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# An $A_4$ flavor model for quarks and leptons in warped geometry

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**ABSTRACT:** We propose a spontaneous  $A_4$  flavor symmetry breaking scheme implemented in a warped extra dimensional setup to explain the observed pattern of quark and lepton masses and mixings. The main advantages of this choice are the explanation of fermion mass hierarchies by wave function overlaps, the emergence of tribimaximal neutrino mixing and zero quark mixing at the leading order and the absence of tree-level gauge mediated flavor violations. Quark mixing is induced by the presence of bulk flavons, which allow for “cross-brane” interactions and a “cross-talk” between the quark and neutrino sectors, realizing the spontaneous symmetry breaking pattern  $A_4 \rightarrow$  nothing first proposed in [X.G. He, Y.Y. Keum and R.R. Volkas, *JHEP* **04** (2006) 039]. We show that the observed quark mixing pattern can be explained in a rather economical way, including the CP violating phase, with leading order cross-interactions, while the observed difference between the smallest CKM entries  $V_{ub}$  and  $V_{td}$  must arise from higher order corrections. Without implementing  $P_{LR}$  (or other versions of) custodial symmetry, the bulk mass parameter of the left-handed quarks in this model is constrained by the  $Zb\bar{b}$  best fits, still allowing for a Kaluza-Klein scale below 2 TeV. Finally, we briefly discuss bounds on the Kaluza-Klein scale implied by flavor changing neutral current processes in our model and show that the residual little CP problem is milder than in flavor anarchic models.

**KEYWORDS:** Large Extra Dimensions, Beyond Standard Model, Quark Masses and SM Parameters

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**1 Introduction**

Warped extra dimensions [1, 2], which have been proposed as an alternative solution to the gauge hierarchy problem, also provide a simple framework in which fermion masses are explained by the overlap of the fermion and Higgs wave functions in the bulk of the warped extra dimension [3–6]. Having the zero mode fermions peaked at different points in the fifth dimension, the exponentially hierarchical masses of quarks and charged leptons can be obtained with a tiny hierarchy of bulk masses and all 5D Yukawa couplings being of order unity [7–10]. However, letting the standard model (SM) fermion content propagate through the bulk generally results in large contributions to electroweak precision observables, such as the Peskin-Takeuchi  $S$ ,  $T$  parameters, unless the lowest Kaluza-Klein (KK) mass scale is unnaturally pushed to values much higher than a TeV. To suppress these contributions, more realistic models involving a bulk custodial symmetry, broken differently at the two branes [11] were constructed. Alternatively, large brane kinetic terms were introduced [12–14]. In both cases a mass of the first KK excited state as low as  $\mathcal{O}(3 \text{ TeV})$ , is now allowed

by electroweak precision data. Another problem arises, this time due to the presence of non degenerate 5D bulk mass parameters, governing the localization of bulk zero modes. The non degeneracy induces new physics (NP) contributions to flavor changing neutral current (FCNC) processes mediated by KK excitations of the gauge bosons and fermions, through gauge interactions in the fermion kinetic terms and 5D Yukawa interactions. In the most general case, without imposing any additional flavor symmetry and assuming anarchical 5D Yukawa couplings, new physics contributions can already be generated at tree level through a KK gauge boson exchange. Even if an RS-GIM suppression mechanism [10, 15, 16] is at work, stringent constraints on the KK scale come from the  $K^0 - \bar{K}^0$  oscillation parameter  $\epsilon_K$  and the radiative decays  $b \rightarrow s(d)\gamma$  [17–19], the direct CP violation parameter  $\epsilon'/\epsilon_K$  [20], and especially the neutron electric dipole moment [17–19], where a mass of the first KK state of  $\mathcal{O}(3 \text{ TeV})$  gives rise to a NP contribution which is roughly twenty times larger than the current experimental bound — a CP problem in itself, referred to as little CP problem. Stringent constraints on the KK scale are also present in the lepton sector [21–25]. Even in the absence of neutrino masses, severe bounds arise from contributions to FCNC processes mediated by tree level KK gauge bosons with anarchical 5D Yukawa couplings [25]. It was also recently observed [26–28] that the mixing between fermion zero modes and KK modes generally induces a misalignment in the 4D effective theory between the SM fermion masses and the Higgs Yukawa couplings. This misalignment leads to Higgs mediated flavor changing neutral currents, once the fermion mass matrix is diagonalized. In particular,  $\epsilon_K$  is found [26–28] to produce stringent combined lower bounds on the KK gluon and the standard model Higgs mass in flavor anarchic models.

Additional flavor symmetries in the bulk can in principle allow to partially or fully remove these constraints, by forbidding or providing a further suppression of tree level FCNCs and one loop contributions induced by the presence of KK modes. One example that removes or suppresses all tree level contributions is the generalization to 5D of minimal flavor violation in the quark sector [29, 30] and in the lepton sector [31, 32]. In these settings, the bulk mass matrices are aligned with the 5D Yukawa matrices as a result of a bulk  $[U(3)]^6$  flavor symmetry that is broken in a controlled manner. In [33] a shining mechanism is proposed, where the suppression of flavor violation in the effective 4D theory on the IR brane is obtained by confining the sources of flavor violation to the UV brane, and communicating its effects through gauge bosons of the gauged bulk flavor symmetry. There, it is also shown that Higgs mediated FCNCs are eliminated to leading order, and a lowest KK scale of about 2-3 TeV seems to be allowed, rendering the model testable at collider experiments [34–38].

All above considerations suggest that a candidate for a realistic model of lepton and quark masses and mixings in a warped setup should possibly be realized with all standard model fields in the bulk, including the Higgs field, a bulk custodial symmetry and an additional flavor symmetry, to avoid large new physics contributions and maintain the KK scale of order a TeV. In [39], see also [40], a bulk  $A_4$  family symmetry [41–44] was used to explain masses and mixings in the SM lepton sector. In this setting, the three left-handed lepton doublets form a triplet of  $A_4$  to generate tribimaximal (TBM) neutrino mixing [45], in agreement with the recent global fit in [46]. In addition, tree-level leptonic FCNCs are absent in this scheme. While the simplest realization of  $A_4$  well describes the lepton sector,

it does not give rise to a realistic quark sector. In this paper, we propose a model based on a bulk  $A_4$  family symmetry, implemented in a slightly different setup, in an attempt to describe both the quark and lepton sectors. In this setup the scalar fields that transform under non trivial representations of  $A_4$ , namely two flavon triplets, reside in the bulk. Consequently, they allow for a complete "cross-talk" [47] between the  $A_4 \rightarrow Z_2$  spontaneous symmetry breaking (SSB) pattern associated with the heavy neutrino sector — with scalar mediator peaked towards the UV brane — and the  $A_4 \rightarrow Z_3$  SSB pattern associated with the quark and charged lepton sectors — with scalar mediator peaked towards the IR brane. As in previous models based on  $A_4$ , the three generations of left-handed quarks transform as triplets of  $A_4$ ; this assignment forbids tree level gauge mediated FCNCs and will allow to obtain realistic masses and almost realistic mixing angles in the quark sector. It will also be instructive to compare this pattern to the case of larger realizations of the flavor symmetry, like  $T'$  [48–53], which are usually associated with a rather richer flavon sector.

An additional feature worth to mention is the constraint on the common left-handed quarks bulk mass parameter implied by the  $Zb\bar{b}$  best fits in our model. The numerical significance of such a constraint has been thoroughly investigated in the minimal version of RS models [54], and it is largely relaxed in models with (extended)  $P_{LR}$  custodial symmetry [55, 56].

The paper is organized as follows. In section 2 we review the basic setup of the model and the various representation assignments. We then present a RS model with custodial symmetry and a bulk  $A_4$  family symmetry, and derive the leading order results for masses and mixings. In section 3 we classify all higher order corrections, including cross-talk and cross-brane operators for leptons and quarks and parametrize their effect. Section 4 contains our numerical analysis and results. In section 5 we discuss the vacuum alignment problem and suggest possible solutions, while in section 6 we briefly discuss constraints from flavor violating processes on the Kaluza-Klein scale in our model. We conclude in section 7.

## 2 The model and leading order results

We adopt the RS1 framework and thus assume the bulk of our model to be a slice of  $AdS_5$ , with the extra dimension,  $y$ , compactified on an orbifold  $S_1/Z_2$  with radius  $R$ . Two 3-branes with opposite tension are located at the orbifold fixed points  $y = 0$ , the UV brane, and  $y = \pi R$ , the IR brane. The resulting bulk geometry is described by the metric

$$ds^2 = dy^2 + e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu, \quad (2.1)$$

where  $k \sim M_{Pl}$  is the  $AdS_5$  curvature scale. The geometric warp factor sequestering the two branes generates two characteristic scales within this setup,  $k$  and  $M_{KK} \equiv k \exp(-k\pi R)$ , the latter referred to as the KK scale.<sup>1</sup> The electroweak scale naturally arises for  $kR \simeq 11$ .

All matter fields of our model, fermions and scalars including the Higgs field, live in the bulk and we allow for arbitrary  $Z_2 \times Z'_2$  orbifold boundary conditions, where  $Z_2$  is the reflection about  $y = 0$  and  $Z'_2$  is the reflection about  $y = \pi R$ . In other words, we allow for

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<sup>1</sup>Notice that the first KK gauge boson mass is about  $2.45 M_{KK}$

discontinuity of the bulk profiles at the orbifold fixed points by the presence of non trivial Scherk-Schwarz twists [57, 58].

The symmetry group in our model is

$$G = G_{\text{SM}}^{\text{cust}} \times A_4 \times Z_2 = \text{SU}(3)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L} \times A_4 \times Z_2. \quad (2.2)$$

The bulk gauge symmetry  $G_{\text{SM}}^{\text{cust}}$  is augmented by an  $A_4$  flavor symmetry plus an auxiliary  $Z_2$ , whose nature and role will be explained below. In addition, the electroweak gauge group is extended to  $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L}$  to incorporate custodial symmetry [11], and thus protect electroweak precision measurements with the lightest Kaluza-Klein mass being as light as  $\mathcal{O}(4\text{TeV})$ . This symmetry is broken down to the SM group  $\text{SU}(2)_L \times \text{U}(1)_Y$  on the UV brane, and down to  $\text{SU}(2)_D \times \text{U}(1)_{B-L}$  on the IR brane.

Both breaking patterns can be realized by orbifold boundary conditions on the gauge fields under  $Z_2 \times Z'_2$  as in [11]. In particular, the complete UV breaking pattern is achieved via an  $\text{SU}(2)_R$  doublet  $(1, 2)_{1/2}$  or a triplet scalar VEV, while a bidoublet  $(2, 2)_0$  Higgs VEV induces the IR breaking. These fields can either be decoupled by taking their infinite mass limit as in higgsless models, or be dynamical and used to generate masses of quarks and leptons, as it is true in our case for the Higgs field. Notice also that, in our case, the bidoublet Higgs field lives in the bulk and it is peaked towards the IR brane.

We introduce two scalar flavons  $\Phi$  and  $\chi$  and a Higgs field transforming under  $G_{\text{SM}}^{\text{cust}} \times A_4$  as

$$\Phi \sim (1, 1, 1, 0) (\mathbf{3}), \quad \chi \sim (1, 1, 1, 0) (\mathbf{3}), \quad H (1, 2, 2, 0) (\mathbf{1}). \quad (2.3)$$

The three families of quarks and leptons are assigned to the following representations:

$$\begin{aligned} Q_L &\sim \left(3, 2, 1, \frac{1}{3}\right) (\mathbf{3}) & \ell_L &\sim (1, 2, 1, -1) (\mathbf{3}) \\ u_R \oplus u'_R \oplus u''_R &\sim \left(3, 1, 2, \frac{1}{3}\right) (\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'') & \nu_R &\sim (1, 1, 2, 0) (\mathbf{3}) \\ d_R \oplus d'_R \oplus d''_R &\sim \left(3, 1, 2, \frac{1}{3}\right) (\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'') & e_R \oplus e'_R \oplus e''_R &\sim (1, 1, 2, -1) (\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'') , \end{aligned} \quad (2.4)$$

where the  $A_4$  notation is explained in the appendix, and the  $G_{\text{SM}}^{\text{cust}}$  notation is standard. The  $Z_2$  assignments will be specified later. Models with similar  $A_4$  assignments for leptons and scalar fields have been considered before [41–44, 47], thus many of the leading order properties are shared with our model. Notice that the right-handed neutrinos are assigned to a  $\mathbf{3}$  of  $A_4$ , whereas the right-handed charged-fermions are each given a  $\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}''$  structure. It is also important to notice that we have a separate  $\text{SU}(2)_R$  doublet for each right handed fermion.

Bulk fermions are assigned specific parities under  $Z_2 \times Z'_2$ , so that zero modes will provide the standard model particle content. As implied by the invariance of the 5D action, the bulk fermion field  $\Psi$  transforms as  $\Psi(y) = Z\gamma_5\Psi(-y)$  and  $\Psi(\pi + y) = Z'\gamma_5\Psi(\pi - y)$ , with  $Z, Z' = \pm 1$ , under  $Z_2$  and  $Z'_2$ , respectively. The 5D fermion corresponds to two Weyl spinors of opposite chirality in 4D

$$\Psi = \begin{pmatrix} \xi \\ \bar{\psi} \end{pmatrix} \quad (2.5)$$

and with opposite  $Z_2$  and  $Z'_2$  parities. Hence, if  $\xi$  has parities  $(Z, Z') = (+, +)$ ,  $\bar{\psi}$  has parities  $(-, -)$ . Even parity at  $y = 0$  or  $y = \pi R$  implies Neumann-like boundary conditions on the bulk fermion profile, while odd parity implies Dirichlet boundary conditions. It follows that a massless zero-mode only exists for a  $(+, +)$  Weyl spinor.

The bulk profile of the would-be zero mode is shaped by the fermionic bulk mass term with mass  $m = \epsilon(y)ck$ , and  $\epsilon(y)$  is the sign function. For  $c > 1/2$  (resp.  $c < 1/2$ ), the zero mode is exponentially localized on the UV (IR) brane. Finally, it is important to notice that since  $SU(2)_R$  is broken on the UV brane, the two components of each  $SU(2)_R$  doublet must have opposite  $Z_2$  parities. To get a massless zero mode from both components, we thus need to double the number of doublets [11], and we do this for quarks and leptons. Taking into account the above considerations, we assign the following boundary conditions to leptons, in the absence of localized mass terms:

$$\ell_L = \left( L \begin{bmatrix} +, + \end{bmatrix} \right) \quad e_R, \mu_R, \tau_R = \begin{pmatrix} \tilde{\nu}_{e,\mu,\tau} & \begin{bmatrix} +, - \end{bmatrix} \\ e_{R, \mu_R, \tau_R} & \begin{bmatrix} -, - \end{bmatrix} \end{pmatrix} \quad \nu_R = \begin{pmatrix} \nu_R & \begin{bmatrix} -, - \end{bmatrix} \\ \tilde{\ell} & \begin{bmatrix} +, - \end{bmatrix} \end{pmatrix}, \quad (2.6)$$

where the parities  $(Z, Z')$  are given for the upper Weyl spinor  $\xi$ , while  $\bar{\psi}$  has the opposite conditions. Hence, there is a left handed zero mode for each left handed doublet in  $\ell_L$ ,  $\ell = e, \mu, \tau$ , and a single right handed zero mode in  $e_R, \mu_R, \tau_R$  and  $\nu_R$  each. These fields have bulk masses, in units of the AdS curvature, given by  $c_{\ell L}, c_{\ell R}$  and  $c_{\nu R}$ , with  $\ell = e, \mu, \tau$ , and we work in the basis where they are real and diagonal. An important restriction is due to the  $A_4(\times Z_2)$  bulk global symmetry: since the three left handed lepton doublets are unified into a triplet of  $A_4$ , they will share one common  $c$  parameter which we label  $c_L^\ell$ .

