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Brief paper

Modeling and passivity properties of multi-producer district heating systems ☆

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ABSTRACT

We propose a comprehensive nonlinear ODE-based thermo-hydraulic model of a district heating system featuring several heat producers, consumers and storage devices which are interconnected through a distribution network of meshed topology whose temperature dynamics are explicitly considered. Moreover, we analyze the conditions under which the hydraulic and thermal subsystems of the model exhibit shifted passivity properties. For the hydraulic subsystem, our claims on passivity draw on the monotonicity of the vector field associated to the district heating system's flow dynamics, which mainly codifies viscous friction effects on the system's pressures. For the temperature dynamics, we propose a storage function based on the *ectropy function* of a thermodynamic system, recently used in the passivity analysis of heat exchanger networks.

1. Introduction

District heating (DH) has been identified as a key technology to enable the heating sector's potential to reduce greenhouse emissions due to the possibility to seamlessly include environmentally friendly energy sources and storage devices (Dominković, Stunjek, Blanco, Madsen, & Krajačić, 2020; Lund et al., 2014; Vandermeulen, van der Heijde, & Helsen, 2018; Werner, 2017). A DH system comprises a network of pipes connecting buildings in a neighborhood, town center or whole city, so that they can be served from varied heat production units (Lund et al., 2014). To further unlock the potential of DH systems, prospective installations will feature multiple, distributed heat sources, *e.g.,* waste-to-energy facilities or solar collectors (Lund et al., 2014), promoting as a consequence heat distribution networks of meshed topology, as opposed to the salient tree-like structure of conventional installations with a single heat source (Vesterlund, Toffolo, & Dahl, 2017; Wang et al., 2017).

Modeling aspects of DH systems with a single heat producer have received considerable attention. Following a rigorous graph-theoretic approach, a control system based on ordinary differential equations (ODE) and focused on the hydraulic layer, is presented in De Persis and Kallesoe (2011). Volume and temperature dynamics are modeled in Scholten, De Persis, and Tesi (2015) for a DH system based on a single stratified water storage tank adjacent to the heat production plant. Comprehensive, partial differential equation (PDE)-based DH pipe models are presented in Hauschild et al. (2020), Krug, Mehrmann, and Schmidt (2021), Rein, Mohring, Damm, and Klar (2021): a fully discretized model is subsequently obtained in Krug et al. (2021) to be used in the design of a nonlinear optimal controller; semi-discrete and PDE-based DH port-Hamiltonian models (van der Schaft & Jeltsema, 2014) are formulated in Hauschild et al. (2020); and model reduction is performed on a semi-discretized DH system model in Rein et al. (2021). Modeling of DH systems with multiple heat producers has been addressed in Alisic, Paré, and Sandberg (2019), Trip, Scholten, and De Persis (2019), Vesterlund et al. (2017), Wang et al. (2017). (Static) steady-state hydraulic and thermo-hydraulic modeling is considered in Wang et al. (2017) and Vesterlund et al. (2017), for the purposes of solving operational optimization problems. Dynamic volume storage modeling (and control) is presented in Trip et al. (2019) for a DH system with multiple storage tanks. A similar model is established in Alisic et al. (2019), which further considers temperature dynamics.[1]

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[1] We refer the reader to Talebi, Mirzaei, Bastani, and Haghighat (2016) and Guelpa and Verda (2019) for surveys on DH modeling and thermal energy storage in DH applications, respectively.

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Passivity analysis within the context of heating networks has been considered in Dong, Li, and Huang (2019), Mukherjee, Mishra, and Wen (2012). In Mukherjee et al. (2012), a (linear) model to describe the temperature dynamics of a multi-zone building is presented and subsequently shown to be passive via a storage function which is quadratic in the rooms' temperatures. In Dong et al. (2019), a general model of a network of heat exchangers is shown to be *shifted* passive using a novel storage function based on the concept of ectropy (Haddad, 2019). It was mentioned in a previous paragraph that port-Hamiltonian formulations of (single producer) DH system models is presented in Hauschild et al. (2020), thus, passivity follows directly under mild assumptions (van der Schaft & Jeltsema, 2014).[2]

Contributions: In this paper, we build on De Persis and Kallesoe (2011), Hauschild et al. (2020), Krug et al. (2021), Rein et al. (2021), Scholten et al. (2015), Wang et al. (2017) and present a simplified ODE-based model that simultaneously describes the hydraulic and thermal dynamic behavior of a DH system featuring multiple heat producers, storage devices and consumers, all of them connected to a common meshed distribution network. The model is highly nonlinear due to the consideration of frictional forces in the flow dynamics and due to the coupling between the flow and temperature dynamics.[3] As a second contribution we analyze the conditions under which flow and thermal dynamics of the proposed model are shifted passive and briefly touch on the implications for the design of decentralized passivity-based controllers with stability guarantees. Our claims on the passivity of the DH system's flow dynamics are based on the observations made in De Persis and Kallesoe (2011), in the single producer setting, about the monotonicity of the associated vector field. On the other hand, following Dong et al. (2019) (see also Hauschild et al., 2020), for the thermal dynamics we propose a quadratic storage function based on the total ectropy, extending the results of Dong et al. (2019), Hauschild et al. (2020) to the multiple heat producers and multiple storage devices case.

Notation. The symbol $\mathbb{R}$ denotes the set of real numbers. For a vector $x \in \mathbb{R}^n$, x_i denotes its ith component, *i.e.*, $x = [x_1, \dots, x_n]^\top$; moreover, $\textbf{sign}(x) = [\text{sign}(x_1), \dots, \text{sign}(x_n)]^\top$, with $\text{sign}(0) = 0$, and $|x| = [|x_1|, \dots, |x_n|]^\top$. An $m \times n$ matrix with all-zero entries is written as $\mathbf{0}_{m\times n}$. An n-vector of ones is written as $\mathbf{1}_n$, whereas the identity matrix of size n is represented by I_n. For any vector $x \in \mathbb{R}^n$, we denote by $\text{diag}(x)$ a diagonal matrix with elements x_i in its main diagonal. For any time-varying signal w, we represent by $\bar{w}$ its steady-state value, if exists. Also, we write time derivatives as $\dot{x}(t)$, and omit the argument t whenever is clear from the context.

2. System setup

We consider a DH system with multiple heat producers, consumers and storage tanks. These devices are interconnected through a common meshed distribution network (see Fig. 1). Water is the medium used to transport thermal energy and we

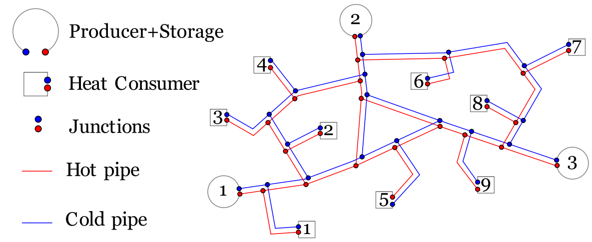

Fig. 1. Sketch of a DH system in which a distribution network interconnects 3 heat producers and 9 consumers (Wang et al., 2017). Each producer is interfaced to the distribution network through a storage tank (see Fig. 3 for details).

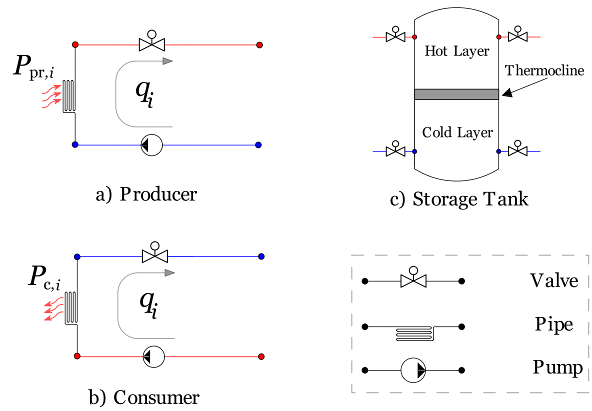

Fig. 2. Topologies of producers, consumers and storage tanks; see De Persis and Kallesoe (2011), Scholten et al. (2015).

assume that temperatures and pressures in the system are such that water is always in liquid state. The distribution network is considered to be conformed by two independent systems of pipelines. They respectively transport hot and cold water: hot water flows from heat producers to consumers via a supply layer and cold water flows in a converse manner through a return layer. It is assumed that the distribution network is symmetric, *i.e.,* the supply and return layers are topologically equivalent, and that the physical interconnection between the two layers is done exclusively via producers, consumers or storage devices. The overall system is assumed to be leak free without any loss of the fluid mass (Talebi et al., 2016). In this paper, producers, consumers and distribution network are conformed by basic hydraulic devices, namely, valves, pipes and pumps (see Fig. 2). Next we provide a summary about the precise composition and operation mode of each producer, consumer and storage tank (see Scholten et al. (2015) for more details). The hydraulic model of valves, pipes and pumps is detailed in Section 3.

