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# ELECTRON SCATTERING IN THE INTERACTING BOSON MODEL

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It is suggested that the interacting boson model be used in the analysis of electron scattering data. Qualitative features of the expected behavior of the inelastic excitation of some $2^+$ states in the transitional Sm–Nd region are discussed.

Present knowledge of the structure of low-lying collective states in nuclei is largely based upon experimental information (electromagnetic moments and transition rates) obtained in the long wavelength limit. The more general information provided by electron scattering has up to now mainly been used for a study of the ground state charge distribution. With the advent of high resolution electron scattering facilities, a detailed study of transition charge densities of low-lying levels in heavy nuclei has become feasible. The measurements of form factors for transitions within the ground state band of axially symmetric deformed nuclei have been interpreted in terms of the projected Hartree–Fock approach [1]. Similar microscopic calculations for the inelastic excitations of other degrees of freedom in deformed nuclei ($\beta$ and $\gamma$ vibrations) and/or inelastic excitations in nuclei which do not have a large (and axially symmetric) permanent deformation are lacking at present. Nor is it clear (even at a lower phenomenological level) how to interpret the new data in a simple and yet detailed way.

In axially deformed nuclei, a simple phenomenological parametrization of the inelastic scattering to the $2^+$ state of the ground state band, $2^+_g$, is given by the geometrical model [2]. Here the deformed surface is characterized by

$$R(\Omega) = R_0(1 + \beta_2 Y_{20}(\Omega)). \tag{1}$$

Assuming a density of the form $\rho(r) = \rho(r - R(\Omega))$ and expanding $\rho(r)$ around $\rho_0(r) = \rho(r, \beta_2 = 0)$, one can calculate the cross section for inelastic excitation of the state $2^+_g$ in terms of $\beta_2$ and $\partial\rho_0/\partial r$. In order to describe additional degrees of freedom (such as $\beta$ and $\gamma$ vibrations) these must be added explicitly to eq. (1), which then becomes [2]

$$R(\Omega, t) = R_0\left[1 + \beta_2 Y_{20}(\Omega) + \sum_{\nu=0,\pm 2} \gamma_{2\nu}(t) Y_{2\nu}(\Omega)\right]. \tag{2}$$

The cross sections for the inelastic excitation of the states $2^+_\beta$, $2^+_\gamma$ are then given in terms of $\gamma_{2,0}$, $\gamma_{2,+2} = \gamma_{2,-2}$ and $\partial\rho_0/\partial r$, $\partial^2\rho_0/\partial r^2$. Although refinements can be made by introducing higher-order terms or by allowing for band mixing, this approach is completely phenomenological and requires the introduction of new parameters for each new level or new nucleus. In order to remove, at least in part, this arbitrariness one has to write down a collective hamiltonian, as it has been done by Gneuss and Greiner [3] in terms of the hydrodynamical model, and by Kumar and Baranger

[4] in a more microscopic approach. However, for transitional nuclei these calculations are rather complex.

In this note we suggest a different approach, which is simple and yet describes the situation in many nuclei, including the transitional regions in addition to the strongly deformed and spherical ones. The approach is based upon the interacting boson approximation (IBA) recently introduced by Arima and Iachello [5]. Here one describes an even–even nucleus as a system of correlated $L = 0$ ("s") and $L = 2$ ("d") nucleon pairs. The E2 transition operator up to first order (to which we restrict our discussion here) can then be written as

$$T^{(E2)}_{\mu} = \alpha(r)(d^{\dagger}s + s^{\dagger}d)^{(2)}_{\mu} + \beta(r)\,5^{-1/2}(d^{\dagger}d)^{(2)}_{\mu}, \quad (3)$$

where, in order to discuss transition densities, we have replaced the constants $\alpha_2$ and $\beta_2$ of [5] by functions $\alpha(r)$ and $\beta(r)$. Although in a subsequent publication [6] we have introduced explicitly proton and neutron pairs, for the purposes of the present note, no distinction will be made between protons and neutrons. Wave functions of states with $N$ pairs outside the closed shells can then be written as [5]