Analogously, the boundary conditions for the quarks are chosen to be:

$$Q_L = \left( Q_L \begin{bmatrix} +, + \end{bmatrix} \right) \quad u_R, c_R, t_R = \begin{pmatrix} u_R, c_R, t_R & \begin{bmatrix} -, - \end{bmatrix} \\ \tilde{d}, \tilde{s}, \tilde{b} & \begin{bmatrix} +, - \end{bmatrix} \end{pmatrix} \quad d_R, s_R, b_R = \begin{pmatrix} \tilde{u}, \tilde{c}, \tilde{t} & \begin{bmatrix} +, - \end{bmatrix} \\ d_R, s_R, b_R & \begin{bmatrix} -, - \end{bmatrix} \end{pmatrix}. \quad (2.7)$$

In this way we have a left handed massless zero mode for the three left handed doublets in  $Q_L$  and a single right handed zero mode in  $u_R, c_R, t_R, d_R, s_R$  and  $b_R$  each. Again, the three left handed quark doublets, being assigned to a triplet of  $A_4$ , share one common bulk mass parameter,  $c_L^q$ . The right handed quarks are assigned to distinct one dimensional representations of  $A_4$ , hence there are 6 different bulk mass parameters entering their zero mode profiles,  $c_i^{u,d}$  and  $i = 1, 2, 3$ .

The  $G$  invariant 5D Yukawa lagrangian at leading order reads

$$\begin{aligned} \mathcal{L}_{\text{Yuk}(5D)} = & \Lambda_{5D}^{-2} [y_u(\bar{Q}_L \Phi)_{\underline{\mathbf{1}}} H u_R + y'_u(\bar{Q}_L \Phi)_{\underline{\mathbf{1}}'} H u''_R + y''_u(\bar{Q}_L \Phi)_{\underline{\mathbf{1}}''} H u'_R \\ & + y_d(\bar{Q}_L \Phi)_{\underline{\mathbf{1}}} \tilde{H} d_R + y'_d(\bar{Q}_L \Phi)_{\underline{\mathbf{1}}'} \tilde{H} d''_R + y''_d(\bar{Q}_L \Phi)_{\underline{\mathbf{1}}''} \tilde{H} d'_R \\ & + y_e(\bar{\ell}_L \Phi)_{\underline{\mathbf{1}}} \tilde{H} e_R + y'_e(\bar{\ell}_L \Phi)_{\underline{\mathbf{1}}'} \tilde{H} e''_R + y''_e(\bar{\ell}_L \Phi)_{\underline{\mathbf{1}}''} \tilde{H} e'_R] \\ & + \Lambda_{5D}^{-1/2} [y_\nu^D(\bar{\ell}_L \nu_R)_{\underline{\mathbf{1}}} H + y_\chi[\bar{\nu}_R(\nu_R)^c]_{\underline{\mathbf{3}}_s} \cdot \chi] + M[\bar{\nu}_R(\nu_R)^c]_{\underline{\mathbf{1}}} + h.c., \quad (2.8) \end{aligned}$$

where  $\tilde{H} \equiv i\tau_{2L} H^*$ , all fields propagate in the bulk and  $\Lambda_{5D}$  is naturally of order  $M_{Pl}$ . The Higgs and  $\Phi$  fields are chosen to be peaked towards the IR brane at  $y = \pi R$ , while

the field  $\chi$  is peaked towards the UV brane at  $y = 0$ , for phenomenological reasons. The lagrangian in eq. (2.8) has a relatively simple structure. Each charged fermion sector  $u, d, e$  has three independent Yukawa terms, all involving the  $A_4$  triplet field  $\Phi$  and the Higgs field. By construction, the neutrino Dirac term is governed by a single coupling constant  $y_\nu^D$  and involves the Higgs only, while the right-handed Majorana sector contains one bare Majorana mass  $M$ , and a single Yukawa coupling  $y_\chi$  to the electroweak singlet and  $A_4$  triplet  $\chi$ . In total there are only twelve, a priori complex, Yukawa parameters to describe masses and mixings of nine Dirac and six Majorana fermions. All of these parameters will be taken to be universal and of  $\mathcal{O}(1)$  in the construction of section 4.

The lagrangian in eq. (2.8) respects an additional  $Z_2$  symmetry, under which  $Q_L, \ell_L, \nu_R$  and  $\Phi$  are odd, while all other fields are even. This non-flavor symmetry ensures that the  $G_{\text{SM}}^{\text{cust}} \times A_4$  invariant term  $\bar{\ell}_L \Phi H \nu_R$  is absent from the Lagrangian.

All the SM fields are identified with the 4D components of the zero modes in the Kaluza-Klein decomposition of the bulk fields. Masses and mixings for leptons and quarks are induced by eq. (2.8) once the flavons  $\Phi, \chi$  and the Higgs have acquired a VEV. The VEVs of  $\Phi$  and  $\chi$  will be responsible for providing two distinct patterns of spontaneous symmetry breaking of  $A_4$ . The VEV profiles for the scalar fields in our model are solutions of the bulk equations of motion with almost vanishing bulk mass for stabilization purposes [59], an IR localized quartic double well potential for  $\Phi$  and the Higgs, and a similar term for  $\chi$  on the UV brane. To leading order in  $\exp(-2\pi kR)$ , they read

$$\Phi_a(y) = v_a e^{4(k|y| - \pi kR)} \quad H(y) = H_0 e^{4(k|y| - \pi kR)} \quad \chi_a(y) = \chi_a (1 - e^{4(k|y| - \pi kR)}), \quad (2.9)$$

where  $a = 1, 2, 3$  denotes the  $A_4$  component. The size of the SM fermion masses is thus determined by the amount of wavefunction overlap of two zero modes of opposite chirality, corresponding to the same Dirac fermion in 4D, together with the VEV profiles of the corresponding scalar fields. This holds as far as the zero mode approximation (ZMA) is a good description of the 4D reduction of our model. In general, the presence of kinetic and potential boundary terms for the scalar fields will, after SSB of  $A_4$  and the electroweak symmetry, lead to boundary conditions that mix all levels of KK fermions. The light modes are however rather insensitive to the presence of these boundary terms and can be treated as a small perturbation. Thus, to leading order, the low energy mass spectrum can be obtained by using the zero mode profiles of all bulk fields. The accuracy of the ZMA in each specific case depends on the lightness of the lowest lying KK states. Since the largest mass present in our model is that of the  $t$  quark,  $m_t = 171.3 \text{ GeV}$ , the ZMA turns out to be as accurate as  $m_t M_{\text{KK}} \simeq 0.03$  for the masses of the zero mode fermions.

Writing out the charged-fermion  $f = u, d, e$  Yukawa invariants of eq. (2.8), and following the rules in the appendix, one finds that each of the three mass matrices has the form

$$M_f = \int_{-\pi R}^{\pi R} dy \sqrt{-g} \frac{H(y)}{\Lambda_{5D}^2} (\bar{f}_{1L}, \bar{f}_{2L}, \bar{f}_{3L}) \begin{pmatrix} y\Phi_1 & y'\Phi_1 & y''\Phi_1 \\ y\Phi_2 & \omega y'\Phi_2 & \omega^2 y''\Phi_2 \\ y\Phi_3 & \omega^2 y'\Phi_3 & \omega y''\Phi_3 \end{pmatrix} \begin{pmatrix} f_R \\ f_R'' \\ f_R' \end{pmatrix} + h.c. \quad (2.10)$$

where the scalar profiles  $H(y)$  and  $\Phi_a(y)$  are from eq. (2.9), the metric factor is  $\sqrt{-g} = \exp(-4k|y|)$ , the Yukawas  $y, y', y''$  have a suppressed subscript  $f$ , and  $f_{L,R}(y)$  are the

fermion bulk profiles. The numerical subscripts 1, 2, 3 denote  $A_4$  components, as in the appendix. For the special VEV pattern of  $\Phi$

$$v_1 = v_2 = v_3 \equiv \Phi_0, \tag{2.11}$$

we define  $v \equiv H_0 \Phi_0 / \Lambda_{5D}^2$ , and each of the above mass matrices translates into the effective 4D mass matrix

$$M_f = U(\omega) \begin{pmatrix} \sqrt{3}\tilde{y}_f v & 0 & 0 \\ 0 & \sqrt{3}\tilde{y}'_f v & 0 \\ 0 & 0 & \sqrt{3}\tilde{y}''_f v \end{pmatrix} \quad \tilde{y}_f = y_f \int_{-\pi R}^{\pi R} \frac{dy}{2\pi R} F(c_{L_f}, c_{R_f}) e^{8k|y|-8k\pi R}, \tag{2.12}$$

where we conveniently introduced the fermion overlap function

$$F(c_{L_i}, c_{R_j}) \equiv \sqrt{-g} f_{L_i}^{(0)}(y) f_{R_j}^{(0)}(y) = k\pi R \sqrt{\frac{(1-2c_{L_i})(1-2c_{R_j})}{(e^{(1-2c_{L_i})\pi kR}-1)(e^{(1-2c_{R_j})\pi kR}-1)}} e^{(-c_{L_i}-c_{R_j})k|y|}, \tag{2.13}$$

product of the 5D profiles for the zero modes of the fermion fields,  $f_{L_i}^{(0)}$  and  $f_{R_j}^{(0)}$ , with  $f = u, d, e$  and the factor  $\sqrt{-g}$  included. This shows that the left-diagonalization matrices  $V_L^{u,d,e}$  for the up-quark, down-quark and charged lepton sectors, respectively, are identical and equal to the unitary trimaximal mixing matrix [47],

$$V_L^{u,d,e} = U(\omega) = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}. \tag{2.14}$$

This diagonalization property of mass matrices is referred to as ‘‘form diagonalizability’’ [60]; in this case the mixing angles are independent of the mass eigenvalues. One can easily see that, at this order and due to the above  $A_4$  assignments, the  $\Phi$  VEV of eq. (2.11) forces the CKM matrix to be the identity:

$$V_{\text{CKM}} = V_L^{u\dagger} V_L^d = U(\omega)^\dagger U(\omega) = 1. \tag{2.15}$$

It also induces the breaking pattern

$$A_4 \rightarrow Z_3 \cong C_3 = \{1, c, a\}, \tag{2.16}$$

where  $\cong$  denotes ‘‘isomorphism’’, see the appendix. The remnant  $Z_3$  flavor subgroup cyclically permutes the three  $A_4$  triplet basis states with no change of signs, see eq. (A.1) and [47]. The  $\mathbf{1}'$  and  $\mathbf{1}''$  representations transform under this subgroup in the same way they do under the full flavor group,  $A_4$ . We will show below that, if the remnant  $Z_3$  symmetry remains unbroken, the CKM matrix will remain trivial to all orders. A further breaking is thus needed in order to produce deviations of the CKM matrix from unity. In [47] it was first suggested that such small deviations can be generated by higher-order effects, able to induce a relatively weak subsequent breaking of the residual  $Z_3$  flavor symmetry, i.e.  $Z_3 \rightarrow \text{nothing}$ .

Our main goal is to implement this idea in our setup to obtain a realistic CKM matrix, without spoiling the results that will be obtained below for the neutrino and charged lepton sector. It is for this purpose that we choose to use bulk flavons. The non vanishing overlap between the profiles of  $\Phi$  and  $\chi$  allows for the presence of higher dimensional operators, which communicate the symmetry breaking pattern associated with  $\langle\chi\rangle$  to the quark sector, as explained in section 3.2. On the other hand, having these profiles exponentially localized on different branes, combined with the internal symmetries of the model, strongly suppresses the mixed interaction terms in the scalar potential  $V(\Phi, \chi)$ , thus allowing to approximately preserve the original alignment between the VEVs of these fields, as discussed in section 5.

### 2.1 Leading order results in the neutrino sector

This is an immediate generalization to our model of the 4D derivation in [47] and the 5D warped model with brane localized scalars on an interval in [39]. The neutrino Dirac mass matrix is derived from the Yukawa term  $\bar{\ell}_L H \nu_R$  in eq. (2.8). Using eq. (A.5), the 4D Dirac mass matrix turns out to be proportional to the identity matrix

$$M_\nu^D = H_0 \tilde{y}_\nu^D \mathbb{1} = \left( \frac{H_0 y_\nu^D}{\Lambda_{5D}^{1/2}} \int_{-\pi R}^{\pi R} \frac{dy}{2\pi R} e^{4k(|y|-\pi R)} F(c_L^\ell, c_{\nu_R}) \right) \cdot \mathbb{1} \equiv m_\nu^D \mathbb{1}. \quad (2.17)$$

In this equation  $\tilde{y}_\nu^D$  is the effective 4D coupling and  $y_\nu^D$  is dimensionless. The right-handed neutrino bare Majorana mass matrix is similarly trivial, being  $M_\nu^M = M \int dy F(c_{\nu_R}, c_{\nu_R}) \cdot \mathbb{1} \equiv \tilde{M} \mathbb{1}$ . It is the coupling to the flavon  $\chi$  that induces a non trivial pattern in the neutrino mass matrix, with contribution

$$M_\nu^X = \tilde{y}_\chi \begin{pmatrix} 0 & \chi_3 & \chi_2 \\ \chi_3 & 0 & \chi_1 \\ \chi_2 & \chi_1 & 0 \end{pmatrix}, \quad \tilde{y}_\chi = \frac{y_\chi}{\Lambda_{5D}^{1/2}} \int_{-\pi R}^{\pi R} \frac{dy}{2\pi R} (1 - e^{4k(|y|-\pi R)}) F(c_{\nu_R}, c_{\nu_R}), \quad (2.18)$$

with  $\chi_a$ ,  $a = 1, 2, 3$  the VEV components of eq. (2.9). We now follow [41–44, 47] and assume the breaking pattern  $A_4 \rightarrow Z_2 = \{1, r_2\}$  induced by the choice of the  $\chi$  VEV

$$\chi_1 = \chi_3 = 0, \quad \chi_2 \equiv \chi_0 \neq 0, \quad (2.19)$$

so that the full  $6 \times 6$  neutrino mass matrix in 4D becomes

$$M_\nu^{\text{total}} = \begin{pmatrix} 0 & 0 & 0 & m_\nu^D & 0 & 0 \\ 0 & 0 & 0 & 0 & m_\nu^D & 0 \\ 0 & 0 & 0 & 0 & 0 & m_\nu^D \\ m_\nu^D & 0 & 0 & \tilde{M} & 0 & M_\chi \\ 0 & m_\nu^D & 0 & 0 & \tilde{M} & 0 \\ 0 & 0 & m_\nu^D & M_\chi & 0 & \tilde{M} \end{pmatrix}, \quad (2.20)$$

where  $M_\chi \equiv \tilde{y}_\chi \chi_0$ , and  $\tilde{M}$  and  $M_\chi$  are in general complex. In the see-saw limit  $|\tilde{M}|, |M_\chi| \gg m_\nu^D$ , and the effective  $3 \times 3$  light neutrino mass matrix is thus

$$M_L^\nu = -M_\nu^D (M_\nu^M + M_\nu^\chi)^{-1} (M_\nu^D)^T = -\frac{(m_\nu^D)^2}{\tilde{M}} \begin{pmatrix} \frac{\tilde{M}^2}{\tilde{M}^2 - M_\chi^2} & 0 & -\frac{\tilde{M}M_\chi}{\tilde{M}^2 - M_\chi^2} \\ 0 & 1 & 0 \\ -\frac{\tilde{M}M_\chi}{\tilde{M}^2 - M_\chi^2} & 0 & \frac{\tilde{M}^2}{\tilde{M}^2 - M_\chi^2} \end{pmatrix}, \quad (2.21)$$

whose diagonalization matrix is

$$V_L^\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix}. \quad (2.22)$$

The MNSP matrix, at this order, is then

$$V_{\text{MNSP}} = V_L^{e\dagger} V_L^\nu = U(\omega)^\dagger V_L^\nu = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{\omega^2}{\sqrt{6}} & \frac{\omega^2}{\sqrt{3}} & -\frac{e^{-i\pi/6}}{\sqrt{2}} \\ -\frac{\omega}{\sqrt{6}} & \frac{\omega}{\sqrt{3}} & \frac{e^{-5i\pi/6}}{\sqrt{2}} \end{pmatrix}, \quad (2.23)$$

which is tribimaximal up to phases and in good agreement with the neutrino oscillation data [46], as already concluded in [47, 61]. Notice that the Jarlskog invariant is vanishing despite the presence of these phases, and consequently CP violation is absent at this order.

