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## Justification by an Infinity of Conditional Probabilities

David Atkinson and Jeanne Peijnenburg

**Abstract** Today it is generally assumed that epistemic justification comes in degrees. The consequences, however, have not been adequately appreciated. In this paper we show that the assumption invalidates some venerable attacks on infinitism: once we accept that epistemic justification is gradual, an infinitist stance makes perfect sense. It is only without the assumption that infinitism runs into difficulties.

### 1 Introduction

Foundationalism and coherentism come in various sorts and sizes, but the difference between the two is clear: foundationalists hold that basic beliefs justify nonbasic beliefs while coherentists maintain that beliefs justify one another and that basic beliefs do not exist. In some versions of foundationalism basic beliefs depend on basic nonbeliefs, and in some versions of coherentism some beliefs are more fundamental than others, but this is a fine structure that for our purposes we may neglect.

To gain a better understanding of the difference, let us consider the simplest of all belief systems, namely, an epistemic chain. In such a chain, (a belief in) a target proposition  $E_0$  is justified by (a belief in) proposition  $E_1$ , which in turn is justified by  $E_2$ , and so on:

$$E_0 \Leftarrow E_1 \Leftarrow E_2 \Leftarrow E_3 \Leftarrow \dots, \quad (1)$$

where  $E_n \Leftarrow E_{n+1}$  means ‘ $E_{n+1}$  justifies  $E_n$ ’. Of course, most belief systems are much richer and more intricate than this. Nevertheless, the chain is a good starting point: it can help us to understand more realistic cases, which have been represented as trees, rafts, pyramids, teepees, houses of cards, cobwebs, or crossword puzzles, all of which have single chains as their elements.

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Now foundationalists will argue that (1) only makes sense if it finally comes to a halt in a basic proposition corresponding to  $E_{s+1}$ , where  $s$  is some finite number:

$$E_0 \Leftarrow E_1 \Leftarrow E_2 \Leftarrow E_3 \Leftarrow \dots \Leftarrow E_{s+1}. \quad (2)$$

Coherentists, on the other hand, characteristically claim that the chain will eventually make a loop, thus supporting itself:

$$E_0 \Leftarrow E_1 \Leftarrow E_2 \Leftarrow E_3 \Leftarrow \dots \Leftarrow E_s \{\Leftarrow E_0\}. \quad (3)$$

Both (2) and (3) presuppose that the chain is finite. This is in accordance with the positions of both foundationalists and coherentists, namely, that chains of infinite length do not make sense. Adherents of infinitism, on the other hand, allow  $s$  to go to infinity; indeed they argue that epistemic chains have the form

$$E_0 \Leftarrow E_1 \Leftarrow E_2 \Leftarrow E_3 \Leftarrow \dots \Leftarrow \infty. \quad (4)$$

In this paper we are especially interested in the viability of infinitism, and so we will focus on chains like (4).

Some see justification as a normative relation, claiming that the expression ' $E_{n+1}$  justifies  $E_n$ ' states something about the logical connection between the justifier and the justified. Others conceive it as an empirical connection, holding that  $E_{n+1}$  brings about  $E_n$  in one way or another. Still others believe that justification is best captured by a double aspect theory, arguing that it incorporates both normative and empirical aspects.

Apart from the question of whether justification is normative or empirical, on which we will remain neutral, there is the issue of strength. How close should the connection between  $E_n$  and  $E_{n+1}$  be in order to say legitimately that the one is *justified* by the other? Should we take an austere position and say that  $E_{n+1}$  only justifies  $E_n$  if it logically implies or singly causes  $E_n$ ? Or could we be more lenient, admitting that justification is a gradual concept, allowing a more or less? Today an increasing number of epistemologists opt for the latter alternative. In an attempt to make the concept of epistemic justification more general and more realistic, they prefer to speak about 'justification' even if  $E_{n+1}$  only probabilistically supports  $E_n$  (to a certain degree above some threshold).<sup>2</sup>

In this paper we follow suit and regard the justification relation as a probability relation. This means that, from now on, we interpret (1) as follows:  $E_0$ , the target proposition, is probabilistically supported by  $E_1$ , which in turn is probabilistically supported by  $E_2$ , and so on. To symbolize this adoption of probability we replace the double arrow  $\Leftarrow$  of justification tout court by a single arrow  $\leftarrow$ , signifying probabilistic support. Thus the chain in which we are interested, namely, (4), becomes

$$E_0 \leftarrow E_1 \leftarrow E_2 \leftarrow E_3 \leftarrow \dots \leftarrow \infty. \quad (5)$$

