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Published in:
Journal of Early Modern Studies

DOI:
10.5840/jems2018712

Publication date:
2018

JEMS 7 (2018), 1: 15–45

Rotating Poles, Shifting Angles and the Use of Geometry (Bond's *Longitude Found* and Hobbes' confutation)

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Abstract: In *The Sea–Mans Kalendar* (1636 [1638?]), Henry Bond predicted that magnetic declination would be 0° in 1657, and would then increase westerly for (at least) 30 years. Based on these predictions, Bond went on to claim in *The Longitude Found* (1676) that, by using his model of magnetism, he can offer a technique for determining longitude. This paper offers an assessment of Bond's method for longitude determination and critically evaluates Thomas Hobbes's so–far neglected response to Bond's proposal in *Decameron physiologicum* (1678), in which Hobbes complains about what he takes to be Bond's implicit natural philosophy and about his use of spherical trigonometry.

Keywords: Thomas Hobbes, Henry Bond, mathematics, natural philosophy, natural sciences, magnetism, longitude

1. Introduction

In 1676, Henry Bond published *The Longitude Found*, a short treatise in which he claims that longitude can be found by magnetic measurements (of variation and inclination[1]). The publication of the treatise was, however, anticlimactic, because many had come to have serious doubts about the feasibility

[1] Geomagnetic variation is the angular difference between the magnetic pole and the geographic pole. Inclination, also known as magnetic dip, is the phenomenon whereby, e.g., a compass held parallel to the Earth's surface will align itself on the vertical axis as well as the expected horizontal axis.

of Bond's method. For instance, by 1673 the Magnetics Committee of the Royal Society was showing serious skepticism about Bond's solution to the longitude problem. The members of the committee (which included the mathematician Seth Ward, John Pell, a good friend of Bond's, but also Robert Hooke, amongst others) complained about the unreliability of the magnetic instruments, as well as about the limited number of empirical measurements on which Bond based his method. Criticism also followed shortly after publication. In 1678, Peter Blackborrow published *The Longitude Not Found*, in which he claims that Bond's account of the tilted dynamic dipole is based on flagrant miscalculations. But criticism of Bond's treatise extended beyond the community of scientists directly interested in the longitude problem. Thomas Hobbes, in *Decameron physiologicum* (1678), explicitly devotes a large part of his discussion on magnetism to the refutation of Bond's treatise, and it is with Hobbes' criticism that this article is concerned.

Despite the negative response—and even despite our now being able to evaluate his solution as clearly mistaken—Bond's approach was not entirely unsuccessful. His work on magnetism sedimented the shift from a conception of geomagnetism in terms of an axial and static magnetic dipole (as defended by William Gilbert in *De magnete*, 1600) to a tilted dynamic dipole. For instance, Hooke's work on magnetism, presented at the Royal Society during the 1660s, built on Bond's tilted dipole, in that it posited rotating magnetic poles with a 10° tilt, completing a full revolution in 370 years.[2] Similarly, Edmund Halley's 1683 account of magnetism shares much with Bond's. Halley posits what is known as the internal globe hypothesis, according to which the Earth contains an internal magnetic globe that rotates independently of the Earth's own rotation; this is a clear development of a Bond–style dynamic–pole account. Where Halley differs from Bond, however, is in moving away from a dipole to a multipole account.[3] Moreover, William Whiston, in *The Longitude and Latitude Found by the Inclinatory or Dipping Needle* (1721), combines Halley's dynamic internal globe hypothesis with geometrical considerations specific to Bond's technique to find the longitude by means of inclinatory (or dipping) needles.

This brief overview of some significant work on magnetism carried after the publication of the *The Longitude Found* gives some indication that Bond's work on magnetism did not die out in a single stroke, with the critique from his contemporaries, but left significant traces. Indeed, these traces started to form long before *The Longitude Found* was published, ever since Bond's first announcements about a possible correlation between magnetism and longitude in the 1630s. And these traces continued to leave their mark long after,

[2] Thomas Birch, *The History of the Royal Society of London*, vol. I, London: A. Millar, 1756, pp. 130–131.

[3] Edmund Halley, "A Theory of the Variation of the Magnetical Compass," *Philosophical Transactions of the Royal Society* 13.148 (1683), pp. 208–221.

well into the eighteenth century. The long–lasting influence of Bond's work still awaits being exposed. I do not pretend, in this paper, to do so exhaustively. To begin with, I won't delve too much into discussing much of the sociological aspects and the many turns Bond's work took, as these have preoccupied historians of science for several decades now.[4] Instead, what I do in this article is, in the first part, to offer a reassessment of Bond's work by focusing on the ways in which he combined empirical and geometrical reasoning in order to substantiate his claims about magnetism and to show why magnetism ought to be used to determine longitude. In the second part of the paper, I zoom in on one of the facets of Bond's reception—namely, Hobbes' relentless criticism of Bond's treatise. Hobbes complains not only about errors of computation in Bond's work, but, more importantly, he claims that Bond misused and misunderstood (1) basic natural philosophical principles and (2) the doctrine of spherical triangles. It is these latter two criticisms that are my focus here. The thread tying this paper together is the undermining of narratives defending the possibility of disentangling *physical* considerations from *geometrical* considerations. No such disentanglement is possible in Bond's work or in Hobbes' reply. In Bond's work on magnetism, the relevant variables are hybrids: they are irreducibly geometrico—physical variables.

2. Dipping needles, rotating poles and a theoric–based calculation of longitude

In *The Sea–Mans Kalendar* (roughly around 1639), Bond claimed that he had discovered the "the true and infallible way of finding the long hidden secret of longitude" by means of magnetic measurements. The "secret" itself was not revealed, but Bond made it clear that the calculations of longitude were based on a mathematical method which (at minimum) involved the determination of the distribution of magnetic variation across the globe. Whatever the rule, Bond predicted that in 1657 in London the magnetic variation will

[4] The history of Henry Bond's account of longitude is discussed in Jim A. Bennett "The Longitude and the New Science," *Vistas in Astronomy* 28 (1986), pp. 219–225; Jim A. Bennett, "Mathematics, Instruments and Navigation, 1600–1800," in Glen Van Brummelen and Michael Kinyon (eds.), *Mathematics and the Historian's Craft: The Kenneth O. May Lectures*, New York: Springer, 2005, pp. 43–57; D.J. Bryden, "Magnetic Inclinatory Needles: Approved by the Royal Society?," *Notes and Records of the Royal Society* 47 (1993), pp. 17–31; Richard J. Howarth, "Fitting Geomagnetic Fields before the Invention of Least Squares: I. Henry Bond's Predictions (1636, 1668) of the Change in Magnetic Declination in London," *Annals of Science* 59 (2002), pp. 391–408; A.R.T. Jonkers, *Earth's Magnetism in the Age of Sail*, Baltimore: The Johns Hopkins University Press, 2003; A.R.T. Jonkers, "The Pursuit of Magnetic Shadows: The Formal–Empirical Dipole Field of Early–Modern Geomagnetism," *Centaurus* 50 (2008), pp. 254–289; S.R.C. Malin and E.C. Bullard, "The Direction of the Earth's Magnetic Field at London 1570–1975," *Philosophical Transactions of the Royal Society, Series A* A299 (1981), pp. 357–423.

be 0° and then would progressively increase westerly for at least fifty years. The edition of the *Kalendar* in which Bond introduced his claims about longitude is lost, but a record of his text is found amongst the Hartlib papers.[5]

2.1. A secret 'way' for longitude

Despite its brevity, the text in the Hartlib papers is packed with bold, but unjustified claims about a relationship between the longitude of any given place and magnetic variation. At this point, however, Bond does not share with the public the method by which the relationship was allegedly found. In fact, he protected his method for longitude determination from becoming public for several decades, despite the fact that, in his writing, he shows tremendous confidence in it. Thus, he claims that what he found was a "manifest way […] to attaine the Longitudes," a way which is "reall and grounded upon as certaine naturall principles as any Mathematicall conclusions whatsoever in Geography and Navigation."[6] Bond proposes his method as an explicit alternative to schemes for longitude determination based on astronomical observations, in particular the more recent scheme which involved using the satellites of Jupiter as a universal clock. This is a method proposed by Galileo as early as 1612, but which had not attracted much interest because it involved constant accurate measurements of the moon's position in the sky, which could hardly be done on a moving ship. To replace such ineffectual methods, Bond proposes not an astronomical method, but one based on magnetism. More specifically, it is a method based on predicating that there "truely […] are magneticall Poles," whose latitude and longitude can be known by means of "concurrent observations and Arithmeticall calculations."[7] The magnetic poles are not static, however, but dynamic, and many of the actual predictions Bond makes are based on his claims to have calculated the exact time it takes for such poles to make a "revolution"—which means that he takes the magnetic poles to be rotating (be they points or bodies), and to be located below the moon. In other words, accepting Bond's techniques for calculating longitude would have also entailed buying into his *specific* theory about the source of the Earth's geomagnetism.

In the same Hartlib Papers text, Bond also claimed that his method can explain why "the variation in many, nay most places is very Irregular," such that "in some places in the same paralell in equall spaces, it altereth much

[5] See Hartlib Papers, 71/16/1A, *A discovery of the True & Infallible way of finding the long hidden Secret of Longitude. By Henry Bond Practitioner in the Mathematickes*, accessible online at: https://www.dhi.ac.uk/hartlib/view?docset=main&docname=71_16_01&term0=transtext_bond#highlight

[6] Hartlib Papers, 71/16/1A.

