the SB product. Thus, our paper is a more realistic analysis of the interplay between manufacturers and retailers.
Our results indicate that when the SB is not introduced, both the manufacturer and the retailer may benefit from the manufacturer's introduction of the online direct channel. Hence, the introduction of the online direct channel may be preferred by both firms. When the online direct channel is not introduced, the introduction of the SB always benefits the retailer but hurts the manufacturer. This occurs because the SB adds competition to the market previously occupied by the NB product. We show that the firms' equilibrium decisions regarding their channel and brand strategies depend on a comparison of the perceived values of the NB product and the SB product. Specifically, if the perceived value of the NB product sold through the online direct channel is lower than that of the SB product sold through the traditional retail channel, two kinds of equilibrium results emerge: the online direct channel is not introduced, and the SB is introduced; and both the online direct channel and the SB are introduced. Otherwise, the only equilibrium result would be that both the online direct channel and the SB are introduced.

Interestingly, when the quality of the SB product is relatively low, both the manufacturer and the retailer must choose to introduce the online direct channel and the SB, respectively, although both of them would be better off by not doing so. This implies a prisoner's dilemma for them in their decisions regarding their channel and brand strategies. Given this case, it is possible that the manufacturer and the retailer will collude with each other by not introducing an online direct channel or an SB.

Furthermore, we find that the manufacturer's choice of channel strategy depends on the retailer's choice of brand strategy. That is, given a sufficiently low acceptance degree of the online direct channel, the manufacturer does not have the incentive to introduce the online direct channel if the SB has not been introduced. However, due to the fact that the retailer always has the incentive to introduce the SB, the manufacturer may have to launch the online direct channel even when the acceptance degree of the online direct channel is sufficiently low. Finally, our main results are robust regardless of whether the manufacturer and the retailer engage in quantity or price competition.

This paper can be extended in a number of ways. First, we consider deterministic market demand in our model. One may incorporate demand uncertainty into the modelling process. Second, it is worth examining advertising strategy or other strategies in the interaction between manufacturer and retailer. Third, in our model, we assume zero production and selling costs of the NB product and the SB product. It would be interesting to investigate the impacts of costs on SB introduction and online direct channel introduction in future research.

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Appendix A. Equilibrium solutions

In this appendix, we solve the dynamic game in the four scenarios described in Section 3.1.

A.1. Online direct channel not introduced and SB not introduced (nn)

Given the sequence of the game, we solve the model by backward induction. In stage 2, the retailer chooses the value of $q_{11}$ to maximise his profit ($\pi_r$).

$$\max_{q_{11}} \pi_r = (p_{11} - w)q_{11}. \tag{A.1}$$

Solving the first-order condition, we get $q_{11} = \frac{1 - w}{2}$. Given the sale quantity chosen by the retailer, the manufacturer decides on the optimal wholesale price to maximise their profit ($\pi_m$).

$$\max_w \pi_m = w q_{11} \tag{A.2}$$

Solving the first-order condition, we get the optimal wholesale price $w^{nn} = \frac{1}{2}$. Then, substitution and calculation yields $q_{11}^{nn} = \frac{1}{4}$, $p_{11}^{nn} = \frac{3}{4}$, $\pi_m^{nn} = \frac{1}{8}$ and $\pi_r^{nn} = \frac{1}{8}$.

A.2. Online direct channel introduced and SB not introduced (dn)

Using backward induction, in stage 3, the manufacturer decides on the sale quantity in the online direct channel.

$$\max_{q_{12}} \pi_m = w q_{11} + p_{11} q_{12} \tag{A.3}$$

Solving the first-order condition, we obtain $q_{12} = \frac{1}{2} q_{11}$. In stage 2, the retailer chooses the optimal quantity of NB product sold through the traditional channel.

$$\max_{q_{11}} \pi_r = (p_{11} - w)q_{11}. \tag{A.4}$$

Solving the first-order condition, we obtain $q_{11} = \frac{1}{2} - \frac{w}{2 - \theta}$. 

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Anticipating the quantity responses from the manufacturer and retailer, in stage 1, the manufacturer chooses the optimal wholesale price to maximise their profit.

$$\max_w \pi_m = wq_{11} + p_{12}q_{12}, \text{ s.t. } p_{12} \geq w$$  \hspace{1cm} (A.5)

Note that by substituting the above expressions of $q_{11}$ and $q_{12}$ into $p_{12} = \partial(1-q_{11} - q_{12})$, the constraint $p_{12} \geq w$ is equivalent to $w \leq \frac{\theta (2-\theta)}{8-\theta^2}$.