It has often been argued that infinitism must be incorrect because an infinite chain of propositions, the one justified by another, does not make sense. In the present paper we will show that such arguments are untenable if the idea that justification comes in degrees is taken seriously. Once justification is interpreted in terms of probabilistic support, then certain venerable arguments against infinitism become questionable. Among these are the argument that as yet no example of an infinite epistemic chain has been found (Black [1], p. 436), the argument that such chains cannot be completed (Klein [10], p. 920; cf. Lehrer [11], p. 155–56), the argument that the support given by an infinite epistemic chain always culminates in zero (Lewis [13],

p. 172; cf. Lewis [12], pp. 327–28), and the argument that “if all justification is conditional. . . , then nothing can be shown to be actually, nonconditionally justified” (Dancy [5], p. 55). A variant of the latter argument has recently been put forward by Gillet, who holds that an infinite regress of deferred justification cannot yield an unconditional, determinate justification for the target proposition (Gillet [8]).

Against all these arguments we will show that the gradual character of justification allows the existence of infinite epistemic chains that may indeed be completed, yielding a final unconditional probabilistic justification that is not zero. In Sections 3 and 4 we will give detailed examples of such infinite chains, showing that they can be perfectly consistent. But first, in Section 2, we explain more carefully what we mean by “probabilistic support,” for that will allow us to formulate our examples with more precision.

## 2 Probabilistic Support

We will say that  $E_{n+1}$  probabilistically supports  $E_n$  if and only if  $E_n$  is more probable if  $E_{n+1}$  is true than if it is false. In other words, the conditional probability of  $E_n$ , given that  $E_{n+1}$  is true, is greater than the conditional probability of  $E_n$ , given that  $E_{n+1}$  is false:

$$P(E_n|E_{n+1}) > P(E_n|\neg E_{n+1}). \quad (6)$$

The notion of probabilistic support is closely allied to that of (probabilistic) confirmation. There are, however, many measures of confirmation currently on the market (Douven and Meijs [6]). A popular one is the Carnap [3] degree of confirmation that  $E_{n+1}$  gives to  $E_n$ . It is defined by

$$D(E_n|E_{n+1}) = P(E_n|E_{n+1}) - P(E_n).$$

Another measure, used by Christensen [4] and Joyce [9], is

$$S(E_n|E_{n+1}) = P(E_n|E_{n+1}) - P(E_n|\neg E_{n+1}). \quad (7)$$

The condition of probabilistic support (6) is equivalent to  $S(E_n|E_{n+1}) > 0$ . In fact, since one can show that  $D(E_n|E_{n+1}) = S(E_n|E_{n+1})P(\neg E_{n+1})$ , it follows that (6) is also equivalent to  $D(E_n|E_{n+1}) > 0$ , provided that  $P(\neg E_{n+1})$  does not vanish, that is, provided that  $P(E_{n+1}) \neq 1$ . Actually all the so-called Bayesian measures of confirmation are positive if (6) is satisfied (Fitelson [7]; Shogenji [16]).

The rule of total probability is

$$P(E_n) = P(E_n|E_{n+1})P(E_{n+1}) + P(E_n|\neg E_{n+1})P(\neg E_{n+1}), \quad (8)$$

and this may be rewritten in the form

$$P(E_n) = P(E_n|\neg E_{n+1}) + S(E_n|E_{n+1})P(E_{n+1}), \quad (9)$$

where  $S(E_n|E_{n+1})$  is the Christensen–Joyce measure (7).