The bulk scalar fields  $\chi$  and  $\Phi$ , which are in charge of the symmetry breaking pattern  $A_4 \rightarrow Z_2$  in the neutrino sector and  $A_4 \rightarrow Z_3$  in the charged-fermion sector, respectively are peaked on different branes. Thus, the two distinct flavor symmetry breaking patterns will be approximately, but not fully sequestered from one another, due to the bulk nature of these fields. Thus, while the leading order lagrangian in eq. (2.8) does not allow for a talking between the two sectors, higher dimensional operators will ensure the complete breaking of the  $A_4$  flavor symmetry through the overlap of bulk scalar fields. It is these effects that we are interested in, in order to account for a realistic CKM matrix.

Higher order effects were naturally divided into two classes [47], those which preserve the flavor subgroups ( $Z_2$  or  $Z_3$ ) of each sector, call them “higher-order”, and those that involve interactions between the two sectors, “cross-talk”. The former preserve  $Z_3$  in the quark and charged lepton sectors, and  $Z_2$  in the neutrino sector. The latter communicate  $Z_3$  breaking to the charged sector via  $\chi$ , and  $Z_2$  breaking to the neutrino sector via  $\Phi$ . In our context, a further way to isolate the dominant contributions amongst all higher order terms is to distinguish between brane localized interactions, UV or IR, and “cross-brane” interactions induced by the overlap of the bulk profiles of  $\Phi$  and  $\chi$ . We will first show the pattern of all dominant higher order corrections to the CKM and MNSP matrices, producing deviations from the trivial and tribimaximal forms, respectively. We will then estimate their structure and size in our model and compare with existing results. We are mainly interested in the quark sector. The reason is that within our setup an almost realistic structure for the CKM matrix can emerge with a fairly restricted choice of the parameters involved.

### 3 Higher order and cross-talk corrections

We first consider charged fermions, to immediately show that higher order contributions in the  $Z_3$ -preserving sector, i.e. no cross-talk, leave unchanged the leading order result for mass and mixing matrices, in particular  $V_L^\ell = U^\dagger(\omega)$  to all orders for charged leptons.

The leading order contribution comes from the operator in eq. (2.8) of the form  $\bar{f}_L \Phi H f_R$ . The only higher order corrections allowed in the  $Z_3$ -preserving sector come from the operators of the type  $\bar{f}_L \Phi^n H f_R$ . After breaking of  $A_4$ , and since the VEV of  $\Phi$  is  $Z_3$  symmetric,  $\Phi^2$  transforms as  $1 + \Phi$ . Given that only the  $A_4$  triplet part of  $\Phi^n$  will contribute, it is clear that the corrections to masses and mixings of the above operators to all orders will be identical in structure to the leading order ones. This shows that the  $Z_3$  symmetry in the charged fermion sector will ensure  $V_L^{\ell,u,d} = U^\dagger(\omega)$  and thus prevent quark mixing, so that the only way to allow for a non trivial CKM matrix is to further break  $Z_3$ . We assume  $V_L^\ell = U^\dagger(\omega)$  in the rest of this section, and postpone to sections 3.2 and 4.4 the analysis of the suppressed cross-talk  $Z_3$  violating contributions.

#### 3.1 Cross-talk and cross-brane effects in the neutrino sector

We now identify all the  $A_4$  symmetric higher dimensional operators contributing to the neutrino sector in our model. These are obtained by additional insertions of the fields  $\chi$  and  $\Phi$  into the leading order terms for neutrinos in eq. (2.8). Cross-talk is induced by all contributions involving  $\Phi$ . We can already anticipate a pattern in the corrections. Given that the VEV of  $\chi$  is  $Z_2$  preserving,  $\chi^3$  transforms effectively as  $\chi$  under  $A_4$ . Hence, the contributions of  $\chi^m$  operators to the (1, 2), (2, 1), (2, 3) and (3, 2) entries of both the neutrino Dirac and right-handed Majorana mass matrices will be zero to all orders. Analogously, since  $\Phi^2$  transforms as  $1 + \Phi$ , the contributions of  $\Phi^{2n}$  operators will be absorbed in the leading order contributions, or effectively amount to the operator with one  $\Phi$  insertion. In general, the size of all higher order contributions will be suppressed with respect to the leading order by powers of the relevant scales  $(VEV)^n / \Lambda_{5D}^n$  and by the amount of overlap of bulk profiles; recall that  $\chi$  is UV peaked, while  $H$  and  $\Phi$  are IR peaked. Hence, a strong suppression is induced by the small overlap in the interference of  $\chi - \Phi$  and  $\chi - H$ . The complete higher order contribution to the neutrino Dirac lagrangian can be written as follows

$$\Delta \mathcal{L}_\nu^D = \frac{1}{\Lambda_{5D}^{(6n+3m+1)/2}} \left[ (\bar{\ell}_L \chi^m H \nu_R)_{m \geq 1} \Big|_{\text{h.o.}} + ((\bar{\ell}_L \Phi^{2n} H \nu_R)_{n \geq 1} + (\bar{\ell}_L \Phi^{2n} \chi^m H \nu_R)_{m,n \geq 1}) \Big|_{\text{cross-talk}} \right], \quad (3.1)$$

where we separated the cross-talk terms from the rest. Notice that odd powers of  $\Phi$  are forbidden by the additional  $Z_2$  in  $G$ . It is immediate to obtain the following textures for the above corrections to the neutrino Dirac mass matrix to all orders:

$$\frac{M_\nu^D}{\tilde{y}_\nu^D H_0} = \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right) \Big|_{\text{l.o.}} + \left( \begin{array}{ccc} \epsilon_{11}^\chi & 0 & \epsilon_{13}^\chi \\ 0 & \epsilon_{22}^\chi & 0 \\ \epsilon_{13}^\chi & 0 & \epsilon_{11}^{\chi*} \end{array} \right) \Big|_{\text{h.o.}} + \left( \begin{array}{ccc} \epsilon_{11}^\Phi + \hat{\epsilon}_{11} & \epsilon_2^\Phi + \hat{\epsilon}_1 & \epsilon_3^\Phi \\ \epsilon_3^\Phi + \hat{\epsilon}_2 & \epsilon_{11}^\Phi + \hat{\epsilon}_{22} & \epsilon_2^\Phi + \hat{\epsilon}_2 \\ \epsilon_2^\Phi & \epsilon_3^\Phi + \hat{\epsilon}_1 & \epsilon_{11}^\Phi + \hat{\epsilon}_{11} \end{array} \right) \Big|_{\text{cross-talk}}, \quad (3.2)$$

where  $\epsilon_{11,22}^\chi \sim \mathcal{O}(\chi_0^2/\Lambda_{5D}^3)$ ,  $\epsilon_{13}^\chi \sim \mathcal{O}(\chi_0/\Lambda_{5D}^{3/2})$ ,  $\epsilon_i^\Phi, \epsilon_{ii}^\Phi \sim \mathcal{O}(\Phi_0^2/\Lambda_{5D}^3)$  and  $\hat{\epsilon}_i, \hat{\epsilon}_{ii} \sim \mathcal{O}(\Phi_0^2\chi_0/\Lambda_{5D}^{9/2})$ . Notice that in the above expression the coefficient  $\epsilon_{11}^\Phi$  can be absorbed into a redefinition of  $\tilde{y}_\nu^D$ , and similarly the coefficients  $\hat{\epsilon}_{11}$  and  $\hat{\epsilon}_{22}$  can be absorbed in  $\epsilon_{11}^\chi$  and  $\epsilon_{22}^\chi$ . The Majorana mass matrix is corrected by the operators

$$\Delta\mathcal{L}_\nu^M = \frac{1}{\Lambda_{5D}^{(3m+6n-2)/2}} \left[ (\chi^m \bar{\nu}_R (\nu_R)^c)_{m \geq 2} \Big|_{\text{h.o.}} + (\Phi^{2n} \chi^m \bar{\nu}_R (\nu_R)^c)_{m \geq 0, n \geq 1} \Big|_{\text{cross-talk}} \right]. \tag{3.3}$$

Using again the transformation properties of  $\Phi^{2n}$  and  $\chi^m$  we obtain the following texture for the Majorana mass matrix,

$$M_\nu^M = \left( \begin{array}{ccc} \tilde{M} & 0 & M_\chi \\ 0 & \tilde{M} & 0 \\ M_\chi & 0 & \tilde{M} \end{array} \right) \Big|_{\text{l.o.}} + \left( \begin{array}{ccc} \epsilon'_{11} & 0 & \epsilon'_{13} \\ 0 & \epsilon'_{22} & 0 \\ \epsilon'_{13} & 0 & \epsilon'_{11} \end{array} \right) \Big|_{\text{h.o.}} + \left( \begin{array}{ccc} 0 & \tilde{\epsilon}_1 + \epsilon_1 & \epsilon_2 \\ \tilde{\epsilon}_2 + \epsilon_2 & 0 & \tilde{\epsilon}_2 + \epsilon_1 \\ \epsilon_1 & \tilde{\epsilon}_1 + \epsilon_2 & 0 \end{array} \right) \Big|_{\text{cross-talk}}, \tag{3.4}$$

where  $\epsilon'_{ij} \sim \mathcal{O}(M_\chi \chi_0^2/\Lambda_{5D}^3)$ ,  $\tilde{\epsilon}_i \sim \mathcal{O}(M_\chi \Phi_0^2 \chi_0/\Lambda_{5D}^{9/2})$  and  $\epsilon_i \sim \mathcal{O}(M_\chi \Phi_0^2/\Lambda_{5D}^3)$ . Notice that  $\epsilon'_{13}$  can be absorbed into a redefinition of  $M_\chi$ , while for simplicity we have neglected diagonal contributions that redefine  $\tilde{M}$ .

If we neglect cross-brane interactions, induced by the overlap of  $\Phi$ ,  $\chi$  and  $H$  bulk profiles, the corrected light neutrino mass matrix,  $M_L^\nu$ , once rotated to the basis of diagonal charged leptons with  $U^\dagger(\omega)$ , and assuming all real input parameters, will be to any order of the following form:

$$M_L^\nu \rightarrow \tilde{M}_L^\nu = \left( \begin{array}{ccc} \delta_1 & \delta_2 & \delta_2^* \\ \delta_2 & \delta_4 & \delta_3 \\ \delta_2^* & \delta_3 & \delta_4^* \end{array} \right) \Big|_{\text{no cross-brane}}, \tag{3.5}$$

where the entries  $\delta_i$  are anyway complex due to the  $\omega$  factors in  $U(\omega)^\dagger$ . This matrix is identical to the one obtained in [39]; this is not surprising, since their model is the limit of our model when  $\chi$  is confined to the UV brane, while  $\Phi$  and the Higgs are confined to the IR brane. One can easily verify that the above matrix is diagonalized with  $\theta_{23} = \pi/4$  and  $\theta_{13} = 0$ , if  $\delta_2$  and  $\delta_4$  are real. We can conclude that for particular realizations of the parameters, it is certainly possible that the leading order values  $\theta_{23} = \pi/4$  and  $\theta_{13} = 0$  remain protected from higher order brane-localized corrections, while in the most general case they will be corrected by naturally suppressed contributions.

The most dominant cross-brane operator is  $\bar{\ell}_L \chi H \nu_R$ . If the bulk mass of  $\chi$  is vanishing, this operator is suppressed only by  $\epsilon_{13}^\chi$ , compared to the leading order Dirac mass term. Keeping the perturbative expansion linear in each of the various  $\epsilon$ 's, one can treat the textures associated with each operator in an additive manner. Therefore, even before we set the exact profile of  $H$  and  $\chi$  and deal with all of the other operators, we can have a good understanding of the deviations it induces to  $\theta_{12}$ ,  $\theta_{13}$  and  $\theta_{23}$ .

We now recall that in general the effective Majorana Mass matrix is a  $3 \times 3$  complex symmetric matrix and thus contains 12 parameters. These parameters are the 3 masses, the 3 mixing angles and 6 phases, out of which 3 can be absorbed in the neutrino fields and

the remaining are 2 Majorana and 1 KM phase. Notice also that, in general, the various  $\epsilon_{ij}^\chi$  of eq. (3.2) are complex. Thus, considering only the contributions of operators of the form  $\bar{\ell}_L \chi^m H \nu_R$ , the left-diagonalization matrix is now corrected to:

$$V_L^\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} 1/2(\sqrt{2} - \epsilon_\chi^2) & 0 & -(1/\sqrt{2} + \epsilon_\chi) \\ 0 & 1 & 0 \\ 1/\sqrt{2} + \epsilon_\chi & 0 & 1/2(\sqrt{2} - \epsilon_\chi^2) \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & e^{i\alpha_3} \end{pmatrix}, \quad (3.6)$$

where  $\epsilon_\chi \sim \mathcal{O}(\chi_0/\Lambda_{5D}^{3/2})$  stands for contributions from  $\epsilon_{13}^\chi$  and  $\epsilon_{11,22}^\chi$  in eq. (3.2) and we have omitted terms of  $\mathcal{O}(\epsilon_\chi^3)$  and higher. In particular,  $\epsilon_{13}^\chi \sim \tilde{y}_\nu^D H_0 \epsilon_\chi$  and  $\epsilon_{11,22}^\chi \sim \tilde{y}_\nu^D H_0 \epsilon_\chi^2$ . The phases  $\alpha_i$  can be absorbed in a rotation of the neutrino fields, while the KM phase  $\delta$ , given by

$$\delta = \text{Arg}(\tilde{M} + |\tilde{M}| \epsilon_{11}^{\chi*}) - \text{Arg}(\tilde{M} + |\tilde{M}| \epsilon_{11}^\chi), \quad (3.7)$$

will contribute to  $CP$  violation in neutrino oscillations. The MNSP matrix, to  $\mathcal{O}(\epsilon_\chi)$ , acquires a simple structure

$$V_{\text{MNSP}} = U(\omega)^\dagger V_L^\nu = \frac{1}{\sqrt{6}} \begin{pmatrix} (1 + e^{i\alpha} \epsilon_\chi) & \sqrt{2} & (e^{i\alpha} - \epsilon_\chi) \\ (1 + \omega e^{i\alpha} \epsilon_\chi) & \sqrt{2} \omega^2 & (\omega e^{i\alpha} - \epsilon_\chi) \\ (1 + \omega^2 e^{i\alpha} \epsilon_\chi) & \sqrt{2} \omega & (\omega^2 e^{i\alpha} - \epsilon_\chi) \end{pmatrix}, \quad (3.8)$$

where the middle column does not receive corrections. A non zero  $\theta_{13}$  is generated, and  $\theta_{12}$  and  $\theta_{23}$  deviate from their leading order bimaximal values. Defining  $\theta = \pi/4 + \epsilon_\chi$ , the Jarlskog invariant turns out to be

$$\text{Im}[V_{11} V_{12}^* V_{21}^* V_{22}] = \frac{\sqrt{3}}{18} (\cos 2\theta - \sin 2\theta \sin \delta), \quad (3.9)$$

where the  $V_{ij}$  denote the entries of  $V_{\text{MNSP}}$ .