Producer (consumer): Each producer (consumer) is represented as a connection in series of a pump, a pipe and a valve. The pipe models a tube through which the cold (hot) stream of a heat exchanger is flowing. We assume that, via the exchanger, the producer (consumer) directly injects (draws) a thermal power $P_{\text{pr},i}$ ($P_{\text{c},i}$) into (out of) the DH system.

Storage tank: A tank stores a mixture of hot and cold water which is perfectly separated by a thermocline. The hot layer is on top and the cold one at the bottom. It is assumed that there is no heat or mass exchange between the mixtures. The device has four valves, two at the top and two at the bottom, which are used as inlets and outlets of hot and cold water, respectively.

The overall DH system is viewed as a connected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ with no self-loops (see, *e.g.*, De Persis & Kallesoe, 2011;

[2] Ectropy is a quadratic function on the total energy of a thermodynamic system and is described in Haddad (2019) as the dual of entropy in the sense that it represents a measure of the tendency of a thermodynamic system to do useful work and grow more organized (see also Willems, 2006). On the other hand, shifted passivity is particularly relevant in these applications by allowing the stability assessment and stabilization of non-trivial equilibria (see Jayawardhana, Ortega, García-Canseco, & Castaños, 2007; Monshizadeh, Machado, Ortega, & van der Schaft, 2019; Monshizadeh, Monshizadeh, Ortega, & van der Schaft, 2019).

[3] Our modeling procedure is based on De Persis and Kallesoe (2011), Hauschild et al. (2020), Krug et al. (2021), Scholten et al. (2015), Vladimarsson (2014), Wang et al. (2017) and every important difference is emphasized throughout our paper (see, *e.g.*, Remarks 5 and 7).

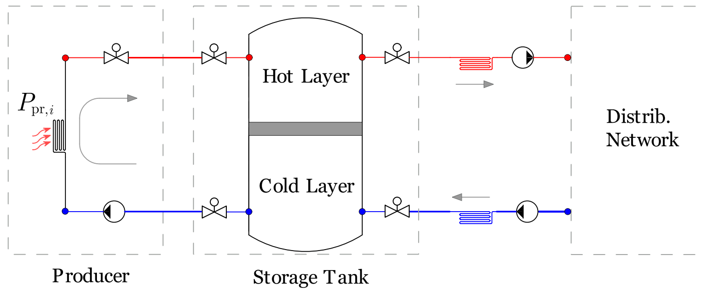

Fig. 3. Every producer in the system is interfaced to the distribution network through a storage tank. The gray arrows indicate the direction of the stream.

Hauschild et al., 2020; Krug et al., 2021; Wang et al., 2017). The set of edges $\mathcal{E}$ contains all two-terminal devices (valves, pumps or pipes), and the set of nodes $\mathcal{N}$ contains all junctions as well as the hot and cold layers of each storage tank. The cardinalities of $\mathcal{N}$ and $\mathcal{E}$ are denoted by n_{N} and n_{E}, respectively. Taking as reference the sketch in Fig. 1, the gray, blue and red lines therein represent edges, whereas colored circles, nodes.

The variables $q_{\mathrm{E},i}$, $V_{\mathrm{E},i}$, $T_{\mathrm{E},i}$ and $p_{\mathrm{E},i}$ denote the flow rate, volume, temperature and pressure of the stream through $i \in \mathcal{E}$. Analogous descriptions follow for the variables $V_{\mathrm{N},k}$, $T_{\mathrm{N},k}$ and $p_{\mathrm{N},k}$, $k \in \mathcal{N}$. Also, we fix an arbitrary orientation to every edge of $\mathcal{G}$. Then, for any $i \in \mathcal{E}$ with end nodes $j, k \in \mathcal{N}$, $j \neq k$, we say that j is the head and k is the tail of i, or vice versa, that j is the tail and k is the head of i. Then, for each node we define the following sets (Hauschild et al., 2020) (see also Krug et al., 2021; Vladimarsson, 2014):

$$\mathfrak{S}_k = \{i \in \mathcal{E} : k \text{ is the tail of } i \in \mathcal{E}\},\ \ k \in \mathcal{N}, \tag{1a}$$

$$\mathfrak{T}_k = \{i \in \mathcal{E} : k \text{ is the head of } i \in \mathcal{E}\},\ \ k \in \mathcal{N}, \tag{1b}$$

We define a constant incidence matrix $\mathcal{B}_0$ associated with the arbitrary orientation we have fixed for the DH system's edges, as follows:

$$(\mathcal{B}_0)_{i,j} = \begin{cases} 1, & \text{if node } i \text{ is the head of edge } j, \\ -1, & \text{if node } i \text{ is the tail of edge } j, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Remark 1. For simplicity of exposition, we introduce the preliminary assumption that the orientation of any edge $i \in \mathcal{E}$ matches the direction of the stream through it. That is, if $j, k \in \mathcal{N}$, $j \neq k$, are the tail and head of any $i \in \mathcal{E}$, respectively, then the stream through i is assumed to flow from j to k and, moreover, $q_{\mathrm{E},i} \geq 0$. However, some adjustments are necessary when modeling the DH system's temperature dynamics.

The following are standing assumptions in this work:

Assumption 1. (i) The density $\rho > 0$ and specific heat $c_{\mathrm{s.h.}} > 0$ of water are spatially uniform and constant in time. (ii) All pipes are cylindrical. (iii) The flow through any edge $i \in \mathcal{E}$ is (spatially) one-dimensional. (iv) Gravitational forces are neglected. (v) The pressure of each $k \in \mathcal{N}$ is spatially uniform and for each tank the pressure of its layers is equal. (vi) Each producer is interfaced to the distribution network through a storage tank as depicted in Fig. 3. (vii) Each device (pipe, valve, pump, storage tank, junction) is completely filled with water all the time. (viii) There are no standalone storage tanks in the system.

Remark 2. Assumption 1(i)–(iii) are fairly common in DH system modeling (see, *e.g.*, Krug et al., 2021; Scholten et al., 2015). Assumption 1(iv) is taken for simplification purposes. We consider Assumption 1(v) to discard inertial and viscous forces in the equations of balance of momentum at each node and thus simplify the modeling procedure. Assumption 1(vi) is taken without loss of generality. In fact, since producers and consumers are comprised by the same type of devices and have the same topology, then the modeling procedure and analysis described in this paper would still hold if we consider producers directly connected to the distribution network. Assumption 1(vii) is also common and, in particular, it implies that for each tank the sum of the volume of the hot and cold layer is constant (see, *e.g.*, Ismail, Leal, & Zanardi, 1997; Scholten et al., 2015). Assumption 1(viii) is mainly technical and current work is underway to relax it.

Remark 3. Storage tanks allow the continuous injection of heat to the system, at least for a period of time, even if the capacity of the associated heat producer is reduced. The design of control strategies for the adequate management of energetic resources in storage units is beyond the scope of this paper and is left as future research. Then, for the results we present in Section 5, we assume that sensible equilibria of the DH system exist, *i.e.*, producers are considered to be able to supply all the time the heat demanded by consumers.

3. Hydraulic model

In this section we present a model to describe the dynamic behavior of the hydraulic variables of the DH system.

