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Compressibility of nuclei in relativistic mean field theory

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Using the relativistic Hartree approximation in the σ–ω model we study the isoscalar giant monopole resonance. It is shown that the ISGMR of lighter nuclei has non-negligible anharmonic terms. The compressibility of nuclear matter is determined using a leptodermous expansion.

Effective relativistic nuclear models like the σ–ω model [1], yield a compressibility of nuclear matter which is only in poor agreement with experimental results $K_{\mathrm{exp}}=210$–300 MeV [2–4]. The σ–ω model used in the relativistic Hartree approximation (RHA) [5] provides us with compressibilities in heavy finite nuclei of about $K_A=330$ MeV [6]. The compressibility of nuclear matter, K_{nm}, is extracted from these finite nuclei results and compared with experiment.

The frequency of the isoscalar giant monopole resonance (ISGMR) in the σ–ω model is found to be approximately constant over a wide range of nuclei [6], whereas experiments give $\omega\simeq 80A^{-1/3}$ MeV [7,8]. Both the experimental and theoretical values result from calculations in which a harmonic oscillator (HO) approximation is assumed to be valid.

Simple estimates show that for lighter nuclei ($A\leqslant 100$) the amplitude of the ISGMR corresponds to a variation of the RMS radius of 5–10%. Our σ–ω model calculations show that for such large deviations from equilibrium the first anharmonic terms in the energy expansion of the ISGMR are not negligible.

The purpose of this letter is to investigate the determination of the excitation energy beyond the HO approximation of the ISGMR. We will also discuss the application of the leptodermous expansion to obtain values for the nuclear matter and surface compressibility.

We follow the authors of ref. [6] and use the σ–ω model in the RHA [5], but we also include the Coulomb interaction. Introducing the single particle wavefunction $U_a(\boldsymbol{r})$ and the single particle energy ε_a the relativistic Hartree equation can be written as

$$\hat{H}U_a(\boldsymbol{r})=\varepsilon_a U_a(\boldsymbol{r})\ , \qquad (1)$$

with

$$\hat{H}=-\mathrm{i}\boldsymbol{\alpha}\cdot\nabla+\gamma_0 m-g_{\mathrm{s}}\gamma_0\phi_0(\boldsymbol{r})+g_{\mathrm{v}}V_0(\boldsymbol{r}) +eA_0(\boldsymbol{r})\ . \qquad (2)$$

The scalar and vector meson fields ϕ_0, V_0, and the Coulomb field A_0 are given by

$$\phi_0(\boldsymbol{r})=g_{\mathrm{s}}\int \mathrm{d}\boldsymbol{r}'\ D_{\mathrm{s}}(\boldsymbol{r}-\boldsymbol{r}')\rho_{\mathrm{s}}(\boldsymbol{r}')\ , \qquad (3)$$

$$V_0(\boldsymbol{r})=g_{\mathrm{v}}\int \mathrm{d}\boldsymbol{r}'\ D_{\mathrm{v}}(\boldsymbol{r}-\boldsymbol{r}')\rho_{\mathrm{b}}(\boldsymbol{r}')\ , \qquad (4)$$

$$A_0(\boldsymbol{r})=\tfrac{1}{2}e(1+\tau_3)\int \mathrm{d}\boldsymbol{r}'\ \frac{\rho_{\mathrm{p}}(\boldsymbol{r}')}{\max(r,r')}\ , \qquad (5)$$

where $D_{\mathrm{s(v)}}$ and $g_{\mathrm{s(v)}}$ denote the Green function and coupling constant of the σ (ω) meson respectively. τ_3 gives the usual isospin factor. The scalar and baryon densities are given by

$$\rho_{\mathrm{s}}(\boldsymbol{r})=\sum_a^{\mathrm{occ}}\bar{U}_a(\boldsymbol{r})U_a(\boldsymbol{r})\ , \qquad (6)$$

$$\rho_{\mathrm{b}}(\boldsymbol{r})=\sum_a^{\mathrm{occ}}U_a^{\dagger}(\boldsymbol{r})U_a(\boldsymbol{r})\ . \qquad (7)$$