For convenience we will frequently make use of the abbreviations

$$\begin{aligned} \alpha_n &= P(E_n|E_{n+1}) \\ \beta_n &= P(E_n|\neg E_{n+1}) \\ \gamma_n &= S(E_n|E_{n+1}) = \alpha_n - \beta_n. \end{aligned} \quad (10)$$

This means that equation (9) takes on the form

$$P(E_n) = \beta_n + \gamma_n P(E_{n+1}), \quad (11)$$

which is a compact way of writing the rule of total probability. Evidently the condition of probabilistic support as expressed in (6) can be written more succinctly in the new notation as

$$\alpha_n > \beta_n, \text{ or equivalently as } \gamma_n > 0.$$

So  $\alpha_n > \beta_n$  together with equation (11) are what we mean by ‘ $E_{n+1}$  probabilistically supports  $E_n$ ’ or  $E_n \leftarrow E_{n+1}$ . To indicate this, we introduce the symbol

$$E_n \xleftarrow{\alpha_n > \beta_n} E_{n+1}.$$

Here  $\alpha_n > \beta_n$  expresses the requirement that  $\gamma_n = \alpha_n - \beta_n$  is positive, the condition that  $E_{n+1}$  probabilistically supports  $E_n$ . With our new symbol, the epistemic chain (5) becomes

$$E_0 \xleftarrow{\alpha_0 > \beta_0} E_1 \xleftarrow{\alpha_1 > \beta_1} E_2 \xleftarrow{\alpha_2 > \beta_2} E_3 \dots \dots \infty. \tag{12}$$

In Sections 3 and 4 we will discuss the viability of (12) in more detail. Our overall aim is to show that, if one takes seriously that justification comes in degrees, infinitism is less eccentric than many people think.

### 3 Finite and Infinite Chains

A standard way of envisaging an infinite chain is first to look at a chain consisting of  $s + 1$  links, where  $s$  is a finite number. So let us consider the finite chain

$$E_0 \xleftarrow{\alpha_0 > \beta_0} E_1 \xleftarrow{\alpha_1 > \beta_1} E_2 \xleftarrow{\alpha_2 > \beta_2} E_3 \dots \xleftarrow{\alpha_s > \beta_s} E_{s+1}.$$

From the rule of total probability in the form of equation (11), we find

$$\begin{aligned} \text{with } n = 0, & \quad P(E_0) = \beta_0 + \gamma_0 P(E_1) \\ \text{with } n = 1, & \quad P(E_1) = \beta_1 + \gamma_1 P(E_2), \end{aligned}$$

and on combining these equations, we obtain

$$P(E_0) = \beta_0 + \gamma_0 \beta_1 + \gamma_0 \gamma_1 P(E_2).$$

The iteration can be continued up to  $n = s$ , with the result

$$P(E_0) = \beta_0 + \gamma_0 \beta_1 + \gamma_0 \gamma_1 \beta_2 + \dots + \gamma_0 \gamma_1 \dots \gamma_{s-1} \beta_s + \gamma_0 \gamma_1 \dots \gamma_s P(E_{s+1}). \tag{13}$$

For a foundationalist,  $E_{s+1}$  would be the basic proposition that forms the starting point for the entire chain. Often that starting point has been seen as the report of a sensation or an impression. Traditional foundationalists such as C. I. Lewis hold that  $E_{s+1}$  is absolutely certain, so that the probability of this proposition equals unity:  $P(E_{s+1}) = 1$ . Many contemporary foundationalists, on the other hand, are what Moser et al. have called *modest*, and what Bonjour has called *moderate* foundationalists (Moser et al. [14], p. 87; Bonjour [2], p. 26). Like their philosophical forbears, they insist that epistemic chains must be rooted in basic propositions, but unlike them they hold that a basic proposition need not have probability one. It is enough if it has a definite (known) probability greater than some threshold of acceptance, say  $P(E_{s+1}) > \tau$ . We might think here of the infamous Gedankenexperiment of Schrödinger, in which  $P(E_0)$  could be the probability that the cat is alive at time  $t_1$ ,  $P(E_1)$  the probability that the hammer breaks the vial of hydrocyanic acid,  $P(E_2)$  the probability that an alpha particle that leaves the radium actually enters the Geiger counter window, and  $P(E_3)$  the probability that at least one radium nucleus decays between  $t_0$ , the beginning of the grisly experiment, and  $t_1$ . Here  $P(E_3)$  is supposed

to have a value that exceeds the threshold  $\tau$ , and it could be calculated from the known half-life of radium and the known number of radium atoms in the sample.

