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Published in:
Physics Letters B

DOI:
10.1016/0370-2693(82)91163-7

A PSEUDO-SPIN SYMMETRY IN THE IBFA MODEL

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Received 1 October 1981

It is shown that the level doubling, observed in certain collective medium heavy odd-mass nuclei can be explained in terms of a pseudo-spin symmetry in the IBFA model. The selection rules for single particle transfer agree with experiment.

In the IBA model [1] the low-lying collective states in a medium mass even–even nucleus are described in terms of a system of s- and d-bosons. In the IBFA model [2–4] the low-lying states in medium heavy odd-mass nuclei are described by coupling the degrees of freedom of a single fermion to the s–d boson system. An exact pseudo-spin symmetry occurs in the IBFA hamiltonian [3] whenever the odd fermion can occupy two single particle (SP) orbits which differ in spin by one unit, have equal SP energies and have equal occupation probabilities. In this case the energy levels occur in doublets, differing in spin by one unit.

Since the present symmetry is related to pseudo-spin it will prove easier to work in an L–S coupling scheme rather than the usual j–j coupling basis. The relation between the two is given by

$$|\alpha,(R,l)L,1/2;J\rangle = \sum_{j=l-1/2}^{l+1/2} (-1)^{R+l+J+1/2}\sqrt{(2L+1)(2j+1)} \times \begin{Bmatrix} R & l & L \\ 1/2 & J & j \end{Bmatrix} |\alpha,R,(l,1/2)j;J\rangle, \quad (1)$$

where R is the boson total angular momentum and α denotes the other quantum numbers necessary to label the boson state uniquely. The l introduced in eq. (1) is not necessarily the real SP orbital angular momentum, but is chosen such that the j-values of the two involved SP orbits can be obtained from it by coupling a pseudo-spin, i.e. $l = (j+j')/2$. The symmetry is based on the fact that in the IBFA model the boson fermion interaction stems from a nucleon-nucleon quadrupole force which acts only on the pseudo-orbital angular momentum. Because of the spin independence of the interaction, the energies depend only on (α, $(R,l)L$) and the levels in the spectrum thus occur in doublets with spins $l = L \pm 1/2$.

The spin independence of the boson–fermion interaction can be demonstrated explicitly. We will outline here only the proof for the quadrupole term, which can be written as [4]

$$V_Q = \gamma \sum_{jj'} (-1)^{j'-1/2}\sqrt{(2j+1)(2j'+1)}\sqrt{5/4\pi} \times \begin{pmatrix} j & 2 & j' \\ -1/2 & 0 & 1/2 \end{pmatrix} Q_B^{(2)} \cdot (a_j^\dagger \tilde{a}_{j'})^{(2)} \quad (2)$$

where $Q_B^{(2)}$ is the boson quadrupole operator and $a_j^\dagger$ (a_j) the fermion creation (annihilation) operator while $\tilde{a}_{j,m} = (-1)^{j-m} a_{j,-m}$. With the use of some angular momentum algebra it can be shown that

$$\langle \alpha,(R,l)L,1/2;JM|V_Q|\alpha',(R',l)L',1/2;JM\rangle = \gamma\delta_{L,L'}\frac{5(2l+1)}{\sqrt{4\pi}}(-1)^{R'+L} \times \begin{Bmatrix} R' & R & 2 \\ l & l & L \end{Bmatrix} \begin{pmatrix} l & l & 2 \\ 0 & 0 & 0 \end{pmatrix} \langle \alpha,R\|Q_B^{(2)}\|\alpha',R'\rangle, \quad (3)$$