[7] *Ibid.*

swifter then in other,"[8] but once again Bond was reluctant to share his explanation for the irregularity of variation. However, it becomes clear that, in Bond's theory, the hotly debated phenomenon of magnetic variation would have had an important role to play in the attempt to determine longitude, and that maps of magnetic variation across the globe could have been generated from Bond's account. Rather abruptly, the text ends with the prediction of magnetic variation, reiterated a few more times in *The Sea–Mans Kalendar*, which made Bond's account a subject of conversation for several decades. Thus, Bond claimed:

> because I am confined to a little roome, I must abruptly end my discourse of Longitude, onely take notice that those that live untill the yeare of our Lord 1657, shall not see any variation at all at London & afterwards it will increase westerly at least for 50 yeares.[9]

In 1657, Bond's prediction came true: the magnetic variation at London was null. Soon after, unsurprisingly, Bond's theory became a subject of interest for the Royal Society. In 1662, the Magnetics Committee of the Royal Society requested Bond to join the committee in order to participate at the annual declination measurement at Whitehall. As recorded by Birch in *The History of the Royal Society*, every year members of the society would attempt to verify Bond's prediction about the western progression of magnetic variation at London, despite his unwillingness to provide any variation tables. This situation changed in 1668 when a table with values of magnetic variation in London up to 1695 was published in the January issue of the *Philosophical Transactions*.[10] The text, written by Henry Oldenburg, makes it clear that Bond was not willing to present the method behind his calculations, despite the fact that, by that point, it was known that it involved a theory of dynamic magnetic poles, whose motion (i.e. period of revolution) Bond claimed to have calculated. Bond finally reveals his method to the public in 1676 when he publishes *The Longitude Found*.

2.2. Bond's dynamic dipole in context

Before moving to an examination of Bond's *Longitude Found*, I propose to situate Bond's work on magnetism in the context of the English tradition of navigational magnetism, a tradition in which geomagnetic effects (declination, variation and inclination) are used to solve cartographical and

[8] *Ibid.*

[9] *Ibid.*

[10] Henry Oldenburg, "The variations of the magnetick needle predicted for many yeares following," *Philosophical Transactions* 3.40 (1668), pp. 789–790.

navigational problems, since Bond's account is not developed in a vacuum, but is part of a long tradition in which observational practices of magnetic effects and mathematical practices (such as spherical trigonometry, logarithms) are co-dependent, with each acting as a constraint on and regulator of the other.

Longitude Found (1676) responds both globally and locally to claims about magnetism, in general, and the application of magnetism to various navigational problems, in particular. This is why in the opening pages of the treatise, Bond engages explicitly with William Gilbert's *De magnete*, Edward Wright's *Certain Errors in Navigations* (first published in 1599, and republished with corrections and additions by Wright in 1610), Robert Norman's *The Newe Attractive* (1581), William Burrows *A Discourse of the Variation of the Compass* (1581), Henry Philips' account of rotating magnetic poles in the *Seamans Kalendar*, and so on.

Bond gives particular relevance to Norman's *Newe Attractive*, as the first systematic treatment of the effect of magnetic inclination (that is, the vertical deviation of a magnetised needle when left to move freely on a fulcrum), because it was this treatise which established inclination as a "genuine" magnetic effect, and not just a mere error produced by faulty construction.[11] He retells Norman's story about the discovery of dip and praises Norman's inclinatory needle.[12] At the same time, Bond denies the Gilbertian view according to which there is a correlation between magnetic inclination and latitude. Instead, he claims that the inclinatory needle can be used as an instrument for *longitude.*[13] Magnetic inclination cannot be used to determine the geographical latitude because a magnetic body aligns itself with the magnetic poles,

[11] On treatments of Norman's account of dip see Alan Chapman, "Gresham College: Scientific Instruments and the Advancement of Useful Knowledge in Seventeenth–Century England," *Bulletin of the Scientific Instruments Society* 56 (1998), pp. 6–13; Laura Georgescu, "One Experiment, Different Uses: Floating Magnetic Bodies in Peregrinus, Norman and Gilbert," *Journal of Early Modern Studies* 2 (2013), pp. 81–103; Rom Harré, *The Method of Science*, London: Wykelham Press, 1970, pp. 28–32; David Watkin Waters, *The Art of Navigation in England in Elizabethan and Early Stuart Times*, New Haven: Yale University Press, 1958.

[12] An inclinatory needle is an instrument used to measure the magnetic dip. Norman at the suggestion of "ceratyne learned and expert men" constructed the first inclinatory needle (also called dip circle) that we have record of in order to measure the value of dip in London, and gave a detailed description of its construction in *The New Attractive* (1581, pp. 14-16). Norman's construction was very much appreciated by the English practitioners of magnetism and navigation as the instrument was well used in the second half of the seventeenth century. At the same time, the unreliability of the inclinatory was well known by those working in navigation and magnetism such that throughout the seventeenth century and well into the eighteenth century there were never ending discussions about how to improve the construction of the instrument to seal it off from perturbations and to reduce friction. The instrument's lack of reliability was one of the reasons why campaigns for gathering magnetic observations across the globe were at the time centred on magnetic variation rather than magnetic inclination. On a brief history of the inclinatory needle at the Royal Society see Bryden, "Magnetic Inclinatory Needles."

[13] Henry Bond, *The Longitude Found*, London: Printed by W. Godbid, 1676, p. 2.

and not with the poles of the Earth as Gilbert's theory would have it such that magnetic inclination could be used alone as a way to map latitude at any location.[14]

In fact, the theoretical context of the English scene in the seventeenth century was such that it was almost taken for granted that measurements of magnetic variation and magnetic inclination are what one needs if one wants to find out how magnetism works, and in order to develop new cartographical methods. This situation is explainable not by the fact that the practitioners were invested in a systematic program for gathering measurements of magnetic effects across the globe, as such empirical measurements were rather scarce, especially in the first decades of the century, but rather by the fact that many of these practitioners were working with the assumption that what you do when you measure a magnetic effect such as variation and inclination is to measure a part of a *magnetic motion*—indeed, of a *regular* motion. It is this assumption that grounds the belief that there must be a regular distribution of these magnetic effects across the globe, and that such a distribution could also be discovered remotely, as it were, with the aid of mathematical methods. That magnetic effects might have a random distribution across the globe was not part of the conversation. The default position was regular distribution, as this was imposed by the theoretical background informing the research programs in play. Any supposition of randomness in the distribution of magnetic effects would need to be proven, just because regular distribution was the default position. This attitude was at odds with what was happening on the Continent, where the dominant corpuscularian accounts of magnetism represented a challenge to such neat global patterns of variation or inclination.

The Cartesian theory of magnetism and its subordination to his physical theories of vortices effectively made the compass needle a useless instrument for a wide range of the purposes it was used for across the channel; and it also put into question the grounds for attempts to establish that magnetic motions took on reliable patterns. The magnetic motions were taken to be noise, whose measured values were created by small disturbance in the flux of particles in vortices.[15] Even when Cartesian vortices were not involved in the debate, on the Continent, the acceptance of a time–dependent variation of magnetic variation did not amount to accepting that the magnetic variation was regular enough as a dipolar magnetic axis would have entailed. In a nutshell, in such accounts, atmospheric changes, the tidal influences of the moon, earthquakes and all sorts of other environmental variables could have

[14] *Ibid.*, p. 1.

[15] For instance, in a letter exchange with Mersenne, Descartes dismisses Gellibrand's alleged discovery of the secular magnetic variation of 1635, claiming that Gellibrand did not give sufficient measurements to show that indeed the variation of magnetic variation follows a pattern rather than just being simply irregular. See e.g. *Œuvres de Descartes*, ed. Charles Adam and Paul Tannery, 11 vols, Paris: J. Vrin, 1996, vol. III, p. 7.

influenced the direction of the magnetic needle, such that randomness became the rule in accounting for such effects. On this understanding, magnetic inclination and magnetic variation became side effects of the media, and not genuine phenomena.

However, by 1639, the story of the neat distribution of variation across the globe got complicated by Gellibrand's alleged discovery of the secular variation (also known as the variation of variation), published in his *A Discourse Mathematical on the Variation of the Magneticall Needle* (1635), which claimed that the deflection of a magnetic needle from the magnetic pole, that is, the magnetic variation, changed over time:[16]

> whereas in the year 1580 Mr. Burrows […] found the variation at Limehouse near London to be 11gr. 15 min. or neere one point of the compass; in the year 1622 Mr. Gunter [...] found the variation in the same place to be but 6gr. 13 min. And myself this present yeare 1634 with some friends had recourse to Dedpford (where Mr. Gunter had therefore made the same observations with those at Limehouse) and found it not much to exceed 4 degrees.[17]

Gellibrand's claim that there was a "sensibile diminution" of magnetic variation at the same place in a relatively short amount of time was quickly accepted by the intellectual community as an established fact even though it directly challenged the then dominant account of variation (at least) in England, that is the Gilbertian account.

According to Gilbert, magnetic variation was not a genuine magnetic effect. Magnetic variation (at any particular location) is the degree of angular movement a needle makes to the east or to the west of the magnetic meridian. It is a deviation from magnetic direction, that is the orientation of a magnetised needle relative to the magnetic polar axis. It also should be recalled here that Gilbert assumes an axial dipole, with no tilt, which also means that, in this account, the Earth's axis of rotation is the same as the Earth's magnetic axis. If so, according to Gilbert, what explains magnetic variation is the distribution of continental landmasses (plus the local effects that deposits of iron and/

[16] For a discussion of Gellibrand's claim that magnetic variation varies with time, see Stephen Pumfrey, "'O tempora, O magnes!' A Sociological Analysis of the Discovery of Secular Magnetic Variation in 1634," *British Journal for the History of Science* 22 (1989), pp. 181–214. Gellibrand explains the changes in the measurement of magnetic declination in London not by positing time *per se* as the relevant variable, but temperature changes. This aspect of Gellibrand's proposal became obscured by contemporary reconstructions of Gellibrand (which would have influenced Bond) as defending the westerly drift hypothesis. Either way, with Gellibrand's discovery, Gilbert's conflation of the geographical and the magnetic poles seemed unlikely, so that, after 1635, the hypotheses of a tilted dipole or of a multipole structure began to be explored.