$$|[N]\phi LM\rangle = \sum_{n_d,\chi} c^{\phi}_{n_d \chi L}(d^{\dagger})^{n_d}_{\chi LM}(s^{\dagger})^{N-n_d}|0\rangle, \quad (4)$$

enabling reduced matrix elements of $T^{(E2)}$ between any initial and final states to be expressed as

$$\langle[N]\phi'L'||T^{(E2)}||[N]\phi L\rangle = \alpha(r)A + \beta(r)B \equiv \rho(r). \quad (5)$$

Here the numbers

$$A = \langle[N]\phi'L'||(d^{\dagger}s + s^{\dagger}d)^{(2)}||[N]\phi L\rangle,$$
$$B = \langle[N]\phi'L'||5^{-1/2}(d^{\dagger}d)^{(2)}||[N]\phi L\rangle, \quad (6)$$

depend on the initial and final angular momenta $L, L'$ and on the quantum numbers $\phi, \phi'$ which distinguish states with same $L, L'$. Finally $\chi$ in eq. (4) denotes the additional quantum numbers which are needed to specify the states of $n_d$ d-bosons [5]. Knowledge of the coefficients (6) then permits a standard DWBA calculation of the E2 electron scattering cross sections in terms of the densities $\alpha(r)$ and $\beta(r)$.

The assumption (3) for $T^{(E2)}$ has several implications. First, it is clear that inelastic scattering to collective $2^+$ states can be calculated in terms of only *two* functions $\alpha(r)$ and $\beta(r)$ which are expected to vary very slowly and smoothly with $N$. Thus there should be a simple, linear relation between the cross sections for inelastic excitation of the $2^+_1, 2^+_2, 2^+_3$ collective states in each nucleus. (Here we have written $2^+_1, 2^+_2, 2^+_3$ instead of $2^+_g, 2^+_\beta, 2^+_\gamma$ because the classification in terms of $\beta$ and $\gamma$ quantum numbers is only appropriate for strongly deformed nuclei.) Assuming further that the variation of the functions $\alpha(r)$ and $\beta(r)$ in a certain mass region is negligible, eq. (3) can be used to relate and thus predict electron scattering cross sections to states in neighbouring nuclei. Alternatively, one can use electron scattering to extract $A$ and $B$ directly from experiment.

By way of illustration we now discuss some qualitative features of the transitional Nd–Sm region. Application of the IBA model to the energies of the low-lying collective states in the nuclei considered, provides a set of parameters for these nuclei [7]. These are in turn used to calculate the coefficients $A, B$ in eq. (6), which are given in table 1. We start with the static $B$(E2) values:

$$B(E2; \phi L \to \phi' L') = \frac{1}{2L+1}\left|\int_0^{\infty} \rho(r) r^4 dr\right|^2, \quad (7)$$

where $\rho(r)$ is given by eq. (5) with $A$ and $B$ appropriate for the transition $\phi L \to \phi' L'$. Because $B$(E2) values are given in terms of

$$\int_0^{\infty} \alpha(r)\, r^4\, dr = \alpha_2, \quad \int_0^{\infty} \beta(r)\, r^4\, dr = \beta_2, \quad (8)$$

their measurement only provides information on $\alpha_2$ and $\beta_2$. A study of electromagnetic transitions in the Sm and Nd isotopes shows that the values $\beta_2/\alpha_2 \approx -1.5$ and $\alpha_2 \approx 14.3\, e\,\mathrm{fm}^2$ give a reasonable description of the known $B$(E2) values in this region [7]. The negative value of $\beta_2/\alpha_2$ brings about a cancellation in the matrix elements $0^+_1 \to 2^+_2$ and $0^+_1 \to 2^+_3$ while enhancing the $0^+_1 \to 2^+_1$ one, and this is in agreement with experiment. We now proceed to electron scattering, which, through measurement of the densities $\alpha(r)$, $\beta(r)$, provides a more detailed test of the IBA wave functions.