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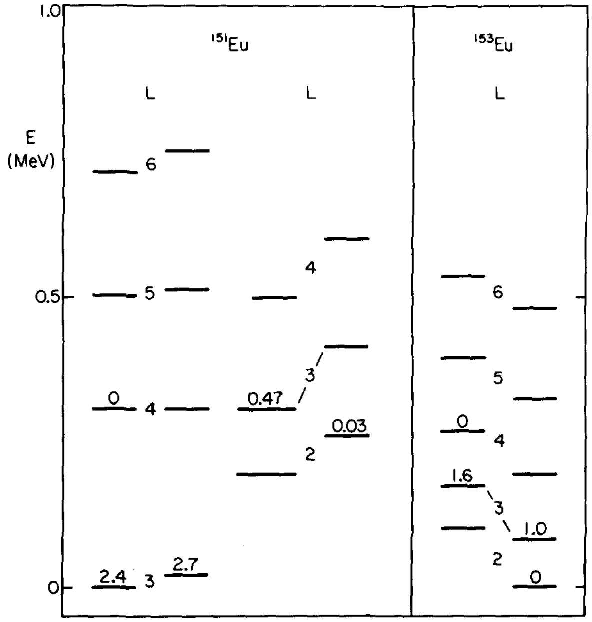

Fig. 1. The levels in the positive parity spectrum of ^{151}Eu [5] and ^{153}Eu [6] have been ordered in doublets on the basis of excitation energies and spin assignments. It is furthermore assumed that the pseudo-angular momentum, L, increases for the levels in an experimentally well-established band. For each doublet the value of L is given, the level with $J = L - 1/2$ is plotted on the left and $J = L + 1/2$ on the right. The value of the $d_{5/2}$ and $g_{7/2}$ spectroscopic factor as measured in (^{3}He, d) [7,8] is also given in the figure.

which is manifestly independent of J.

Although unequal occupation probabilities or unequal SP energies break the symmetry, still several examples can be found. In fig. 1 the positive parity spectra of ^{151}Eu and ^{153}Eu are shown, in which the odd proton occupies the close lying $1d_{5/2}$ and $0g_{7/2}$ orbits.

The present symmetry dictates also selection rules for transition operators. In the IBFA model the operator for single particle pick-up, leading from an even–even to an odd-A nucleus can be given by [3]

$$A_j^\dagger = A_0 a_j^\dagger + A_1 \sum_{j'} (-1)^{j-1/2} \sqrt{2j'+1} \sqrt{5/4\pi} \times \begin{pmatrix} j' & 2 & j \\ -1/2 & 0 & 1/2 \end{pmatrix} s^\dagger (\tilde{d} a_{j'}^\dagger)^{(j)} \tag{4}$$

in the case of equal occupancies for the SP levels. The spectroscopic amplitude for transfer from the GS, $|\alpha_0, 0\rangle$, of the even–even nucleus to the ith state in the odd-A nucleus, $|\psi_i\rangle = \Sigma_{\alpha,R}\, C^i_{\alpha,R} |\alpha, (R,l)L, 1/2; JM\rangle$ is given by

$$\langle \psi_i \| A_j^\dagger \| \alpha_0, 0\rangle = \delta_{j,J} \delta_{l,L} \sqrt{2j+1} \sum_{\alpha,R} C^i_{\alpha,R} \times \Big\{ A_0 \delta_{\alpha,\alpha_0} \delta_{R,0} + A_1 \sqrt{(2l+1)/4\pi} \times \begin{pmatrix} l & l & 2 \\ 0 & 0 & 0 \end{pmatrix} \delta_{R,2} \langle \alpha, R \| (s^\dagger \tilde{d})^{(2)} \| \alpha_0, 0\rangle \Big\} . \tag{5}$$

In single particle pick-up, levels with $L \neq l$ are thus not excited. As an example the measured spectroscopic factors for ^{151}Eu and ^{153}Eu are given in fig. 1. No state with $L \neq l = 3$ is strongly excited. The nicest example is the GS of ^{153}Eu which is not excited in $d_{5/2}$ transfer although it is the lowest $5/2^+$ state.

In the symmetry discussed here the actual structure of the even–even case does not enter, it is entirely related to the odd-particle. In this respect it should be distinguished from the super-symmetries discussed in ref. [9]. The symmetry discussed here can be generalized from SU(2) to SU(n) by taking a quadrupole operator similar to the one used by Ginocchio [10].

I wish to thank Professor F. Iachello for interesting discussions. This work is supported by the National Science Foundation under Grants PHY79-27054 and PHY77-27084, and the “Stichting voor Fundamenteel Onderzoek der Materie” (FOM) with financial support of the “Stichting voor Zuiver Weten schappelijk Onderzoek ” (ZWO).

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