[17] Henry Gellibrand, *A Discourse Mathematical on the Variation of the Magneticall Needle. Together with its admirable diminution lately discovered*, London: Printed by William Jones, 1635, p. 7.

or lodestone would have). A perfectly homogeneous magnetic sphere would display only magnetic direction (i.e. alignment with the magnetic pole). This account makes magnetic variation a "perverted" magnetic movement.[18] But more importantly, it makes it intractable. It blocks the requirement of sustained global–scale observations of magnetic variation by effectively explaining away magnetic variation.

Given Gilbert's theory, Gellibrand's secular variation was a direct challenge to it, but the former was far from being proven. To claim a "sensible diminution" of variation, Gellibrand had to trust that the practice of measurement, and the instruments used by both Burrows and Gunter were reliable, since his own account directly built on their measurements. Gellibrand also disregarded all sorts of other potential factors which might have affected the result – from potential atmospheric influences to the presence of a different quantity of iron in the neighbourhood, factors which otherwise were consistent with Gilbert's account of the non–magnetic nature of magnetic variation. Irrespective of such quibbles, it seems that by 1639, Bond, like many of the English practitioners, took secular variation as an established fact.

This does not mean however that Bond took Gellibrand's findings uncritically. He wanted to explain what caused secular variation, something about which Gellibrand remained silent. In a letter from John Pell to Marin Mersenne in 1640, we are told that Bond believed he had an explanation for the secular variation, but was hesitant to make it public. A similar attitude is reported in the 1668 issue of the *Philosophical Transactions*. Oldenburg received the table from Bond and published it as a new "conjecture […] to excite new thoughts, and to make farther Inquiries concerning […] the doctrine of the *Magnet* and *Magnetical motions.*"[19] Oldenburg explicitly states that the purpose of publishing the table is "to the end that Inquisitive Men may everywhere from time to time make Observations accordingly" in order to verify Bond's hypothesis, according to which "the *Variations* of the *Magnetick needle* will fall out for the years therein specified; which he conceives to be now *Westward*, and to have been so for some years last past; whereas they were formerly *Eastward*."[20]

It is important to note that Oldenburg classifies Bond's calculation of the table of magnetic variation as based on "a certain hypothesis," "designed to

[18] See William Gilbert, *On the Magnet, Magnetick Bodies Also, and On the Great Magnet the Earth, a New Physiology, Demonstrated by Many Arguments & Experiments* [Translated by S.P. Thompson], London: Chiswick Press, 1900, Book IV.

[19] Oldenburg was enthusiastic about Bond's theory. For instance, according to Martha Baldwin, *Kircher and the magnetical philosophy*, Chicago: University of Chicago (Department of History), unpublished PhD, 1987, pp. 26–27, Oldenburg asked his friends, Petit in France, Richard Stafford in Bermuda, and John Winthrop in Boston, to take measurements of magnetic variation and compare them to Bond's findings as a way to verify the theory.

[20] Oldenburg, "The variations of the magnetick needle," p. 790.

be confirm'd" but which Bond "thinks not yet fit to declare to the World."[21] Presumably, then, the hypothesis at stake here refers to the *cause* of magnetic variation. This means that for most of his career, Bond was unwilling to make public the explanation of secular variation, but had no hesitation to advertise his predictions about the rate of change of magnetic variation in London and across the globe. As already mentioned, the earliest table of predictions he published was in the *Sea–Mans Kalendar*, shortly after Gellibrand's findings, as well as in the *Philosophical Transactions*. Thus, rather than seeing Bond as having taken Gellibrand's findings for granted, we also have the option of seeing Bond as putting a possible cause (the secular variation of magnetic variation) into practice, in order to see if it produces the effects it is supposed to. Verification of Bond's table of predictions (as long as the initial observational and the mathematical translation of these observations would have been deemed reliable) would mean a verification of a cause–effect nexus, namely a dynamics of the magnetic poles and the local effects that such dynamics will produce. If so, then Bond's *Longitude Found* is the place in which the exact description of the cause–effect nexus is given. It is to this treatise that we turn next.

2.3. A theoric–based calculation of longitude

According to Bond, *The Longitude Found* was written in order to "find the longitude, as well by night as by day," with the help of which "the Geography and Hydrography of the whole world" will be "rectified and settled."[22] After its publication, it became clear that Bond's solution to longitude did not rely on magnetic variation alone. Magnetic inclination was also necessary:

> the *Longitude* of any place from the Magnetical colure, may be found by the Inclination, and by that the Difference of Longitude between any two places on Earth.[23]

What remained to be established is the exact relation between magnetic inclination, magnetic variation and longitude. Bond's answer is a geometrical construction devised such that it can be used to either find the magnetic inclination of a place or the longitude of a place. It is what in historiography is known as a theoric (or nomograph), and at the time the use of theorics as a problem–solving technique for navigational matters was common practice. Johnston shows that as early as 1576 Thomas Digges attempts to explain and predict the magnetic variation across the globe by making use of a "theoric of variation." The trend did not stop with Digges: in *De magnete*, William

[21] *Ibid.*, p. 790.
[22] Bond, *Longitude Found*, The epistle to the reader.
[23] *Ibid.*, p. 18.

Gilbert used a theoric in his account of magnetic inclination, in order to calculate the values of the inclination at various locations across the globe.

Theorics are geometrical constructions used initially in Ptolemaic astronomy to determine equants and epicycles, but also made use of in Copernican astronomy; from the sixteenth century onwards, they were exported from astronomy into a large variety of mathematical disciplines, such as "surveying, navigation, warfare, architecture, gnomonics [...] as well as for drawing and calculation."[24] Theorics combine observational results with geometrical considerations in a construction meant to be used as a predictive tool. They can be seen as purely instrumental, without making use of relations between physical variables, or they can be taken to instantiate actual relations between physical variables.[25] Given the latter, a theoric is simultaneously theoretical and practical: it is theoretical insofar as it encompasses an explanation (of magnetism in our case) and it is practical because the predictions can be used for practical uses (in Bond's case, for determination of longitude). But if the theoric instantiates physical relations, then it becomes a test case. If the predictions hold, the posited cause holds.[26] If the predictions fail, the natural philosophical prediction fails. Bond's theoric of longitude would have been a solution to the problem of longitude as long as the calculated magnetic values in his tables would have corresponded to the empirical value measured at the actual sites, and if the correspondence would have held, that would have meant that the magnetic motion assumed in the geometrical model corresponds to the true magnetic motion.

2.4. Tilted rotating dipoles

Bond posits a tilted dipole on supposedly empirical grounds, which means that they are the cause of the behaviour of a magnetised needle, namely its

[24] Stephen Johnston, "Theory, Theoric, Practice: Mathematics and Magnetism in Elizabethan England," *Journal de la Renaissance* 2 (2004), pp. 53–62.

[25] For a discussion of the functions theorics played in history of science, see Jim A. Bennett, "Knowing and Doing in the Sixteenth Century: What Were Instruments For?," *British Journal for the History of Science* 36.2 (2003), pp. 129–150. In this text, Bennett defends the claim that, in Gilbert's *De magnete*, there is a "crucial move from mathematical theoric to natural philosophy," such that the mixed mathematical practices and the natural philosophical practices are blended into a new way of doing natural philosophy. See also Jim A. Bennett, "Modern Mathematical Instruments," *Isis* 102. 4 (2011), pp. 697–705. For a response to Bennett's claims, see Johnston, "Theory, Theoric, Practice."

[26] The reader might believe that, strictly speaking, this is a violation of *modus ponens*, but, this is not the case because the hypothesis is here taken to be constitutive of the model. The model *is what it is* because it incorporates the said hypothesis in its *construction.* With a different hypothesis, the actual constructed geometrical model would have to look different.

horizontal (magnetic variation) and vertical (magnetic inclination) orientations.[27] Assuming that the magnetic poles move with a geometrically describable motion, then finding such a description entails that the pattern of distribution of variation and inclination across the globe is regular, and thus predictable. If so, then these values can be used to chart latitude and longitude. Of course, this is a much simplified account of magnetism. But it is also what Bond is after: he wants to find out the location of the magnetic poles, "their Latitude, their longitude, and their Annual Motion."[28]

To do so, he takes the historical measurements of variation and inclination ("Eighty five Observations") available in tables printed in navigation textbooks to be reliable, even though the instruments alone (that is, the magnetic compass and the inclinatory needle) were known to be unreliable because of either technical limitations or all kinds of perturbations at the location where the measurement was taken. Bond himself does not show any sign of being interested in contesting the values, or even of taking them with a grain of salt. Of course, one possible explanation of this is simply sloppiness in his practice; another might very well be that he was following the norm of practice at the time—after all, one is somewhat obliged to trust the past measurement if the magnetic variation is time–dependent, since one simply cannot go back in time to check. Yet another way at making sense of Bond's practice is to speculate that perhaps he was not interested in the specific values themselves, but rather in giving a mathematical account of the rate of their change. It was this rate of change which Bond could then use to describe the motion of the magnetic poles. If so, then Bond's practice requires the following assumptions: (1) that there is a *regular* rate of change of sorts, such that looking for it is justified; (2) that the rate of change is time–dependent, such that it tracks the motion of the magnetic poles itself rather than a different variable; (3) that Gellibrand's variation of variation is a well–established empirical fact. Taking these assumptions on board secured the possibility of further investigating a possible correlation between geomagnetism and longitude.

