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G-FACTORS IN THE NEUTRON–PROTON INTERACTING BOSON APPROXIMATION

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Recent accurate measurements of g-factors of the first excited 2^+ states in collective even–even nuclei show substantial deviations from the value Z/A. We investigate the implications that these deviations have for our understanding of collective states in nuclei.

In the conventional collective model, the g-factors of all states are expected to be equal to $g_R = Z/A$, corresponding to the fact that in this model all protons and neutrons in a nucleus are assumed to take part equally in the collective motion [1]. Until recently, the accuracy of the measurements was not good enough to test unambiguously this property. However, in the last few years, accurate measurements of g-factors of the first excited 2^+ states in several medium-mass even–even nuclei have become available [2–4]. The experiments show substantial and systematic deviations from the value Z/A, and in this letter we want to investigate the implications of these deviations for an understanding of collective states in nuclei.

We begin by noting that, since magnetic properties of protons and neutrons are very different from each other, an accurate determination of g-factors for a series of isotopes (or isotones) offers a unique possibility for studying proton–neutron degrees of freedom in collective states of nuclei. This was noted several years ago by Greiner [5], who suggested that differences in proton and neutron quadrupole deformations are responsible for most magnetic properties of collective states. A natural framework for studying proton–neutron degrees of freedom in collective states of nuclei is provided by the proton–neutron interacting boson model (IBA–2) [6] and in this letter we wish to present an analysis of the recently obtained data in medium-mass nuclei in terms of this model. Here the collective magnetic (M1) operator can be written as [7]:

$$\boldsymbol{\mu} = g_\pi \boldsymbol{J}_\pi + g_\nu \boldsymbol{J}_\nu \tag{1}$$

where g_π (g_ν) is the g-factor of the correlated proton (neutron) pairs and $\boldsymbol{J}_\pi$ ($\boldsymbol{J}_\nu$) is the corresponding angular momentum operator. According to the microscopic foundation of the model [6], g_π (g_ν) is expected to depend, in first approximation, on proton (neutron) boson number N_π (N_ν) only, i.e., $g_\pi = g_\pi(N_\pi)$ and $g_\nu = g_\nu(N_\nu)$.

It is then clear that two effects contribute to the dependence of the magnetic moments on proton and neutron number: the dependence of g_π and g_ν on proton and neutron number and the variation of the matrix elements of the operators $\boldsymbol{J}_\pi$ ($\boldsymbol{J}_\nu$) with N_π and N_ν. As will be better shown below, the former effect is related to the shell structure of the orbits, while the latter is related to the average number of proton and neutron pairs taking part in the collective motion. The calculation of the magnetic moments for a series of isotopes, then, requires the determination of the functions g_π and g_ν in combination with the wavefunctions of the nuclei in the framework of IBA–2. Although all the results discussed below have been obtained by evaluating the matrix elements of $\boldsymbol{J}_\pi$ and $\boldsymbol{J}_\nu$ numerically, it is interesting to note that, for the first excited 2^+ state, these matrix elements are approximately proportional to $N_\pi/(N_\pi + N_\nu)$ and $N_\nu/(N_\pi + N_\nu)$, respectively, and thus directly related to the number of active proton (N_π) and neutron (N_ν) pairs. This leads to the approximate expression

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$$g_{2_1^+}(N_\pi, N_\nu) \qquad (2)$$
$$\approx g_\pi(N_\pi)\, N_\pi/(N_\pi + N_\nu) + g_\nu(N_\nu)\, N_\nu/(N_\pi + N_\nu)\,.$$

We note that the qualitative behaviour of $g_{2_1^+}$ as predicted by eq. (2) will, in general, be considerably different from that of Z/A which shows a monotonic decrease throughout.

In order to make this statement more quantitative, we have analyzed the available data in the region around proton number 50. We have taken the wave functions of the 2_1^+ states in the $_{54}$Xe, $_{56}$Ba, $_{46}$Pd and $_{44}$Ru isotopes from previous calculations [8,9]. One of us (M.S.) has completed the study of this region by extending the calculations to the $_{48}$Cd and $_{52}$Te isotopes. The functions g_ν and g_π have been determined in the following way. It is tempting to assume that the 2_1^+ states in the Sn isotopes (although the IBA model is not expected to describe, in general, properties of semi-magic nuclei) can be characterized by neutron d-boson number $n_{d\nu} = 1$, in which case $g_\nu(N_\nu)$ could be obtained directly from the experimental g-factors of the 2_1^+ states in these isotopes. Since the experimental g-factors have large uncertainties [10] no attempt has been made to fit them in detail. Instead a smooth behaviour of $g_\nu(N_\nu)$ between negative values for $N \approx 74$ and positive values for the lighter isotopes has been assumed. Values of $g_\nu(N_\nu)$ for other values of N_ν and the function $g_\pi(N_\pi)$ have been obtained by requiring an overall fit to the experimental $g_{2_1^+}$ factors in the neighbouring isotopes with $44 \leqslant Z \leqslant 56$, assuming a smooth variation with N_ν and N_π. The resulting functions g_ν and g_π are shown in fig. 1 and the results for the calculated values of $g_{2_1^+}$ are given in fig. 2.