By substituting the above expressions of $q_{11}$ and $q_{12}$ into $\pi_m$ and letting $\frac{\partial \pi_m}{\partial w} = 0$, we obtain the optimal prices $w^{ds} = \frac{8 + \theta^2 - 6\theta}{16 - 10\theta}$ and $w^{ns} = \frac{\theta(2-\theta)(6-5\theta)}{4(4-3\theta)^2}$. But the constraint requirement $p_{12} \geq w$ would not be satisfied.

Since $p_{12} \geq w$, we have $w^{ds} = \frac{\theta (2-\theta)}{8-\theta^2}$. Substituting $w^{ds} = \frac{\theta (2-\theta)}{8-\theta^2}$ back, we obtain $q_{11}^{ds} = \frac{2-\theta}{4-\theta}$ and $q_{12}^{ds} = \frac{\theta}{8-\theta}$.

A.3. Online direct channel not introduced and SB introduced (ns)

Using backward induction, in stage 2, the retailer decides on the sale quantity of the NB product and SB product.

$$\max_{q_{11}, q_{21}} \pi_r = (p_{11}-w_{11})q_{11} + p_{21}q_{21}$$  \hspace{1cm} (A.6)

The Hessian matrix of $\pi_r$ is given by $H = (-\partial^2 - \partial^2 - 2\theta^2)$. Because $H_{11} = -2 < 0$ and $|H| = -4\theta^2 - 2\theta^2 > 0$, the Hessian matrix of $\pi_r$ is a negative definite for all values of $q_{11}$ and $q_{21}$. Hence, $\pi_r$ is strictly jointly concave in $q_{11}$ and $q_{21}$. Jointly solving the first-order conditions, we obtain the equilibrium quantities $q_{11}^{ns} = \frac{1}{2} \frac{w}{2(1-\theta)}$ and $q_{21}^{ns} = \frac{w}{2(1-\theta)}$. Anticipating the quantity responses from the retailer, in stage 1, the manufacturer chooses the optimal wholesale price to maximise their profit.

$$\max_w \pi_m = wq_{11}$$  \hspace{1cm} (A.7)

Solving the first-order condition yields the equilibrium wholesale price $w^{ns} = \frac{1-\delta}{2}$. Then, after substitution and calculation, we obtain $q_{11}^{ns} = \frac{1}{4}$, $q_{21}^{ns} = \frac{1}{4}$, $p_{11}^{ns} = \frac{\theta}{2}$, $p_{21}^{ns} = \frac{\delta}{4}$, $\pi_r^{ns} = \frac{1-\delta}{16}$ and $\pi_m^{ns} = \frac{3-\delta}{16}$.

A.4. Online direct channel introduced and SB introduced (ds)

A.4.1. The case where $\theta < \delta < 1$

Using backward induction, in stage 3, the manufacturer decides on the sale quantity of the NB product sold through the online direct channel.

$$\max_{q_{12}} \pi_r = (p_{11}-w_{11})q_{12}$$  \hspace{1cm} (A.8)

By solving the first-order condition, we obtain $q_{12} = \frac{1}{2} \frac{-p_{12}-q_{11}}{w}$.

In the second stage, the retailer makes decisions regarding the sale quantities of the NB product and SB product in the traditional retail channel.

$$\max_{q_{11}, q_{21}} \pi_r = (p_{11}-w_{11})q_{11} + p_{21}q_{21}$$  \hspace{1cm} (A.9)

The Hessian matrix of $\pi_r$ is given by $\frac{\partial^2}{\partial^2 - \partial^2 - 2\theta^2}$. Because $H_{11} = 0 < 0$ and $|H| = 1-\delta(4\theta^2-2\theta^2) > 0$, the Hessian matrix of $\pi_r$ is a negative definite for all values of $q_{11}$ and $q_{21}$. Hence, $\pi_r$ is strictly jointly concave in $q_{11}$ and $q_{21}$. Jointly solving the first-order conditions, we obtain the equilibrium quantities $q_{11}^{ds} = \frac{1}{2} \frac{w}{2(1-\delta)}$ and $q_{21}^{ds} = \frac{w}{2(1-\delta)}$. In the first stage, the manufacturer decides on the wholesale price $w$.

$$\max_w \pi_m = wq_{11} + p_{12}q_{12}, \text{ s.t. } p_{12} \geq w$$  \hspace{1cm} (A.10)

Note that the constraint $p_{12} \geq w$ is equivalent to $w \leq \frac{\theta}{4}$ by substituting $q_{11}$ and $q_{12}$ into $p_{12} = \partial(1-q_{11} - q_{12})$.