According to Bond, the magnetic poles are not locatable on Earth, but "in or above in the air," in a magnetic sphere that circumscribes the Earth itself.[29] In order to account for the purported change in variation between the apparent locations of the magnetic poles and the locations of the geographical poles over time, Bond's magnetic sphere moves as well, but with a motion slower than that of the Earth's, because the sphere is "a substance that hath not solidity to keep pace with the motion of the Earth, but loseth in its motion."[30] Its direction is also different: if the axial rotation of the Earth is from west to east, the rotation of this magnetic sphere is taken to be from east to west. The

[27] Bond, *Longitude Found*, pp. 2–3.
[28] *Ibid.*, p. 3.
[29] *Ibid.*, pp. 6–7.
[30] *Ibid.*, p. 7.

magnetic poles rotate from east to west with a period of 600 years, and a yearly motion of "six tenth of a degree, that is, thirty six minutes," along a parallel at a co–latitude of 8° 30' relative to the geographical pole. Imagining the "magnetick sphere" as a *sphere*, specifically, is not inconsequential:

> […] as there is a Magnetick Sphere, that hath two Poles, so there are other circles belonging to it, as to the other spheres, an AEquator, two Colures, and Parallel Circles to the Equator [i.e. lines of latitude].[31]

There is no clear cut distinction between the magnetic sphere *qua physical object* and the magnetic sphere *qua geometrical object.* This move is not peculiar to Bond. It is frequently used in the mathematical treatises of the seventeenth century, especially treatises of navigation and astronomy. Take for instance John Seller in *Practical Navigation*, a textbook on navigation republished many times during the seventeenth century, in which the student of navigation is given the conceptual repertoire and a toolkit of problem–solving techniques: here, as with Bond, we are told that any sphere has privileged circles, and the list of the privileged circles of the sphere follows Bond's closely.[32] What makes these circles privileged is not that they can be used to describe a mathematical relation (such as the perimeter or the volume of the sphere), but rather the fact that they are used to describe the *physical* interactions a *spherical body* enters into. The nature of the object we are dealing with in such situation is of a peculiar mixture. It cannot be a mere geometrical object because some of its privileged properties (i.e. the poles, privileged circles, etc.) are articulated as part of a *physics* and not a mere science of geometry. It cannot be a mere *physical* object because some of its parts have fundamentally non–physical parts. If a *physical object* is essentially to be an *extended object* then each of its parts have to be extended, each has to have breadth, depth, length; they can hardly be points and lines. Such considerations show that Bond's magnetic sphere is neither simply geometrical nor simply physical. It is a geometrico–physical object, such that it makes little sense to attempt to disentangle the physical aspects from the geometrical. They are blended into a unit. What one can do, and what Bond (and others) does is to use the *geometrical properties* of the mathematical objects as a constraint to describe the *physical* properties and the *physical interactions* an object enters into. In this sense, the geometry dictates the relevant properties of the magnetic sphere, such that what we are left with a strategy of a physicalization of the geometrical object, rather than a geometrization of the physical object. To make these points clearer, consider a sphere's *colure*. Here is how Seller defines it:

[31] *Ibid.*, p. 12.

[32] John Seller, *Practical Navigation*, London: Printed by J. Darby, 1669, p. 6.

> The two *Colures* are two great Circles passing through both Poles of the World, which are two prime Meridians crossing one another at right Angles in the Poles of the World, dividing the Equinoctial into 4 equal parts, making thereby the four seasons of the year. the one called […] the Solstitial Colure [….]. The other Colure [….] called the Equinoctial Colure.[33]

Defined this way, the *colure* is neither purely geometrical, nor purely physical. It is a geometrico–physical object, which instantiates physical relations (the seasons) in virtue of some geometrically describable properties. Similar analyses can be given for all the other privileged circles and points of a sphere (Bond's magnetical sphere, in our case). After all, for Bond, the location of the magnequator is as much the diameter of a *circle* as it is the place in which magnetic inclination is supposed to be null.[34]

Magnetic motions (magnetic direction, variation and inclination) depend only on the rotation of the magnetic poles from east to west on the same axis. In which case, on the same meridian (that is, the same line of longitude), the variation registered by the magnetic needle should be the same. But, as measurements show, this is not actually the case, such that Bond's theory cannot be empirically corroborated (as Halley will show at length[35]). Bond is, however, aware of this limitation to his proposal, but does not see it as a fatal argument against his account, because he denies that the variation of declination (as registered by the horizontal needle) is a genuine magnetic phenomenon, and sees it as the result of some "accidental cause in the Earth."

Practically, according to Bond, the longitude of any place could be found by "the Needles Inclination, and that Table [i.e. of inclination], and the latitude of the place."[36] By 1676, however, few recordings of magnetic inclination were available since inclinatory needles were rarely taken on board of ships. These values were Norman's 1576 measurement of 71° 50'; those of Wright and Ridley (1613), measuring the angular value of inclination at 72°; and Bond's own measurement of 73° 47'. The differences in value are small. All sorts of factors could have been posited to explain these differences: the unreliability of the dipping instrument (especially at sea), changes of the location at which the measurements were taken, changes in temperature, atmospheric changes, and so on. For instance, Bond's contemporaries found

[33] *Ibid.*, p. 8.

[34] See Bond's discussion of inclination in *Longitude Found*, pp. 4–5.

[35] Edmund Halley, "A Theory of the Variation of the Magnetical Compass," p. 214.

[36] Henry Bond, "The undertakings of Mr. Henry Bond Senior, a famous teacher of the art of navigation in London, concerning the variation of the variation of the magnetical compass and the inclination of the inclinatory needle; as the result and conclusion of 38. Years magnetical study," *Philosophical Transactions of the Royal Society* 8.95 (1673), pp. 6065–6066, at p. 6066.

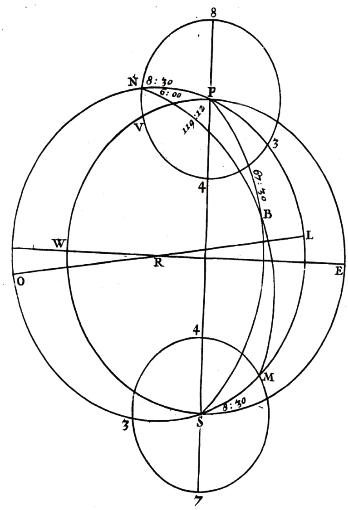

Figure A.

the unreliability of the dipping needle to be a major shortcoming of Bond's proposal.[37] There are many technical concerns regarding the dipping needle, but the insurmountable technical problem seemed at the time to be securing its proper balance: this is challenging because it is difficult to make the needle's centre of gravity coincide with the centre of its axis of rotation so that wobbling and measurement errors are avoided—especially at sea, where the motion of the ship would constantly move the instrument away from the horizontal. Bond, I suspect, has some theoretical (and methodological) reasons to set aside these concerns. From his published material, it seems Bond only uses a few inclination values to construct his theoric. Once it was constructed, he would be able to predict inclination values: the values would, of course, be amenable to empirical rectification, but the misreadings of the dipping needle would be correctable from the table as well (eventually). What matters to Bond is to have a way to geometrically determine either the inclination or the longitude under the assumption of the magnetic poles rotating with "thirty six minutes" a year.

2.5. A theoric for longitude

What Bond offers in order to determine where you are at sea is a paper device: a pre–calculated table of inclinations with a corresponding table of longitudes and a geometrical construction to be used for the verification of the value in the table, or to fill in the table. This means that the values in the table are computed based on the geometrical construction. This geometrical construction (and its corresponding table) is, then, the theoric (see Figure A).[38]

[37] See D.J. Bryden, "Magnetic Inclinatory Needles" for responses of Bond's contemporaries. The dipping needle is so unreliable that even in the twentieth century, local magnetic inclination was not measured empirically, but was deduced from the total magnetic intensity of the Earth's magnetic field.

[38] What Bond offers for longitude is similar to what Wright (working within the theoretical parameters of Gilbert's magnetical philosophy) was claiming to offer for latitude in his corrected version of the Edward Wright, *Certaine Errors in Navigation, Arising either of the ordinarie erroneous making or vsing of the sea Chart, Compasse, Crosse staffe, and Tables of declination of the Sunne, and fixed Starrs detected and corrected*, London: Printed by Joseph Moxon, 1610, chapter 13. For a detailed reconstruction of the geometrical details informing Gilbert's (and by extension Wright's) theoric see, Thomas Sonar, "Navigation on Sea: Topics in the History

Bond's construction of the theoric was made possible *through* his theory of moving dipoles. What Bond's theoric is supposed to help him find out is the location of the magnetic poles. Once their location is given, then one can use the theoric to determine the longitude–inclination correlation. But just being shown the theoric is not sufficient. One needs to understand what the rule that this theoric is supposed to have shown is in order to use it. However, even knowing the rule the theoric is supposed to show is insufficient. To use Bond's theoric, one has to know (a) how to use a Mercator map and (b) how to use spherical trigonometry, because the theoric involves computations of either angular values or sides of triangles on the surfaces of spheres. Moreover, one also has to understand the rationale Bond uses, not so much in his computation, but in the *manner* by which he proceeds with the construction of the theoric. This is why much of Bond's treatise is filled with examples of how to use the theoric to determine either the longitude or the magnetic inclination at a given location. The examples are not there to *prove* his theory, but to *teach* the reader how to use the theoric. In fact, Bond seemed to be not at all preoccupied with proving his account of magnetism. He expends very little ink substantiating his claims about the revolution of the magnetic poles, for instance.

I shall not go in the specifics of the construction here, but shall point out instead that the 8° 30' tilt and the period of the revolution of the magnetic poles relative to a fixed standard (which Bond takes to be the meridian at London in 1657 because the magnetic variation was 0°) are crucial to the construction. Any changes to these values entail changes in the calculated tables. To show then either (a) that the magnetic poles are not tilted or (b) that they simply do not exist would be sufficient to break down Bond's solution to longitude and his account of magnetism in one blow. This is precisely what Thomas Hobbes will attempt to do, and this is what we turn to in the next section.

3. *From theorics to natural philosophy and spherical triangles*

Hobbes' confutation of Bond is given in the final part of Chapter IX of his *Decameron physiologicum* (hereafter *Decameron*).[39] The *Decameron* is written in the form of a dialogue between a student (A) and a teacher (B), with B representing Hobbes' opinion. In 1678, when the *Decameron* is published, Bond's theory of a tilted dynamic magnetic dipoles is the most direct attack on theories of the axial dipole, which Hobbes embraces. Thus, the ultimate goal of Hobbes' argument against Bond's theory is to show that the geographic

of Geomathematics," in W. Freeden, M. Z. Nashed, & T. Sonar (eds.), *Handbook of Geomathematics*, Berlin: Springer, 2010, pp. 43–68.