Since we are interested in chains of infinite length, it proves profitable to consider further examples, namely, ones in which the length of the chain,  $s$ , is regarded as a variable, instead of being fixed, as it was in the case of the cat (there  $s = 2$ ). The advantage of having  $s$  as a variable is that we will be able to investigate what happens as  $s$  becomes very large and ultimately tends to infinity. Below we shall in fact provide three examples with variable  $s$ . In the first example the conditional probabilities are uniform, that is,  $\alpha_n \equiv \alpha$ , independently of  $n$ , and similarly  $\beta_n \equiv \beta$  and so  $\gamma_n \equiv \gamma = \alpha - \beta$ . In the second example there is nonuniformity:  $\alpha_n$  and  $\beta_n$  do depend nontrivially on  $n$ , although  $\gamma_n$  does not, while in the third  $\alpha_n$ ,  $\beta_n$  and  $\gamma_n$  all depend in an essential way on  $n$ .

Here is the first example. Imagine colonies of a bacterium growing in a stable chemical environment known to be favorable to a particular mutation of practical interest. The bacteria reproduce asexually so that only one parent, the ‘mother’, produces ‘daughters’. The probability that a mutated daughter descends from a normal, not mutated, mother is known to be very small (say, 0.02), but the probability that a mutated daughter descends from a mutated mother is, on the other hand, high (say, 0.99). Let  $E_n$  be the proposition: ‘the ancestor in generation  $n$ , reckoned backward from the present, was a mutant’. In this case the conditional probabilities are  $\alpha_n = P(E_n|E_{n+1}) = 0.99$ ,  $\beta_n = P(E_n|\neg E_{n+1}) = 0.02$  and so  $\gamma_n = \alpha_n - \beta_n = 0.97$ . We are told further that each batch develops from a single, mutant ancestor. In this situation, in which all the  $\beta_n$  are equal to one another, and likewise all the  $\gamma_n$ , equation (13) reduces to a finite geometric series that can be summed explicitly:

$$\begin{aligned} P(E_0) &= \beta \left[ 1 + \gamma + \gamma^2 + \dots + \gamma^s \right] + \gamma^{s+1} P(E_{s+1}) \\ &= \beta \frac{1 - \gamma^{s+1}}{1 - \gamma} + \gamma^{s+1} P(E_{s+1}). \end{aligned} \quad (14)$$

Note that  $\gamma^{s+1}$  is not the same as  $\gamma_{s+1}$ . The former means “ $\gamma$  raised to the power  $s + 1$ ”, while the latter is the Christensen-Joyce confirmation that  $E_{s+2}$  confers on  $E_{s+1}$ —see equations (9) and (10).

Now imagine a batch to be sampled after, shall we say, 150 generations. Then  $s = 149$ , and with the above values of  $\beta$  and  $\gamma$ , we obtain

$$P(E_0) = 0.667 \left[ 1 - (0.97)^{150} \right] + (0.97)^{150} P(E_{150}). \quad (15)$$

The original great-great-grandmother, in generation 150 before the generation to be sampled, is known to be a mutant, so  $P(E_{150}) = 1$ , and therefore  $P(E_0)$  is perfectly well defined: it works out to be 0.670.

We will come back to the example of the bacteria in Section 4, but for the moment we may note that  $P(E_{150}) = 1$  is not, as strong foundationalists would insist, a necessary requirement in order that equation (15) determine  $P(E_0)$ . It may not be known if the original bacterium was mutated, but it could have been obtained from a naturally occurring sample, in which the probability of mutation had been determined by earlier observation of many samples. In this case  $P(E_{150})$  would be set equal to this known probability: the situation formally resembles the Schrödinger

cat example, where the originating probability, associated with the decaying radium nucleus, was known and unequal to unity.

A more extreme situation, and one that even more clearly flies in the face of strong foundationalists like C. I. Lewis, is when  $P(E_{150})$  vanishes, which would be so if it were known that the great-great-grandmother in the 150th generation was certainly *not* a mutant. It seems that here a strong foundationalist has no choice but to deny any justification for the mutation in generation 0. However, if  $P(E_{150}) = 0$ , then  $P(E_0) = 0.667 [1 - (0.97)^{150}] = 0.660$ . Here is a case that fits into the foundationalist's requirement of linear finitude, but in which the 'ultimate ground', in this case the mutation of the originating bacterium, is absent, although the probability that is built up from the conditional probabilities is perfectly definite—and nonzero, of course.