The proton density ρ_{p} is identical to the baryon density ρ_{b} with the summation carried out over the proton states only. Eqs. (1)–(7) are solved self-consistently for all particle levels, assuming the nuclear ground state to be spherically symmetric. The total energy of the nucleus is given as

$$E_0 = \int \mathrm{d}\boldsymbol{r}\, \varepsilon(\boldsymbol{r})$$
$$= \sum_a^{\mathrm{occ}} \varepsilon_a - \tfrac{1}{2} \int \mathrm{d}\boldsymbol{r} [-g_s \rho_s(r) \phi_0(r) + g_v \rho_b(r) V_0(r)$$
$$+ e\rho_p(r) A_0(r)] . \quad (8)$$

The ISGMR is induced by the response to an external static potential. We follow the authors of ref. [6] in taking a quadratic, harmonic potential causing a change in the nuclear mean square radius R defined by

$$R^2 = \frac{1}{A} \int \mathrm{d}\boldsymbol{r}\, \rho_b(r) r^2 . \quad (9)$$

The number of nucleons is denoted by A. The RH equation now reads

$$[\hat{H} - \lambda(r^2 - R_0^2)] U_a(\boldsymbol{r}) = \varepsilon_a U_a(\boldsymbol{r}) . \quad (10)$$

The RMS radius of the unperturbed nucleus is denoted by R_0. Both the mean square radius R and the energy of the nucleus now depend on the parameter λ. Instead of using this parameter, we introduce the collective variable s,

$$s = \frac{R}{R_0} - 1 , \quad (11)$$

for which the energy E of the nucleus can be expanded in terms of s:

$$E(s) = E_0 + \tfrac{1}{2} A K_A s^2 + \tfrac{1}{2} A K_3 s^3 + \tfrac{1}{2} A K_4 s^4 + \ldots . \quad (12)$$

The coefficient K_A is identified to be the compressibility of the nucleus. In fact, for a nucleus with a sharp surface and a homogeneous density distribution it can be shown easily that K_A corresponds to the nuclear matter compressibility K_{nm}. The mass or inertia parameter B of the monopole vibration results from an expansion of E in terms of the time derivative of the collective variable, $\dot{s}$ [9]. Defining the velocity field $\boldsymbol{v}(\boldsymbol{r}, s, \dot{s})$ by

$$\boldsymbol{j}(\boldsymbol{r}, s, \dot{s}) = \rho_b(r, s) \boldsymbol{v}(\boldsymbol{r}, s, \dot{s}) , \quad (13)$$

where $\boldsymbol{j}(\boldsymbol{r}, s, \dot{s})$ is the nuclear current resulting from the monopole vibration, the current conservation law yields

$$\frac{\partial \rho_b(r)}{\partial t} = \dot{s} \frac{\partial \rho_b(r)}{\partial s} = -\nabla \cdot [\rho_b(r) \boldsymbol{v}(\boldsymbol{r}, s, \dot{s})] , \quad (14)$$

eq. (14) suggests a radial velocity field $\boldsymbol{v}$ of the form [6]

$$\boldsymbol{v}(\boldsymbol{r}, s, \dot{s}) = -\dot{s} u(r) \hat{\boldsymbol{r}} . \quad (15)$$

The velocity distribution $u(r)$ can be determined from eq. (14). It can now easily be shown [6] that

$$B = \frac{1}{A} \int \mathrm{d}\boldsymbol{r}\, u^2(r) \varepsilon(r) . \quad (16)$$

In our actual σ–ω model calculations, we use the parameters of Horowitz and Serot [5]. Table 1 summarizes the results for K_A and B obtained for six spherical nuclei, ^{16}O, ^{40}Ca, ^{48}Ca, ^{90}Zr, ^{114}Sn and ^{208}Pb, using the previously discussed method. Since higher order contributions in the expansion of E (eq. (12)) turn out to be of significant importance, we also list the coefficient of the first anharmonic terms in the expansion eq. (12), K_3 and K_4. Our calculations indicate that these corrections properly reflect the anharmonicities in eq. (12).

