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ONSET OF STATIC QUADRUPOLE DEFORMATION IN THE LIGHT ACTINIDES

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We examine the transition from spherical to deformed structure in even-even Ra and Th nuclei as a function of the number of valence neutron and proton pairs. While the onset of collectivity is delayed, the onset of static quadrupole deformation occurs for values of the product of valence neutron and proton numbers similar to those in other mass regions.

The heavier actinide nuclei ($A \geq 230$) have long been known to be well characterized as prolate deformed rotors with deformation $\beta_2 \approx 0.25$ and possibly with hexadecapole and/or $\beta_6$ deformations. In contrast, the structure of nuclei near doubly magic $^{208}$Pb can be well described [1] in terms of a few spherical particle orbitals that the valence protons and neutrons occupy. In fact, recent studies [2,3] have shown that even the $N = 128$ isotones, $^{214}$Rn-$^{215}$Fr, with four and five valence protons, respectively, can be understood to a great extent in terms of reasonably simple shell model configurations.

One measure of the collective structure of a nucleus is the energy ratio

$$R(4/2) = \frac{E(4^+_1)}{E(2^+_1)}.$$ 

This quantity is only 1.64 in $^{214}$Rn, much less than the collective spherical quadrupole vibrator value of 2.00. That is, even in a system with valence protons and neutrons, not only is there no sign of static quadrupole deformation at low excitation energies, but the system does not even show a pattern of excitation indicative of collective quadrupole motion. This seems to contradict the suggestion [4] of Talmi that collectivity arises from a strong valence proton—neutron interaction. For singly magic nuclei, such as the $^{146}$Ba and $^{150}$Sn isotopes, the $2^+_1$ state energies are large and constant while with valence neutrons and protons $^{146}$Ba is deformed.

Recently Casten has suggested [5] that the properties of low-lying states in transitional regions can be understood primarily in terms of a simple hamiltonian having a single free parameter which varies as a function of the product of the number of valence protons and neutrons $N_p N_n$. In particular, he showed that the quantities $R(4/2)$, $E(2^+_1)$ and $B(E2: 0^+_1 \rightarrow 2^+_1)$ could be described very well for the mass regions $A \approx 100$, $A \approx 130$ and $A \approx 150$. In these regions the collectivity seems to arise from the filling of neutron and proton orbitals with a large radial overlap.

It is only very recently that the structure of light actinides with $A \approx 220$ has been measured [2,6–10] and that the change in structure from $N \approx 126$ isotones to $A \approx 230$ nuclei could be examined. For the Ra and Th nuclei complete systematics for $N \approx 126$–140 are known and we shall concentrate our discussion on these elements; in addition more limited information [11] on Rn and Po isotopes will be examined.

We concentrate on two simple signatures of static quadrupole deformation: the $2^+_1$ state at low excitation and the energy ratio $R(4/2)$, which should be 3.33
for a good rotor. In figs. 1a, 1c we plot the systematics [2,6–11] of these two quantities as a function of $N_n$ or, equivalently $N_\nu$, the neutron boson number, which counts the number of valence neutron pairs.

For Ra and Th isotopes, the $2^+_1$ state energy drops rapidly, while the $R(4/2)$ value approaches the value for a rotor, labelled as SU(3) in fig. 1a. For the Rn and Po isotopes the $2^+_1$ state energy decreases as the number of valence neutrons is increased, but the onset of deformation has not yet been observed. For the heavier Rn and Po isotopes, the available data [11] come from alpha-particle decay studies. These nuclei are more difficult to study because of the limited number of target–projectile combinations available and in many cases only the $2^+_1$ state energies are known.

The $R(4/2)$ value can serve as a convenient measure of the collectivity, since $R(4/2) = 2.00$ corresponds to a collective spherical quadrupole vibrator. The onset of collectivity then takes place in the Ra–Th nuclei when $N_\nu \approx 2–3$. In other words in this region one needs $N_\pi \approx 3–4$ valence proton pairs and $N_\nu \approx 2–3$ neutron pairs for collective behavior to set in.

To see more clearly the effect of the neutron–proton interaction on the onset of collectivity, we have plotted in figs. 1b, 1d the $E(2^+_1)$ and $R(4/2)$ values for these same nuclei as a function of the product $N_\pi N_\nu$, one-fourth the product of valence particles $N_\pi N_\nu$. Collectivity sets in at $N_\pi^2 N_\nu \approx 7$, while the $N_\pi^2 N_\nu$ values for the Po and Rn isotopes for which data exist are far from this value. This suggests that

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**Fig. 1.** Systematics of $E(2^+_1)$ and $R(4/2)$ values in the light actinides as a function of neutron boson number ($N_n$) in parts (a) and (c) and the product $N_\pi N_\nu$ in parts (b) and (d). The data were taken from refs. [2,6–11]. The solid lines serve only to guide the eye. The dotted line represents results of IBA calculations described in the text.
it is not only the number of valence neutrons that is important, but it is the number of valence proton—neutron combinations that determines the structure. When prolate axial deformation is setting in (at \( R(4/2) > 3.00 \)) the curves become identical for the Ra, Th and U isotopes. This universality of collective behavior as a function of \( N_\pi N_\nu \) has already been noted [5,12] by Casten.