To account for all possible deviations from tribimaximal mixing we should consider the fully perturbed effective neutrino mass matrix to first order in all the  $\epsilon$ 's defined in eqs. (3.2) and (3.4). However, considering the scales and wavefunction overlaps associated with  $\hat{\epsilon}_i$  in eq. (3.2) and  $\tilde{\epsilon}_i$  in eq. (3.4), it is clear that their contributions are of characteristic strength  $\mathcal{O}(10^{-3})$ . Consequently, the deviations from TBM induced by these effect are negligible and actually below the model theoretical error. To complete our analysis, the only contributions that need to be further studied are those encoded in  $\epsilon_i$  of eq. (3.4) and those associated with complex  $\epsilon_i^\Phi$  parameters in eq. (3.2). The resulting expressions for the deviations induced by these effects turn out to be quite cumbersome, yet the largest deviations are still almost an order of magnitude smaller than those described in eq. (3.8). These contributions will anyway be taken into account in the estimations of section 4.4.

### 3.2 Cross-talk and cross-brane effects in the charged fermion sector

As suggested in [47],  $Z_3$  breaking cross-talk effects should produce deviations of the CKM matrix from unity. The details of the cross-talk depend on the specific dynamics and in our case on the wavefunction overlap of  $\Phi$  and  $\chi$ . After introducing the higher dimensional

cross-talk operators for quarks, we will set the values of  $\Phi$ ,  $\chi$  and  $H$  VEVs to obtain the physical quark masses while maintaining the Yukawa couplings  $y_{u^i, d^i} \simeq 1$ .

The higher dimensional 5D operators relevant for quark mixing are generically suppressed by  $\Lambda_{5D}^{7/2}$ . Schematically, they are of the form

$$\begin{aligned} \bar{Q}_L u_R H \Phi \chi, \quad \bar{Q}_L u'_R H \Phi \chi, \quad \bar{Q}_L u''_R H \Phi \chi, \\ \bar{Q}_L d_R \tilde{H} \Phi \chi, \quad \bar{Q}_L d'_R \tilde{H} \Phi \chi, \quad \bar{Q}_L d''_R \tilde{H} \Phi \chi, \end{aligned} \quad (3.10)$$

The VEV of the UV peaked  $\chi$  communicates  $Z_3$  breaking to the quarks through these operators, and they are suppressed by an amount  $\delta \simeq \mathcal{O}(\chi_0/\Lambda_{5D}^{3/2})$  compared to the leading order. They are also suppressed by the small overlap between the bulk profiles of  $\chi$  and  $\phi$ , a suppression that amounts to a multiplicative factor, slightly different for the various operators.

We focus on the  $A_4$  textures associated with the above operators and conveniently disregard  $H$  in what follows, since it transforms trivially under  $A_4$ . Each operator in eq. (3.10) represents two independent  $A_4$  invariants; for example  $\bar{Q}_L \Phi \chi u_R$  schematically denotes the independent terms

$$[(\bar{Q}_L \Phi)_{\mathbf{3}_s} \chi]_{\mathbf{1}} u_R \quad \text{and} \quad [(\bar{Q}_L \Phi)_{\mathbf{3}_a} \chi]_{\mathbf{1}} u_R. \quad (3.11)$$

Writing these terms according to the decomposition in the appendix, and after  $A_4$  breaking by the VEVs in eqs. (2.11) and (2.19), we obtain the general corrections to the 4D quark mass matrices, as also in [47]

$$\Delta M_{u,d} = \begin{pmatrix} x_1^{u,d} & x_2^{u,d} & x_3^{u,d} \\ 0 & 0 & 0 \\ y_1^{u,d} & y_2^{u,d} & y_3^{u,d} \end{pmatrix}, \quad (3.12)$$

where all entries are in general complex, and the leading order mass matrix is linearly corrected to,  $q = u, d$

$$\begin{aligned} M_q + \Delta M_q &= U(\omega) \sqrt{3} \begin{pmatrix} \tilde{y}_{q1} v + (x_1^q + y_1^q)/3 & (x_2^q + y_2^q)/3 & (x_3^q + y_3^q)/3 \\ (x_1^q + \omega y_1^q)/3 & \tilde{y}'_{q2} v + (x_2^q + \omega y_2^q)/3 & (x_3^q + \omega y_3^q)/3 \\ (x_1^q + \omega^2 y_1^q)/3 & (x_2^q + \omega^2 y_2^q)/3 & \tilde{y}''_{q3} v + (x_3^q + \omega^2 y_3^q)/3 \end{pmatrix} \\ &\equiv U(\omega) V_L^{u,d} \begin{pmatrix} m_{u,d} & 0 & 0 \\ 0 & m_{c,s} & 0 \\ 0 & 0 & m_{t,b} \end{pmatrix} V_R^{u,d\dagger}, \end{aligned} \quad (3.13)$$

with  $x_i, y_i \sim \mathcal{O}(\tilde{y}_{qi} v \chi_0 / \Lambda_{5D}^{3/2})$ . The left-diagonalization matrices are promoted to  $U(\omega) V_L^{u,d}$ , where  $V_L^{u,d}$  are nearly diagonal for  $x_i^q$ 's and  $y_i^q$ 's small enough compared to the mass eigenvalues  $\sim \tilde{y} v$ . Consequently, the CKM matrix will be given by

$$V_{\text{CKM}} = \left( [U(\omega) V_L^u]^\dagger [U(\omega) V_L^d] \right) = (V_L^{u\dagger} V_L^d) \neq 1. \quad (3.14)$$

There should be enough freedom in  $V_L^{u,d}$  to fit the observed CKM matrix, while still explaining why it is nearly the identity. However, it is not obvious that an appealing

solution can be found, that is an economical one, satisfying all perturbativity constraints with a minimal amount of fine tuning. To account for the more detailed features of the CKM matrix, like the hierarchy of  $\theta_{12}$ ,  $\theta_{23}$  and  $\theta_{13}$ , we will first obtain the effective 4D couplings of the above operators,  $x_i^{u,d}$  and  $y_i^{u,d}$ , in terms of the 5D ones,  $\tilde{x}_i^{u,d}$  and  $\tilde{y}_i^{u,d}$ . Secondly, we search for the minimal number of parameter assignments possible, in order to obtain a realistic CKM matrix.

At this point, with all scales fixed in the quark sector, we will need to estimate the size of the analogous cross-talk effects back in the charged lepton sector, mediated by operators of the form  $\bar{\ell}_L \Phi H \chi e_R (e'_R, e''_R)$ , and the way they affect the neutrino mixing matrix. Clearly, we want to preserve the appealing neutrino mixing pattern described in the previous section, at least within the  $1\sigma$  range of the experimental values for the neutrino mixing angles [46]. The maximal deviations from TBM will be discussed in section 4.4.

#### 4 Numerical results for fermion masses and mixings

We first proceed to set all VEVs and mass scales, according to the observed mass spectrum and neutrino oscillation data. Once these scales are set, we will be able to quantify the various higher order contributions, starting from the quark sector.

We take the fundamental 5D scale to be  $k \simeq \Lambda_{5D} \simeq M_{Pl}$ , where  $M_{Pl} \simeq 2.44 \times 10^{18}$  GeV is the reduced Planck mass. To keep the scale of the IR brane in reach of future collider experiments, but satisfying the constraints from the observed S and T parameters, we will always use values around  $k\pi R \simeq 34$ , such that the mass of the first KK excitation is of a few TeV. It is natural to expect all of the scalars in our theory, being bulk fields, to acquire a VEV of order  $M_{Pl}$ . Using the known mass of the  $W$  boson we can set the amplitude of  $H$  in terms of the 5D weak gauge couplings to be  $H_0 = 0.396 M_{Pl}^{3/2}$ .

By exploiting the warped geometry, we can match all the observed 4D fermion masses by taking the bulk fermion parameters  $c_{q,l,\nu}$  to be all of order one, a well known pleasing feature of the warped scenario [7–9]; a large mass hierarchy is seeded by a tiny hierarchy of the  $c$  parameters.

In this setting, fermion masses are determined by the overlap integrals in the corresponding Yukawa terms, involving the fermion zero mode profiles and the scalar VEV profiles of eqs. (2.9) and (2.13). Therefore, if we take the 5D Yukawa couplings to be universal and set them to one, all bulk parameters can be matched to the observed mass spectrum. We remind the reader that there is only one bulk parameter  $c_L^{q,l}$  for each left-handed quark and lepton doublet, being it a triplet of  $A_4$ . Also, the  $c_L^{q,l}$  are essentially free parameters, since we can always set the mass of each fermion by tuning the bulk mass of the corresponding right handed  $A_4$  singlet. For the same reasons, the scalar VEV  $\Phi_0$  is essentially also a free parameter and we set it to be  $\Phi_0 = 0.577 M_{Pl}^{3/2}$ . However, all fermion zero mode wave functions associated with the various  $c$  parameters are still constrained by a few important perturbativity bounds [17–19] and by precision measurements. The most stringent constraint is on the quark left-handed bulk parameter  $c_L^q$  and comes from the bottom sector, in particular the combined best fits for the ratio of the Z-boson decay

width into bottom quarks and the total hadronic width  $R_b^0$ , the bottom-quark left-right asymmetry,  $A_b$ , and the forward-backward asymmetry,  $A_{\text{FB}}^{0,b}$  [62].

#### 4.1 The quark and charged lepton sector

A thorough comparison of the combined best fits for  $R_b^0$ ,  $A_b$ ,  $A_{\text{FB}}^{0,b}$  with the tree-level corrections to the  $Zb\bar{b}$  couplings in the minimal (non custodial) RS model can be found in [54], while the case of  $P_{\text{LR}}$  custodial symmetry and extended  $P_{\text{LR}}$  has been recently considered [56]. These analyses<sup>2</sup> show that the allowed window for new physics corrections to the SM prediction — and consequently the window for the corresponding bottom bulk parameter  $c_L^b$  — is severely constrained by the  $Zb\bar{b}$  best fits, unless an extended custodial symmetry such as  $P_{\text{LR}}$  custodial, or extended  $P_{\text{LR}}$  custodial is in place. The model considered here is an intermediate example, which embeds non-extended custodial symmetry, and it is thus expected to be subject to constraints on the left-handed profile  $c_L^q$  more severe than in the  $P_{\text{LR}}$  custodial case and possibly close to the minimal RS model. On the other hand, the model also differs from the cases analyzed in [54, 56] in two respects. The additional  $A_4$  flavor symmetry might modify the non orthonormality properties of bulk fermions and eventually suppress contributions due to mixing of zero modes with KK states. Secondly, a bulk Higgs instead of a brane localized Higgs generally allows for further suppressions via overlap in the presence of mass insertions. We defer to future work the analysis of these two aspects in the context of  $Zb\bar{b}$ .

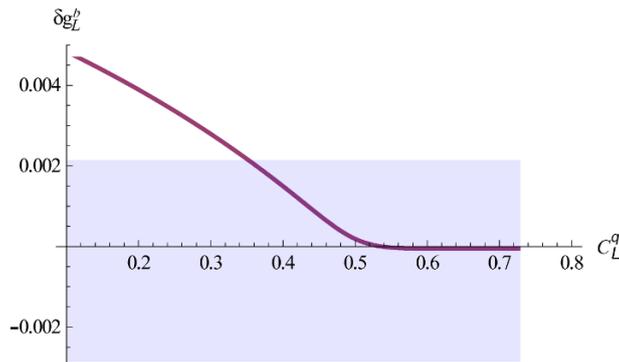
For the purpose of model building we provide here an approximate estimate of the allowed range for  $c_L^q$  in our model, at tree level and in the ZMA, based on the recent calculations in [54, 56]. We correct the contributions to the  $Zb\bar{b}$  couplings  $g_{L(R)}^b$  in the minimal RS model of [54], with the dominant  $\omega_Z^{bL,R} \neq 1$  corrections due to the presence of custodial symmetry [56]. Analogously to [54], we conveniently define the functions  $f(c)$  in terms of the canonically normalized fermion zero mode wave functions  $f^{(0)}(y)$  of eq. (2.13) evaluated at the IR brane

$$f^{(0)}(y = \pi R) = \sqrt{k\pi R} e^{\frac{3}{2}k\pi R} f(c). \quad (4.1)$$

We obtain

$$\begin{aligned} g_L^b &= \left( -\frac{1}{2} + \frac{s_w^2}{3} \right) \left[ 1 - \frac{m_Z^2}{2M_{\text{KK}}^2} \frac{f^2(c_L^q)}{3 - 2c_L^q} \left( \omega_Z^{bL} \cdot k\pi R - \frac{5 - 2c_L^q}{2(3 - 2c_L^q)} \right) \right] \\ &\quad + \frac{m_b^2}{2M_{\text{KK}}^2} \left[ \frac{1}{1 + 2c_b} \left( \frac{1}{f^2(c_b)} - 1 + \frac{f^2(c_b)}{3 - 2c_b} \right) + \sum_{i=d,s} \frac{|(Y_d)_{3i}|^2}{|(Y_d)_{33}|^2} \frac{1}{1 + 2c_i} \frac{1}{f^2(c_b)} \right] \\ g_R^b &= \frac{s_w^2}{3} \left[ 1 - \frac{m_Z^2}{2M_{\text{KK}}^2} \frac{f^2(c_b)}{3 - 2c_b} \left( \omega_Z^{bR} \cdot k\pi R - \frac{5 - 2c_b}{2(3 - 2c_b)} \right) \right] \\ &\quad - \frac{m_b^2}{2M_{\text{KK}}^2} \left[ \frac{1}{1 + 2c_L^q} \left( \frac{1}{f^2(c_L^q)} - 1 + \frac{f^2(c_L^q)}{3 - 2c_L^q} \right) + \sum_{i=d,s} \frac{|(Y_d)_{3i}|^2}{|(Y_d)_{33}|^2} \frac{1}{1 + 2c_L^q} \frac{1}{f^2(c_L^q)} \right] \end{aligned} \quad (4.2)$$

<sup>2</sup>Notice that the analyses of [54, 56] take into account contributions induced by the complete summation over KK states and their non orthonormality. These corrections were not considered in previous bounds and turn out to be numerically significant in the case of  $Zb\bar{b}$  constraints.