First, we present a proposition where we establish a hydraulic model written as a differential algebraic equation (DAE). To streamline its presentation, we note that since the pressure is uniform in each tank (see Assumption 1(v)), then we find it convenient to reduce the DH system's graph $\mathcal{G}$ by viewing each tank as an individual node. More precisely, let $\mathcal{N}_{\mathrm{sh}}$ and $\mathcal{N}_{\mathrm{sc}}$ denote the storage tanks' hot and cold layers', respectively, and $\mathcal{N}_{\mathrm{sj}}$ all the simple junctions. Then, the graph $G = (N, \mathcal{E})$, with $N = \mathcal{N}_{\mathrm{ST}} \cup \mathcal{N}_{\mathrm{sj}}$, $\mathcal{N}_{\mathrm{ST}} := \mathcal{N}_{\mathrm{sh}} = \mathcal{N}_{\mathrm{sc}}$, represents a reduced version of $\mathcal{G}$ and $|N| = |\mathcal{N}| - n_{\mathrm{ST}} =: n_{\mathrm{n}}$. G has the same edges as $\mathcal{G}$ and we assume they have the same orientation. Also, we associate to G an incidence matrix B_0 which is analogous to (2). With some abuse of notation, we also define sets analogous to (1), and use the same symbols. Then, by construction G is connected.

Proposition 1. *Let Assumption 1 hold. Let p_{n} and V_{n} denote the vector of pressures and volumes of the nodes of G, respectively. Then, the overall flow and volume dynamics of the DH system is determined by the following DAE:*

$$-B_0^\top p_{\mathrm{n}} = \mathrm{diag}(J_{\mathrm{E}})\dot{q}_{\mathrm{E}} + f_{\mathrm{E}}(q_{\mathrm{E}}) - w_{\mathrm{E}}, \tag{3a}$$

$$0 = B_0 q_{\mathrm{E}}, \tag{3b}$$

where $J_{\mathrm{E},i} = \rho\ell_{\mathrm{E},i}/A_{\mathrm{E},i}$ if $i \in \mathcal{E}$ is a pipe and zero otherwise, $f_{\mathrm{E},i}(q_{\mathrm{E},i}) = K_{\mathrm{E},i}|q_{\mathrm{E},i}|q_{\mathrm{E},i}$, with $K_{\mathrm{E},i} > 0$ if $i \in \mathcal{E}$ is either a pipe or a valve (and zero otherwise), and if $i \in \mathcal{E}$ is a pump, then $w_{\mathrm{E},i}$ is an external input representing the differential pressure it produces between its terminals, otherwise $w_{\mathrm{E},i} = 0$.

Proof. We begin by establishing (3a). Let $i \in \mathcal{E}$ be an arbitrary pipe. Since ρ is constant (see Assumption 1(i)), then $q_{\mathrm{E},i}$ is a function of time only (Hauschild et al., 2020; Krug et al., 2021). It follows that the momentum balance at i can be written as (see Machado, Cucuzzella, & Scherpen, 2020 for details)

$$p^{\mathrm{in}}_{\mathrm{E},i} - p^{\mathrm{out}}_{\mathrm{E},i} = J_{\mathrm{E},i}\dot{q}_{\mathrm{E},i} + K_{\mathrm{E},i}|q_{\mathrm{E},i}|q_{\mathrm{E},i}, \tag{4}$$

where the second term in the right-hand side is the Darcy–Weisbach formula for frictional forces, and $K_{\mathrm{E},i} = \theta_{\mathrm{E},i}\rho\ell_{\mathrm{E},i}/2D_{\mathrm{E},i}A^2_{\mathrm{E},i}$ (see Grosswindhager, Voigt, and Kozek (2011), Hauschild et al. (2020)).[4]

Analogously, if $i \in \mathcal{E}$ is a valve, we use (4) to model the pressure drop through it. Following De Persis and Kallesoe (2011), we neglect the inertial forces by taking $J_{\mathrm{E},i} = 0$. Also $K_{\mathrm{E},i} > 0$ is in this case taken as an unknown constant whose value depends on generic parameters, *e.g.*, the flow capacity and opening degree of the valve (Wang et al., 2017).

Similarly, for any pump $i \in \mathcal{E}$ we model the pressure difference between inlet and outlet by $p^{\mathrm{in}}_{\mathrm{E},i} - p^{\mathrm{out}}_{\mathrm{E},i} = -w_{\mathrm{E},i}$, which is analogous to (4) (with $J_{\mathrm{E},i} = K_{\mathrm{E},i} = 0$) and where $w_{\mathrm{E},i}$ is the pressure difference produced by the pump.

From these developments, we can write the momentum balance equation at all edges in vector form as $p^{\mathrm{in}}_{\mathrm{E}} - p^{\mathrm{out}}_{\mathrm{E}} = \mathrm{diag}(J_{\mathrm{E}})\dot{q}_{\mathrm{E}} + f_{\mathrm{E}}(q_{\mathrm{E}}) - w_{\mathrm{E}}$; see De Persis and Kallesoe (2011). To match the left-hand side of this equation with that of (3a), we bring the constraints $p^{\mathrm{in}}_{\mathrm{E},i} = p_{\mathrm{n},k}$ and $p^{\mathrm{out}}_{\mathrm{E},j} = p_{\mathrm{n},k}$, for all $i \in \mathfrak{S}_k$, $j \in \mathfrak{T}_k$ and $k \in N$, which guarantee that the pressure profile throughout the DH system is continuous in space (Hauschild et al., 2020) (see also Krug et al., 2021; Vladimarsson, 2014). Then, if we use the incidence matrix, we get that $p^{\mathrm{in}}_{\mathrm{E}} - p^{\mathrm{out}}_{\mathrm{E}} = -B_0^\top p_{\mathrm{N}}$ (see Vladimarsson, 2014), which leads to (3a).

To establish (3b), we note that in view of Remark 1, the mass balance equation at any node $k \in N$ can be written as (see Hauschild et al., 2020; Krug et al., 2021; Vladimarsson, 2014)

$$\rho\dot{V}_{\mathrm{n},k} = \rho\sum_{i\in\mathfrak{T}_k} q_{\mathrm{E},i} - \rho\sum_{i\in\mathfrak{S}_k} q_{\mathrm{E},i}, \quad k \in N. \tag{5}$$

According to Assumption 1(vii), the left-hand side of (5) is zero for all $k \in N$. Moreover, if we express it in vector form, the right-hand side of the set of all Eqs. (5) is equivalent to $B_0 q_{\mathrm{E}}$. Therefore (3b) is established. ∎

Following De Persis and Kallesoe (2011), we proceed to identify a set of independent flows from which the entire hydraulic state of the DH system can be determined. These flows are associated with a selected collection of *pipes* that generate fundamental loops of G.

To be precise, we recall that G is connected, then it has a (possibly non-unique) spanning tree (see Bollobás, 1998; De Persis & Kallesoe, 2011), which we denote by $S = (N, \mathcal{E}')$. Any $i \in \mathcal{E} \setminus \mathcal{E}'$ is referred to as a *chord* of S and the set of all chords is denoted by $\mathcal{C} = \mathcal{E} \setminus \mathcal{E}'$. We denote by $n_{\mathrm{f}} = n_{\mathrm{E}} - (n_{\mathrm{n}} - 1)$ the cardinality of $\mathcal{C}$. Moreover, a fundamental loop, denoted here by $\mathcal{L}_i$, is the sequence of edges associated with the loop that is formed when a chord is added to the spanning tree S. It is assumed that each $\mathcal{L}_i$ has an orientation matching that of the chord that generates it. Then, a fundamental loop matrix, denoted by $F \in \mathbb{R}^{n_{\mathrm{f}}\times n_{\mathrm{E}}}$, is defined by components as $F_{i,h} = 1$, if $h \in \mathcal{L}_i$ and orientations agree, $F_{i,h} = -1$, if $h \in \mathcal{L}_i$ and orientations disagree and $F_{i,h} = 0$ if $h \notin \mathcal{L}_i$ (see De Persis & Kallesoe, 2011). We note that since G is connected, then F is a full-rank matrix (see De Persis & Kallesoe, 2011).

[4] We have assumed that the Darcy's friction factor $\theta_{\mathrm{E},i}$ is uniform along the pipe's axis and constant in time (see, *e.g.*, Krug et al., 2021; Rein et al., 2021).