In order to contrast the systematics in the actinides with those of another region, in fig. 2 we repeat the systematics already considered [12] by Casten et al., for the transitional nuclei above the doubly magic \(^{132}\text{Sn}\). This region was chosen because both the \( Z = 50 \) and \( N = 82 \) shells are closed, as opposed to \(^{146}\text{Gd}\), for example, where only a proton subshell closure is operative. However, to properly examine the onset of deformation in \( A \approx 140 \) nuclei, we have made the same assumption as Casten et al., that \( Z = 64 \) is magic [13] for \( N \leq 88 \).

In the \( A \approx 140 \) nuclei, the onset of collectivity occurs quickly at \( N_\pi N_\nu \approx 3 \), while \( N_\pi N_\nu \approx 7 \) for collectivity to occur for the \( A \approx 230 \) region. On the other hand, the onset of deformation (\( R(4/2) > 3.00 \)) occurs in both regions for the same value of the \( N_\pi N_\nu \) product, namely \( \approx 16 \).

To model the change in structure in translational nuclei, Casten and coworkers [5] have considered a simple Hamiltonian in the interacting boson approximation (IBA) model:

\[
H = e\hat{n}_d - \kappa Q^{(2)} \cdot Q^{(2)},
\]

where

\[
Q^{(2)}_\mu = (s^\dagger d + d^\dagger s)^{(2)}_\mu + (\chi/\sqrt{3})(d^\dagger d)^{(2)}_\mu.
\]
In the earlier calculation the parameters $\kappa$ and $\chi = -0.9$ MeV were kept constant for all regions, and the d-boson energy $\varepsilon$ followed an exponentially decreasing dependence on $N_\pi N_\nu$ [5]:

$$\varepsilon = \varepsilon_0 \exp[-\theta(N_\pi N_\nu)],$$

where again $\theta = 0.0405$ was kept constant and $\varepsilon_0$ was kept constant within each mass region. The structure is strongly dependent on the ratio $\varepsilon/\kappa$. To reproduce the $2^+_1$ state energies in the light actinides, we found it necessary to reduce the value of $\kappa$, but kept the $\varepsilon/\kappa$ ratio constant with respect to the parameters of ref. [5]. The values for these parameters in the present calculations are $\kappa = 0.012$ MeV and $\varepsilon_0 = 0.4194$ MeV. The trends for the light actinides with $N_\pi N_\nu \geq 12$ are well reproduced, as illustrated in figs. 1b, 1d. On the other hand, we were unable to reproduce the sharp onset of deformation in the actinides with such a simple Hamiltonian and stringent parameter choices.

Another difference between the $A \approx 140$ and $A \approx 230$ regions comes from the importance of the number of valence neutrons. In fig. 3 we show the same data as presented in fig. 2, but the $a$ and $c$ parts are plotted as a function of $N_\nu$ as opposed to $N_\pi$. (In other words, the same dependence as used in figs. 1a, 1c.) The behavior of $A \approx 140$ nuclei appears to be governed by the number of valence neutrons as opposed to the actinide region where there is a more marked importance on the number of valence proton—neutron combinations.

In conclusion, we have examined the recent data on light Ra and Th nuclei and have shown that while the onset of collectivity is delayed in this region, when compared to the systematics above $^{132}$Sn, for example, the onset of quadrupole deformation occurs for values of $N_\nu N_\pi$ similar to those observed in other regions of the periodic table and arises from a strong interaction between neutrons and protons. The onset of deformation has frequently been suggested [5,12,14] to arise from the strong proton—neutron interaction between spin—orbit partners. In the actinide region, these partners would correspond to the $p_{13/2} - n_{11/2}$ configurations. The delay in the onset of collectivity may be because the first orbitals above $^{208}$Pb have high angular momentum and because they do not have good radial overlap, namely the $1h_{9/2}$ orbital for the protons and $2g_{9/2}$ orbital for the neutrons. Nature may require that the lowest lying orbitals above the shell gap be fairly full before collectivity can set in, or in this mass region $7-8$ valence particles of each kind, which would give $N_\pi N_\nu$ of $\approx 14$, reproducing the observed systematics. Microscopic calculations are needed to confirm this speculation.

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References


R.F. Casten, W. Frank and P. von Brentano, to be published.


W. Bonin, thesis Technische Hochschule Darmstadt (1983), unpublished;