**Figure 1.** An illustrative example of the new physics contribution  $\delta g_L^b$  as a function of  $c_L^q$ , with  $M_{\text{KK}} = 1.8 \text{ TeV}$ , the down-type 5D Yukawa couplings in eq. (4.2) set to 1 in magnitude and the right-handed bulk parameters set to the conservative value  $c_b = 0.58$ , and  $c_d = c_s = 0.5$ . The shaded horizontal band corresponds to the 99% probability interval allowed by the best fit values for  $g_{L(R)}^b$  with the SM prediction computed at  $m_H = 150 \text{ GeV}$  as in [54, 56].

where  $\omega_Z^q = c_w^2(T_L^{3q} + T_R^{3q})/(T_L^{3q} - s_w^2 Q_q)$  in the case of equal  $SU(2)_L$  and  $SU(2)_R$  gauge couplings. It is instructive to compare the values for  $\omega_Z^{b_{L,R}}$  in the three different setups. The minimal RS model has  $\omega_Z^{b_L} = \omega_Z^{b_R} = 1$ , the  $P_{\text{LR}}$  custodial has  $\omega_Z^{b_L} = 0$  and  $\omega_Z^{b_R} = 3c_w^2/s_w^2 \sim 10$  for  $s_w^2 \approx 0.23$ , while the custodial case as in our model has  $\omega_Z^{b_L} = c_w^2/(1 - 2s_w^2/3) \sim 0.9$  and  $\omega_Z^{b_R} = -3c_w^2/2s_w^2 \sim -5$ . Notice that the latter is negative, and would go in the right direction to solve the  $A_{\text{FB}}^{0,b}$  anomaly. We have however verified that its numerical impact is limited, as it is its positive contribution in the minimal RS setup.

In the estimate provided by eq. (4.2) we have disregarded corrections to the  $m_b$  dependent terms due to the admixture of the KK partners in the zero modes of the  $SU(2)_R$  doublets; the exact form of the corrections depends on the symmetries of the model, also  $A_4$  flavor in our case, and weak isospin assignments of bulk fermions. The  $m_b$  dependent contribution as in eq. (4.2) is strongly dependent on the right-handed bottom bulk parameter  $c_b$ . However, it can be effectively suppressed in the particular case of extended  $P_{\text{LR}}$  custodial symmetry due to degeneracy of right-handed profiles [56].

We used the combined best fit values for the couplings  $(g_L^b)_{\text{exp}} = -0.41918$  and  $(g_R^b)_{\text{exp}} = 0.090677$  [54] and the predicted SM values  $(g_L^b)_{\text{SM}} = -0.42114$  and  $(g_R^b)_{\text{SM}} = 0.077345$  [62] at the reference Higgs mass of 150 GeV, to constrain the new physics contributions defined as  $\delta g_{L(R)}^b = g_{L(R)}^b - (g_{L(R)}^b)_{\text{SM}}$ . We disregarded the new physics corrections to the SM contributions in the light sector as in [54, 56].

For a Higgs mass of 150 GeV, a KK scale  $M_{\text{KK}} = 1.8 \text{ TeV}$ , and imposing a 99% probability interval for the left-handed coupling given by  $-0.424 \lesssim g_L^b \lesssim -0.419$ , we obtain the constraint  $c_L^q > 0.35$ , as it can be inferred from figure 1. Notice that eq. (4.2) has a strong dependence on  $c_b$  and only a mild dependence on  $c_{d,s}$ . Thus, lowering the value of  $c_b$  towards 0.5 will allow for significantly larger windows for  $c_L^q$ , at the price of higher Yukawa couplings. The corrections to the SM prediction for  $g_R^b$  satisfy  $\delta g_R^b \lesssim 10^{-4}$  in the region of interest and have been safely neglected. The perturbativity constraint on the 5D top Yukawa coupling

$|y_t| \lesssim \pi$ , together with the matching to the  $\overline{\text{MS}}$  top mass at the KK scale  $m_t(1.8 \text{ TeV}) \simeq 140 \text{ GeV}$  [63] and the constraint  $c_t \gtrsim -0.2$ ,<sup>3</sup> implies instead the upper bound  $c_L^q < 0.42$ . Notice that by requiring  $|y_t| \lesssim 4\pi/\sqrt{N}$  [64], with  $N = 1$ , the bound on  $c_L^q$  changes significantly to  $c_L^q < 0.52$ . Hence, one obtains  $0.35 \lesssim c_L^q \lesssim 0.42$  in the most conservative case. As expected, the allowed minimal value for  $c_L^q$  is severely constrained by the  $Zb\bar{b}$  best fits.

For  $c_L^q = 0.40$ , all the right-handed quark bulk parameters are chosen in order to yield the  $\overline{\text{MS}}$  quark masses at the KK scale of 1.8 TeV. Their values are  $c_u = 0.790$ ,  $c_d = 0.770$ ,  $c_s = 0.683$ ,  $c_c = 0.606$ ,  $c_b = 0.557$  and  $c_t = -0.17$ , corresponding to  $m_u = 1.5 \text{ MeV}$ ,  $m_d = 3 \text{ MeV}$ ,  $m_s = 50 \text{ MeV}$ ,  $m_c = 550 \text{ MeV}$ ,  $m_b = 2.2 \text{ GeV}$  and  $m_t = 140 \text{ GeV}$ . All 5D Yukawa couplings have been set to 1 in magnitude, while the top Yukawa coupling slightly breaks universality with  $|y_t| \simeq 2.8$ .

It should be added that lower values for  $c_L^q$  become accessible for a KK scale higher than  $M_{\text{KK}} = 1.8 \text{ TeV}$ . On the other hand, a KK scale as low as 1 TeV would force  $c_L^q \gtrsim 0.45$  and the top Yukawa coupling to values  $|y_t| \sim 7$ . Another possibility is the one of a heavier Higgs. A Higgs mass larger than 150 GeV would bring the model prediction closer to the best fit values for  $g_L^b$  and  $g_R^b$ , thus allowing for a larger range for  $c_L^q$ . For example, a mass  $m_H = 300 \text{ GeV}$  would imply a lower bound  $c_L \gtrsim 0.32$ , within our estimate and using the shifts  $\Delta g_L^b = 1.77 \cdot 10^{-3} \ln(m_H/150 \text{ GeV})$  and  $\Delta g_R^b = 0.92 \cdot 10^{-2} \ln(m_H/150 \text{ GeV})$  [56] induced by  $m_H \neq 150 \text{ GeV}$ . However, a heavier Higgs mass in the custodial setup easily induces conflicts with electroweak precision measurements and a careful estimate of the actually allowed range of values for  $m_H$  should be produced in each version of the custodial setup. We defer this point to future work on phenomenological applications.

For the charged leptons we make the choice  $c_L^l = 0.52$ ,  $c_e = 0.803$ ,  $c_\mu = 0.635$  and  $c_\tau = 0.5336$ , which reproduce the experimental value of the corresponding masses  $m_e = 0.511 \text{ MeV}$ ,  $m_\mu = 105.6 \text{ MeV}$  and  $m_\tau = 1.776 \text{ GeV}$ .

## 4.2 The neutrino sector

To obtain the neutrino masses we first recall the leading order structure of the light neutrino mass matrix after see-saw,  $M_L^\nu$  in eq. (2.21), for which the eigenvalues are given by:

$$M_L^{diag.} = -(m_\nu^D)^2 \times \left[ \frac{1}{\tilde{M} + M_\chi}, \frac{1}{\tilde{M}}, \frac{1}{\tilde{M} - M_\chi} \right]. \quad (4.3)$$

We now write the neutrino mass-squared splittings, for afterwards we would like to impose the observed values of  $\Delta m_{\text{atm}}^2$  and  $\Delta m_{\text{sol}}^2$ , in order to constrain the possible choices of  $\tilde{M}$  and  $M_\chi$ . Given

$$\Delta m_{12}^2 \equiv |m_1|^2 - |m_2|^2 = \left| \frac{(m_\nu^D)^2}{(\tilde{M})} \right|^2 \left[ \frac{1}{(1+q)^2} - 1 \right], \quad (4.4)$$

$$\Delta m_{23}^2 \equiv |m_2|^2 - |m_3|^2 = \left| \frac{(m_\nu^D)^2}{(\tilde{M})} \right|^2 \left[ 1 - \frac{1}{(1-q)^2} \right], \quad (4.5)$$

---

<sup>3</sup>This constraint ensures that the contribution through mixing from  $\tilde{b}$ , the partner of the  $\text{SU}(2)_R$  top, is sufficiently suppressed in  $Zb\bar{b}$  [11, 17–19].

where  $q = M_\chi/\tilde{M}$ ,  $|\Delta m_{12}^2| = \Delta m_{\text{sol}}^2$  and  $|\Delta m_{23}^2| = \Delta m_{\text{atm}}^2$ , we obtain the following cubic equation for  $q$ :

$$q^3 - 3q - 2 \left( \frac{x-1}{x+1} \right) = 0, \quad (4.6)$$

where  $x = \Delta m_{\text{sol}}^2/\Delta m_{\text{atm}}^2$  for  $|q| < 2$ , and  $x \rightarrow -x$  for  $|q| > 2$ . Given  $q$ , the ratio  $\tilde{M}/(m_\nu^D)^2$  can be extracted either in terms of  $\Delta m_{\text{atm}}^2$  or  $\Delta m_{\text{sol}}^2$

$$\tilde{M} = \frac{(m_\nu^D)^2}{\sqrt{\Delta m_{\text{sol}}^2}} \times \left( \left| \frac{1}{(1+q)^2} - 1 \right| \right)^{-1/2}. \quad (4.7)$$

Imposing the measured values [46] for the mass splittings,  $\Delta m_{\text{sol}}^2 \simeq 7.67 \times 10^{-5} \text{eV}^2$  and  $\Delta m_{\text{atm}}^2 \simeq 2.39 \times 10^{-3} \text{eV}^2$ , we find four possible solutions to the above equation,  $q \simeq \{-2.02, -1.99, 0.79, 1.2\}$ , where the first two correspond to inverted hierarchy, while the second two correspond to normal hierarchy. In addition, once we set  $c_{\nu_R}$ , we can constrain  $\tilde{M}$  and  $M_\chi$  from the same data. This is a nice feature of the light neutrino mass matrix obtained in all  $A_4$  models with similar assignments [39, 41–44, 47]. Since at this stage only the overall light neutrino mass ratios,  $(m_\nu^D)^2/\tilde{M}$  and  $M_\chi/\tilde{M}$  are constrained by the observed splittings, we choose not to set  $c_{\nu_R}$  and extract  $M_\chi$  from the quark mixing data. It will be possible afterwards to set  $c_{\nu_R}$  to a natural value  $\sim 1/2$  to match the neutrino mass spectrum.

### 4.3 Obtaining the CKM matrix and fixing the scale

We now analyze the CKM matrix resulting from the contributions introduced in eq. (3.13). Using standard perturbative techniques, the left diagonalization matrix  $V_L^{u,d}$  is obtained by the unitary diagonalization of  $(M + \Delta M)_{u,d}(M + \Delta M)_{u,d}^\dagger$ , and the right diagonalization matrix  $V_R^{u,d}$  is analogously obtained by the unitary diagonalization of  $(M + \Delta M)_{u,d}^\dagger(M + \Delta M)_{u,d}$ . The entries of the CKM matrix are then derived in terms of the  $x_i^{u,d}$ ,  $y_i^{u,d}$  parameters defined in eq. (3.13), to leading order in perturbation theory. The up and down left-diagonalization matrices turn out to be:

$$V_L^q = \begin{pmatrix} 1 & m_2^{-1}(x_2^q + y_2^q) & m_3^{-1}(x_3^q + y_3^q) \\ -m_2^{-1}(\bar{x}_2^q + \bar{y}_2^q) & 1 & m_3^{-1}(x_3^q + \omega y_3^q) \\ -m_3^{-1}(\bar{x}_3^q + \bar{y}_3^q) & -m_3^{-1}(\bar{x}_3^q + \omega^2 \bar{y}_3^q) & 1 \end{pmatrix}, \quad (4.8)$$

with  $q = u, d$ ,  $m_i = m_{u_i, d_i}$ , and  $\bar{x}(\bar{y})$  stands for the complex conjugate. In the above matrices, for simplicity, we have redefined the  $x_i^{u,d}$  and  $y_i^{u,d}$  to absorb the relative factor of  $1/\sqrt{3}$  compared to the unperturbed mass eigenvalues  $m_i = \sqrt{3}v\tilde{y}_{u_i, d_i}$ . Furthermore, we have omitted contributions that are suppressed by quadratic quark mass ratios and still linear in  $x_i^{u,d}$ ,  $y_i^{u,d}$ . Terms of this kind will be included in the complete expressions derived below, for each of the interesting CKM matrix elements.

It is now straightforward to extract the estimations for the upper off-diagonal elements of the CKM matrix out of  $V_L^{u\dagger}V_L^d$ , in order to eventually match the three mixing angles

and the CP violating phase  $\delta_{\text{CKM}}^{13}$ . To leading order in  $x_i, y_i$ , these elements turn out to be

$$V_{\text{us}} \simeq -V_{\text{cd}}^* \simeq \frac{m_u(\bar{x}_1^u + \omega^2 \bar{y}_1^u) + m_c(x_2^u + y_2^u)}{m_u^2 - m_c^2} + \frac{m_d(\bar{x}_1^d + \omega^2 \bar{y}_1^d) + m_s(x_2^d + y_2^d)}{m_s^2 - m_d^2}, \quad (4.9)$$

$$V_{\text{cb}} \simeq -V_{\text{ts}}^* \simeq \frac{m_c(\bar{x}_2^u + \omega \bar{y}_2^u) + m_t(x_3^u + \omega y_3^u)}{m_c^2 - m_t^2} + \frac{m_s(\bar{x}_2^d + \omega \bar{y}_2^d) + m_b(x_3^d + \omega y_3^d)}{m_b^2 - m_s^2}, \quad (4.10)$$

$$V_{\text{td}} \simeq -V_{\text{ub}}^* \simeq \frac{m_u(\bar{x}_1^u + \omega \bar{y}_1^u) + m_t(x_3^u + y_3^u)}{m_u^2 - m_t^2} + \frac{m_d(\bar{x}_1^d + \omega \bar{y}_1^d) + m_b(x_3^d + y_3^d)}{m_b^2 - m_s^2}, \quad (4.11)$$

where it is important to observe that the first equality is exact to leading order in  $x_i$  and  $y_i$ . The diagonal elements of the CKM matrix remain unchanged at this order and equal to one.