[39] Thomas Hobbes, *Decameron physiologicum*, London: Printed by J.C. for W. Crook, 1678, pp. 110–122.

and magnetic poles are the same. But the attack is multilayered, ranging over charges of improper geometrical construction, of assent to false natural philosophical principles and of misusing the doctrine of spherical triangles. In this section, I look at Hobbes' criticisms targeting Bond's natural philosophical principles (section 2.1) and geometrical reasoning (section 2.2.). As will become clear, while some of Hobbes' criticisms are right on point, others miss the mark because they are built on assumptions that Bond would not have taken on board.

3.1. Hobbes's confutation of Bond's implicit natural philosophy

Let's begin with Hobbes' assessment of Bond's natural philosophical principles. As we have seen above, according to Bond, the Earth is surrounded by a magnetic sphere, whose poles are at 8° 30 min. co–latitude from the geographical poles. The east–west rotation of the magnetic sphere is slower than the axial rotation of the Earth because the magnetic sphere is a body of a lesser solidity than the Earth. This account is, according to Hobbes, flawed because it is built on a confused conception of (1) what a body is and (2) how the alleged motion of the magnetic sphere works. Let's begin with (2). For Hobbes, if the magnetic sphere moves, it moves because of an external cause; it is an externally imprinted motion. It cannot move by an intrinsic motion because Hobbes denies the possibility of self–motion altogether. Despite the fact that Bond does not discuss it explicitly, there are textual reasons to conclude that he would have also denied the possibility of self–motion for the magnetic sphere. For instance, he claims that the existence of rotating magnetic poles can be used as an argument in favour of the Earth's axial rotation.[40] If we do deny the possibility of self–motion, then, as Hobbes reasons, the magnetic motion can "have no Motion at all but what the Earth and the Air give it."[41] Despite the copula "and" used here, I take Hobbes to consider and discuss the following four possibilities of the causal source for the magnetic sphere's motion: (1) the stream of air itself; (2) the stream of air and the Earth's axial motion; (3) the Earth's axial motion; and (4) the motion of the Earth's poles, or the precession of the equinoxes. In what follows, I discuss each separately, at greater length than in Hobbes' original rather condensed treatment.

(1) If the magnetic sphere is moved by the stream of air, then the motion of the magnetic sphere would vary when the stream of air varies, and since the latter happens in an irregular manner and rather frequently, then the motion of the magnetic sphere would itself have to be irregular, which in return entails that it cannot be used as an instrument to determine longitude. If this were the case, there would be no pattern against which to determine longitude

[40] Bond, *Longitude Found*, p. 6.

[41] Hobbes, *Decameron*, p. 117.

and latitude. Hobbes's reasoning against (1) also works for (2) because, despite the regularity of the Earth's axial motion, the stream of air is a perturbing factor such that, if involved, it would distort any regular dynamics of the magnetic sphere.

Let's move now to (3) and (4). According to Hobbes, if the magnetic sphere is moved by the Earth itself, the motion the Earth gives to the magnetic sphere is "no other Motion than what it gives to its own Poles by the precession of the Equinoctial points" because the axis of motion of the magnetic sphere "is always at 8° 30 min. distance from the Pole of the Earth."[42] Here Hobbes denies the possibility of (3) on Bond's own grounds. The reasoning goes as follows: suppose the motion of the magnetic sphere is caused by the axial rotation of the Earth. In its daily rotation, the Earth carries along the magnetic sphere, but because the magnetic sphere is not solid but appears, on the contrary, to be fairly rare, it must move more slowly. But, Hobbes notices that if the axial rotation were the cause, the magnetic dipole would not always be tilted at 8° 30 min. from the geographical pole because the geographical poles themselves shift position each year (the precession of the equinoxes), a factor that Bond does not take into account since he considers the co–latitude between the geographical pole and the magnetic pole to be constant. If Bond is to take into account that the Earth's motion also entails an annual precession of the poles of 36 sec, then each year the co–latitude of the magnetic pole would have to shift by 36 sec. This ought to have factored into Bond's calculations, but there is no indication that it did. In effect, Bond's geometrical demonstrations of how to calculate longitude or determine magnetic variation or inclination presupposes the co–latitude of the magnetic poles to be immutable.

We are thus left with (4): that is, the motion of the magnetic sphere is caused by the motion of the geographical poles as a result of the precession of the equinoxes—otherwise, the co–latitude of the magnetic and geographical poles could not remain at 8° 30 min. But Hobbes identifies a flaw in this scenario as well. Hobbes takes the problem to be the "vast difference […] between the period of the Motion of the Equinoctial points, which are about or near 36000 years according to Copernicus (lib. 3 cap. 6) […] and the period of the Magnetical poles Motion, which is but 600 years."[43] I take Hobbes to formulate here a *reductio* argument: according to Bond, it takes 600 years for the magnetic sphere to make a complete revolution,[44] and it takes 36000 years for the geographical poles to make a complete revolution, which would mean that the magnetic sphere moves faster than the motion of the poles, but this goes directly against Bond's claim that the magnetic sphere moves slower. In evaluating these four possibilities, Hobbes shows that Bond does not have a

[42] *Ibid.*, p. 117.
[43] *Ibid.*, p. 117.
[44] Bond, *Longitude Found*, p. 7.

consistent account of the cause of the motion of the magnetic sphere. But he also goes a step further and shows that Bond's concept of the magnetic sphere is itself confused.

As we have already seen, according to Bond, the magnetic sphere encompasses (surrounds) the Earth. This is the conception that Hobbes argues against, as he takes Bond to violate a basic principle in natural philosophy, namely that "two bodies cannot be in one place" at the same time. In effect, Hobbes' strategy is to show that the magnetic sphere cannot be a body. And if it cannot be a body, then it just cannot be at all (given Hobbes' materialism). Hobbes' reasoning is straightforward. If the magnetic sphere "encompasses the Earth," since two bodies cannot be in the same place, then there is no real distinction between the magnetic sphere and the Earth itself. If so, then the Earth itself is a giant magnet, and the poles of the magnetic sphere are in fact the poles of the Earth.

The other possibility Hobbes considers is that, for Bond, the magnetic sphere might not in fact be a body but a virtue, such that it is this magnetic virtue that causes the magnetic effects, and not a body. Hobbes deals with this possibility from within his framework. He claims that: "[..] if the Body of the Magnet be not there, the virtue then is the virtue of the Earth; and so again the Poles of the Earth are Magnetick Poles."[45] Hobbes' strategy here is not to deny the existence of virtues altogether, but to implicitly use a *reductio* argument. Virtues are properties of bodies. Since there is no body of the magnet there (on pain of the violation of the natural–philosophical principle that no two bodies occupy the same place at the same time) then the virtue cannot be the virtue of the magnet, but has to be the virtue of the body that is *there*, namely the Earth. From here, Hobbes moves on to conclude that if the magnetic virtue is the virtue of the Earth, then the poles of the Earth simply are the magnetic poles.

This latter argumentative move, however, is easy to reject for a philosopher ready to entertain an ontological distinction between bodies and virtues, since this would entail that bodies and their virtues can (at least, in principle) have distinct properties, such that the implication that the poles of the Earth and the magnetic poles coincide need not follow. To make this point clear, let's consider that, according to several seventeenth–century natural philosophers, there is an ontological distinction between the spatial location of a body and that body's virtue.[46] The spatial location of a body's virtue is the distance at

[45] *Ibid.*, p. 116.

[46] The possibility I consider here can be reconstructed in Bacon's account of a body's virtue. For such reconstructions, see Silvia Parigi, "Effluvia, Action at a Distance, and the Challenge of the Third Causal Model," *International Studies in the Philosophy of Science* 29.4 (2015), pp. 351–368; Dana Jalobeanu, "'Boders,' 'Leaps' and 'Orbs of Virtue:' A Contextual Reconstruction of Francis Bacon's Extension–Related Concepts," in Koen Vermeir and Jonathan Regier

which the virtue is causally efficacious, whereas the body's own spatial location refers to its boundaries as discerned by our perceptual apparatus. I take it that, in such accounts, the principle that two bodies cannot occupy the same place at the same time is reworked considerably. Given the previous distinction, it is the case that *stricto sensu* two bodies cannot occupy the same physical location at the same time, but in the case of a body's virtue (or a body's virtues), things get much murkier, since multiple virtues can share the same location simultaneously. This is because locations are now determined by the measurement of the virtues' range of causal efficacy; they are not determined prior to such measurements.

We see in this way that, in this particular case, Hobbes' arguments would not hit home if Bond were working, for instance, with Gilbert's virtue–based account of magnetism. The magnetic sphere that surrounds the Earth acts as the Earths' virtue (or one of its virtues that happens to be magnetic, and which affects only those bodies that are recipients of the magnetic virtues, that is, iron and loadstone, for instance). It is not easy to establish, however, whether Bond does follow Gilbert in this respect. Bond does not seem overly concerned with philosophical concepts of body, and he says virtually nothing on the subject, such that it is not clear whether he held any particular position. Of course, this does not preclude what I take to be Hobbes' criticism here, namely that Bond operated with a confused notion of body. At the same time, it points to an important distinction between the levels at which Bond's and Hobbes' accounts operate. Hobbes' reading of Bond's account operates at the level of interaction between physical material bodies—bodies, that is, whose properties are first and foremost material. Bond's account is, I take it, much less about such a concept of body. As already discussed in section 2.3, for Bond, the relevant bodies are geometrico–physical, such that relevant aspects of the body can be treated as points, arcs and geometrical manipulations on paper. It is not necessary for relevant properties of a geometrico–physical body to show depth or breadth, or to stand for a *stricto sensu* physical body (as Hobbes would have it). What matters for Bond is to use well–grounded geometrical (mostly trigonometrical) relations in order to make manifest previously unknown connections. What makes the latter goal possible is that the objects at stake are hybrids of geometrical and physical considerations.