Solving the profit maximisation problem of the manufacturer, we obtain the equilibrium results as follows.

1. When $\delta > \theta \geq 2(1-\delta)$, the interior solution of the optimal wholesale price is $w^{ds} = \frac{1}{2} \frac{\theta}{2}$. By substitution, we derive other equilibrium results as follows: $q_{11}^{ds} = \frac{1}{4}$, $q_{12}^{ds} = \frac{1}{4}$, $\pi_r^{ds} = \frac{1}{16}$, $\pi_m^{ds} = \frac{3-\delta}{32(1-\delta)}$ and $\pi_r^{ds} = \frac{1}{4} \frac{\theta^2}{64(1-\delta)}$.

2. When $\delta < \min[2(1-\delta), \theta]$, the corner solution of the optimal wholesale price is $w^{ds} = \frac{\theta}{2}$.

Using backward induction, we derive other equilibrium results as follows: $p_{12}^{ds} = \frac{1}{2} \frac{\theta}{8}$, $p_{21}^{ds} = \frac{\delta}{2} \frac{\theta}{4}$, $q_{11}^{ds} = \frac{1}{2} \frac{\theta}{8(1-\delta)}$, $q_{12}^{ds} = \frac{\delta}{8(1-\delta)}$, $q_{21}^{ds} = \frac{1}{4}$, $\pi_r^{ds} = \frac{\delta}{32(1-\delta)}$ and $\pi_m^{ds} = \frac{3-\delta}{32(1-\delta)}$.

A.4.2. The case where $\delta < \theta < 1$

Using backward induction, in stage 3, the manufacturer decides on the sale quantity of the NB product sold through an online direct channel.
By some algebraic computation, it is straightforward to obtain (i) \( w^{dn} \leq w^{mn} \), \( p_1^{dn} \leq p_1^{mn} \), (ii) \( \theta > 0.6 \), \( p_1^{dn} - w^{dn} < p_1^{mn} - w^{mn} \), (iii) \( \theta < 0.6 \), \( p_1^{dn} - w^{dn} > p_1^{mn} - w^{mn} \); and (iv) \( \theta > 0.8 \), \( q_1^{dn} < q_1^{mn} \), \( \theta < 0.8 \), \( q_1^{dn} > q_1^{mn} \).

**Proof of Proposition 3:** It is straightforward to obtain the results.

**Proof of Proposition 4:** By some algebraic computation, it is straightforward to obtain \( q_2^{dn} = q_2^{mn} \), \( P_1^{dn} < P_1^{mn} \), \( w^{dn} < w^{mn} \) and \( P_1^{mn} - w^{mn} > P_1^{mn} - w^{mn} \).

**Proof of Proposition 5:** It is straightforward to obtain \( p_2^{dn} > p_2^{mn} \).

**Proof of Proposition 6** and **Proposition 7:** By some algebraic computation, it is straightforward to obtain that when \( \delta > \theta \geq (1-\delta) \), \( w^{dn} = w^{mn} \), \( q_2^{dn} = q_2^{mn} \), \( q_2^{dn} = q_2^{mn} \), \( P_2^{dn} < P_2^{mn} \), \( P_2^{dn} = P_2^{mn} \), \( \pi_{m}^{dn} < \pi_{m}^{mn} \), \( \pi_{m}^{dn} < \pi_{m}^{mn} \), \( \pi_{m}^{n} > \pi_{m}^{n} \), \( \pi_{m}^{n} > \pi_{m}^{n} \), and \( \pi_{m}^{n} > \pi_{m}^{n} \).

**Proof of Proposition 8:** Since \( \theta < \min(2(1-\delta), \delta) \), we obtain \( q_2^{dn} - q_2^{dn} = \frac{1}{4} \frac{(2(1-\delta) - \theta)}{2(1-\delta)} > 0 \), \( q_2^{dn} - q_2^{dn} = \frac{1}{4} \frac{(2(1-\delta) - \theta)}{2(1-\delta)} < 0 \), \( w^{dn} - w^{mn} = \frac{1}{4} [\theta - 2(1-\delta)] < 0 \), and \( \pi_{m}^{dn} - \pi_{m}^{mn} = \frac{6(1-\delta) - \theta^2}{32(1-\delta)} > 0 \).