We recall that  $x_i^{u,d}$  and  $y_i^{u,d}$  correspond to the (1*i*) and (3*i*) entries of  $\Delta M^{u,d}$  in the interaction basis, respectively. This tells us which fermionic wave function overlaps enter the integral for each of the above parameters. For the 4D couplings we thus obtain

$$x_i^{u,d}(y_i^{u,d}) = \left( \frac{H_0 \Phi_0 \chi_0}{\Lambda_{5D}^{7/2}} \int_{-\pi R}^{\pi R} \frac{dy}{2\pi R} F(c_L^q, c_i^{u,d}) e^{8(k|y| - \pi R)} (1 - e^{4(k|y| - \pi R)}) \right) \tilde{x}_i^{u,d}(\tilde{y}_i^{u,d}). \quad (4.12)$$

To narrow down the parameter space, we choose all the  $\tilde{x}_i^{u,d}(\tilde{y}_i^{u,d})$  to be universal and equal to one in magnitude, while relative phases between these parameters will be allowed. Hence, due to the mass hierarchy of the quarks, which keeps the denominators in eqs. (4.9)–(4.11) proportional to just one quark mass to a good approximation, the resulting corrections to each of the CKM matrix elements are of the generic form,

$$\frac{m_i^{u,d}(x_i^{u,d} + \omega^n y_i^{u,d})}{(m_j^{u,d})^2 - (m_k^{u,d})^2} \Rightarrow \frac{((m_i^{u,d})^2 C_\chi f_\chi^i)(\tilde{x}_i^{u,d} + \omega^n \tilde{y}_i^{u,d})}{\pm \max[(m_j^{u,d})^2, (m_k^{u,d})^2]}, \quad (4.13)$$

with  $n = 0, 1, 2$ , and where  $C_\chi = \chi_0/M_{Pl}^{3/2}$ ,  $f_\chi^i = 4/(12 - c_L^q - c_i^{u,d})$  and  $i = j$  or  $k$ . Let us first set  $C_\chi$  according to the experimental value of  $V_{\text{us}}$ ; using eq. (4.9) and taking into account eq. (4.13) we obtain:

$$V_{\text{us}} \simeq \left( (\tilde{x}_2^d + \tilde{y}_2^d) f_\chi^s - (\tilde{x}_2^u + \tilde{y}_2^u) f_\chi^c + \mathcal{O}(m_d^2/m_s^2) \right) C_\chi \simeq 0.2257 \Rightarrow C_\chi \simeq 0.155. \quad (4.14)$$

in which we have fixed  $\tilde{x}_2^d, \tilde{y}_2^d, \tilde{x}_2^u$  and  $\tilde{y}_2^u$  to be 1 in magnitude, with a relative phase,  $\delta_2^u = \pi$ , between the contributions from the up and down sectors. We want to see if the above value of  $C_\chi$  together with a minimal number of relative phases between the remaining  $\tilde{x}$  and  $\tilde{y}$  parameters, are enough to account for the magnitudes of the remaining observed CKM elements. Yet, we first want to check the consistency of the scale associated with the above value of  $C_\chi$  with the neutrino mass splittings and the bare Majorana mass scale. It turns out that for  $c_{\nu R} = 0.408$  it is possible to satisfy the constraints in both sectors, namely to have a realistic  $V_{\text{us}}$ , while at the same time having a realistic neutrino mass spectrum with a normal or inverted hierarchy. At this level the neutrino mixing matrix is obviously tribimaximal with small deviations, as we saw in section 3. The numerical results are reported in table 1, once the scales are fixed by  $C_\chi$ . The dominant contributions to  $V_{\text{cb}}$  (and  $V_{\text{ts}}$ ) are given by

$$V_{\text{cb}} \simeq \left( (\tilde{x}_3^d + \omega \tilde{y}_3^d) f_\chi^b - (\tilde{x}_3^u + \omega \tilde{y}_3^u) f_\chi^t + \mathcal{O}(m_s^2/m_b^2) \right) C_\chi \simeq 0.004 \Leftarrow (\delta_3^u = 0). \quad (4.15)$$

$q$	$m_1$	$m_2$	$m_3$	$\tilde{M}/M_{Pl}$	$C_\chi$
-2.02	50.7	51.8	17.1	-0.077	0.155
-1.99	52.3	51.8	17.3	-0.077	0.155
0.79	5.8	10.5	50	0.202	0.155
1.19	4.3	9.4	49.4	0.135	0.155

**Table 1.** Approximate numerical values of the neutrino masses and relevant UV scales, where  $c_{\nu_R} = 0.408$  has been chosen to match the four solutions for  $q \equiv M_\chi/\tilde{M}$  to eq. (4.6) with the constraint  $C_\chi = 0.155$  arising from  $V_{us}$ . The masses are given in units of  $10^{-3}$  eV.

Up till now, we introduced only one phase,  $\delta_2^u$ , to match  $V_{us}$  (and  $V_{cd}$ ), but failed to match  $V_{cb}$  (and  $|V_{ts}|$ ) to their central experimental values  $|V_{ts}| \simeq |V_{cb}| = 0.0415$ . The leading order contribution to  $V_{ub}$ , without any relative phase assignments, and with the  $\tilde{x}_3^{u,d}$ 's and  $\tilde{y}_3^{u,d}$ 's set to 1, is

$$V_{ub} \simeq \left( (\tilde{x}_3^d + \tilde{y}_3^d) f_\chi^b - (\tilde{x}_3^u + \tilde{y}_3^u) f_\chi^t + \mathcal{O}(m_d^2/m_b^2) \right) C_\chi \simeq 0.007 \Leftarrow (\delta_3^u = 0). \quad (4.16)$$

We see that  $V_{ub}$ , and thus  $V_{td}$ , turn out correctly to be of order  $\lambda_{CKM}^3$ , but still outside the experimental error.

The next to leading order corrections to the various CKM elements enter at  $\mathcal{O}((x_i^{u,d}, y_i^{u,d})^2)$ . In general, one may still expect them to modify the relatively small values of  $V_{ub}$  and  $V_{cb}$ , especially in the presence of strong cancellations at leading order, and given that each independent contribution is effectively suppressed by  $f_\chi^i C_\chi \approx 0.05$  compared to the corrections linear in  $x_i^{u,d}$  and  $y_i^{u,d}$ . We are going to elaborate on this possibility once we made an attempt to obtain an almost realistic CKM matrix, using the first order results.

The only way to obtain a realistic prediction for the independent magnitudes of  $V_{ub}$  and  $V_{cb}$ , and the related values of  $|V_{td}|$  and  $|V_{ts}|$ , is to break the universality assumption for the Yukawa couplings,  $\tilde{x}_3^{u,d}$ ,  $\tilde{y}_3^{u,d}$  and find their values that match the experimental data. However, we are interested in the smallest possible deviations from the universality assumption which assumes all of the Yukawa couplings to be of order one, so that contributions to various flavor violating processes, which are present in any RS-flavor setup [15], will not be arbitrarily modified. Therefore, while trying to find the minimum number of assignments in the  $\tilde{x}_3^{u,d}$ ,  $\tilde{y}_3^{u,d}$  parameter space, we still require small deviations, in general complex, from the  $\mathcal{O}(1)$  universality assumption. It is obvious that assignments in terms of one parameter will yield results proportional to those of eq. (4.15) and (4.16), and will hence fail again to account for realistic values of  $V_{cb}$  and  $V_{ub}$ . The minimal viable choice consists of at least 2 parameter assignments. Namely, we will have to break the universality assumption for two out of the four  $\tilde{x}_3^{u,d}$ ,  $\tilde{y}_3^{u,d}$  parameters.

In addition the only choice of parameter assignments that maintains all coefficients of  $\mathcal{O}(1)$  is to break universality for  $\tilde{x}_3^u$  and  $\tilde{y}_3^d$ , while setting  $\tilde{x}_3^d = \tilde{y}_3^u = 1$ . Solving eqs. (4.16)

and (4.15) for  $\tilde{x}_3^u$  and  $\tilde{y}_3^d$  we obtain:

$$\tilde{x}_3^u \simeq 0.67 - 0.19i, \quad \tilde{y}_3^d \simeq 0.60 - 0.23i. \quad (4.17)$$

These parameters are almost degenerate, in particular if one considers the overall accuracy of the zero mode approximation, and substituting these values in eqs. (4.15) and (4.16), we obtain

$$|V_{cb}| = 0.0415 \quad |V_{ub}| = 0.0039, \quad (4.18)$$

which are the central experimental values [63]. Interestingly, we are able to match also the CP violating phase  $\delta_{13}$ , using the same assignment. We find it to be  $\delta_{13} \simeq 1.2$ , which is well within the experimental error.<sup>4</sup> Obviously, at this order  $|V_{ts}| = |V_{cb}|$  holds exactly, and this gives a value for  $|V_{ts}|$  close to the central experimental value  $|V_{ts}| = 0.0407$ . We also obtain  $|V_{td}| = |V_{ub}|$ , and thus fail to match the central experimental value  $|V_{td}| = 0.0087$ . Notice, however, that corrections of the size of the smallest CKM entries, i.e.  $O(\lambda_{\text{CKM}}^3)$  are below the model theoretical error induced by the zero mode approximation. A realistic value for  $V_{td}$  can easily arise from the subleading corrections in  $\tilde{x}_i^{u,d}$  and  $\tilde{y}_i^{u,d}$  and from higher dimensional operators beyond the zero mode approximation. We remind that the corrections quadratic in  $\tilde{x}_i^{u,d}$  and  $\tilde{y}_i^{u,d}$  are generically suppressed by a factor  $f_\chi^i C_\chi \approx 0.05$  with respect to the linear contributions.

We have thus shown that at leading order in the VEV expansion the model predicts  $V_{\text{CKM}}$  to be the unit matrix, a rather good first step in the description of quark mixing. At the next to leading order, cross-brane and cross-talk operators induce deviations from unity, parametrized in terms of twelve complex parameters  $\tilde{x}_i^{u,d}$  and  $\tilde{y}_i^{u,d}$ , with  $i = 1, 2, 3$ . We have shown that, linearly in these parameters, realistic values of the CKM entries can be obtained within the model theoretical error in a finite portion of the parameter space, with all the  $\tilde{x}_i$  and  $\tilde{y}_i$  of order one and two non zero relative phases; however at this order, no corrections to the diagonal unit entries is produced and the two smallest entries  $V_{ub}$  and  $V_{td}$  are degenerate.

We notice that the possibility of producing hierarchical CKM entries, of order  $\lambda_{\text{CKM}}$ ,  $\lambda_{\text{CKM}}^2$  and  $\lambda_{\text{CKM}}^3$ , with all parameters of order one stems from the presence of built-in cancellations induced by the hierarchical masses. The presence of the  $A_4$  induced phase  $\omega$  also produces a pattern in the corrections. In this framework, subleading corrections of order  $(\tilde{x}_i^{u,d}, \tilde{y}_i^{u,d})^2$  and contributions beyond the ZMA, must be responsible for the deviation from one of the CKM diagonal entries and the non degeneracy of  $V_{ub}$  and  $V_{td}$ . The first is of order  $10^{-2}$ , the latter of order  $10^{-3}$ , hence a cancellation pattern must again be in place.

It is instructive to compare this  $A_4$  pattern with other flavor symmetry groups, in particular  $T'$  [48–53]. The Wolfenstein parametrization would suggest the existence of an expansion parameter to be naturally identified with  $\lambda_{\text{CKM}}$ , offering an elegant and simple description of quark mixing in the standard model. On the other hand, flavor models based on otherwise appealing discrete flavor symmetries such as  $A_4$  and  $T'$  must rely on more complicated patterns to produce a realistic CKM matrix. In the case of  $T'$  one

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<sup>4</sup>At a higher level of accuracy one should obviously satisfy the full set of constraints implied by the measured Jarlskog invariant.

needs to postulate a hierarchy of Yukawa couplings to distinguish between  $O(\lambda_{\text{CKM}})$  and  $O(\lambda_{\text{CKM}}^2)$  entries, and a hierarchy of specific VEVs to induce the splitting between  $V_{\text{ub}}$  and  $V_{\text{td}}$ . Similar hierarchies can in principle also be postulated in the  $A_4$  case, at the price of an increased fine tuning of the input parameters.

#### 4.4 Estimation of cross-talk and cross-brane contributions in the lepton sector

To complete our analysis we want to ensure that the deviations from TBM induced by cross-talk operators of the characteristic forms  $\bar{\ell}_L \Phi H \chi e_R (e'_R, e''_R)$  and  $\bar{\ell}_L H \chi \nu_R$  are suppressed and keep the predictions for the neutrino mixing parameters within the  $2\sigma$  range of the experimental error. We first consider the cross-brane operator  $\bar{\ell}_L H \chi \nu_R$ , already explored in section III. Recall that deviations from  $\theta_{13} = 0$  and  $\theta_{23} = \pi/4$  are induced only for complex  $\epsilon_{ij}^\chi$  parameters, and therefore we expect the maximal deviations from TBM when they are purely imaginary. All said, we estimate the magnitude of the  $\epsilon_{ij}^\chi$  coefficients based on the results of the previous section. As we are interested only in the dominant contributions, associated with one insertion of  $\chi$ , we are only interested in  $\epsilon_{13}^\chi$  as defined in eq. (3.2). This provides

$$\epsilon_{13}^\chi \simeq \tilde{y}_\nu H_0 C_\chi \frac{4}{8 - c_L^e - c_{\nu_R}} \Rightarrow \epsilon_\chi \simeq 0.07, \quad (4.19)$$

where  $\epsilon_\chi$  was introduced in eq. (3.6). Given  $\epsilon_\chi$  we also obtain  $\epsilon_{11}^\chi$ . Taking its phase to be  $\pi/2$  we get the largest possible contribution to the phase  $\delta$  defined in eq. (3.7) to be  $\delta \simeq -0.11$ . Using eq. (3.8) we can now estimate the deviations from TBM arising from the operator  $(1/\Lambda_{5D}^2) \bar{\ell}_L H \chi \nu_R$  in the worst scenario where its coefficient is purely imaginary. These turn out to be

$$\Delta\theta_{13} \simeq 0.05 \quad \Delta\theta_{23} \simeq 0.04 \quad \Delta\theta_{12} \simeq 0. \quad (4.20)$$

All of these deviations are within  $1\sigma$  from their experimental values. The other class of operators  $(1/\Lambda_{5D}^{7/2}) \bar{\ell}_L \Phi H \chi e_R (e'_R, e''_R)$  have the same structure as the operators in charge of quark mixing. The perturbative diagonalization procedure can thus be carried out as it is done in the quark sector, in order to determine the deviations from  $U(\omega)$  of the rotation matrix for the left handed charged leptons. Generically, these operators will induce perturbations to the mass matrix in the form of eq. (3.13) and with characteristic strength  $\epsilon_\ell \simeq C_\chi f_\chi (c_L^e, c_\ell) \simeq 0.028$ , which is of the same order of the model theoretical error. We could have proceeded to explicitly write all of these corrections as we did for the quarks, yet since the structure is practically identical in both cases we can easily deduce the effect of these small terms. For example, in analogy with eqs. (4.14) and (4.16) and using no additional phase assignments for  $\tilde{x}_i^e$  and  $\tilde{y}_i^e$ , we get that the contribution to  $\Delta\theta_{13}$  is vanishing, while the contributions to  $\Delta\theta_{12}$  and  $\Delta\theta_{23}$  are of approximate strength  $|\Delta\theta_{12}| \simeq |\Delta\theta_{23}| \simeq 0.04$ , comparable to the model theoretical error. Higher order cross-talk operators are not considered, as their contribution lies safely below 1%. We can conclude that the most significant deviations from TBM induced by cross-talk and cross-brane operators on the three mixing angles stay within the experimental errors for these quantities. This can be obtained without making any further assumption on the parameters of the model and maintaining all of them naturally of order one.