In the considered DH system's setup, it can be verified that a suitable, yet not unique, selection of the set of chords is given by $\mathcal{C} = \mathcal{P}_{\mathrm{c}} \cup \mathcal{P}_{\mathrm{pr}} \cup \mathcal{P}_{\mathrm{d}} \cup \mathcal{P}_{\mathrm{ST}}$, where $\mathcal{P}_{\mathrm{c}}$ and $\mathcal{P}_{\mathrm{pr}}$ comprise all pipes associated with consumers and producers, respectively. Also, $\mathcal{P}_{\mathrm{d}}$ are all pipes that generate (fundamental) loops in the distribution network. Each element in $\mathcal{P}_{\mathrm{ST}}$ corresponds to the pipe at the hot water outlet of each storage tank (see Fig. 3), with the exception of one, which is chosen to belong to the spanning tree S (see Wang et al. (2017)). The latter consideration ensures that the spanning tree S is *connected* (see De Persis and Kallesoe (2011)). In particular, we assume that the elements of $\mathcal{P}_{\mathrm{pr}}$ and $\mathcal{P}_{\mathrm{ST}}$ are oriented towards the supply layer of the DH system. Conversely, the elements of $\mathcal{P}_{\mathrm{c}}$ are considered to point towards the return layer.

We are in position to state this section's main result:

Theorem 1. *Let Assumption 1 hold. Also, let us assume there is an independently controlled pump adjacent to each $i \in \mathcal{C}$. Let $\mathcal{W}_{\mathrm{f}} \subset \mathcal{E}$ and $w_{\mathrm{f},i}$ denote the set of these pumps and the pressure difference produced by each of them, respectively. If the orientation of each pump in $\mathcal{W}_{\mathrm{f}}$ matches that of its adjacent chord, then the following holds true:*

(I) The overall DH system's flows are determined by the flows through the chords. More precisely, $q_{\mathrm{E}} = F^\top q_{\mathrm{f}}$, where $q_{\mathrm{f}} \in \mathbb{R}^{n_{\mathrm{f}}}$ is the vector of chords' flows.
(II) All solutions of (3) are determined by the ODE

$$\mathcal{J}_{\mathrm{f}}\dot{q}_{\mathrm{f}} = -f_{\mathrm{f}}(q_{\mathrm{f}}) + w_{\mathrm{f}} + B_{\mathrm{b}}w_{\mathrm{b}}, \tag{6}$$

where $\mathcal{J}_f = F\mathrm{diag}(J_{\mathrm{E}})F^\top > 0$, $f_{\mathrm{f}}(q_{\mathrm{f}}) = Ff_{\mathrm{E}}(F^\top q_{\mathrm{f}})$. Also, $B_{\mathrm{b}}w_{\mathrm{b}}$, with $(B_{\mathrm{b}})_{\alpha,\beta} \in \{-1, 0, 1\}$, codifies the effect on q_{f} of any other pump $i \in \mathcal{E} \setminus \mathcal{W}_{\mathrm{f}}$.
(III) There exists $W \in \mathbb{R}^{n_{\mathrm{ST}}\times n_{\mathrm{f}}}$, with entries in $\{0, 1\}$, such that the dynamics of the storage tanks' volumes are given by $\dot{V}_{\mathrm{sh}} = Wq_{\mathrm{f}}$ and $\dot{V}_{\mathrm{sc}} = -Wq_{\mathrm{f}}$, for the hot and cold layers, respectively.

Proof. The proof procedure of claims (I) and (II) is akin to the developments in De Persis and Kallesoe (2011), but we present some details out of completeness. Claim (I) is directly established by invoking the analogous versions of Kirchhoff's voltage and current laws for hydraulic networks. Following De Persis and Kallesoe (2011) (see also Desoer & Kuh, 1969), they can be respectively stated as

$$FB_0^\top p_{\mathrm{n}} = 0, \quad q_{\mathrm{E}} = F^\top q_{\mathrm{f}}. \tag{7}$$

These expressions represent also the starting point to establish claim (II). Indeed, by combining (7) with (3a) we obtain

$$F\mathrm{diag}(J_{\mathrm{E}})F^\top\dot{q}_{\mathrm{f}} = -Ff_{\mathrm{E}}(F^\top q_{\mathrm{f}}) + Fw_{\mathrm{E}}. \tag{8}$$

Moreover, (3b), which represents the mass balance equations at the nodes of G, holds too. Indeed, both equations in (7) imply that $B_0 q_{\mathrm{E}} = B_0 F^\top q_{\mathrm{f}} = 0$.

We note that (8) matches (6), except for the terms related to the pumps' pressures. To see that $Fw_{\mathrm{E}} = w_{\mathrm{f}} + B_{\mathrm{b}}w_{\mathrm{b}}$, it is more convenient to assume, without loss of generality, that $\mathcal{E}$ is ordered as $\mathcal{C} \cup \mathcal{W}_{\mathrm{f}} \cup \mathcal{E}''$, where $\mathcal{E}''$ comprises the remaining pipes, pumps and valves. This has two implications. On the one hand, by recalling that the edges in $\mathcal{C}$ and $\mathcal{W}_{\mathrm{f}}$ have the same orientation, then F can be split as

$$F = \left[I_{n_{\mathrm{f}}}\ I_{n_{\mathrm{f}}}\ B_{\mathrm{b}}\right], \tag{9}$$

where B_{b} is as in the theorem's statement. On the other hand, $w_{\mathrm{E}} = \left[\mathbf{0}^\top_{n_{\mathrm{f}}},\ w_{\mathrm{f}}^\top,\ w_{\mathrm{b}}^\top\right]^\top$, which proves our assertion. The splitting in (9) is also beneficial to show that $\mathcal{J}_{\mathrm{f}} > 0$, however we omit this step for brevity.

Regarding claim (III), let us assume that the nodes $\mathcal{N}$ of the primal DH system's graph $\mathcal{G}$ are ordered as $\mathcal{N}_{\rm sh} \cup \mathcal{N}_{\rm sc} \cup \mathcal{N}_{\rm sj}$. Then, the set of mass balance equations at each node in $\mathcal{N}$ is equivalent to $\dot{V}_{\rm N} = \mathcal{B}_0 q_{\rm E}$, which combined with (7) produces $\dot{V}_{\rm N} = \mathcal{B}_0 F^\top q_{\rm f}$. Pre-multiplying both sides of this equation by $W' = \left[I_{n_{\rm ST}}, \quad 0_{n_{\rm ST}\times(n_{\rm N}-n_{\rm ST})}\right]$ leads to $\dot{V}_{\rm sh} = Wq_{\rm f}$, where $W = W'\mathcal{B}_0 F^\top$. To see that $\dot{V}_{\rm sc} = -Wq_{\rm f}$, we recall that (3b), which holds due to (7), implies that $\dot{V}_{\rm sh} + \dot{V}_{\rm sc} = 0_{n_{\rm ST}}$, *i.e.*, the sum of the volume of the hot and cold layer of each tank is constant (see Assumption 1(vii) and Remark 2).[5] ■

Remark 4.

(i) Concerning the set of pumps $\mathcal{W}_{\rm f}$, we note that it is common that independently controlled pumps are placed at heat producers (Wang et al., 2017), consumers (De Persis & Kallesoe, 2011; Gong et al., 2019), and storage units (Guelpa & Verda, 2019). Although some consumers or storage units might alternatively have control valves (Gong et al., 2019; Wang et al., 2017), in this work we restrict ourselves to the multi-pump case only. Moreover, we imply that there is a pump adjacent to each element in $\mathcal{P}_{\rm d}$. In Wang et al. (2017), there are no control valves (nor pumps) placed at these locations, where the elements in $\mathcal{P}_{\rm d}$ are referred to as *residual branches*. Thus, we are currently exploring the full extent of the implications of relaxing this assumption in our work.
(ii) Costs associated with the system operation, which are related to pump power, may change in accordance to which setup and scheme is used to control system pumps and valves (Gong et al., 2019; Wang et al., 2017). Such analysis is beyond the scope of our manuscript and is left as future research.
(iii) Following De Persis and Kallesoe (2011), we refer to any other pump in the system as a *booster pump*. For simplicity we assume that each of them produces a constant pressure difference across its terminals; see De Persis and Kallesoe (2011) for more details.

Remark 5.