Hence, we can obtain that when \( \theta < \min(2(1-\delta), \delta) \), \( q_2^{dn} > q_2^{mn} \), \( q_2^{dn} > q_2^{mn} \), \( w^{dn} > w^{mn} \), and \( \pi_{m}^{dn} > \pi_{m}^{mn} \).
Appendix C. Price competition

C.1. Online direct channel introduced and SB not introduced (dn)

The demand functions are \( q_1 = 1 - \frac{p_1 - p_2}{1 - \theta} \), \( q_1 = \frac{\theta p_1 - p_2}{1 - \theta} \).

In this scenario, the sequence of the game is as follows: in stage 1, the manufacturer determines the optimal wholesale price of the NB product sold through the traditional retail channel. In stage 2, the retailer decides on the price of the NB product. In stage 3, the manufacturer decides on the price of the NB product sold through the online direct channel. Here, the online direct channel is treated as a complementary marketing channel.

Using backward induction, in stage 3, the manufacturer decides on the retail price in the online direct channel.

\[
\max_{p_{12}} \pi_m = w q_1 + p_{12} q_1 \tag{C.1}
\]

Solving the first-order condition, we obtain \( p_{12} = \frac{1}{2} \theta (p_1 + w) \).

In stage 2, the retailer chooses the optimal quantity of the NB product sold through the traditional channel.

\[
\max_{p_{11}} \pi_r = (p_1 - w) q_1 \tag{C.2}
\]

Solving the first-order condition, we obtain \( p_{11} = 1 - \frac{1 - w}{2 - \theta} \).

Anticipating the quantity responses from manufacturer and retailer, in stage 1, the manufacturer chooses the optimal wholesale price to maximise their profit.

\[
\max_w \pi_m = w q_1 + p_{12} q_1 \text{ s.t. } p_{12} \geq w \tag{C.3}
\]

Note that by substituting the above expressions of \( p_1 \) into \( p_{12} \), the constraint \( p_{12} \geq w \) is equivalent to \( w \leq \frac{\theta}{4 - \theta} \).

Substituting the above expressions of \( p_1 \) and \( p_{12} \) into \( \pi_m \) and letting \( \pi_m^{dn} = 0 \), we obtain the optimal prices \( w_{dn} = \frac{1 - 3 \theta + \theta^2}{8 - 5 \theta + \theta^2} \) and \( p_{12}^{dn} = \frac{\theta (5 - 4 \theta + \theta^2)}{8 - 5 \theta + \theta^2} \). But the constraint requirement \( p_{12} \geq w \) would not be satisfied because it can be proved that \( p_{12}^{dn} - w_{dn} = \frac{(2 - \theta) (1 - \theta)}{8 - 5 \theta + \theta^2} < 0 \).

Since \( p_{12} \geq w_{dn} \), we have the optimal wholesale price \( w_{dn} = \frac{\theta}{4 - \theta} \). Substituting \( w = \frac{\theta}{4 - \theta} \) back, we obtain \( p_{11}^{dn} = \frac{2 - \theta}{4 - \theta} \), \( p_{12}^{dn} = \frac{\theta}{4 - \theta} q_1^{dn} = \frac{2 - \theta}{4 - \theta} q_2^{dn} = \frac{1}{4 - \theta} \), \( \pi_m^{dn} = \frac{(3 - \theta) \theta}{(4 - \theta)^2} \) and \( \pi_r^{dn} = \frac{2 (2 - \theta + \theta^2)}{(4 - \theta)^2} \).

**Proposition C1.** Suppose that \( K = dn \). Compared with the scenario where \( K = nn \), both the optimal wholesale price and the retailer’s optimal selling price decrease, and the retail margin decreases if \( \theta > \frac{1}{2} \) and otherwise increases. In addition, the retailer’s optimal sale quantity lowers if \( \theta > 0.75 \) and otherwise increases.

After the introduction of an online direct channel by the NB manufacturer, both the wholesale price and the retail price decrease. In the meantime, the effect of double marginalisation is weakened. When the value of the acceptance degree of the online direct channel is higher than \( \frac{1}{2} \), the retail margin decreases. Otherwise, the retail margin increases. When the value of the acceptance degree of the online direct channel is higher than 0.75, the sale quantity in the retailer channel decreases. Otherwise, the sale quantity increases. In sum, the high acceptance of an online direct channel by consumers can cause a lower retail margin and sale quantity in the traditional retail channel. In this situation, the introduction of an online direct channel causes fierce competition to the retailer. From Proposition 1, it can be further inferred that the introduction of an online direct channel will hurt the retailer’s profit when the value of the acceptance degree of the online direct channel is higher than 0.75.