The qualitative features of these results can be understood on the basis of eq. (2). The systematically low values of the g-factors in the middle of the neutron shell for fixed proton numbers are related to the fact that here the number of active neutrons is maximal [and thus $N_\pi/(N_\pi + N_\nu)$ is minimal], and that the effective g-factor of the neutron pairs g_ν is small. Superimposed on the smooth behaviour of the terms $N_\pi/(N_\pi + N_\nu)$ and $N_\nu/(N_\pi + N_\nu)$, there appear in the data also fluctuations presumably due to shell effects. These fluctuations are quite visible in $_{52}$Te and to some extent in $_{46}$Pd, where, as will be discussed also below, the decrease of the $g_{2_1^+}$ at neutron number 56 is probably related to the increasing influence of the $d_{5/2}$ orbit.

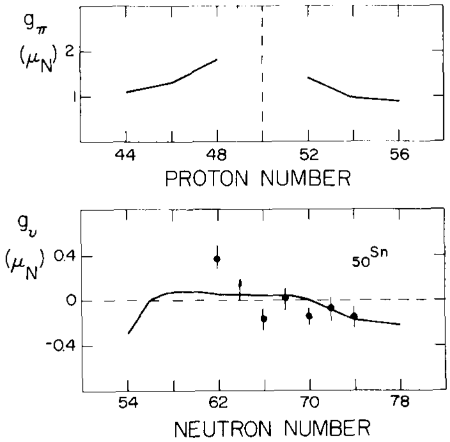

Fig. 1. Dependence of the functions g_π and g_ν on the proton and neutron number, respectively. The points indicate the experimental $g_{2_1^+}$ values of the Sn isotopes [10].

An analogous decrease is also predicted in ^{112}Cd and ^{114}Cd.

With respect to the behaviour of the functions g_π and g_ν we remark that one of us (M.S.) has made an attempt to derive them from a shell model picture [12]. The preliminary results appear to be consistent with the phenomenological values of $g_\pi(N_\pi)$ and $g_\nu(N_\nu)$. In particular for the lighter Sn isotopes ($N \sim 52, 54$),

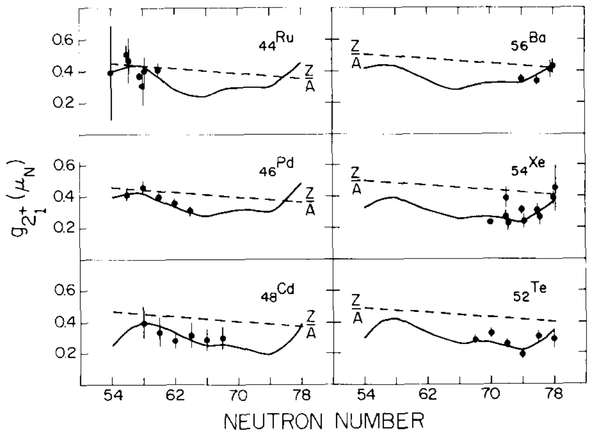

Fig. 2. The g-factors of the 2_1^+ states (in μ_N) as a function of neutron number for various isotopes around $Z = 50$. The solid line represents the results of the present paper. The dashed line indicates the values Z/A. The experimental points are taken from refs. [4,11].

negative g-factors for the 2_1^+ states are predicted due to the dominance of the $\nu d_{5/2}$ orbital (for a pure $\nu d_{5/2}^2$ $L = 2$, configuration $g = -0.76\ \mu_N$). In the region around $N = 60$ an increase of $g_\nu(N_\nu)$ towards positive values is found mainly because of the contribution of the $g_{7/2}$ orbital (for a $\nu g_{7/2}^2$ $L = 2$, configuration $g = +0.42\ \mu_N$). In the heavier isotopes, instead, the influence of mostly the $\nu h_{11/2}$ orbital (for a $\nu h_{11/2}^2$ $L = 2$ configuration $g = -0.35\ \mu_N$) again reduces $g_\nu(N_\nu)$ towards negative values. Also in the case of the large positive value of g_π for the Cd isotopes ($Z = 48$) can qualitatively be ascribed to the contribution of the $\pi g_{9/2}$ orbital.