(i) Claim (III) of Theorem 1 implies that the total mass in the DH system remains constant all the time. Thus, tanks will operate at full capacity provided this condition is met at the initial time instant. Critically, this does not guarantee that the volumes of the hot and cold layer of any storage are always simultaneously positive nor constant. Henceforth, we assume that the volume of each layer of each storage tank is positive all the time. This sensible assumption is needed to guarantee that diag($V_{\rm th}$) in Theorem 4 is positive definite.
(ii) Considering Assumption 1, it can be shown that two decoupled dynamical systems can be obtained from (6), one for the flow vector of $\mathcal{P}_{\rm pr}$ and the other for the flow vector of $\mathcal{P}_{\rm c} \cup \mathcal{P}_{\rm d} \cup \mathcal{P}_{\rm ST}$. More details can be found in Machado et al. (2020).
(iii) In the single producer case treated in De Persis and Kallesoe (2011), the orientation of the edges can be arranged such that a matrix analogous to $B_{\rm b}$ in (9) has components only in $\{0, 1\}$. This cannot be guaranteed in our system setup (multi-producer, meshed distribution network). One salient implication of this is that (component-wise) positive solutions $q_{\rm f}$ of (6) do not imply that $q_{\rm E}$ in (3) remains in the positive orthant $\mathbb{R}^{n_{\rm E}}_{\geq 0}$. In the next section, this is taken into account when defining a matrix which is central in the DH system's thermal dynamic modeling (see Eq. (16)).
(iv) Differently from Theorem 1, the DH system's flow dynamics in Hauschild et al. (2020), Krug et al. (2021), Rein et al. (2021) are not formulated as an ODE.

[5] More details about this proof appear in Machado et al. (2020).

4. Thermal model

In this section we present a model that describes the dynamic behavior of the overall DH system's temperatures. Standing assumptions behind this model, which complement those in Assumption 1, are the following (*c.f.*, Ismail et al., 1997; Scholten et al., 2015; van der Heijde et al., 2017):

Assumption 2. (i) Kinetic, potential and flow energies are negligible compared to the internal energy of water. (ii) Heat conduction is neglected. (iii) Dissipation due to friction is negligible. (iv) The internal energy of water linearly depends on the temperature. (v) The rate at which work is done by external forces that act on the (moving) boundary of any tank's layer is negligible. (vi) The temperature $T_{{\rm E},i}$ ($T_{{\rm N},k}$) of each $i \in \mathcal{E}$ ($k \in \mathcal{N}$) represents a spatially averaged quantity over its control volume.

We note that Assumption 2(i)–(iv) holds for all edges and nodes, while Assumption 2(v) only for the layers of the storage tanks. Additionally, by neglecting heat conduction, the heat transfer through the boundaries between edges and nodes is discarded (as well as the heat transfer towards the environment); for storage tanks this also means that there is no heat transfer through the thermocline. Also, Assumption 2(v) is introduced to take into consideration the fact that if k represents a layer of a storage tank, then $\dot{V}_{{\rm N},k}$ is not necessarily zero all the time, *i.e.*, its boundary (the thermocline) might move. Then, when we write later the energy balance at k, which is a deforming control volume (see, *e.g.*, Sonin, 2001), we discard the effects of the boundary forces; volume-varying nodes are not considered in Hauschild et al. (2020), Krug et al. (2021), Rein et al. (2021). Assumption 2(vi) may bring inaccuracies in the description of the temperature profile of long pipes. Note however that in such cases a pipe can be viewed as a series connection of shorter pipes of similar properties; see Hauschild et al. (2020) for a rigorous treatment on spatial discretization of PDE-based pipe models.

4.1. Edges

Let Assumptions 1 and 2 hold and let $i \in \mathcal{E}$ be an arbitrary pipe of the DH system. For simplicity of exposition, we assume that the orientation of i matches the direction of the stream through it, then, $q_{{\rm E},i} \geq 0$ all the time. Then, the energy balance at i can be written as follows:

$$\rho c_{\rm s.h.} V_{{\rm E},i} \dot{T}_{{\rm E},i} = \rho c_{\rm s.h.} q_{{\rm E},i} \left(T^{\rm in}_{{\rm E},i} - T^{\rm out}_{{\rm E},i}\right), \tag{10}$$

where $T^{\rm in}_{{\rm E},i}$ ($T^{\rm out}_{{\rm E},i}$) is the temperature at the inlet (outlet) of the pipe.

Based on (10), we model the heat exchangers (which are viewed as pipes) at producers and consumers as follows (Scholten et al., 2015)

$$\rho c_{\rm s.h.} V_{{\rm E},i} \dot{T}_{{\rm E},i} = \rho c_{\rm s.h.} q_{{\rm E},i} \left(T^{\rm in}_{{\rm E},i} - T^{\rm out}_{{\rm E},i}\right) + P_{{\rm pr},i} - P_{{\rm c},i}, \tag{11}$$

where $P_{{\rm pr},i}$ is the thermal power the ith producer transfers into the pipe's stream and $P_{{\rm c},i}$ is the thermal power the consumer extracts from it. If $i \in \mathcal{E}$ is associated with a producer, we take $P_{{\rm c},i} = 0$, and $P_{{\rm pr},i} = 0$ if i is associated with a consumer.

Remark 6.

(i) We also use the model (11) to describe the thermal behavior of all valves and pumps in the DH system. Then, (11) describes the temperature dynamics of any edge $i \in \mathcal{E}$. Note that $P_{{\rm pr},i}$ and $P_{{\rm c},i}$ are associated with producers and consumers' pipes, then we take them as zero for any other type of edge.

(ii) In the following section we describe the thermal dynamics of the DH system's nodes. Therein, we write a relationship between the temperature $T^{\text{in}}_{\text{E},i}$ at the inlet of any edge i with the temperature of the node from which the stream through i sources from.

4.2. Nodes

Again, let Assumptions 1 and 2 hold and let $k \in \mathcal{N}$ be an arbitrary node of the DH system. For simplicity we assume again and without loss of generality that the orientation of any edge $i \in \mathcal{E}$ matches the direction of the stream through it, *i.e.*, we assume that $q_{\text{E},i} \geq 0$ (see Remark 7). Considering Assumptions 1 and 2, it is possible to write the energy balance at any $k \in \mathcal{N}$ as follows (*c.f.*, Hauschild et al., 2020; Krug et al., 2021):

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\rho c_{\text{s.h.}} V_{\text{N},k} T_{\text{N},k}\right) = \sum_{j\in\mathfrak{T}_k} \rho c_{\text{s.h.}} q_{\text{E},j} T^{\text{out}}_{\text{E},j} - \sum_{j\in\mathfrak{S}_k} \rho c_{\text{s.h.}} q_{\text{E},j} T^{\text{in}}_{\text{E},j}. \tag{12}$$

The term in the left-hand side represents the thermal energy stored at node k whereas the terms in the right-hand side are the sum of the thermal energies of the streams that target and source from k, respectively.

Based on the (upwind) semi-discretization scheme discussed in Hauschild et al. (2020) and on the nodal constraints described in Krug et al. (2021), we impose on (11) and (12) the following conditions:

$$T^{\text{in}}_{\text{E},j} = T_{\text{N},k},\ j \in \mathfrak{S}_k,\ k \in \mathcal{N}, \tag{13a}$$

$$T^{\text{out}}_{\text{E},j} = T_{\text{E},j},\ j \in \mathcal{E}. \tag{13b}$$

In particular, these conditions allow us to write (12) in a simpler, equivalent form. Indeed, since the mass balance equation at node $k \in \mathcal{N}$ can be written as $\rho \dot{V}_{\text{N},k} = \sum_{j\in\mathfrak{T}_k} \rho q_{\text{E},j} - \sum_{j\in\mathfrak{S}_k} \rho q_{\text{E},j}$, then (12) is equivalent to

$$\rho c_{\text{s.h.}} V_{\text{N},k} \dot{T}_{\text{N},k} = \sum_{j\in\mathfrak{T}_k} \rho c_{\text{s.h.}} q_{\text{E},j} T_{\text{E},j} - \left(\sum_{j\in\mathfrak{T}_k} \rho c_{\text{s.h.}} q_{\text{E},j}\right) T_{\text{N},k},\ \ k \in \mathcal{N}. \tag{14}$$

For simplicity we henceforth let $\rho = c_{\text{s.h}} = 1$.