Traditionally, the frequency of the ISGMR is defined as the harmonic oscillator frequency:

$$\omega_{\mathrm{HO}} = \hbar c \sqrt{K_A / B} . \quad (17)$$

A simple calculation shows however that the corresponding amplitudes of the harmonic oscillation are in the range from $s = 0.02$ for ^{208}Pb up to about $s = 0.1$ for ^{16}O. It is immediately clear from eq. (12) that the anharmonic terms are not negligible for these amplitudes. We therefore conclude that eq. (17) does not satisfy to calculate the frequency of the ISGMR. Instead we propose to obtain this frequency using the energy solutions of a Schrödinger equation for a potential well, described by

$$V(s) = \tfrac{1}{2} A K_A s^2 + \tfrac{1}{2} A K_3 s^3 + \tfrac{1}{2} A K_4 s^4 , \quad (18)$$

with the mass parameter taken from table 1. The frequencies ω_A, resulting from this analysis, as well as the HO frequencies are listed in table 2. The full potential (18) leads to results that are only slightly different from the HO approximation. Inspite of this apparent agreement the anharmonic corrections are rather large, especially for lighter nuclei. This is illustrated in fig. 1 for the case of ^{16}O. With respect to the HO well the full potential shows a steeper behaviour

Table 1
Compressibilities, anharmonicity coefficients and mass parameters of spherical nuclei.

Nucleus	K_A (GeV)	K_3 (GeV)	K_4 (GeV)	B (GeV fm^2)	$BA^{-2/3}$ (GeV fm^2)
^{16}O	0.093	−0.63	1.59	9.02	1.421
^{40}Ca	0.163	−1.04	2.81	13.69	1.170
^{48}Ca	0.202	−1.21	3.20	13.37	1.012
^{90}Zr	0.254	−1.30	3.54	18.26	0.909
^{114}Sn	0.272	−1.26	2.83	20.31	0.863
^{208}Pb	0.328	−1.36	1.89	28.79	0.820

Table 2
The frequency of the ISGMR.

Nucleus	ω_A (MeV)	$\omega_A A^{1/3}$ (MeV)	ω_{HO} (MeV)	$\omega_{HO} A^{1/3}$ (MeV)
^{16}O	18.0	45.3	20.1	50.6
^{40}Ca	19.0	64.8	21.5	73.6
^{48}Ca	22.2	80.7	24.3	88.2
^{90}Zr	22.7	101.8	23.3	104.2
^{114}Sn	22.5	109.1	22.9	110.8
^{208}Pb	21.0	124.2	21.1	124.8

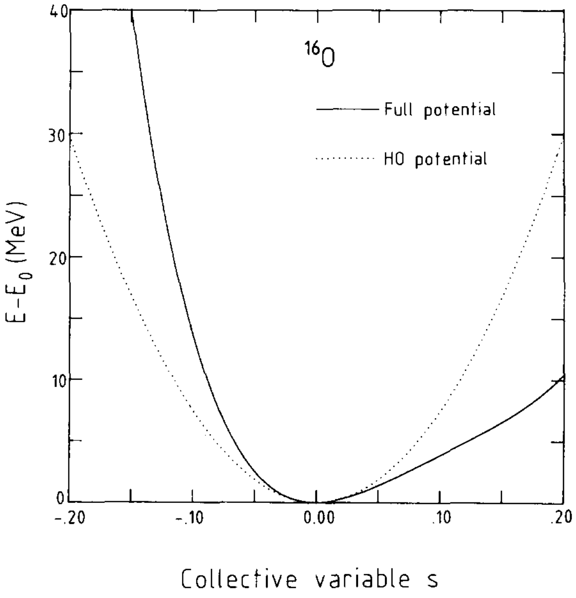

Fig. 1. The total energy of ^{16}O as a function of the collective variable s (solid line). The dashed line represents the corresponding HO potential.

inside the nucleus whereas it is much softer outside. The difference between the two approaches also becomes pronounced by observing the mean values and the RMS values of the collective variable s. The mean value of s in the harmonic oscillator model vanishes naturally since the wavefunctions are (anti)symmetric. In this model $\langle s^2 \rangle$ is determined analytically by

$$\langle s^2 \rangle_{HO} = \frac{\omega_{HO}(\frac{1}{2}+n)}{AK_A}, \quad n=0, 1, 2, \dots . \tag{19}$$

In table 3 the results of the HO and full potential are compared, both for the ground state and the first excited state of the ISGMR. We observe that inclusion of anharmonic terms leads to a significant increase of the radius of lighter nuclei, having possibly consequences for the process of sub-barrier fusion (see e.g. ref. [10]). In contrast, the HO model gives $\langle R^2 \rangle \approx R_0^2$. As expected, the difference between the two models gets less for heavy nuclei.

The HO approach (eq. (17)) is customarily used to extract the compressibility coefficient from experiment. Replacing B by mR_0^2, where m is the nucleon mass, one has [11]

$$K_{exp} = mR_0^2 \omega_{exp}^2 / (\hbar c)^2 . \tag{20}$$

Anharmonic corrections will not drastically change K_{exp}.