3.2. Shifting spherical angles

Leaving behind Bond's implicit natural philosophical commitments, Hobbes also criticises Bond's geometrical reasoning, and it is to these that

(eds.), *Boundaries, Extents and Circulations: Space and Spatiality in Early Modern Natural Philosophy*, Dordrecht: Springer, 2016, pp. 229–254.

we return here. The goal of Hobbes' criticism is twofold: to show that the geometrical reasoning that led Bond to an ontological distinction between magnetic and geometrical poles is unfounded, and to show that Bond has a fundamentally flawed understanding of spherical triangles ("he knoweth not sufficiently the Doctrine of Spherical Triangles"[47]). The present subsection is focused mainly on the latter point. It aims to show that the goal of Hobbes' criticism is not Bond's own use of geometry, but an entire mathematical tradition which relates angles and chords in spherical triangles.[48]

Part of Hobbes' critique is the result of a misunderstanding of Bond's diagrammatic representation of his geometrical reasoning, and part of it has to do with Hobbes' idiosyncratic account of basic mathematical objects. Hobbes' criticism, however, misses the mark in a few ways, the most important of which is that his response to Bond relies on an account of angles that Bond would not have accepted. A close examination of Hobbes' response reveals incompatibilities about their respective views on the conditions that make certain geometrical moves possibilities (or vice versa).

Hobbes's response to Bond's use of geometry targets the geometrical demonstration by which Bond takes himself to have determined the location of the magnetic poles (Figure B). To understand Hobbes' criticism, let's first introduce and then briefly discuss Bond's geometrical reasoning:

> For finding the Latitude of the [magnetic] Poles, we have in the Triangle LPV, the co–latitude of London LP 38deg28m and PV, the co–latitude of *Vaygates*, 20d.00m and the angle LPV, sere 58d.00m the Difference of Longitude between *London* and *Vaygates.* To find the Side LV, the Distance between *London* and *Vaygates,* and the angles PLV, and PVL, from PVL subtract 11d15m. and there remains the Angle VLM. And substract 8d38m. from PVL, and there will remain the angle LVM. Then in the Triangle LMV, we have the Sides LV, and the Angles LMV, and MVL, to find the Sides LM and VM, the distances of the Magnetical Pole from the Zenith of London, and the Zenith of Vaygats: And lastly, we have in two Triangles, vis. PLM and PVM, two Sides and one Angle included, to find PM, the distance of the Magnetical Pole from the Pole of the Earth, 8d.30M.[49]

[47] Hobbes, *Decameron*, p. 110.

[48] I will not discuss here in great detail Hobbes' philosophy of mathematics. On this, see Douglas Jesseph, *Squaring the Circle: The War Between Hobbes and Wallis*, Chicago: University of Chicago Press, 1999; Douglas Jesseph, "The Decline and Fall of Hobbesian Geometry," *Studies in the History and Philosophy of Science* 30 (1999), pp. 425–453; Katherine Dunlop, "Hobbes's Mathematical Thought," in A. P. Martinich and Kinch Hoekstra (eds.), *The Oxford Handbook of Hobbes*, Oxford: Oxford University Press, 2016, pp. 76–105.

[49] Bond, *Longitude Found*, p. 5.

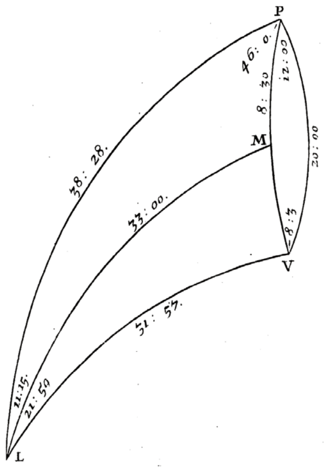

Figure B

The diagram is a geometrical construction for finding the location of the magnetic pole (M in Figure B).[50] The diagram is used in order (1) to determine the value of MP, and in this way to find the location of the magnetic pole, and (2) to show that the motion of the magnetic sphere *could* be from east to west. Some of the values Bond takes to be given are: (1) the magnetic variation at London (the angle PLM = 11° 15 min.), a value which he reports as the value calculated by William Burrowe in his *Discourse on the Variation of the Compass (*1581)[51] (2) the co–latitude of London = 38° 28 min; (3) the co–latitude of Vaygates = 20°; and (4) the difference of longitude between London and Vaygates = 58°. This is particularly clear if we consider that, in Bond's time, the difference of longitude was given by the number of degrees that separate the observer from the initial meridian, which always referred to the default starting position. Bond takes the London meridian to be the initial meridian. Here again (as in the case of his use of magnetic measurements), we see Bond ready to use questionable knowledge preliminarily, with the goal of constructing a theoric to answer the problem at stake. Given that Bond takes values (1) to (4) and a measurement of magnetic variation from 1580 as instrumental in determining the position of the magnetic pole for 1676, it is safe to conclude that Bond must have already accepted that the co–latitude of the magnetic pole from the geographical pole remains constant. In short, he does not allow for changes in the tilt of the magnetic pole. Had the tilt shifted,

[50] Alongside Hobbes, Peter Blackborow, *The Longitude Not Found*, London: Harford, 1678, also criticises the same construction. Indicating once more the obscurity of Bond's text and explanation, Blackborow's reconstruction of the mathematical considerations behind the diagram is different from Hobbes's.

[51] Burrowe's table for magnetic variation at London is reprinted in most textbooks of navigation at the time, so it is not too surprising that Bond takes it to be an uncontroversial value. However, it should be pointed out that the value of 11° 15 min is rather a mean of different values of variation that Burrowe recorded over a single day, as Burrowe's calculations of magnetic variation also took into consideration the elevation of the Sun at the moment of the measurement, since the east–west displacement of the magnetic compass was calculated against the variation of the Sun's shadow: the values of magnetic variation that Burrowe calculates range from between 11° 11 min 30 sec to 11° 30 min, which makes Bond's approximation in line with Burrowe's own calculations (William Burrowe, *A Discourse of the Variation of the Compass*, London: Printed by Richard Ballard, 1581, page not numbered).

past values of magnetic variation could not have been used instrumentally to find the actual tilt.

With these values given, all that was left was to apply the doctrine of spherical triangles in order to determine the value of MP. Bond considers the trigonometrical property of the following spherical triangles. First, there is the triangle LVP, in which he knows the values of LP, VP and the angle LPV. He uses these to calculate the arch LV and then the angles VLP and LVP. Given that the value of the angle PLM is known, he can then also determine the value of the angle MLV, which then allows him to calculate the sides and angles of, second and third, the triangles MLV and PLM. In this way, he can find the value of the arc MP.

But things are not quite so straightforwardly clear. The visual representation of the diagram in Figure B is indeed confusing. PMV is represented as if it were a single chord. But if we consider what the points P, M and V stand for, it becomes evident that they are not in fact points on the same arc. P is the geographical pole, V is the location of Vaygates and M is the magnetic pole. This means that PM measures some distance between the magnetic pole and the geographic pole, while VM is the distance between Vaygates and the magnetic pole, such that the angle MVP represents the magnetic variation at V. So the diagram does not represent a cross–section of the same plane. While PLV is a representation in the same plane, the triangles PLM and MLV are different planes, because M, the magnetic pole, is tilted from the geographical pole.

Given this, it should be clear that the diagram is not intended to establish the *existence* of the magnetic pole M, nor *that* it is tilted from P, but merely to determine the *amount* of tilt. That distinct magnetic poles exist ought to be inferred from the *success* of the predictions of the inclination–longitude theoric that Bond takes himself to have calculated. And because the *tilted dipole* is constitutive for the *actual* construction of the theoric, then its existence can be predicated on the success of the theoric.

Now, Bond is not explicit about the different cross–sections that ought to be considered in the diagram, and by not specifying them he seems to have confused some of his readers. This is certainly true of Hobbes, who claims that "if PM be 8deg.30min. and VM 8deg. 38 min the whole arc PMV will be 17deg.8min which should be 20deg."[52] We see here Hobbes taking PMV to be an arc rather than an intersection of two distinct arcs.

Hobbes' criticism however does not restrict itself to a misunderstanding of Bond's diagram. Rather, Hobbes uses Bond's use of spherical triangles in order to show that fundamental aspects of the enterprise are flawed. This explains the easiness with which Hobbes shifts from the criticism of Bond's reasoning to a more general criticism:

[52] Hobbes, *Decameron*, p. 114.

> A. And I see besides that he takes the Superficies that lyeth between the sides LP and LM for an Arch, which is the quantity of an Angle; and is a Line, and cannot be taken out of a Superficies […]
> B. […] A corner (in vulgar speech) and an Angle (in the language of geometry) are not the same thing. But it is easy even for a learned Man sometimes to take them for the same, as this Author now has done;and proceeding he saith, *substract 8deg 38min from the angle PVL, and there remains the angle VLM.*
> A. That again is false, because impossible. What was it that deceived him now?
> B. The same misunderstanding of the nature of a spherical angle. […] when he knew the arc VP was part of a great circle, he thought PM (which he maketh 8deg. 30min.) were also part of a great circle; which is manifestly false. For two great Circles (because they pass through the Center) do cut each other into halves. But PV is not half a Circle.[53]

This long passage is the cornerstone of Hobbes' claim that Bond had not understood the doctrine of spherical angles, and consequently misapplied it in the demonstrations. Let's disentangle what Hobbes aims to get at in his criticism by first explaining why he claims that Bond misunderstands "the nature of a *spherical angle.*" What Hobbes is saying here is that Bond assumes a spherical angle—which can be addressed by spherical trigonometry—where there is no spherical angle. The troublesome angle is the one corresponding to the arc $\widehat{PM}$. The problem is that spherical trigonometry only works with arcs of so–called "great circles," but, according to Hobbes, $\widehat{PM}$ is part of a small circle. Great circles are circles that pass through the center of a sphere, and because they divide the sphere in half, they will always be the biggest circles you can draw within the sphere (hence "great"). What this means is that one can use the properties of the sphere itself to calculate great circles: where great circles cross each other, you get angles that can be calculated with spherical trigonometry. On the other hand, small circles, are those which do not pass through the center of the sphere, and this is why they cannot be calculated in the same way. If Hobbes thinks $\widehat{PM}$ is not part of a great circle, then he thinks it's part of a small circle, which means that Hobbes takes the entire geometrical construction that Bond presents to be mistaken, because if $\widehat{PM}$ and $\widehat{VM}$ are parts of small circles, they should not be addressed using spherical trigonometry, which is precisely what Bond does. But Hobbes' criticism here is again based on a misunderstanding of Bond's construction: where Hobbes takes the points P, M and V to be on the same arc, for Bond, VM, MP and PV are not even on the same plane, never mind the same arc. Hobbes misconstrues Bond's diagram because the apparent curve between PMV misleads him into thinking it is a single arc, and because his natural–philosophical commitments force him to see what Bond thinks of as the magnetic sphere as coinciding with sphere of the Earth itself. Bond, however, takes the magnetic

[53] *Ibid.*, pp. 113–114.

sphere to be fully separate from the terrestrial sphere, which means that M is on a different sphere entirely, and PM and MV are really two sides of a spherical triangle rather than one single arc.