Lest it be thought that the above considerations rest essentially on the requirement of uniformity, let us briefly sketch our second example, that is, a generalization in which the conditional probabilities are not the same from generation to generation. Suppose that an effect of an increasing pollution of the nutrient, as a result of the growing mass of bacteria in it, is that the probability of mutation of a daughter increases as time goes on, whether or not the mother is a mutant. For example, if  $\alpha_n = P(E_n|E_{n+1}) = a + b^{n+1}$  and  $\beta_n = P(E_n|\neg E_{n+1}) = b^{n+1}$ , where  $a$  and  $b$  are positive numbers such that  $a + b < 1$ , then  $\alpha_n$  and  $\beta_n$  are different from generation to generation, although  $\gamma_n = a$  is constant. Equation (13) once more reduces to a finite geometric series that can be summed:

$$\begin{aligned} P(E_0) &= b \left[ 1 + ab + (ab)^2 + \dots + (ab)^s \right] + a^{s+1} P(E_{s+1}) \\ &= b \frac{1 - (ab)^{s+1}}{1 - ab} + a^{s+1} P(E_{s+1}). \end{aligned} \tag{16}$$

A more sophisticated abstract example, our third and last, is one in which  $\gamma_n$ , as well as  $\alpha_n$  and  $\beta_n$ , depend nontrivially on  $n$ , and, moreover, in which the resulting infinite series is not geometric:

$$\beta_n = \frac{1}{n+3} \qquad \gamma_n = \frac{n+1}{n+2} = 1 - \frac{1}{n+2}, \tag{17}$$

so that

$$\alpha_n = \beta_n + \gamma_n = 1 - \frac{1}{n+2} + \frac{1}{n+3}.$$

Then

$$\begin{aligned} \gamma_0 \gamma_1 \dots \gamma_s &= \frac{1}{2} \times \frac{2}{3} \times \dots \times \frac{s}{s+1} \times \frac{s+1}{s+2} = \frac{1}{s+2} \\ \gamma_0 \gamma_1 \dots \gamma_{s-1} \beta_s &= \frac{1}{s+1} \times \frac{1}{s+3} = \frac{1}{2} \left( \frac{1}{s+1} - \frac{1}{s+3} \right), \end{aligned}$$

so equation (13) reduces to

$$\begin{aligned} P(E_0) &= \frac{1}{3} + \frac{1}{2} \left( \frac{1}{2} - \frac{1}{4} \right) + \frac{1}{2} \left( \frac{1}{3} - \frac{1}{5} \right) + \dots + \frac{1}{2} \left( \frac{1}{s+1} - \frac{1}{s+3} \right) + \frac{1}{s+2} P(E_{s+1}) \\ &= \frac{1}{3} + \frac{1}{2} \left( \frac{1}{2} + \frac{1}{3} - \frac{1}{s+2} - \frac{1}{s+3} \right) + \frac{1}{s+2} P(E_{s+1}). \end{aligned} \tag{18}$$

Like the geometric series, this series can thus be summed, and the result written down in explicit form. In the next section we will investigate what happens to our three examples when the number of terms, that is, the number of links in the epistemic chain, becomes infinite.

#### 4 An Infinity of Conditional Probabilities Can Yield a Determinate Probability

During the past several years, Klein has repeatedly stressed that epistemic justification has the character of infinite chains such as (5) or (12). He has defended infinitism as the one true faith against attacks of foundationalists, who have traditionally argued that infinite chains do not make sense. As Klein sees it, there is nothing problematic about an infinite linear chain, the reason being that such a chain need not be completed. What is more, the requirement that an infinite chain must be completed “would be tantamount to rejecting infinitism” ([10], p. 920). The only thing that an infinitist à la Klein requires is that for every proposition  $E_n$  in the chain, there is a proposition  $E_{n+1}$  such that the conditional probability  $P(E_n|E_{n+1})$  is known to be greater than  $P(E_n|\neg E_{n+1})$ , for *that* is the condition under which we can properly say that  $E_{n+1}$  probabilistically justifies  $E_n$ .

We fully agree with Klein that an infinite linear chain need not trouble us, but our reasons are somewhat different from his. For if such a chain is approached by way of a finite chain in the limit as  $s$  tends to infinity, then we are able to show that the chain *can* be completed. In the present section we first give an example of a probabilistic chain that, although infinite, can be completed in the sense that its value can be computed. This example not only shows that infinite chains of reasons can exist (contra Black [1]), but it also illustrates that such chains might culminate in an unconditional justification that yields a number other than zero (contra Lewis [13] and [12]). Last but not least, the example shows that an infinite regress of deferred conditional justification can result in a determinate, unconditional justification for the target proposition (contra Dancy [5] and Gillet [8]).