**Proposition C2.** The manufacturer benefits from the introduction of the online direct channel if \(-16 + 32 \theta - 9 \theta^2 > 0 \) and is otherwise hurt from the introduction of an online channel. The retailer benefits from the introduction of the online direct channel if \(48 - 88 \theta + 31 \theta^2 > 0 \) and is otherwise hurt from the introduction of an online channel.

Due to the complexity of the expressions in Proposition 2, we cannot clearly obtain managerial insights by analytical results. Hence, we resort to numerical analysis and find that when \( \theta \in [0.7, 0.9] \), the manufacturer’s profit is enhanced and the retailer’s profit is reduced by the introduction of an online direct channel.

C.2. Online direct channel introduced and SB introduced (ds)

Once the online direct channel is introduced by the manufacturer and the SB product is introduced by the retailer, there are two different cases: the case where \( \delta < \theta < 1 \) and the case where \( \theta < \delta < 1 \). In the first case, the perceived value of an NB product sold through an online direct channel is higher than that of an SB product sold through a retailer channel, while in the second case, the perceived value of an NB product sold through an online direct channel is lower than that of an SB product sold through a retailer channel.

In this scenario, the sequence of the game is as follows: in stage 1, the manufacturer determines the optimal wholesale price of the NB product sold through the traditional retail channel. In stage 2, the retailer decides on the retail price of the NB product and the SB product. In stage 3, the manufacturer decides on the retail price of the NB product sold through the online direct channel.

Here, following the same logic in Section 3.2, the online direct channel is treated as a complementary marketing channel.
C.2.1. The case where $\theta < \delta < 1$

In this case, the demand functions are given by $q_{11} = 1 - \frac{p_{11} - p_{21}}{1 - \delta}$, $q_{21} = \frac{p_{11} - p_{21}}{1 - \delta}$, and $q_{12} = \frac{p_{21} - p_{12}}{1 - \delta}$.

Using backward induction, in stage 3, the manufacturer decides on the price of the NB product sold through the online direct channel.

$$\max_{p_{12}} \pi_m = w q_{11} + p_{12} q_{12}$$  \hspace{1cm} (C.4)

By solving the first-order condition, we obtain $p_{12} = \frac{p_{11} \delta}{2 \delta}$.

In the second stage, the retailer decides the price of the NB product and SB product in the traditional retail channel.

$$\max_{p_{11}, p_{21}} \pi_r = (p_{11} - w) q_{11} + p_{21} q_{21}.$$  \hspace{1cm} (C.5)

The Hessian matrix of $\pi_r$ is given by $(\frac{2}{1 + \delta}, \frac{2}{2 \delta - \theta})$. Because $H_{11} = \frac{2}{1 + \delta} < 0$ and $|H| = \frac{45 - 2\theta}{(1 - \delta)(\delta - \theta)} > 0$, the Hessian matrix of $\pi_r$ is a negative-definite matrix for all values of $p_{11}$ and $p_{21}$. Hence, $\pi_r$ is strictly jointly concave in $p_{11}$ and $p_{21}$. Jointly solving the first-order conditions, we obtain the equilibrium prices as follows:

$$p_{11} = \frac{1}{2} + \frac{\delta}{2} \frac{\delta^2}{2 \delta - \theta} + \frac{w}{2}, \quad p_{21} = \frac{\delta (\delta - \theta)}{2 \delta - \theta}$$  \hspace{1cm} (C.6)

By substitution, we can easily obtain the optimal price $p^{ds}_{12} = \frac{(\delta - \theta) \delta}{2 \delta - \theta}$. In the first stage, the manufacturer decides on the wholesale price $w^{ds}$.

$$\max_w \pi_m = w q_{11} + p_{12} q_{12} \text{ s. t. } p_{12} \geq w$$  \hspace{1cm} (C.7)

Solving the profit maximisation problem of the manufacturer, we gain the equilibrium results as follows.