As it emerges from the above discussion, accurate measurements of g-factors open the way to a detailed study of magnetic properties of collective states in nuclei giving rise to a wealth of predictions. Here we mention only two examples. First, the determination of the values of g_π and g_ν allows one to estimate the size of the M1 matrix elements leading from the ground state to the predicted 1^+ states which are an antisymmetric combination of the collective proton and neutron degrees of freedom [13]. For instance using the values given in fig. 1 one obtains $B(\mathrm{M1}; 0_1^+ \to 1_1^+) = 1.67\ (\mu_N)^2$ in $^{134}_{56}$Ba. The existence of such a magnetic state which carries orbital M1 strength has also been proposed in a different context for deformed nuclei by Lo Iudice and Palumbo [14]. We note that this strength could possibly be detected in an inelastic magnetic electron scattering experiment.

Secondly, the determination of $g_\pi(N_\pi)$ and $g_\nu(N_\nu)$ makes it also possible to estimate the contribution of the proton–neutron effect to the M1 transitions between low-lying states. These transitions vanish in the IBA-1 model, in which no distinction is made between neutron and proton degrees of freedom, if the lowest order operator is used (it has been shown [15] that in that case M1 transitions can be described if higher-order terms are added to the operator). In IBA-2, instead, these transitions are non-zero because of the contributions to the matrix elements of the M1 operator [eq. (1)] of components in the wave functions which are not symmetric in the proton–neutron degrees of freedom. Limiting ourselves to the 2_i^+ ($i = 1$, 2, 3) states, we find that the off-diagonal M1 matrix elements are rather sensitive to possible admixtures of an antisymmetric state of the type

$$\psi_a \sim (d_\pi^\dagger s_\pi - d_\nu^\dagger s_\nu)|s_\nu^{N_\nu} s_\pi^{N_\pi}\rangle \ .$$

In the IBA-2 approach the unperturbed energy of this state is controlled by the strength, ξ_2, of the so-called Majorana force [8]. Defining

$$\Delta_i = \langle 2_i^+ \| \mathrm{E2} \| 2_1^+ \rangle / \langle 2_i^+ \| \mathrm{M1} \| 2_1^+ \rangle$$

we find that for small values of ξ_2 (~0.10 MeV) (as used in refs. [8,9]) there are appreciably antisymmetric components present in the 2_2^+ states, resulting in rather large M1 matrix elements and consequently small values of Δ_2. For larger values of ξ_2 the antisymmetric 2^+ state is pushed to higher energies thus reducing its components in the 2_2^+ states, resulting in larger values of Δ_2 (and smaller values of Δ_3). The available experimental data on Δ_i in the $Z = 50$ region [16] show that in general $|\Delta_3| \ll |\Delta_2|$. This information can conveniently be used to fix the value of ξ_2 which cannot accurately be determined in other ways. For example, recent accurate measurements of Δ_i in the ^{106,108}Ru isotopes [17] can be reproduced rather well with the choice $\xi_2 = 0.18$ (MeV): the calculated values in ^{106}Ru and ^{108}Ru are $\Delta_2 = 9.7$, 12.9 and $\Delta_3 = 0.43$, 0.66 (in eb/μ_N), respectively, compared to the experimental ones, $\Delta_2 = 17.9^{+4.8}_{-2.2}$, $11.1^{+2.2}_{-1.6}$ and $\Delta_3 = 0.28 \pm 0.15$, $1.03^{+0.66}_{-0.46}$ (in eb/μ_N). This value of ξ_2 is larger than the value adopted in refs. [8,9], however, we have checked that this hardly affects the energies of low-lying states.

In conclusion, the new, accurate, measurements of g-factors of 2_1^+ states in even–even nuclei appear to provide a unique tool for studying magnetic properties of collective states in nuclei. The present experimental information in medium-mass nuclei appears to be consistent with the analysis in terms of the proton–neutron interacting boson model and corroborates the importance of the proton–neutron degrees of freedom, as suggested by Greiner. From this point of view, the large deviations from the value Z/A are not surprising, since the latter is obtained within the framework of a model in which the proton–neutron degrees of freedom are not included.

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