Consider again the bacterial colonies of Section 3, and particularly the first line of equation (14), with its remainder term, the product  $\gamma^{s+1}P(E_{s+1})$ . As  $s$  becomes larger, the number of terms within the square braces increases, while at the same time  $\gamma^{s+1}$  becomes smaller (recall that  $\gamma$  is strictly smaller than one, and therefore that  $\gamma^{s+1}$ , the  $(s + 1)$ st power of  $\gamma$ , is even smaller). In the limit that  $s$  is taken to infinity,  $\gamma^{s+1}$  disappears. Since  $\gamma^{s+1}$  multiplies  $P(E_{s+1})$ , which is a probability and so can never be greater than one, we see that the remainder term vanishes in the limit that  $s$  is taken to infinity. In this same limit the number of terms within the square braces has become infinite, and so we obtain

$$P(E_0) = \beta(1 + \gamma + \gamma^2 + \dots). \quad (19)$$

Although equation (19) is a geometric series with an infinite number of terms, it can nevertheless be completed, in the sense that it can be computed exactly. An explicit form for its sum can be read off from the second line of equation (14), if we use the fact that, in the infinite limit,  $\gamma^{s+1}$  disappears. It is

$$P(E_0) = \frac{\beta}{1 - \gamma}. \quad (20)$$

If we now take, as in the bacterium case,  $\alpha = 0.99$  and  $\beta = 0.02$ , then  $\gamma = 0.97$  and  $P(E_0) = \frac{2}{3}$ . In other words, even though the value of  $P(E_0)$  is based on an infinite number of terms, we are still able to calculate it (cf. Peijnenburg[15]).<sup>3</sup> This suffices to show what happens when the number of terms in the first example of Section 3 becomes infinite. The second example (16) similarly has a well-defined limit as  $s$  tends to infinity, namely,  $b/(1 - ab)$ .



At this juncture, a foundationalist objecting to infinite chains might argue that our story about the bacterial colonies is not an example of infinitism at all. For no bacterium has an infinite number of ancestor bacteria, if only because of the fact of evolution from more primitive algal slime, which had evolved from earlier life forms, which sprang from inanimate matter, which originated in a supernova explosion, and so on, back to ... to what? To the Big Bang? But it seems that the Big Bang may well not represent a beginning, in view of the deformation of spacetime. The whole point here is precisely the question whether or not there was a starting point. The foundationalist's postulate that in the bacterial case there was a start begs the question.

The fact that we can compute a probability from an infinite chain might be good news for the infinitist, who may now relax the requirement that infinite chains cannot be completed. But could not the same fact serve as grist to a foundationalist mill? After all, it seems that a foundationalist might argue that, since the infinite chain converges to a number that we can compute, an ultimate ground must exist. Could not the foundationalist claim that it is the *limit itself* that constitutes the foundation? The answer is in the negative. For the limit of *what* exactly is supposed to provide the ground? It cannot be the limit of the remainder term,  $\gamma_0\gamma_1 \dots \gamma_s P(E_{s+1})$ , for as we have seen this term goes to zero as  $s$  goes to infinity, and something that fades away into nothingness can scarcely support an edifice. What about the limit of the entire infinite sequence? This cannot serve the foundationalist purpose either. The limit of the sequence is the sum of an infinite number of terms—in our example it is  $\frac{2}{3}$ , being the value of  $P(E_0)$ . This value is, however, the *result* of the concatenation of an unending sequence of terms describing probabilistic support; it is not the *initiator* of it, for the purported initiator was precisely the remainder term  $\gamma_0\gamma_1 \dots \gamma_s P(E_{s+1})$ .<sup>4</sup>

The above considerations are not restricted to cases in which the infinite series is geometric, as it was in our first two examples. They also apply to our third example. In the model (17), if one lets  $s$  tend to infinity in equation (18), the terms involving  $\frac{1}{s+2}$  and  $\frac{1}{s+3}$  vanish, including of course the last term, which carries the probability  $P(E_{s+1})$  as a factor. The result is  $P(E_0) = \frac{1}{3} + \frac{1}{2}(\frac{1}{2} + \frac{1}{3}) = \frac{3}{4}$ . Here then is a case in which there is no uniformity, but in which the unconditional probability of the target proposition can be calculated from an infinite, nonconvergent series of conditional probabilities.