Remark 7.

(i) The models (11)–(14) need to be adjusted if the orientation of a given edge $i \in \mathcal{E}$ does not match the direction of the stream through it, *i.e.*, if $q_{\text{E},i}(t) \leq 0$ for certain t. In this case it is necessary to substitute $q_{\text{E},i}$ by its absolute value and re-define the sets $\mathfrak{S}_k$ and $\mathfrak{T}_k$ as follows (Hauschild et al., 2020; Krug et al., 2021), (*c.f.*, Vladimarsson, 2014)

$$\begin{aligned}\mathfrak{S}_k = \{i \in \mathcal{E} :\ &\big(k \text{ is the tail of } i \text{ and } q_{\text{E},i} \geq 0\big) \\ &\text{or } \big(k \text{ is the head of } i \text{ and } q_{\text{E},i} < 0\big)\},\end{aligned} \tag{15a}$$

$$\begin{aligned}\mathfrak{T}_k = \{i \in \mathcal{E} :\ &\big(k \text{ is the tail of } i \text{ and } q_{\text{E},i} < 0\big) \\ &\text{or } \big(k \text{ is the head of } i \text{ and } q_{\text{E},i} \geq 0\big)\}.\end{aligned} \tag{15b}$$

In the sequel we say that node k is the source of the stream through any $i \in \mathfrak{S}_k$ and that k is the target of the stream through any $i \in \mathfrak{T}_k$.

4.3. Thermal model in vector form

To write in vector form the system of equations (11), (13) and (14), we find it convenient to introduce a flow-dependent, node-edge incidence matrix. We denote this matrix by $\mathcal{B}(q_{\text{E}}) \in \mathbb{R}^{n_{\text{N}} \times n_{\text{E}}}$ and define it as follows: $\mathcal{B}_{ij}(q_{\text{E}}) = 1$, if the flow through $j \in \mathcal{E}$ targets $i \in \mathcal{N}$; $\mathcal{B}_{ij}(q_{\text{E}}) = -1$, if the flow through $j \in \mathcal{E}$ originates from $i \in \mathcal{N}$; and $\mathcal{B}_{ij}(q_{\text{E}}) = 0$, otherwise. Following Vladimarsson (2014), this matrix can be compactly written as

$$\mathcal{B} = \mathcal{B}_0 \text{diag}(\textbf{sign}(q_{\text{E}})), \tag{16}$$

where $\mathcal{B}_0$ is given in (2). To our purposes, the positive and negative parts of $\mathcal{B}$ are more relevant, since they respectively identify (for each edge) its target and source nodes (see Vladimarsson, 2014). These matrices are respectively defined as follows (see Vladimarsson (2014)):

$$\mathcal{T} = \frac{1}{2}\left(\mathcal{B} + |\mathcal{B}|\right),\quad \mathcal{S} = \frac{1}{2}\left(|\mathcal{B}| - \mathcal{B}\right). \tag{17}$$

We note that $\mathcal{B}$, $\mathcal{T}$ and $\mathcal{S}$ are time-varying in general.

Consider then the following theorem (its proof can be established via direct computations and is therefore omitted for brevity).

Theorem 2. *The system (11), (13) and (14) can be equivalently represented in vector form as follows:*

$$\text{diag}(V_{\text{E}}, V_{\text{N}}) \begin{bmatrix} \dot{T}_{\text{E}} \\ \dot{T}_{\text{N}} \end{bmatrix} = \mathcal{A}(q_{\text{E}}) \begin{bmatrix} T_{\text{E}} \\ T_{\text{N}} \end{bmatrix} + \begin{bmatrix} \mathbf{P}_{\text{pr}} - \mathbf{P}_{\text{c}} \\ 0_{n_{\text{N}}} \end{bmatrix}, \tag{18}$$

where

$$\mathcal{A}(q_{\text{E}}) = \begin{bmatrix} -\text{diag}(|q_{\text{E}}|) & \text{diag}(|q_{\text{E}}|)\mathcal{S}^{\top} \\ \mathcal{T}\text{diag}(|q_{\text{E}}|) & -\text{diag}(\mathcal{T}|q_{\text{E}}|), \end{bmatrix} \tag{19}$$

and $\mathbf{P}_{\text{pr}} = [P_{\text{pr},1}, \ldots, P_{\text{pr},n_{\text{E}}}]^{\top}$ *and* $\mathbf{P}_{\text{c}} = [P_{\text{c},1}, \ldots, P_{\text{c},n_{\text{E}}}]^{\top}$.

The model (18) represents an ODE and is nonlinear due to the dependency of $\mathcal{A}$ on q_{E}, and due to the time-varying behavior of the volumes associated with the layers of the storage tanks.

We note that due to the small volume of valves, pumps and junctions, it is reasonable to assume they have a negligible thermal inertia compared to pipes and storage tanks. If we take $V_{\text{E},i} = 0$ for any $i \in \mathcal{E}$ associated with a valve or pump, and $V_{\text{N},j} = 0$ for any $j \in \mathcal{N}$ associated with a simple junction, then thermal inertia of these devices is neglected. This reduces the number of differential variables of (18), but turns it into a system of semi-explicit DAEs. In Machado et al. (2020) we discuss an assumption for this system to have differentiation index 1, which facilitates the computation of its trajectories. However, its verification is not direct in general.

An alternative approach consists in defining a reduced DH system's graph $\tilde{\mathcal{G}} = (\tilde{\mathcal{N}}, \tilde{\mathcal{E}})$ obtained by *contracting* all the edges associated with valves and pumps in $\mathcal{G}$, provided they are assumed to have zero volume. To prevent that $\tilde{\mathcal{G}}$ has self-loops, we make the following, practically sensible assumption:

Assumption 3. Each valve and pump of the overall DH system's graph $\mathcal{G}$ is in series with a pipe.

Let q_{r} denote the flows through the pipes of $\tilde{\mathcal{G}}$. Thus, by virtue of Assumption 3, the components of q_{r} are those of q_{E} in (7) associated with the (same) pipes in $\mathcal{G}$. In the following corollary, whose proof is in Machado et al. (2020), any variable, set or matrix defined for $\mathcal{G}$ and the system (18) (*e.g.*, the matrix $\mathcal{A}$, or the sets $\mathfrak{S}_k$ and $\mathfrak{T}_k$) is analogously defined for $\tilde{\mathcal{G}}$ and identified with the symbol $\tilde{(\cdot)}$.

Corollary 1. *Let Assumptions 1–3 hold and assume that all valves, pumps and simple junctions in the DH system have zero volume. Let* $\tilde{\mathcal{N}}_0 \subset \tilde{\mathcal{N}}$ *denote the set of simple junctions in* $\tilde{\mathcal{G}}$ *and assume that* $\sum_{k\in\tilde{\mathfrak{T}}_j} |\tilde{q}_{\text{E},k}| > 0$ *for each* $j \in \tilde{\mathcal{N}}_0$. *Then, the state of the temperatures in all the system's pipes and storage tanks' layers is determined by* *a vector T_{th} comprising the temperatures of each $k \in \tilde{\mathcal{E}} \cup (\tilde{\mathcal{N}} \setminus \tilde{\mathcal{N}}_0)$, which satisfies a dynamic equation of the form*

$$\mathrm{diag}(V_{\mathrm{th}})\dot{T}_{\mathrm{th}} = A_{\mathrm{th}}(q_{\mathrm{r}})T_{\mathrm{th}} + B_{\mathrm{pr}}P_{\mathrm{pr}} - B_{\mathrm{c}}P_{\mathrm{c}}, \tag{20}$$

where $A_{\mathrm{th}}(q_{\mathrm{r}})$ is the Schur complement of an appropriate sub-matrix of $\tilde{\mathcal{A}}(q_{\mathrm{r}})$, and $(B_{\mathrm{pr}})_{\alpha,\beta}, \in \{0,1\}$ and $(B_{\mathrm{c}})_{\alpha,\beta}, \in \{0,1\}$ are properly sized, full (column) rank matrices.

Remark 8.