## 5 Vacuum alignment

We are still left with one important challenge: to ensure that the alignments of the VEVs  $\langle \Phi \rangle$  and  $\langle \chi \rangle$  in eqs. (2.11) and (2.19) are approximately preserved also in the presence of higher order corrections. These VEV patterns were the key ingredient in obtaining all of the above results. In this section, we briefly discuss the challenge and suggest possible solutions. It is obvious that in the scenario of [39], which is the limit of our model when  $\chi$  and  $\Phi$  are strictly localized on the UV and IR brane respectively, the vacuum alignment problem is eliminated *tout court*, as  $\Phi$  and  $\chi$  are completely sequestered. In our model, the cross-brane interactions between  $\Phi$  and  $\chi$  are essential in order to obtain realistic results in the quark sector. The question is whether their modifications to the scalar potential can be sufficiently suppressed, while still protecting the results in the lepton and quark sectors.

The complete G-invariant scalar potential in  $\Phi$  and  $\chi$  up to quartic order is displayed below, and we conveniently separate its terms as follows

$$V = V(\Phi) + V(\chi) + V(\Phi, \chi), \quad (5.1)$$

with the single contributions also derived in [47]

$$\begin{aligned} V(\Phi) = & \mu_{\Phi}^2 (\Phi\Phi)_{\underline{1}} + \lambda_1^{\Phi} (\Phi\Phi)_{\underline{1}} (\Phi\Phi)_{\underline{1}} + \lambda_2^{\Phi} (\Phi\Phi)_{\underline{1}'} (\Phi\Phi)_{\underline{1}''} \\ & + \lambda_3^{\Phi} (\Phi\Phi)_{\underline{3}_s} (\Phi\Phi)_{\underline{3}_s} + \lambda_4^{\Phi} (\Phi\Phi)_{\underline{3}_a} (\Phi\Phi)_{\underline{3}_a} \\ & + i\lambda_5^{\Phi} (\Phi\Phi)_{\underline{3}_s} (\Phi\Phi)_{\underline{3}_a}. \end{aligned} \quad (5.2)$$

$$\begin{aligned} V(\chi) = & \mu_{\chi}^2 (\chi\chi)_{\underline{1}} + \delta^{\chi} (\chi\chi\chi)_{\underline{1}} + \lambda_1^{\chi} (\chi\chi)_{\underline{1}} (\chi\chi)_{\underline{1}} + \lambda_2^{\chi} (\chi\chi)_{\underline{1}'} (\chi\chi)_{\underline{1}''} \\ & + \lambda_3^{\chi} (\chi\chi)_{\underline{3}} (\chi\chi)_{\underline{3}}. \end{aligned} \quad (5.3)$$

$$\begin{aligned} V(\Phi, \chi) = & \delta_s^{\Phi\chi} (\Phi\Phi)_{\underline{3}_s} \chi + i\delta_a^{\Phi\chi} (\Phi\Phi)_{\underline{3}_a} \chi + \lambda_1^{\Phi\chi} (\Phi\Phi)_{\underline{1}} (\chi\chi)_{\underline{1}} \\ & + \lambda_2^{\Phi\chi} (\Phi\Phi)_{\underline{1}'} (\chi\chi)_{\underline{1}''} + \lambda_2^{\Phi\chi*} (\Phi\Phi)_{\underline{1}''} (\chi\chi)_{\underline{1}'} \\ & + \lambda_3^{\Phi\chi} (\Phi\Phi)_{\underline{3}_s} (\chi\chi)_{\underline{3}} + i\lambda_4^{\Phi\chi} (\Phi\Phi)_{\underline{3}_a} (\chi\chi)_{\underline{3}}. \end{aligned} \quad (5.4)$$

Notice that the additional interactions with the Higgs field  $V(H, \Phi, \chi)$  can be omitted in this context, since  $H$  is a singlet under  $A_4$  and gives rise to contributions that can be absorbed in the corresponding coefficients of eq. (5.4). The self interaction terms are assumed to be confined on the two branes as in [59]. It is then easy to check that eq. (2.11) is a global minimum of  $V(\Phi)$ , and that eq. (2.19) is a global minimum of  $V(\chi)$ . This situation drastically changes once interactions between  $\Phi$  and  $\chi$  are switched on via  $V(\Phi, \chi)$ . The problem is that the extremal conditions yield a larger number of independent equations than there are unknown VEVs, as was demonstrated in [65].

To solve this problem extremely hierarchical fine tuning has to be imposed on the various parameters of the scalar potential. The most direct approach to avoid this fine tuning, adopted by [65] and many others, was to prohibit the problematic  $V(\Phi, \chi)$  interaction terms by construction. These models are usually supersymmetric and make use of the Froggatt-Nielsen (FN) mechanism [66] to explain the fermion mass hierarchy. Explaining the observed features of fermion mixings within these setups usually requires a

significantly larger flavon content, and additional higher permutation symmetries, typically  $Z_n$ , in order to eliminate problematic terms from the superpotential. The drawback is that one usually ends up with a large parameter space without avoiding the need to make ad hoc assumptions to account for experimental data. In this case, using no further assumptions, we can try to exploit the suppressed overlap of the flavons bulk profiles in order to reduce the impact of the interaction terms in eq. (5.4) while preserving the results in the quark and lepton sectors. The most problematic cross-brane term is  $\delta_{\Phi\chi}\Phi^2\chi$ , which is not eliminated by the additional  $Z_2$  symmetry we imposed. We know that this term is typically suppressed by an overlap factor  $a_\chi \simeq C_\chi/3 \simeq 0.05$  compared to the self interaction term  $\Phi^2$ . If we switch on the bulk mass of  $\chi$ , its VEV profile will become more sharply localized on the UV brane, an effect that we can parametrize by a multiplicative factor  $C_{\mu_\chi} = e^{-\mu_\chi}$  in front of the original expression in eq. (2.9). This factor can easily suppress the induced shift of the scalar VEV below the model theoretical error. The same factor enters all the calculations performed so far, however its effect only amounts to modify the numerical value of  $C_\chi$  and accordingly, the matching of  $c_{\nu_R}$ . It turns out that to make  $a_\chi = (1/3)C_\chi C_{\mu_\chi}$  smaller than the theoretical error we only need to suppress  $C_\chi$  by a factor of 1/2, which can be compensated by a global rescaling of the various  $x_i^{u,d}$  and  $y_i^{u,d}$  parameters by at most a factor two, without breaking the universality assumption. Quartic and higher  $V(\Phi, \chi)$  interaction terms are obviously further suppressed and lie safely below the model theoretical error, originated by the zero mode approximation.

### 5.1 Alternative solutions by model modifications

We offer two alternative setups, in which the desired vacuum alignment is protected by forbidding cubic mixed interaction terms, and allowing only for quartic (and higher) interactions in  $V(\Phi, \chi)$ . In the first suggested setup, we use an additional  $A_4$  singlet  $\eta$  and modify the external  $Z_2$  symmetry into  $Z_8$ . The singlet  $\eta$  is in charge of the bare Majorana mass term. Assigning the  $Z_8$  charges of  $\Phi, \chi, H$  and  $\eta$  to be  $\alpha^4$ , with  $\alpha = e^{2\pi i/8}$ , we assign the rest of the fields according to

$$(\bar{Q}_L, u'_R, u''_R, d_R, d'_R, d''_R) \Rightarrow \alpha^4, \quad (\bar{\ell}_L, \nu_R) \Rightarrow \alpha^2, \quad (e_R, e'_R, e''_R) \Rightarrow \alpha^6. \quad (5.5)$$

The above assignments allow the presence of all the operators in the Yukawa lagrangian of eq. (2.8), while they forbid the unwanted  $\bar{\ell}_L H \Phi \nu_R$  and  $\bar{\ell}_L H \chi \nu_R$  terms. Most importantly, the dangerous cubic  $\Phi^2 \chi$  interaction term is also prohibited in this setting. The operators in charge of quark mixing are now supplemented with an extra insertion of  $\Phi$  to preserve  $Z_8$ , giving rise to the operators  $\bar{Q}_L \Phi^2 \chi u_R(u'_R, u''_R, d_R, d'_R, d''_R)$ . However, we have an extra source of quark mixing allowed by the above assignments, arising from cross-brane operators of the form  $\bar{Q}_L \chi H(u_R, u'_R, u''_R, d_R, d'_R, d''_R)$ .

The latter will obviously dominate over the contributions of operators involving  $\Phi^2 \chi$  interactions. This is clearly so, given that the above operators are of the same dimension as the leading order operators in the Yukawa lagrangian of the quark sector. However, being cross-brane terms, they will still be suppressed compared to the IR dominated contributions of  $\bar{Q}_L \Phi H(u_R, u'_R, u''_R, d_R, d'_R, d''_R)$ . After the scale  $C_\chi$  is set according to the experimental

value of  $|V_{us}|$ , we are able to tell if this perturbative expansion is justified. Due to the  $Z_3$  preserving VEV of  $\Phi$  and the  $Z_2$  preserving VEV of  $\chi$ , the above operators involving  $\chi H$  will generate the same kind of contributions as those involving  $\Phi\chi H$  to the CKM elements. However, the corrections induced by these operators will enter in the second row of the up and down mass matrices in the interaction basis, differently from eq. (3.12). Each of these contributions will be characterized by one coefficient which we define as  $a_{\chi,i}^{u,d}$ , where  $i$  specifies the generation. When matching the observed magnitudes of the CKM elements and the CP violating phase,  $\delta_{13}^{\text{CKM}}$ , it turns out that we still need a relative phase  $\delta_{\chi,2}^{\text{ud}} = \pi$  between  $a_{\chi,2}^u$  and  $a_{\chi,2}^d$ , to account for  $V_{us}$ . This results in  $C_\chi \simeq 0.045$ , which in turn validates the perturbative expansion, where  $\epsilon_{mod.} \simeq C_\chi/C_\Phi \simeq 0.045/0.577 \simeq 0.08$  acts as the small expansion parameter. The contributions to quark mixing arising from the operators involving  $\Phi^2\chi$  are suppressed by  $\mathcal{O}(10^{-2})$  compared to the ones associated with  $a_{\chi,i}^{u,d}$ . It turns out that we need two independent and non degenerate complex parameter assignments for the above coefficients, in order to obtain an almost realistic CKM matrix at leading order as in section 4.3. This means that, in total, we have 4 non degenerate real parameters governing the mixing data in the quark sector, a less appealing situation than the one with a  $Z_2$  extra discrete symmetry, where the two complex parameters turn out to be degenerate to a good approximation.

The second solution we suggest is based on the simplifying assumption that the field  $\Phi$  can also play the role of the Higgs as in many previous works on  $A_4$  [41–44, 47, 67–69]. It is not clear, at this stage, to which extent this assumption can be justified in the warped setup we use, however it seems possible to identify the lightest mode associated with  $\Phi$  in the 4D effective theory with the SM Higgs.

We again assign a  $Z_8$  discrete symmetry in this slightly simplified setup. Given that  $\Phi$ ,  $\chi$ , and  $\eta$  transform again as  $\alpha^4$ , we choose the other fields to transform according to

$$(\bar{Q}_L) \Rightarrow \alpha^4, \quad (\nu_R) \Rightarrow \alpha^2, \quad (\ell_L, \phi) \Rightarrow \alpha^3, \quad (e_R, e'_R, e''_R) \Rightarrow \alpha, \quad (5.6)$$

and importantly the scalar sector is now supplemented with an additional  $A_4$  singlet  $\phi$  which is in charge of the neutrino Dirac mass term. We naturally expect  $\eta$  and  $\phi$  to be UV and IR localized, respectively. All terms in the Yukawa lagrangian of eq. (2.8) are still allowed by the above assignment with  $H$  being swallowed into  $\Phi$ , and the Dirac mass term for the neutrinos of the form  $\bar{\ell}_L\phi\nu_R$ . The dominant higher order corrections in the Dirac neutrino mass matrix arise from operators of the form  $\bar{\ell}_L\phi\Phi^2\nu_R$  and  $\bar{\ell}_L\phi\chi^2\nu_R$ , for which the associated contributions were already inspected in section III. The dominant higher order corrections to the heavy Majorana mass matrix will consist of  $\chi^3\nu_R(\nu_R)^c$  and  $\Phi^2\eta\nu_R\nu_R^c$ , for which the resulting textures were also inspected in the same section.

Turning back to the quark sector we see that the most dominant cross-talk interactions, leading to quark mixing, are of the form  $\bar{Q}_L\Phi^2\chi q_R^{u,d}$  and  $\bar{Q}_L\Phi\chi^2 q_R^{u,d}$ , both of which give similar contributions to those described in section 3.2. Consequently, we can match the CKM matrix as we already did in section 4. Differences will stem from redefinitions of  $\Phi_0$  and  $C_\chi$  and consequent rescaling of fermion bulk masses. The resulting mixing data in the quark sector will be now governed by  $\tilde{x}_i^{u,d}$ ,  $\tilde{y}_i^{u,d}$  and  $\tilde{z}_i^{u,d}$ , with the newly defined  $\tilde{z}_i^{u,d}$  parameters entering at the second row of the up and down mass matrices in the

interaction basis. A matching of an almost realistic CKM matrix by 4 real parameter assignments analogous to the one in section 4 can be performed, yet the parameter space is still larger. As already said another possible drawback of this proposal may lie in the identification of the 5D flavon  $\Phi$  with the bulk Higgs.

We should also add that in both solutions offered above to protect the desired vacuum alignment, the  $G_{\text{SM}}^{\text{cust}} \times A_4 \times Z_8$  invariant interaction terms in the scalar potential with insertions of the new fields  $\eta$  and  $\phi$  are irrelevant to the vacuum alignment problem, since they are both flavor singlets. Further constructions with additional fields and more complicated flavor symmetries are obviously possible at the price of an increased arbitrariness of the model.

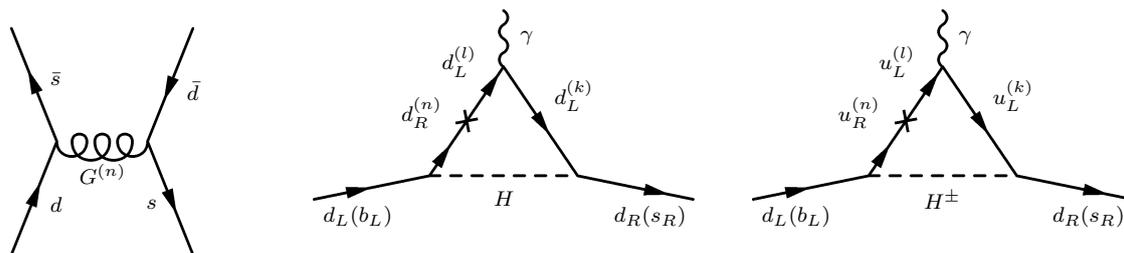
## 6 Flavor violation and the Kaluza-Klein scale

All models with extra dimensions will have to face the presence of mixing between the degrees of freedom of the effective 4D theory and their KK excitations. One of the crucial tasks in the construction of these models, is thus to guarantee that corrections induced by this mixing do not spoil the agreement with observations for a natural value of the lowest KK scale of order a few TeV. In other words, once new physics (NP) contributions induced by the exchange of the KK excitations are taken into account, the agreement with observations will force a lower bound on the KK scale, and we demand it be naturally of order a few TeV.

It has been shown [11] that custodial symmetry in the bulk of RS warped models is able to reduce the lower bound on the first KK mass imposed by electroweak precision measurements from  $\sim 10$  TeV to  $\sim 4$  TeV. A generalization of that analysis to a wider set of scenarios and including higher order corrections can be found in [70]. More recently, it has been observed [17–20, 26–28] that FCNC processes can in general produce more stringent bounds than the observed S, T parameters on the KK scale, and that a residual CP problem remains in the form of excessive contributions to  $\epsilon_K$  [17–19], the direct CP violation parameter  $\epsilon'/\epsilon_K$  [20] and the neutron electric dipole moment (EDM) [17–19].