Let's move on to the next criticism, according to which "the Superficies that lyeth between the sides LP and LM form an Arch, which is the quantity of an Angle; and is a Line, and cannot be taken out of a Superficies."[54] To make sense of Hobbes' claim we have to begin by recalling that, in taking the science of geometry not as abstracted away from the physical properties of corporeal extended objects, but as co–extensive with them, Hobbes sees himself obliged to challenge the Euclidian definitions of many mathematical objects, such as points, lines, angles and so on. This is not inconsequential. Let's take the concept of a line, which Hobbes defines as follows:

> the shortest line is that, whose extreme points cannot be drawn farther asunder without altering the quantity, that is, without altering the proportion of that line to any other given line.[55]

Given this definition, there is a difference between a right line and curved line, since, in the case of a curved line, the endpoints can be drawn further apart while maintaining the same length, while the right line is a limit case of a line in which its termini are as far apart as they can be without changing the actual length of the line. Notice, however, that this definition makes the relative position of the line's termini points just as relevant as the length of the line. The termini points can be moved closer together or further apart, and such motion, although it might not make a geometrical difference in terms of the length of the line, can make a physical difference: consider that closer or further are relevant in relation with a given physical magnitude! Hobbes' discussion of a line shows just how much of his geometrical considerations are borne out of physical considerations. Thus, Hobbes claims that

> between two points given, there can be understood but one strait line; because there cannot be more than one least interval or length between the same points.[56]

Here, Hobbes does not care so much for consideration about geometrical magnitude, but rather connects the concept of the right line with the least

[54] Hobbes, *Decameron*, p. 114.

[55] For Latin see Thomas Hobbes, *Elementorum philosophiae sectio prima De corpore*, 2.14.1, London: Printed for Andrew Crooke, 1655, p. 106. For the English translation see the anonymous translation, published under the title *Elements of philosophy the first section, concerning body written in Latine by Thomas Hobbes of Malmesbury; and now translated into English; to which are added Six lessons to the professors of mathematicks of the Institution of Sr. Henry Savile, in the University of Oxford*, London: Printed by R. & W. Leybourn for Andrew Crooke, 1656, p. 129. The citations will be given as Hobbes, *De corpore*, and pages for both Latin and English editions in this order.

[56] Hobbes*, De corpore*, p. 107; p. 130.

path travelled between two intervals of termini. I take such considerations to be the rationale behind Hobbes' insistence that the distance between the geographical pole and the magnetic pole should be a *line*, that is a *straight line*, a *right line*. If the magnetic pole is to *always remain* at the same distance relative to the geographical pole and not move further or closer to the actual geographical pole, thus shifting and changing the values of magnetic variation, the distance between the magnetic pole and the geographical pole ought to have been given as a right line and not an arc of a circle.

A similar analysis reveals why Hobbes claims that "A corner (in vulgar speech) and an Angle (in the language of Geometry) are not the same thing."[57] One might believe that the criticism is about the limitation of the reduction of the physical to the mathematical. But this is not what is at stake. Hobbes is not claiming here that *corners* as physical objects cannot be represented or abstracted as mathematical objects, because the angle they form is unavoidably curved or ragged—and therefore representable geometrically with a loss. No: given the context in which Hobbes introduces the criticism, his complaint lies in what he (again) sees as Bond's *conflation* of fundamentally different kinds of angles.

In his philosophy of mathematics, Hobbes introduces different kinds of angles because they have different causal origins (i.e., their construction entails different operations). I will not delve into all of the angular distinctions he proposes, but will briefly discuss the ones which I take to be relevant in Bond's context. Thus, according to Hobbes, an angle made by "the concurrence of lines" is a superficial angles, and an angle made by "the concurrence of superficies, […] it is called a solid angle."[58] Now, superficial angles are separated into two classes, depending on how the two lines diverge from one another: if when separating the two lines, the lines remain straight and of the same quantity then what you get is "an angle simply called" (or a simple angle). However, if the two lines are separated by "continual flexion or curvation in every imaginable point," then, "the quantity of this separation is that which is called an Angle of Contingence."[59] This means then that the quantity of an angle of contingence is the extent of the respective "curvation." Because they are differently constructed, and ultimately ought to be differently measured, for Hobbes, the simple angles and the angles of contingence are incommensurable. We see thus Hobbes claiming that:

> An angle of Contingence, if it be compared with an Angle simply so called how little soever, has such proportion to it as a Point has to a Line; that is, no proportion at all, nor any quantity. For first, an angle of contingence is made by continual flexion, so that in the generation of it there is no circular motion

[57] Hobbes, *Decameron*, p. 113.
[58] Hobbes, *De corpore*, p. 110; pp. 134–135.
[59] *Ibid.*, p. 110; p. 135.

> at all, in which consists the nature of an Angle simply so called; and therefore it cannot be compared with it according to Quantity.[60]

This is the context explaining why Hobbes claims that whereas a simple angle is measured by the ratio of the arc to the whole perimeter because it is constructed by the rotation of a line around a terminus, an angle of contingence is measured by the quantity of its curvature because it is constructed by the bending of a right line.

The spherical angles that Hobbes discusses in analysing Bond's geometrical reasoning are—given his definitions—angles of contingence. Hobbes' point against Bond is that the latter treats the angles as simple angles, when in fact he should have treated them as angles of contingence. Given that the measure of a *simple angle* is "arch or circumference of a circle, determined by its proportion to the whole perimeter" if the sides of the angle are right lines,[61] one can use angular values to determine the measure of the arcs (and the other way around, of course).

But the same cannot be said for the angles of contingence, for which "it is impossible to subtract an Arc of Circle out of a Spherical angle."[62] The construction of angles of contingence does not entail "any circular motion"; they are constructed by the "divergence of two lines which concur in one sole point" (the corner) and whose measure or "quantity consists in greater and less flexion."[63] Angles of contingence are to be measured in terms of their degree of curvature. This entails that for angles formed by curved (or crooked) lines, the "quantity of the angle is to be taken in the least distance from the centre, or from their concurrence."[64]

The distinction between simple angles and angles of contingence is one that Bond does not assent to. But Hobbes, of course, does. And this means that, for Hobbes, the measures of angles such as angles VLM, MLP or VLP cannot simply be added or subtracted (they are formed by curved lines). These are angles of contingence, not simple angles. And the measurement of angles of contingence entails the determination of their curvature, which in return means that their individual values are not additive. One cannot subtract the angle PLM out of the angle PLV in order to obtain the value of MLV, something which Bond appears to be doing, since he directs his reader to "subtract 8deg38min. from the angle PLV, and there remains the angle VLM."[65] This is Hobbes' strategy against Bond—to show that Bond takes incommensurable geometrical objects to be homogeneous. He uses this strategy in many

[60] *Ibid.*, p. 117, p. 143.
[61] *Ibid.*, p. 111; p. 136.
[62] Hobbes, *Decameron*, p. 113.
[63] Hobbes, *De corpore*, p. 117; p. 143.
[64] *Ibid.*, p. 111; p. 136.
[65] Hobbes, *Decameron*, quoting Bond, p. 113.

different places: "lines cannot be taken out of superficies"; "spherical angles are not very angles," and so on.

From Bond's vantage point, Hobbes' critique here would have missed its mark. Bond was using principles of the doctrine of spherical triangles well established in the tradition of navigational mathematics, in which the determination of the sides and the angular values of spherical triangles entailed trigonometrical calculations.[66] For the construction of such geometrical tools, experts in navigation would use tables of secants, sine and cosine, and logarithms in order to find the missing variables. Bond seems to have used the exact same strategy; it was the default way of dealing with navigational problems at the time. Given Hobbes' account of angles of contingence, the whole enterprise of the doctrine of spherical triangles would have to be rethought, as most of its axioms and ways of problem solving would break down. Hobbes knew this very well. It is one of the consequences of his taking mathematics to derive out of his philosophy of bodies. For Hobbes, intersections of figures always have to be causally productive. Cross–sections or intersections of planes are causally efficacious, not causally inert, operations, such that the mathematical properties of newly resulting objects are altered by the very fact of plane interaction. Bond's theorics do not work in this manner. They remain incommensurable to Hobbes' criticism because the foundations on which Bond's and Hobbes use of mathematics rely are entirely different.