1. When $\theta - \delta^2 > 2\delta - 2\delta^2$ and $\theta < \delta < 1$, we have the interior solution of the optimal wholesale price $w^{ds} = \frac{\theta - \delta^2}{2 \delta - \theta}$. By substitution, we derive other equilibrium results as follows: $p^{ds}_{11} = \frac{1}{2} + \frac{\delta}{2} \frac{\delta^2}{2 \delta - \theta} + \frac{w}{2}$, $q^{ds}_{11} = \frac{1}{2}$, $q^{ds}_{12} = \frac{1}{2}$, $q^{ds}_{21} = \frac{\delta (\delta - \theta)}{2 \delta - \theta}$ and $\pi^{ds}_{m} = \frac{1}{16} \frac{\delta^3}{2(2 \delta - \theta)}$.

**Proposition C3.** Suppose that $K = ds$, $\theta - \delta^2 > 2\delta - 2\delta^2$ and $\theta < \delta$. Compared with the scenario where $K = ns$, the optimal wholesale price and the retailer’s optimal sale quantities of the NB product and the SB product remain unchanged, and the retailer’s optimal selling prices of the NB product and the SB product decrease.

**Proposition C4.** Suppose that $\theta - \delta^2 > 2\delta - 2\delta^2$ and $\theta < \delta$. Compared with the scenarios where $K = ns$ and $K = nn$, the retailer’s optimal sale quantity of the NB product remains unchanged when $K = ds$. Among the three scenarios where $K = ds$, $K = ns$ and $K = nn$, the manufacturer’s profit is the highest when $K = nn$ and is the lowest when $K = ns$. The retailer’s profit is the highest when $K = ns$ and is the lowest when $K = nn$. The scenario where $K = ds$ is the second-best scenario for both the manufacturer and the retailer.

2. When $\theta - \delta^2 < 2\delta - 2\delta^2$ and $\theta < \delta < 1$, we gain the corner solution of the optimal wholesale price $w^{ds} = \frac{\theta - \delta^2}{2 \delta - \theta}$. By substitution, we derive other equilibrium results as follows: $p^{ds}_{11} = \frac{\theta - \delta^2}{2 \delta - \theta} - \frac{\delta (\delta - \theta)}{2 \delta - \theta}$, $p^{ds}_{21} = \frac{\theta - \delta^2}{2 \delta - \theta} - \frac{\delta (\delta - \theta)}{2 \delta - \theta}$, $q^{ds}_{11} = \frac{\theta (\theta - \delta)}{2 \delta - \theta}$, $q^{ds}_{12} = \frac{\theta (\theta - \delta)}{2 \delta - \theta}$, $q^{ds}_{21} = \frac{\theta (\theta - \delta)}{2 \delta - \theta}$ and $\pi^{ds}_{m} = \frac{16}{(1 - \delta)(\delta - \theta)^2} \frac{\theta (\theta - \delta)^2}{(16(1 - \delta)(\delta - \theta)^2)}$.

**Proposition C5.** Suppose that $K = ds$, $\theta - \delta^2 < 2\delta - 2\delta^2$ and $\theta < \delta$. Compared with the scenario where $K = ns$, the optimal wholesale price decreases while the retailer’s optimal sale quantity of the NB product increases; the optimal sale quantity of the SB product decreases.

**Proof.** Because $\theta - \delta^2 < 2\delta - 2\delta^2$, we have $w^{ds} - w^{ns} = \frac{-2 \delta^2 + 2 \delta^2 + \theta - \theta^2}{2(2 \delta - \theta)} < 0$, $q^{ds}_{11} - q^{ns}_{11} = \frac{-2 \delta + 2 \delta^2 + \theta - \theta^2}{4(1 - \delta)(\delta - \theta)^2} < 0$ and $q^{ds}_{21} - q^{ns}_{21} = \frac{2 \delta^2 + \theta - \theta^2}{4(1 - \delta)(\delta - \theta)^2} < 0$. \hfill $\square$

C.2.2. The case where $\delta < \theta < 1$

When $\delta < \theta < 1$, the demand functions are given by $q_{11} = 1 - \frac{p_{11} - p_{12}}{1 - \delta}$, $q_{12} = \frac{p_{11} - p_{12}}{1 - \delta}$, $q_{21} = \frac{p_{21} - p_{12}}{1 - \delta}$ and $q_{22} = \frac{p_{21} - p_{12}}{1 - \delta}$. Using backward induction, in stage 3, the manufacturer decides on the sale quantity of the NB product sold through the online direct channel.

$$\max_{p_{12}} \pi_m = w q_{11} + p_{12} q_{12}$$  \hspace{1cm} (C.8)

By solving the first-order condition, we obtain $p_{12} = \frac{p_{11} \theta}{2} + \frac{w (1 - \theta) (p_{11} - p_{12} + w)}{2 (1 - \delta)}$.