(i) Henceforth we view each $P_{\mathrm{pr},i}$ as a control input and each $P_{\mathrm{c},i}$ as an external disturbance.
(ii) By directly working with (18), we avoid imposing conditions on the flows as we do to define (20). Besides the main drawback of having a higher order, some values of $V_{\mathrm{E},i}$ and $V_{\mathrm{N},j}$ in (18) could be uncertain, as it could be difficult to accurately measure them for all valves, pumps and junctions in the DH system. Nonetheless, the shifted passivity properties that we present in Section 5 for (20) can be analogously established for (18). Notably, knowledge about the system elements' volumes would not be necessary for that purpose, as is the case for the passivity-based controllers discussed in Remark 9.

5. Passivity properties

In this section we establish that the DH system's flow and thermal dynamics are shifted passive subject to the assumption that there is a distinct time scale separation between these subsystems. The plausibility of this hypothesis stems from the fact that the flows through the system reach a steady-state in a matter of seconds, whereas temperature changes at the heat production sites need hours to affect consumer stations (Gross-windhager et al., 2011, Section 2.4), Pálsson, Larsen, Bohm, Ravn, and Zhou (1999, Section 2.3) and Benonysson, Bohm, and Ravn (1995, Section 2).

To improve the readability, consider the following.

Definition 1 (*van der Schaft, 2000*)**.** Consider a dynamic system $\Sigma : \dot{x} = f(x,u)$, with input u and output $y = h(x,u)$, of the same size. Let $(\bar{x},\bar{u})$ denote an equilibrium pair of Σ and define $\bar{y} = h(\bar{x},\bar{u})$. Then, Σ is shifted passive if there exists a non-negative scalar function $\mathcal{H}(x)$ such that, along any solution x of Σ, it satisfies $\dot{\mathcal{H}}(x) \le (u-\bar{u})^\top(y-\bar{y})$.

In Remark 9 we discuss some of the benefits for control design and stability analysis of identifying that flow and thermal dynamics are shifted passive.[6]

5.1. Passivity of the flow dynamics

We first focus on the flow dynamics (6), which we remark are independent with respect to the DH system's temperatures. Before presenting the main result of this subsection, consider the following lemma, where an important property of the mapping f_{f} is introduced (see Theorem 1). The proof is omitted since it can be established by following the proof's procedure of De Persis, Jensen, Ortega, and Wisniewski (2014, Lemma 3.1). Nonetheless, it is important to note that each mapping $f_{\mathrm{E},i}(q_{\mathrm{E},i})$ in (3) is a continuously differentiable monotonic function; the latter means that the following results would hold for any other friction model $f_{\mathrm{E},i}$ as long as it satisfies such properties.

[6] The definition of *passivity* follows from Definition 1 by taking $\bar{u} = \bar{y} = \mathbf{0}$.

Lemma 1. *The mapping f_{f} that appears in the right-hand side of the flow dynamics (6) is monotone with respect to q_{f}.*[7]

We are in a position to prove the following theorem:

Theorem 3. *The DH system's flow dynamics (6) are shifted passive with storage function $\mathcal{H}_{\mathrm{f}}(q_{\mathrm{f}}) = \frac{1}{2}(q_{\mathrm{f}}-\bar{q}_{\mathrm{f}})^\top \mathcal{J}_{\mathrm{f}}(q_{\mathrm{f}}-\bar{q}_{\mathrm{f}})$ and shifted passive output $y_{\mathrm{f}} - \bar{y}_{\mathrm{f}} = q_{\mathrm{f}} - \bar{q}_{\mathrm{f}}$, where $(\bar{q}_{\mathrm{f}}, \bar{w}_{\mathrm{f}})$ is any equilibrium pair of the system.*

Proof. Since $(\bar{q}_{\mathrm{f}}, \bar{w}_{\mathrm{f}})$ denotes any equilibrium pair of the flow dynamics (6), then the identity $\mathcal{J}_{\mathrm{f}}\dot{\bar{q}}_{\mathrm{f}} = \mathbf{0} = -f_{\mathrm{f}}(\bar{q}_{\mathrm{f}}) + \bar{w}_{\mathrm{f}} + B_{\mathrm{b}}w_{\mathrm{b}}$ holds. Therefore, (6) can be equivalently written as follows (recall that w_{b} is constant):

$$\mathcal{J}_{\mathrm{f}}\dot{q}_{\mathrm{f}} = -(f_{\mathrm{f}}(q_{\mathrm{f}}) - f(\bar{q}_{\mathrm{f}})) + w_{\mathrm{f}} - \bar{w}_{\mathrm{f}}. \tag{21}$$

Consider the mapping $\mathcal{H}_{\mathrm{f}}$ as a candidate storage function to prove that (21) is shifted passive; clearly $\mathcal{H}_{\mathrm{f}}$ is continuously differentiable and bounded from below by zero and, moreover, along the solutions of system (21), its time derivative satisfies $\dot{\mathcal{H}}_{\mathrm{f}}(q_{\mathrm{f}}) = (q_{\mathrm{f}}-\bar{q}_{\mathrm{f}})^\top(-(f_{\mathrm{f}}(q_{\mathrm{f}}) - f_{\mathrm{f}}(\bar{q}_{\mathrm{f}})) + w_{\mathrm{f}} - \bar{w}_{\mathrm{f}})$. From Lemma 1, the mapping f_{f} is monotone with respect to q_{f}, yielding that $-(q_{\mathrm{f}}-\bar{q}_{\mathrm{f}})^\top(f_{\mathrm{f}}(q_{\mathrm{f}}) - f_{\mathrm{f}}(\bar{q}_{\mathrm{f}})) \le 0$. Consequently, $\dot{\mathcal{H}}_{\mathrm{f}}(q_{\mathrm{f}}) \le (y_{\mathrm{f}}-\bar{y}_{\mathrm{f}})^\top(w_{\mathrm{f}}-\bar{w}_{\mathrm{f}})$. ■

5.2. Passivity of the thermal dynamics

Here we focus on the DH system's thermal dynamics (20), however analogous results hold also for (18), as it will be clear from the proofs of the following results.

Let $\bar{q}_{\mathrm{f}}$ denote an equilibrium of (6), and let us assume that the flow dynamics (6) have a much faster response than the thermal model (20). Then, in (20) we can take $q_{\mathrm{r}} = \bar{q}_{\mathrm{r}}$ all the time, where $\bar{q}_{\mathrm{r}}$ is an equilibrium flow depending on $\bar{q}_{\mathrm{f}}$ (see the definition of q_{r} just above Corollary 1).

Lemma 2. *The coefficient matrix $A_{\mathrm{th}}(q_{\mathrm{r}})$ of the thermal dynamics (20) is negative semi definite at any equilibrium $\bar{q}_{\mathrm{r}}$.*

Proof. Let us begin by showing that $\tilde{\mathcal{A}}(\bar{q}_{\mathrm{r}})$ is a Kirchhoff Convection Matrix (KCM) and thus negative semi definite (see Hangos, Alonso, Perkins, and Ydstie (1999, Appendix)). We recall that $\tilde{\mathcal{A}}$ is analogous to $\mathcal{A}$ in (19), but defined for the reduced graph $\tilde{\mathcal{G}}$ instead of $\mathcal{G}$.

Let us treat the case in which each component of $\bar{q}_{\mathrm{r}}$ is different from zero. Following Hangos et al. (1999, Appendix), $\tilde{\mathcal{A}}(\bar{q}_{\mathrm{r}})$ is a KCM if and only if the following four conditions hold: (i) $\tilde{\mathcal{A}}_{i,i}(\bar{q}_{\mathrm{r}}) < 0$; (ii) $\tilde{\mathcal{A}}_{i,j}(\bar{q}_{\mathrm{r}}) \ge 0$; (iii) $\tilde{\mathcal{A}}(\bar{q}_{\mathrm{r}})\mathbf{1} = \mathbf{0}$; and $\mathbf{1}^\top\tilde{\mathcal{A}}(\bar{q}_{\mathrm{r}}) = \mathbf{0}^\top$. Condition (i) holds since each $\tilde{\mathcal{A}}_{i,i}(\bar{q}_{\mathrm{r}})$ corresponds to some element in the main diagonal of $-\mathrm{diag}(|\bar{q}_{\mathrm{r}}|)$ or $-\mathrm{diag}(\tilde{\mathcal{T}}|\bar{q}_{\mathrm{r}}|)$, which are clearly negative. Condition (ii) can be established using similar arguments. Indeed, simply note that each off-diagonal element $\tilde{\mathcal{A}}_{i,j}(\bar{q}_{\mathrm{r}})$, $i \ne j$, corresponds to either one of the components of $\mathrm{diag}(|\bar{q}_{\mathrm{r}}|)\tilde{\mathcal{S}}^\top$ or $\tilde{\mathcal{T}}\mathrm{diag}(|\bar{q}_{\mathrm{r}}|)$, which are all non negative. Conditions (iii) and (iv) can be established by lengthy but direct computations which we omit due to page limit, however all details can be found in Machado et al. (2020).