These bounds can slightly be improved if the Higgs field is allowed to propagate in the bulk. In this case all zero mode fermions can be pushed further towards the UV brane, preserving the same 4D mass and having a reduced overlap with the IR localized KK modes. As a consequence the 5D Yukawa couplings can be raised without violating perturbativity constraints. As we already observed, this is particularly relevant for the top quark, being it the heaviest fermion zero mode and the most IR localized. For these reasons, our model realizes custodial symmetry with a bulk Higgs, in addition to an  $A_4$  discrete bulk flavor symmetry.

The predictions and constraints derived in [17–20] apply to the general case of flavor anarchical 5D Yukawa couplings. The conclusions may differ if a flavor pattern of the Yukawa couplings is assumed to hold in the 5D theory due to bulk flavor symmetries. They typically imply an increased alignment between the 4D fermion mass matrix and the Yukawa and gauge couplings, thus suppressing the amount of flavor violation induced by the interactions with KK states. In our case, the most relevant consequence of the  $A_4$  flavor symmetry is the degeneracy of the left-handed fermion bulk profiles  $f_Q$ , i.e.  $\text{diag}(f_{Q_1, Q_2, Q_3}) = f_Q \times \mathbb{1}$ . In addition, the distribution of phases, CKM and Majorana-like, in the mixing matrices might



**Figure 2.** Contribution to  $\epsilon_K$  from a KK gluon exchange (left). Dipole contributions to the neutron EDM from KK down-quarks (center) and KK up-quarks (right). The same type of dipole diagrams also contribute to  $b \rightarrow s(d)\gamma$  and  $\epsilon'/\epsilon_K$ .

induce zeros in the imaginary components of the Wilson coefficients contributing to CP violating quantities. We can straightforwardly identify a few properties of our model and state some results, while we defer to a separate work a more complete study. Following the spurion analysis in [17–19], we observe the following facts, consequence of the degenerate left-handed fermion profiles. First, the new physics contribution to  $\epsilon_K$  coming from a KK gluon exchange, the leftmost diagram in figure 2 vanishes, since

$$\epsilon_K^{\text{NP}} \propto \text{Im} \left[ V_L^{d\dagger} \text{diag}(f_{Q_1, Q_2, Q_3}^{-2}) V_L^d \right]_{12}^2 = f_Q^{-4} \text{Im}(V_L^{d\dagger} V_L^d)_{12}^2 = 0, \quad (6.1)$$

in the basis where the fermion profiles are real and diagonal. We notice that the amplitude itself vanishes and not only its imaginary part. Second, and for the same reason, the left-handed rotation matrix  $V_L^d$  will disappear from all down-type contributions to the dipole effective operators of the form  $O_{\gamma, (g)}^{ij} = \bar{d}_L^i \sigma_{\mu\nu} d_R^j F(G)^{\mu\nu}$ , center diagram in figure 1. A pleasing result is the vanishing of the down-type new physics contributions to the neutron electric dipole moment, induced by the same diagram with external  $d$  quarks. Again following [17–19], this contribution vanishes as follows

$$\begin{aligned} \text{Im} C_{d_n}^{d\text{-type}} &\propto \text{Im} \left[ V_R^{d\dagger} \text{diag}(f_{d_1, d_2, d_3}^2) V_R^d \text{diag}(m_{d, s, b}) V_L^{d\dagger} \text{diag}(f_{Q_1, Q_2, Q_3}^2) V_L^d \right]_{11} \\ &= f_Q^2 m_d \text{Im} \left[ V_R^{d\dagger} \text{diag}(f_{d_1, d_2, d_3}^2) V_R^d \right]_{11} = 0. \end{aligned} \quad (6.2)$$

Contributions from up-type KK fermion exchange, rightmost diagram in figure 2, involve left- and right-handed matrices and are thus expected to be non zero, generally of the same size as in any flavor anarchic model. However, a bulk flavor symmetry might induce an interesting cancellation of observable phases, so that dominant new physics contributions to the neutron electric dipole moment and, or, to  $\epsilon'/\epsilon_K$  will vanish. We leave this analysis and the study of Higgs mediated FCNCs for future work. We conclude here that the presence of a bulk  $A_4$  flavor symmetry can only improve upon the residual CP violation problem that in general affects warped models with flavor anarchy. We have seen that the constraint induced by tree level KK gluon exchange to  $\epsilon_K$  is released, leaving  $b \rightarrow s(d)\gamma$ ,  $\epsilon'/\epsilon_K$ , the neutron EDM and Higgs mediated FCNCs as possible candidates to produce the

most stringent lower bounds on the KK scale. In the worst scenario, a milder lower bound from the EDM and  $\epsilon'/\epsilon_K$  is expected in our model due to the vanishing of down-type dipole contributions. On the other hand, the degeneracy of left-handed fermion bulk profiles is expected to not be sufficient to produce a suppression of Higgs mediated FCNCs at tree level. This might require additional constraints on the right-handed fermion profiles.

Alternatively, a cancellation of observable phases, and a consequent vanishing of the imaginary parts of flavor violating amplitudes such as the new physics contributions to the EDM,  $\epsilon_K$  and  $\epsilon'/\epsilon_K$ , would obviously be a welcomed feature of the model; it would release the most stringent lower bounds on the KK scale from CP violating observables and solve the residual CP problem.

## 7 Conclusions

We have constructed a warped extradimensional realization of an  $A_4$  flavor model for quarks and leptons, and implemented the flavor symmetry breaking pattern  $A_4 \rightarrow$  nothing first suggested in [47]. In this construction all standard model fields, including the Higgs field, propagate in the bulk and a bulk custodial symmetry is broken in two different ways [11] on the UV and IR brane by orbifold boundary conditions. The spontaneous symmetry breaking of the  $A_4$  flavor symmetry is induced by the VEVs of two bulk flavon fields  $\Phi$  and  $\chi$ :  $\Phi$  is responsible of the breaking pattern  $A_4 \rightarrow Z_3$  in the charged fermion sector, while  $\chi$  is responsible of the breaking pattern  $A_4 \rightarrow Z_2$  in the neutrino sector. By taking the two flavons to be peaked on different branes, we approximately sequester the two sectors and the associated symmetry breaking patterns: neutrinos with the UV-peaked  $\chi$  on one side, charged fermions with the IR-peaked  $\Phi$  on the other. If the two sectors do not communicate, that is when the interactions of  $\Phi$  with neutrinos and  $\chi$  with charged fermions are switched off, tribimaximal mixing for neutrinos is exactly reproduced, while no quark mixing is generated. In our model the two flavons propagating in the bulk are responsible for cross-brane interactions and a complete cross-talk between the charged fermion and neutrino sectors. As a consequence, quark mixing on the IR brane is generated by contributions which are naturally suppressed by the warped geometry with respect to the leading order pattern in the quark and lepton sectors [41–44, 47].

Using this realization we have obtained an almost realistic CKM matrix, including its CP violating phase, with almost degenerate order one complex Yukawa couplings. The large hierarchy of standard model fermion masses is generated by a tiny hierarchy in the bulk fermion mass parameters, a well known pleasing feature of warped constructions. At the same time the contributions of all cross-talk and cross-brane effects do not spoil the tribimaximal mixing pattern in the neutrino sector, where they produce deviations within  $1\sigma$  from the experimental values, with small non zero contributions also to  $\theta_{13}$ .

The cross-talk/brane induced quark mixing, to leading order in the perturbative diagonalization of the mass matrices, is expressed in terms of six complex parameters in the up and down sectors,  $\tilde{x}_i^{u,d}, \tilde{y}_i^{u,d}$ , with  $i = 1, 2, 3$ . It turns out that, with all these parameters of order one and allowing for at least two relative phases, one obtains an

almost realistic CKM matrix within the model theoretical error. At this order the diagonal CKM entries remain equal to one, and the two smallest entries  $V_{ub}$  and  $V_{td}$  are degenerate.

We have also noticed that the possibility of producing hierarchical CKM matrix elements, of order  $\lambda_{\text{CKM}}$ ,  $\lambda_{\text{CKM}}^2$  and  $\lambda_{\text{CKM}}^3$ , with all parameters of order one stems from the presence of built-in cancellations induced by the hierarchical masses. The presence of the  $A_4$  induced phase  $\omega$  also produces a pattern in the corrections. Analogous cancellations are expected to occur at higher orders. It is instructive to compare this  $A_4$  pattern with other flavor symmetry groups, in particular  $T'$  [48–53]. The Wolfenstein parametrization would suggest the existence of an expansion parameter to be naturally identified with  $\lambda_{\text{CKM}}$ , offering an elegant and simple description of quark mixing in the standard model. On the other hand, flavor models based on otherwise appealing discrete flavor symmetries such as  $A_4$  and  $T'$  seem to rely on more complicated patterns in order to produce a realistic CKM matrix.

A bulk  $A_4$  flavor symmetry is also welcomed in order to suppress the amount of flavor violation induced by the mixing of the standard model particles — the zero modes of the 5D theory — with their Kaluza-Klein excitations. The degeneracy of left-handed fermion bulk profiles, due to having assigned the left-handed fermions to triplets of  $A_4$ , implies that the tree level contribution from a KK gluon exchange to  $\epsilon_K$  vanishes. For the same reason, all down-type dipole contributions to the neutron electric dipole moment and  $\epsilon'/\epsilon_K$  also vanish. The situation is different for Higgs mediated FCNCs [26–28] and their contribution to  $\epsilon_K$  at tree level, since they involve both left- and right-handed fermion profiles. Their suppression might require further constraints on the right-handed sector, to be explored in future work. Even in the presence of non vanishing amplitudes, an  $A_4$  induced cancellation of observable phases and the consequent vanishing of new physics contributions to the EDM,  $\epsilon_K$  and  $\epsilon'/\epsilon_K$ , would obviously be a welcomed feature of the model, removing the most stringent bounds on the KK scale and resolving the little CP problem [17–19].

Finally, the presence of cross-brane interactions of the flavon fields  $\Phi$  and  $\chi$  inevitably induces deviations from the VEVs that realize the two breaking patterns  $A_4 \rightarrow Z_3$  and  $A_4 \rightarrow Z_2$ , leading to the well known vacuum alignment problem. However, in this case such corrections are naturally suppressed, being the two flavons peaked on different branes. In particular, we have shown that the contribution from the most dominant term in the interaction potential  $V(\Phi, \chi)$  can be pushed below the model theoretical error by introducing a bulk mass for the UV-peaked  $\chi$  field. Obviously, this implies the need to rescale by a global amount all the  $\tilde{x}_i^{u,d}$ ,  $\tilde{y}_i^{u,d}$  parameters entering quark mixing, a rescaling that should anyway maintain all parameters of order one and satisfy all perturbativity bounds. On the other hand, employing a  $Z_8$  symmetry setup, as suggested in section VI, directly forbids the most dangerous terms and seems to provide a more elegant solution.

The  $A_4$  flavor symmetry still appears to be the most elegant and economical way to account for the nearly tribimaximal mixing pattern of neutrinos. We have shown here that it is also possible to obtain an almost realistic quark mixing, using a rather simple embedding of  $A_4$  in a warped extra dimension and with minimal field content. The main advantage of this construction remains the one of having cross-brane interactions and cross-talk effects sufficiently large to account for the observed quark mixing, without affecting the other results, or spoiling the vacuum alignment. A dynamical completion

of this, as of other flavor models involving a discrete flavor symmetry would certainly be desirable. Possible scenarios already described in the literature include a spontaneous symmetry breaking of a continuous flavor symmetry [71] or having  $A_4$  as a remnant spacetime symmetry of a toroidal compactification scheme of a six-dimensional spacetime [72].

It is worth to mention two additional points. It is appealing to explore the effects of a heavier Higgs in this context. Some of the model predictions would get closer to the best fits for certain observables, such as  $Zb\bar{b}$  ratios and asymmetries, allowing for a larger parameter space. However, as discussed in the literature, a heavier Higgs mass in a custodial setup easily induces conflicts with electroweak precision measurements and a more careful estimate of the allowed range of values for  $m_H$  should be produced in each version of the custodial setup. The final point concerns possible extensions of the warped  $A_4$  model. It is interesting and phenomenologically relevant to consider the possibility to embed  $P_{LR}$  (or other versions of) custodial symmetry [55] into warped  $A_4$ . This would release the most stringent constraint on the model parameter space due to the  $Zb\bar{b}$  best fits and could provide new appealing features alternative to flavor anarchic models.

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### A Basic $A_4$ properties

The alternating group of order four, denoted  $A_4$ , is defined as the set of all twelve even permutations of four objects and is isomorphic to  $T$ , the tetrahedral group. It has a real three-dimensional irreducible representation  $\mathbf{3}$ , and three inequivalent one-dimensional representations  $\mathbf{1}$ ,  $\mathbf{1}'$  and  $\mathbf{1}''$ . The representation  $\mathbf{1}$  is trivial, while  $\mathbf{1}'$  and  $\mathbf{1}''$  are non-trivial and complex conjugates of each other.

The twelve representation matrices for  $\mathbf{3}$  are conveniently taken to be the  $3 \times 3$  identity matrix  $1$ , the reflection matrices  $r_1 \equiv \text{diag}(1, -1, -1)$ ,  $r_2 \equiv \text{diag}(-1, 1, -1)$  and  $r_3 \equiv \text{diag}(-1, -1, 1)$ , the cyclic and anticyclic matrices

$$c = a^{-1} \equiv \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad a = c^{-1} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \tag{A.1}$$

respectively, as well as  $r_i c r_i$  and  $r_i a r_i$ . Under the group element corresponding to  $c(a)$ ,  $\mathbf{1}' \rightarrow \omega(\omega^2)\mathbf{1}'$  and  $\mathbf{1}'' \rightarrow \omega^2(\omega)\mathbf{1}''$ , where  $\omega = e^{i2\pi/3}$  is a complex cube root of unity, with both representations being unchanged under the  $r_i$ .

The basic non-trivial tensor products are

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{3}_s \oplus \mathbf{3}_a \oplus \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'', \quad \text{and} \quad \mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}'', \tag{A.2}$$

where  $s(a)$  denotes symmetric (antisymmetric) product. Let  $(x_1, x_2, x_3)$  and  $(y_1, y_2, y_3)$  denote the basis vectors for two  $\mathbf{3}$ 's. Then

$$(\mathbf{3} \otimes \mathbf{3})\mathbf{3}_s = (x_2 y_3 + x_3 y_2, x_3 y_1 + x_1 y_3, x_1 y_2 + x_2 y_1), \tag{A.3}$$

$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{3}}_a} = (x_2y_3 - x_3y_2, x_3y_1 - x_1y_3, x_1y_2 - x_2y_1), \quad (\text{A.4})$$

$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{1}}} = x_1y_1 + x_2y_2 + x_3y_3, \quad (\text{A.5})$$

$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{1}}'} = x_1y_1 + \omega x_2y_2 + \omega^2 x_3y_3, \quad (\text{A.6})$$

$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{1}}''} = x_1y_1 + \omega^2 x_2y_2 + \omega x_3y_3, \quad (\text{A.7})$$

in an obvious notation.

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