In the second stage, the retailer decides the sale quantities of the NB product and the SB product in the traditional retail channel.

$$\max_{p_{11}, p_{21}} \pi_r = (p_{11} - w) q_{11} + p_{21} q_{21}.$$  \hspace{1cm} (C.9)
The Hessian matrix of \( \pi \) is given by \( \begin{pmatrix} -2 + \delta + \theta & 1 \\ 1 & -2 + \delta + \theta \end{pmatrix} \). Because \( H_{11} = -\frac{2 + \delta + \theta}{(1 - \delta)(1 - \theta)} < 0 \) and \( \det(H) = \frac{2(2 - \delta - \delta)}{(1 - \delta)(1 - \theta)} > 0 \), the Hessian matrix of \( \pi \) is a negative definite for all values of \( p_{11} \) and \( p_{21} \). Hence, \( \pi \) is strictly jointly concave in \( p_{11} \) and \( p_{21} \). Jointly solving the first-order conditions, we obtain the equilibrium quantities as follows:

\[
\begin{align*}
\pi_{11} &= \frac{2(2 - \delta - \delta)(1 - \delta + w_{11})}{2(2 - \delta - \delta)} \\
\pi_{21} &= \frac{\delta(1 - \delta + w_{11})}{2(2 - \delta - \delta)}.
\end{align*}
\]

(C.10)

In the first stage, the manufacturer decides on the wholesale price \( w_{11} \).

\[
\max_{w} \pi_{m} = wq_{11} + p_{11}q_{11}, \quad s.t. \quad p_{11} \leq w > 0
\]

(C.11)

Note that the constraint \( p_{11} \geq w \) is equivalent to \( w \leq \frac{1}{2} \left( 1 - \delta(\delta - \delta) \right) \).

Solving the profit maximisation problem of the manufacturer, we have the equilibrium results as follows.

(1) When \( \begin{pmatrix} 1 + \delta(1 - \delta + \delta) + (2 - \delta - \delta)(2 + 2\delta - \delta^2) + \delta(1 - \delta + \delta)(4 - 2\delta + \delta^2) \end{pmatrix} \geq 0 \), we have the interior solution of the optimal wholesale price

\[
w_{0} = \frac{4(1 + \delta + \delta(1 - \delta + \delta)(4 - 2\delta + \delta^2))}{2(2 - \delta - \delta)(2 + 2\delta - \delta^2)}.
\]

Then, we obtain other equilibrium outcomes by substitution. Because the equilibrium results are very complicated, we did not list them here, and we resorted to numerical analysis for managerial insights in the following section.

(2) When \( \begin{pmatrix} 1 + \delta(1 - \delta + \delta) + (2 - \delta - \delta)(2 + 2\delta - \delta^2) + \delta(1 - \delta + \delta)(4 - 2\delta + \delta^2) \end{pmatrix} < 0 \), we have the corner solution of the optimal wholesale price

\[
w_{0} = \frac{2(2 - \delta - \delta)(1 - \delta + \theta)}{\delta(4 - \delta + \delta^2)}.
\]

Other equilibrium outcomes can be obtained by substitution. We did not list them here because the results are very complicated.

C.3. Equilibrium decisions of channel strategy and brand strategy

C.3.1. The case where \( \theta < \delta < 1 \)

In the case where \( \theta < \delta < 1 \), we discuss the equilibrium decisions of channel strategy and brand strategy both when \( \theta - \delta^2 \geq 2\delta - 2\delta^2 \) and \( \theta - \delta^2 < 2\delta - 2\delta^2 \).

(1) \( \theta - \delta^2 \geq 2\delta - 2\delta^2 \)

By an analytical analysis, we have \( \frac{1}{8} \leq \delta < \frac{9}{8} \). By a numerical analysis of the complicated profit functions, we have \( 0.7^2 < 1.8 < 0.5 \). Hence, when \( \theta - \delta^2 \geq 2\delta - 2\delta^2 \), there is only one equilibrium result, in which both the online direct channel and the SB product are introduced (i.e., \( K = ds \)). Please see Fig. C1 for analysis details.

C.3.2. The case where \( \theta - \delta^2 < 2\delta - 2\delta^2 \)

By a numerical analysis of the complicated profit functions, if the acceptance degree of the online direct channel is low and the quality of the SB product is relatively low, we have \( \pi_{m}^m < \frac{1}{8} \). If the acceptance degree of the online direct channel is high and the quality of the SB product is relatively high, we have \( \pi_{m}^m > \frac{1}{8} \). Hence, there are two possible equilibrium results depending on the comparison of the manufacturer’s profits, which are \( K = rs \) and \( K = ds \). Please see Fig. C2 for analysis details.