In view of the above, $\tilde{\mathcal{A}}(\bar{q}_{\mathrm{r}})$ is a KCM, and hence negative semi definite (Hangos et al., 1999, Lemma 7); *c.f.*, Rein et al. (2021, Theorem 1). Since $A_{\mathrm{th}}(\bar{q}_{\mathrm{r}})$ is the Schur complement of an

[7] A mapping $\mathcal{F} : \mathbb{R}^n \to \mathbb{R}^n$ is said to be monotone if $(u-v)^\top(\mathcal{F}(u) - \mathcal{F}(v)) \ge 0$, $\forall u, v \in \mathbb{R}^n$, Ryu and Boyd (2016, Section 4). In the case that $\mathcal{F}$ is differentiable then a necessary and sufficient condition for monotonicity is given by $\nabla\mathcal{F}(x) + \nabla\mathcal{F}(x)^\top \ge 0$, $\forall x \in \mathbb{R}^n$, Ryu and Boyd (2016, p.12).

appropriate sub-matrix of $\tilde{\mathcal{A}}(\bar{q}_{\rm r})$, then it is also negative semi definite.[8] ∎

The main result of this subsection is the following:

Theorem 4. *The DH system's thermal dynamics (20) are shifted passive with storage function* $\mathcal{H}_{\rm th}(T_{\rm th}) = \frac{1}{2}(T_{\rm th}-\bar{T}_{\rm th})^\top {\rm diag}(\bar{V}_{\rm th})(T_{\rm th}-\bar{T}_{\rm th})$ *and shifted passive output* $y_{\rm th}-\bar{y}_{\rm th} = B_{\rm pr}{}^\top(T_{\rm th}-\bar{T}_{\rm th})$, *under the assumption that* $q_{\rm r} = \bar{q}_{\rm r}$ *and* $P_{\rm c}$ *are constant all the time.*

Proof. Let us assume that the consumers' power $P_{\rm c}$ is constant. Let $(\bar{q}_{\rm r}, \bar{T}_{\rm th}, \bar{P}_{\rm pr})$ be an associated equilibrium triple for (20), *i.e.*, the identity ${\rm diag}(\bar{V}_{\rm th})\dot{\bar{T}}_{\rm th} = \mathbf{0} = A_{\rm th}(\bar{q}_{\rm r})\bar{T}_{\rm th} + B_{\rm pr}\bar{P}_{\rm pr} - B_{\rm c}P_{\rm c}$ holds, where ${\rm diag}(\bar{V}_{\rm th})$ is constant and positive definite (see Remark 5(i)). Since $q_{\rm r} = \bar{q}_{\rm r}$ all the time, then (20) is equivalent to

$$ {\rm diag}(\bar{V}_{\rm th})\dot{T}_{\rm th} = A_{\rm th}(\bar{q}_{\rm r})\left(T_{\rm th}-\bar{T}_{\rm th}\right) + B_{\rm pr}\left(P_{\rm pr}-\bar{P}_{\rm pr}\right). \quad (22) $$

Let us consider the mapping $\mathcal{H}_{\rm th}$ as a candidate storage function. Clearly $\mathcal{H}_{\rm th}$ is continuously differentiable and bounded from below by zero. Moreover, along the solutions of (22), its derivative can be shown to satisfy $\dot{\mathcal{H}}_{\rm th}(T_{\rm th}) \le (y_{\rm th}-\bar{y}_{\rm th})^\top(P_{\rm pr}-\bar{P}_{\rm pr})$, where we have used the fact that matrix $A_{\rm th}(\bar{q}_{\rm r})$ is negative semi definite (see Lemma 2). This concludes the proof. ∎

Remark 9.

(i) In view of the potential time-scale separation between hydraulic (fast) and thermal (slow) dynamics, and under certain conditions (see Jardón-Kojakhmetov and Scherpen (2017, Theorem 1) and Khalil (2002, Theorem 11.4)), singular perturbation theory (Kokotovic, Khalil, & O'Reilly, 1986) is useful to describe the qualitative behavior of the overall slow–fast system by separately analyzing each subsystem. This is the main reason why we directly analyze the slow subsystem assuming that $q_{\rm r} = \bar{q}_{\rm r}$ all the time.
(ii) The storage function of the flow dynamics (6) corresponds to the total kinetic energy of the stream through each pipe in the system, shifted with respect to an equilibrium value. Also, the passive output is a vector stacking the flows through the DH system's chords.
(iii) The proposed storage function $\mathcal{H}_{\rm th}$ in Theorem 4 is the DH system's shifted ectropy (see Haddad, 2019), which is quadratic in the total energy of the system (Haddad, 2019, Chapter 3); see also Dong et al. (2019) and Hauschild et al. (2020, Theorem 3.1). To see this, we recall that potential, kinetic and flow energies were neglected to establish the DH system's thermal dynamics and that we assumed that the internal energy of any element $i \in \mathcal{E} \cup \mathcal{N}$ is linear with respect to its temperature. Also, we recall that we took $\rho = c_{\rm s.h.} = 1$ to simplify notation. Moreover, considering the definition of the matrix $B_{\rm pr}$ in Corollary 1, then the passive output $y_{\rm th} = B_{\rm pr}^\top T_{\rm th}$ corresponds to a vector stacking the (shifted) internal energies of the producers' heat exchangers (see (11)) to which the power $P_{{\rm pr},i}$ is delivered.
(iv) Based on the results of Jayawardhana et al. (2007), De Persis and Kallesoe (2011), in Machado et al. (2020) we discuss the usefulness of the shifted passivity properties identified in Theorems 3 and 4 for the design of decentralized, passivity-based PI controllers to regulate the passive outputs towards desired, constant setpoints. The conditions to guarantee closed-loop asymptotic stability, which rely on verifying a detectability condition for the passive output (see van der Schaft, 2000), are mentioned as well. Moreover, the results are supported by numerical simulations on the model (20), which due to space constraints, could not fit in the present paper. System pressures and storage volume regulation are addressed in De Persis et al. (2014) and Machado, Cucuzzella, Pronk, and Scherpen (2021), respectively.
(v) As we specified at the beginning of this section, we would like to remark that Lemma 2 and Theorem 4 hold also for (18); notice that $\mathcal{A}(\bar{q}_{\rm E})$ has the same structure and properties of $\tilde{\mathcal{A}}(\bar{q}_{\rm E})$.

[8] By invoking (Hangos et al., 1999, Lemma 8) in Machado et al. (2020) we arrive to the same conclusion for the case in which some components of $\bar{q}_{\rm r}$ are zero.

6. Conclusions

In this paper we have addressed the modeling of a district heating system containing multiple heat producers, storage devices and consumers. All these elements are assumed to be interconnected through a common distribution network of meshed topology. The derived model is highly nonlinear, uncertain and explicitly considers the temperature dynamics of the pipes in the distribution network. We have analyzed the conditions under which this model is shifted passive, opening the possibility for the design of passivity-based controllers of simple structure that additionally offer closed-loop stability guarantees. Future research directions include: (i) the relaxation of some of our modeling assumptions, *e.g.*, the consideration that there are no stand-alone storage tanks; (ii) the consideration of more general heat demand profiles from consumers (*c.f.*, Scholten et al., 2015); and (iii) the study of the usefulness of the established passivity properties for the design of distributed or possibly hierarchical passivity-based control schemes such that the flows and temperatures are regulated towards optimal setpoints (see Vandermeulen et al., 2018, Section 6.3).

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