Once you take seriously that justification admits degrees, then the bullet must be bitten: in general there is no difficulty with infinite series of probabilistic support. The reason is that the remainder term usually vanishes in the limit that  $s$  is taken to infinity. Our point is precisely that a perfectly definite probability can be built up from an infinite number of conditional probabilities, without any need, or room for an initiating proposition.

## 5 Afterword: The Bucket Brigade

Our conclusion that an infinite series of conditional probabilities can yield a definite, unconditional probability value for the target proposition might seem counterintuitive, to say the least. How can we ever justify a proposition if its justification is forever postponed?

The apparent oddity of our result can be illustrated by the saga of the bucket brigade.<sup>5</sup> Suppose there is a fire and Abby has to get her water from Boris, and

Boris has to get it from Chris, and Chris from Dan, and so on ad infinitum. It would seem that the fire will never be put out, since there is no first member of the brigade who actually dips his bucket into the lake.

However, this problem would only arise if we drop the assumption that justification comes in degrees. If instead we retain the assumption, and hold that justification *is* probabilistic, then the matter is entirely different. Under this assumption, the proposition ‘Abby gets water’,  $A$ , is only partially justified. In fact, we can calculate the exact probability value of  $A$  by applying the rule of total probability that we cited earlier:

$$P(A) = P(A|B)P(B) + P(A|\neg B)P(\neg B), \quad (21)$$

where  $B$  reads ‘Boris gets water’. Of course, whether Boris gets water is also merely probable, and its probability depends on whether Chris gets water, and so on. We face here an infinite nesting of probability values calculated via the rule of total probability. As we have seen, we are perfectly able to compute the outcome of this infinite series in a finite time: with the numbers that we used in the bacterium example, the probability of  $A$  is  $\frac{2}{3}$ . Note that this is completely independent of whether we embrace an objective interpretation of probability (assuming, for example, that the firefighters have propensities for handing over the water only now and then) or a subjective interpretation (in which we specify our degree of belief in  $A$ ). Both the objective and the subjective interpretation are bound by the rule of total probability, and that is the thing that counts here.

All four probabilities on the right-hand side of equation (21), the conditional as well as the unconditional ones, are supposed to have values strictly between zero and one (in the interesting cases). Now the traditional view, in which justification is *not* gradual, can be modeled by restricting all four ‘probabilities’ to be 0 or 1. Within this nonprobabilistic approach, Abby either gets water or she does not get water. The foundationalist moral of the saga about the bucket brigade is precisely that she does not get water—if the number of brigadiers is infinite. Because this is unacceptable, the foundationalist thinks that there must be a first firefighter who starts off the whole rescue operation.

In the probabilistic scenario the existence of a primordial firefighter is not needed, since the problem that it is supposed to solve does not arise in the first place. If we take seriously that epistemic support comes in degrees, then the probability that Abby extinguishes the fire can have a precise value that we are able to calculate, despite the infinite number of her teammates. As in the examples that we considered above, this unconditional value is a function of all the conditional probabilities.

## Notes

1. Recently, Turri has argued that not only infinitists, but foundationalists, too, can make sense of infinite chains of reasons (Turri, J., “On the Regress Argument for Infinitism,” forthcoming in *Synthese*). In our view Turri’s argument is flawed. We will not go into the details here, but see endnote 4.
2. Note that interpreting justification as probabilistic support is itself neutral with respect to the normative-empirical debate. Defenders of the view that justification is normative will be inclined to a subjective view of probability. Their more empirically-minded counterparts will tend to an objective and even frequentistic interpretation. But both

factions are bound by the Kolmogorovian axioms that underlie subjective and objective interpretations of probability.

3. Equation (20) is the fixed point of a Markov process. Indeed, the stochastic matrix governing this process is regular, and so the infinite iteration of

$$P(E_n) = \beta + \gamma P(E_{n+1}),$$

is guaranteed by Markov theory to converge to  $P^* = \beta + \gamma P^* = \beta/(1 - \gamma)$ . This quick route to the answer only works in examples employing uniform  $\alpha$  and  $\beta$ ; in the general case Markov theory does not help.

4. This is one of the reasons why Turri's argument fails (see endnote 1).
5. We thank an anonymous referee for suggesting this illustration.

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