![Fig. C1. The game matrix under \( \theta - \delta^2 > 2\delta - 2\delta^2 \).](image-url)

At equilibrium, the manufacturer’s profit decreases and the retailer’s profit increases compared with when both the online direct channel and the SB product are not introduced (i.e., \( K = nn \)).

(2) \( \theta - \delta^2 < 2\delta - 2\delta^2 \)

By a numerical analysis of the complicated profit functions, if the acceptance degree of the online direct channel is low and the quality of the SB product is relatively low, we have \( \pi_{m}^m < \frac{1}{8} \). If the acceptance degree of the online direct channel is high and the quality of the SB product is relatively high, we have \( \pi_{m}^m > \frac{1}{8} \). Hence, there are two possible equilibrium results depending on the comparison of the manufacturer’s profits, which are \( K = rs \) and \( K = ds \). Please see Fig. C2 for analysis details.
We also find that in both the equilibrium cases, the manufacturer’s profit decreases and the retailer’s profit increases compared with when both the online direct channel and the SB product are not introduced (i.e., $K = nn$).

**C.3.2. The case where $\delta < \theta < 1$**

\[
\begin{align*}
(1) \quad & (1 - \theta)(1 - \theta) + 2(1 - \theta) \theta^2 + \theta^2(2 + 2\theta - 2\theta^2) + \theta (8 + 30 \theta + 50 \theta^2) \\
& \quad \geq 0
\end{align*}
\]

We find that the constraint condition is not satisfied on the interval $[0.3, 0.9]$.

\[
\begin{align*}
(2) \quad & (1 - \theta)(1 - \theta) + 2(1 - \theta) \theta^2 + \theta^2(2 + 2\theta - 2\theta^2) + \theta (8 + 30 \theta + 50 \theta^2) \\
& \quad < 0
\end{align*}
\]

By a numerical analysis of the complicated profit functions, we have $\pi_{mm}^{nn} > \pi_{mm}^{ns}$, $\pi_{mm}^{nn} > \pi_{mr}^{ns}$, $\pi_{mr}^{nn} < \pi_{mr}^{ds}$ and $\pi_{mr}^{nn} < \pi_{mr}^{ds}$. Hence, when $\delta < \theta < 1$ and $\frac{(1 - \theta)(1 - \theta) + 2(1 - \theta) \theta^2 + \theta^2(2 + 2\theta - 2\theta^2) + \theta (8 + 30 \theta + 50 \theta^2)}{2(1 - \theta)(1 - \theta) + 2(1 - \theta) \theta^2 + \theta^2(2 + 2\theta - 2\theta^2) + \theta (8 + 30 \theta + 50 \theta^2)} < 0$, there is only one equilibrium result, in which both the online direct channel and the SB product are introduced (i.e., $K = ds$). Please see Fig. C3 for analysis details.

Compared with the scenario $K = nn$, we can find that in the equilibrium case, (i) when the perceived value of SB is relatively low, both the manufacturer’s profit and the retailer’s profit decreases, and (ii) when the perceived value of the SB is relatively high, the manufacturer’s profit decreases and the retailer’s profit increases.

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**Fig. C2.** The game matrix under $\delta - \theta^2 < 2\delta - 2\theta^2$.

<table>
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<tr>
<th>Not introducing SB products</th>
<th>Introducing SB products</th>
</tr>
</thead>
<tbody>
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<td>$\pi_{mm}^{nn} = \frac{1}{8}$</td>
<td>$\pi_{mm}^{nn} = \frac{1}{8}$</td>
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<td>$\pi_{mm}^{ds} = \frac{3\delta}{16}$</td>
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<table>
<thead>
<tr>
<th>Introducing an online direct channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{mr}^{nn} = \frac{(3 - \theta)(\theta - 3\theta)}{(4 - \theta)^2}$</td>
</tr>
<tr>
<td>$\pi_{mr}^{ds} = \frac{2(2 - 3\theta + \theta^2)}{(4 - \theta)^2}$</td>
</tr>
</tbody>
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**Fig. C3.** The game matrix under $\delta < \theta < 1$ and $4\theta^2 - 4\theta + 2\delta - 5\delta < 0$.

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**References**