The compressibility of a nucleus can be expressed as [2,12]

$$K_A = K_{nm} + K_s A^{-1/3} + K_\tau \left(\frac{N-Z}{A}\right)^2 + K_{Coul} Z^2 A^{-4/3} + K_{curv} A^{-2/3} , \tag{21}$$

usually referred to as the leptodermous expansion.

Table 3
Mean square values of R and s in the ground state and first excited state of the ISGMR.

	Nucleus	R_0 (fm)	Harmonic oscillator		Full potential		
			$\langle s^2\rangle^{1/2}$	$\langle R^2\rangle^{1/2}$ (fm)	$\langle s\rangle$	$\langle s^2\rangle^{1/2}$	$\langle R^2\rangle^{1/2}$ (fm)
ground state	^{16}O	2.622	0.0819	2.631	0.0515	0.1000	2.766
	^{40}Ca	3.367	0.0406	3.370	0.0143	0.0450	3.418
	^{48}Ca	3.491	0.0354	3.493	0.0096	0.0378	3.527
	^{90}Zr	4.243	0.0226	4.244	0.0024	0.0229	4.254
	^{114}Sn	4.571	0.0192	4.572	0.0012	0.0193	4.577
	^{208}Pb	5.543	0.0124	5.544	−0.0004	0.0125	5.542
first excited state	^{16}O	2.622	0.1419	2.648	0.0891	0.1640	2.878
	^{40}Ca	3.367	0.0704	3.375	0.0450	0.0867	3.528
	^{48}Ca	3.491	0.0612	3.498	0.0325	0.0717	3.611
	^{90}Zr	4.243	0.0391	4.246	0.0102	0.0409	4.290
	^{114}Sn	4.571	0.0332	4.574	0.0063	0.0341	4.603
	^{208}Pb	5.543	0.0215	5.545	0.0016	0.0217	5.553

The number of neutrons (protons) is denoted by N (Z). One mostly uses the so called scaled compressibility K_A^s as input for this expansion, but K_A^s differs only slightly from K_{exp} determined with eq. (20) [12]. The coefficients in eq. (21) are not uniquely determined by experiment. The nuclear matter compressibility K_{nm} e.g., runs from about 210 MeV [2] up to 300 MeV [4]. The discussion on the correct K_{nm} value is still going on [13]. We apply the leptodermous expansion to our theoretical K_A values. It is believed that this approach can be used to extract the nuclear matter compressibility since we observe that the transition density for ^{208}Pb in our model shows a behaviour which is in qualitative agreement with the transition density in the scaling model [12], for which the leptodermous expansion holds (see e.g. ref. [14]). Due to the limited number of spherical nuclei we do not expect very accurate values for the neutron-excess coefficient K_τ, the Coulomb coefficient K_{Coul} and the curvature term K_{curv}. Using a least squares fit we obtained values for K_{nm} and K_s. Firstly (i) we put the three other parameters to zero. Secondly (ii), we estimate K_{Coul} taking advantage of the analytical expression by Blaizot [2]. The value of K_{curv}, which is especially important for light nuclei, is fixed at a value of 375 MeV [4]. Finally (iii), K_{nm} and K_s have been determined using the data of ref. [6]. The results are listed in table 4. These results give a justification for the use of the leptodermous expansion to extract the nuclear matter compressibility from finite nuclei calculations. We rather accurately reproduce the K_{nm} value of the σ–ω model being $K_{nm}=545$ MeV [15] if we take all terms in the leptodermous expansion into account (case ii).

Table 4
Parameters of the leptodermous expansion.

	K_{nm} (MeV)	K_s (MeV)	K_τ (MeV)
(i)	485	−1023	0
(ii)	533	−1258	680
(iii)	506	−972	–

The conclusions of this paper can be summarized in the following way. Anharmonic corrections to the energy expansion do not affect the frequency of the ISGMR determined from a simple HO analysis severely. In contrast, they do have a significant effect on the radius of light nuclei. Finite nuclei compressibilities resulting from the σ–ω model offer the opportunity to extract a value for K_{nm} using this leptodermous expansion; the number of doubly closed nuclei limits the reliability of the other coefficients in this expansion. More realistic relativistic models [16,17] are needed to explain the experimental data.

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