For the example of $^{150}$Nd, we have extracted the functions $\alpha(r)$ and $\beta(r)$ by using the data for the $0^+_1 \to 2^+_1$ and $0^+_1 \to 2^+_3$ transitions [8], and the IBA calcula-

Table 1
The values of $A$ and $B$ for some transitions in the Sm–Gd region as given by the IBA model. Also shown are: the experimental $B$(E2) values [11]; (a) the predicted $B$(E2) using the IBA values of $A$ and $B$ given in the preceding columns; (b) the $B$(E2) values obtained from a fit to the electron scattering data [8].

| | | | | $B(E2\uparrow)$ ($e^2fm^4$) | | |
|---|---|---|---|---|---|---|
| Nucleus | Final state | $A$ | $B$ | Exp | (a) | (b) |
| $^{150}$Nd | $2_1^+$ | 10.02 | −1.01 | 27200 ± 400 | 27196 | 26100 ± 900 |
| | $2_2^+$ | 1.68 | 0.58 | 76 ± 36 | 134 | 2300 ± 400 |
| | $2_3^+$ | 2.73 | 0.65 | 690 ± 90 | 629 | 520 ± 60 |
| $^{150}$Sm | $2_1^+$ | 9.26 | −0.34 | | | |
| $^{152}$Sm | $2_1^+$ | 10.87 | −0.89 | | | |
| $^{154}$Sm | $2_1^+$ | 12.01 | −1.32 | | | |

tion for the numbers $A, B$ (table 1). This has been done by fitting the experimental cross sections in DWBA with functions $\alpha(r)$, $\beta(r)$ parametrized in the form $\Sigma_n a_n r^n e^{-\gamma r^2}$. Cross sections taken at fixed angle (90°) but for various electron energies, have conveniently been transformed to a common energy $E_0$ (the highest measured one) and are calculated as a function of an effective momentum transfer

$$q_{\text{eff}} = q\left(1 + \frac{3Z\alpha\hbar c}{2E_0R_0}\right).$$

In fig. 1 we show the extracted E2 form factors and the corresponding boson transition densities $\alpha(r)$ and $\beta(r)$. Sign ambiguities in the former allow several choices for $\alpha(r)$ and $\beta(r)$ and we have chosen the set which gives $\beta_2/\alpha_2 \approx -1.5$. The function $\alpha(r)$ has the shape characteristic for a collective transition density peaked at the surface, while $\beta(r)$ has a more complicated structure and a significantly larger transition radius. Using these $\alpha(r)$ and $\beta(r)$ and the wave functions of table 1 one can now calculate both the $B$(E2) value and the form factor for the transition $0_1^+ \to 2_2^+$. This calculation reproduces very well the observed $B$(E2) value (table 1) but it does not fit the reported cross section [8] (fig. 2a). Viceversa, we can use the electron scattering data and assumption (3) and thus express the transition density as a linear combination of $\rho_1$ and $\rho_3$ in order to extract the coefficients $A$ and $B$ for the transition $0_1^+ \to 2_2^+$. We then find that the corresponding form factor fits approximately the experiment (fig. 2b), but not the known $B$(E2). This inconsistency may indicate that either assumption (3) for form factors and transition densities is incorrect, or that the $0_1^+ \to 2_2^+$ experimental cross section contains admixtures for the excitation of unresolved states. In this particular case one expects from energy systematics a $J^\pi = 1^-$ state around 900 keV, and in fact it turns out that the cross section can be fitted well with an $L = 1$ transition density. More experimental information is needed to clarify this point: we discussed the case above mainly in order to show an example of consistency checks to which the experimental results can be subjected.