4. Conclusion

Thomas Hobbes makes the doctrine of spherical triangles useless for the investigation of physical phenomena, while Bond makes it instrumental in deciphering the causal structure of such physical phenomena. Their radically different attitudes come down to orders of priority. For Hobbes, geometry is built on his materialist philosophy of body. No geometrical definition can violate and be inconsistent with his natural–philosophical considerations about the nature of bodies, and of bodies in motion. For Bond, the method of spherical triangles and theoric construction have priority. With their help,

[66] We do not know what textbook(s) of trigonometry Bond used, but we do know that he was well acquainted with Wright's *Certaine Errors*. While discussing the method for the calculation of the secant, Wright is explicit about his use of Bartholomew Pitiscus's very popular *Trigonometria: sive de solutione triangulorum tractatus brevis et perspicuus*, 1595, which was republished in 1600 in Latin and got translated soon after into English, in 1614, with two other new editions shortly afterward. It is not unlikely that Bond was familiar with Pitiscus' text, but there is no historical evidence to this effect. What matters here is that similar instructions for solving physical problems with the use of spherical triangles in similar ways to those used by Bond can be found in, e.g., Wright's *Certaine Errors*, Seller's *Practical Navigation*, see especially chapter XVIII, pp. 101–113), and so on. Unlike Hobbes, whose idiosyncratic mathematics is at odds with the practitioner of the art of navigation, Bond's is continuous with it.

the empirical findings are concatenated and placed into an ordered relation. The armchair geometrical manipulations of intersections of meridians and spherical motions are not just instruments of calculation: *through* them, the relevant phenomena are constructed and brought into the space of methodical examination. And, in doing this, Bond is playing an old game. A game in which magnetism, navigation and fundamentally astronomical techniques of problem solving come together as they had done in the English tradition of the mathematical sciences for over a century. What is interesting to notice in both Bond's and Hobbes' distinct mixtures of geometry and physics is that in neither cases can geometry and physics be easily disentangled (if at all). For Hobbes, physical and geometrical definitions are deliberately blended into a unit, and he is ready to sacrifice well–established traditions of geometrical reasoning to make his blend feasible. For Bond, the intelligibility of the physical is given through geometrical relations. Thus, for neither is geometry a mere abstraction from the phenomena, nor a representation of them. For each, the physical "phenomenon" is instantiated through the geometrical, but it is so in different ways.

Works Cited:

Martha Baldwin, *Kircher and the magnetical philosophy*, Chicago: University of Chicago (Department of History), PhD, 1987.

Jim A. Bennett, "The Longitude and the New Science," *Vistas in Astronomy* 28 (1985), pp. 219–225.

——— "Knowing and Doing in the Sixteenth Century: What Were Instruments For?," *British Journal for the History of Science* 36.2 (2003), pp. 129–150.

——— "Mathematics, Instruments and Navigation, 1600–1800," in Glen Van Brummelen and Michael Kinyon (eds.), *Mathematics and the Historian's Craft: The Kenneth O. May Lectures*, New York: Springer, 2005, pp. 43–57

——— "Modern Mathematical Instruments," *Isis* 102.4 (2011), pp. 697–705.

Thomas Birch, *The History of the Royal Society of London*, vol. I, London: A. Millar, 1756.

Peter Blackborow, *The Longitude Not Found or, An answer to a treatise written by Henry Bond, Senior, shewing a way to find the longitude by the magnetical inclinatory needle wherein is proved that the longitude is not, nor cannot be found by the magnetical inclinatory needle*, London: Printed for Robert Harford, 1678.

Henry Bond, "The undertakings of Mr. Henry Bond Senior, a famous teacher of the art of navigation in London, concerning the variation of the variation of the magnetical compass and the inclination of the inclinatory needle; as the result and conclusion of 38 Years of magnetical study," *Philosophical Transactions of the Royal Society* 8.95 (1673), pp. 6065–6066

——— *The Longitude Found: Or, A Treatise Shewing An Easie and Speedy way, as well by Night as by Day, to find the Longitude, having but the Latitude of the Place,* *and the Inclination of the Magnetical Inclinatorie Needle*, London: Printed by W. Godbid, 1676.

D.J. Bryden, "Magnetic Inclinatory Needles: Approved by the Royal Society?," *Notes and Records of the Royal Society* 47 (1993), pp. 17–31.

William Burrowe, *A Discourse of the Variation of the Compass*, London: Printed by Richard Ballard, 1581.

Alan Chapman, "Gresham College: Scientific Instruments and the Advancement of Useful Knowledge in Seventeenth–Century England," *Bulletin of the Scientific instruments Society* 56 (1998), pp. 6–13.

René Descartes, *Œuvres de Descartes*, ed. Charles Adam and Paul Tannery, 11 vols, Paris: J. Vrin, 1996.

Katherine Dunlop, "Hobbes's Mathematical Thought," in A. P. Martinich and Kinch Hoekstra (eds.), *The Oxford Handbook of Hobbes*, Oxford: Oxford University Press, 2016, pp. 76–105.

Laura Georgescu, "One Experiment, Different Uses: Floating Magnetic Bodies in Peregrinus, Norman and Gilbert," *Journal of Early Modern Studies* 2 (2013), pp. 81–103.

Henry Gellibrand, *A Discourse Mathematical on the Variation of the Magneticall Needle. Together with its admirable diminution lately discovered*, London: Printed by William Jones 1635.

William Gilbert, *On the Magnet, Magnetick Bodies Also, and On the Great Magnet the Earth, a New Physiology, Demonstrated by Many Arguments & Experiments* [Translated by S.P. Thompson], London: Chiswick Press, 1900.

Edmund Halley, "A Theory of the Variation of the Magnetical Compass," *Philosophical Transactions of the Royal Society* 13.148 (1683), pp. 208–221.

Hartlib Papers 71/16/1A, *A discovery of the True & Infallible way of finding the long hidden Secret of Longitude. By Henry Bond Practitioner in the Mathematickes*, 1639. Accessible online: https://www.dhi.ac.uk/hartlib/view?docset=main&docname=71_16_01&term0=transtext_bond#highlight

Rom Harré, *The Method of Science*, London: Wykelham Press, 1970.

Thomas Hobbes, *Elementorum philosophiae sectio prima De corpore*, London: Printed for Andrew Crooke, 1655.

——— *Elements of philosophy, the first section concerning Body, written in Latine by Thomas Hobbes of Malmesbury; and now translated into English; to which are added Six lessons to the professors of mathematicks of the Institution of Sr. Henry Savile, in the University of Oxford*, London: Printed by R. & W. Leybourn for Andrew Crooke, 1656.

——— *Decameron physiologicum*, London: Printed by J.C. for W. Crook, 1678.

Richard J. Howarth, "Fitting Geomagnetic Fields before the Invention of Least Squares: I. Henry Bond's Predictions (1636, 1668) of the Change in Magnetic Declination in London," *Annals of Science* 59 (2002), pp. 391–408.

Dana Jalobeanu, "'Boders,' 'Leaps' and 'Orbs of Virtue:' A Contextual Reconstruction of Francis Bacon's Extension–Related Concepts," in Koen Vermeir and Jonathan Regier (eds.), *Boundaries, Extents and Circulations: Space and Spatiality in Early Modern Natural Philosophy*, Dordrecht: Springer, 2016, pp. 229–254.

Douglas Jesseph, *Squaring the Circle: The War Between Hobbes and Wallis*, Chicago: University of Chicago Press, 1999.

——— "The Decline and Fall of Hobbesian Geometry," *Studies in the History and Philosophy of Science* 30 (1999), pp. 425–453.

Stephen Johnston, "Theory, Theoric, Practice: Mathematics and Magnetism in Elizabethan England," *Journal de la Renaissance* 2 (2004), pp. 53–62.

A.R.T. Jonkers, *Earth's Magnetism in the Age of Sail*, London: The Johns Hopkins University Press, 2003.

——— "The Pursuit of Magnetic Shadows: The Formal-Empirical Dipole Field of Early–Modern Geomagnetism," *Centaurus* 50 (2008), pp. 254–289.

S.R.C. Malin & E.C. Bullard, "The Direction of the Earth's Magnetic Field at London 1570–1975," *Philosophical Transactions of the Royal Society, Series A*, A299 (1981), pp. 357–423.

Robert Norman, *The Newe Attractive, Contayning a Short Discourse of the Magnes or Loadstone*, London: Printed by Richard Ballard, 1581.

Henry Oldenburg, "The variations of the magnetick needle predicted for many yeares following," *Philosophical Transactions* 3.40 (1668), pp. 789–790.

Silvia Parigi, "Effluvia, Action at a Distance, and the Challenge of the Third Causal Model," *International Studies in the Philosophy of Science* 29.4 (2015), pp. 351–368.

Bartholomew Pitiscus, *Trigonometria: sive de solutione triangulorum tractatus brevis et perspicuus*, 1595 [translated into English by Ralph Handson, *Trigonometry: Or the doctrine of triangles*, London: Printed by Edw. Allde for Iohn Tap, 1614].

Stephen Pumfrey, "Mechanising Magnetism in Restoration England—the Decline of Magnetic Philosophy," *Annals of Science* 44 (1987), pp. 1–22.

——— "'O tempora, O magnes!' A Sociological Analysis of the Discovery of Secular Magnetic Variation in 1634," *British Journal for the History of Science* 22 (1989), pp.181–214.

John Seller, *Practical Navigation*, London: Printed by J. Darby, 1669.

Thomas Sonar, "Navigation on Sea: Topics in the History of Geomathematics," in W. Freeden, M. Z. Nashed, & T. Sonar (eds.), *Handbook of Geomathematics*, Berlin: Springer, 2010, pp. 43–68.

David Waters, *The Art of Navigation in England in Elizabethan and Early Stuart Times*, New Haven: Yale University Press, 1958.

William Whiston, *The Longitude and Latitude Found by the Inclinatory or Dipping needle*, London: Printed for J. Senex and W. Taylor, 1721.

Edward Wright, *Certaine Errors in Navigation, Arising either of the ordinarie erroneous making or vsing of the sea Chart, Compasse, Crosse staffe, and Tables of declination of the Sunne, and fixed Starrs detected and corrected*, London: Printed by Joseph Moxon, 1610.