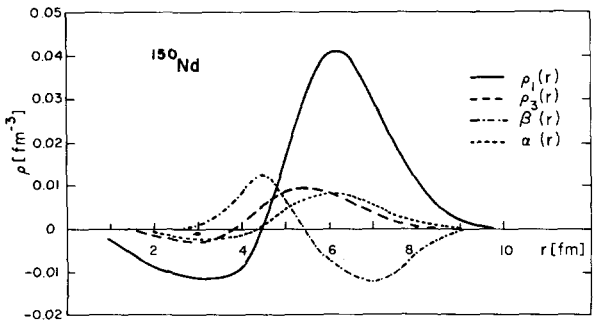


Fig. 1. The transition densities $\rho_1(r)$ and $\rho_3(r)$ for the $0_1^+ \to 2_1^+$ and $0_1^+ \to 2_3^+$ transitions in $^{150}$Nd, and the extracted boson densities $\alpha(r)$ and $\beta(r)$.

Another interesting and straightforward application of the IBA approach is the possibility to describe the variation in the $0_1^+ \to 2_1^+$ transition densities in the Sm isotopes 144–154 currently being measured in Saclay [9] and in Mainz [10]. It is then convenient to introduce the transition radii $R_t$:

$$R_t^2 = \int_0^\infty \rho(r) r^6 \, dr \Big/ \int_0^\infty \rho(r) r^4 \, dr. \qquad (9)$$

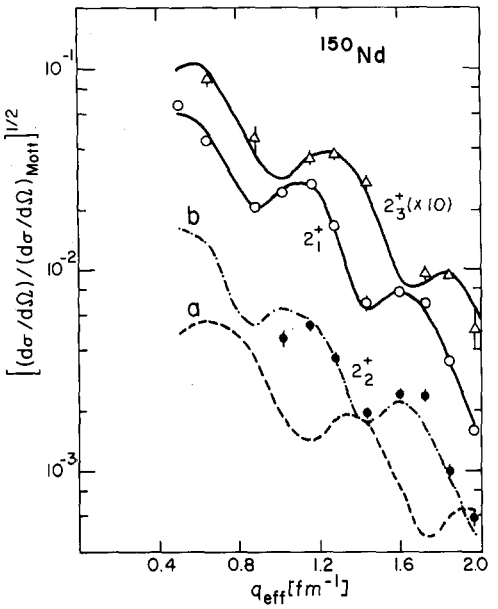


Fig. 2. The experimental form factors $|F(q_{eff})| = (\sigma/\sigma_{Mott})^{1/2}$ for the inelastic excitation of the lowest three $2^+$ states in $^{150}$Nd [8]. The full lines show the fits to $F_{2_1^+}$ and $F_{2_3^+}$ from which the densities $\rho_1(r)$ and $\rho_3(r)$ of fig. 1 were extracted. The broken lines are: (a) the predicted $F_{2_2^+}$ using the IBA wave functions of table 1; (b) the predicted $F_{2_2^+}$ treating the values $A$ and $B$ as free parameters (see text).

Within the framework of the IBA these are given in terms of the boson transition radii

$$\alpha_t = \frac{1}{\alpha_2}\int_0^\infty \alpha(r)\, r^6\, dr, \quad \beta_t = \frac{1}{\beta_2}\int_0^\infty \beta(r)\, r^6\, dr. \qquad (10)$$

The observed increase of collectivity in the $0_1^+ \to 2_1^+$ transition with increasing neutron number is described in the IBA model by a simultaneous increase in $A$ and in the ratio $|B/A|$ (table 1). From the extracted value $\beta_t/\alpha_t \approx 1.2$ in $^{150}$Nd, an increase in $R_t$ for the Sm isotopes of the order of 1% per isotope is expected.

In conclusion we emphasize that (i) the interacting boson model may provide a convenient parametrization of the electron scattering data, and (ii) if, in addition, one has calculated the wave functions it also provides predictions for cross sections. Finally, in a more ambitious program, the functions $\alpha(r)$ and $\beta(r)$ can be calculated from a microscopic theory, and directly related to the two nucleon pairs which determine the structure of the bosons s and d [6]. We also note that the same parametrization can be used for other transitions such as E0, E3 and E4 [5].

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