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Niiniluoto, I.; Cevolani, Gustavo; Kuipers, Theodorus

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INTRODUCTION

Approaching probabilistic truths: introduction to the Topical Collection

Ilkka Niiniluoto[1] · Gustavo Cevolani[2] · Theo Kuipers[3]

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Abstract

After Karl Popper's original work, several approaches were developed to provide a sound explication of the notion of verisimilitude. With few exceptions, these contributions have assumed that the truth to be approximated is deterministic. This collection of ten papers addresses the more general problem of approaching probabilistic truths. They include attempts to find appropriate measures for the closeness to probabilistic truth and to evaluate claims about such distances on the basis of empirical evidence. The papers employ multiple analytical approaches, and connect the research to related issues in the philosophy of science.

The idea that science, and human knowledge more generally, aims at approaching the truth about the world is quite widespread among scientists, philosophers, and laypeople. Indeed, many more-or-less realist philosophers of science think that scientific progress consists in approach towards truth or increasing verisimilitude (or truthlikeness). A typical example of such position is the fallibilist program of Karl Popper (1963), who emphasized that scientific theories are always conjectural and

✉ Ilkka Niiniluoto
ilkka.niiniluoto@helsinki.fi

Gustavo Cevolani
gustavo.cevolani@imtlucca.it

Theo Kuipers
t.a.f.kuipers@rug.nl

[1] Department of Philosophy, History, and Art Studies, University of Helsinki, Unioninkatu 40 B, P. O. BOX 24, 00014 Helsinki, Finland

[2] IMT School for Advanced Studies Lucca, Piazza San Francesco 19, 55100 Lucca, Italy

[3] University of Groningen, Groningen, The Netherlands

corrigible, but still later theories may be "closer to the truth" than earlier ones. Making that idea precise, however, has proven to be a rather exacting task. After the debunking of Popper's own definition of verisimilitude by David Miller (1974) and Pavel Tichý (1976), several approaches were developed to provide a sound explication of this notion. Such approaches face two, related challenges. The first is the so called "logical" problem of verisimilitude, which consists in finding an optimal definition of closer to the truth, possibly based on an adequate notion of distance from the given truth. The second is the "epistemic" problem of verisimilitude, which consists in evaluating claims of truth approximation in the light of empirical evidence and non-empirical features of relevant theories or statements, even when the truth is unknown. The main results of this thriving research program, both philosophical and technical, are summarized in such works as Niiniluoto (1987, 1998, 1999, 2018), Kuipers (1987, 2000, 2019), Oddie (1986, 2016), and Zwart (2001).

With few exceptions, contributions to this line of research have always assumed some kind of "deterministic" truth to be approached. This target could be the descriptive or factual truth about some domain of reality or the "nomic" truth about what is physically or biologically possible. Despite their important differences, all these approaches, including most of the recent ones, agree about the assumption that "the truth" is a deterministic truth. Given the widespread use of probabilistic and statistical methods in all branches of both theoretical and applied science, however, it seems clear that adequate theories of truth approximation should be able to deal also with the problem of approaching probabilistic truths. Here the truth may concern a collection of statistical facts, or the objective probability distribution of some process, or fully probabilistic laws. Relaxing the deterministic assumption requires an extension of current theories of verisimilitude, and raises again, on a new level, both the logical and the epistemic problems: the task becomes to find appropriate measures for the closeness of theories to probabilistic truths and to evaluate claims about such distances on the basis of empirical evidence.

To explore the prospects of such a project, Theo Kuipers proposed to Ilkka Niiniluoto the idea of a meeting among truthlikeness theorists. This led to the symposium on "Approaching Probabilistic Truths, in Comparison with Approaching Deterministic Truths", jointly organized by Kuipers and Niiniluoto in Prague in August 2019, on the occasion of the 16th International Congress of Logic, Methodology and Philosophy of Science and Technology (CLMPST 2019). The symposium promoted a lively debate on how to tackle the issue of probabilistic truth approximation, featuring both proposals based on existing theories of truthlikeness and new ideas from related approaches. In view of this successful discussion, and of the many open problems left on the table, the present authors promoted a call for papers in order to reach a wider audience and collect new proposals from the community.

The present Topical Collection collects both the six papers originally scheduled at the Prague symposium and four other papers contributed later (precisely, the second, sixth, ninth and tenth paper surveyed below). It brings together approaches, methods and perspectives from philosophy of science, formal epistemology, and other related disciplines, thus providing the first systematic exploration of probabilistic truth approximation, which we hope will stimulate further separate and comparative

research. In the following, we offer a brief overview of the content of the ten papers in the Collection, highlighting some of the main conceptual relations with each other.

In the first paper, "Approaching Probabilistic Laws", Ilkka Niiniluoto addresses the problem of legisimilitude in a probabilistic context. This is the problem of defining truth approximation when "the truth" is defined by some relevant law (Cohen 1980), either universal (or deterministic) or probabilistic. The latter case was first discussed by Rosenkrantz (1980) and then briefly addressed by Niiniluoto himself in his 1987 book on *Truthlikeness*. In this paper, the author develops these earlier contributions in different directions, advancing the study of probabilistic legisimilitude on both the technical and the philosophical level. In particular, he points to some limitations of the well-known Kullback–Leibler divergence for measuring the distance between probability distributions, an issue discussed also in other papers of the collection. Niiniluoto also shows how to tackle the epistemic problem of truthlikeness in a probabilistic context, by extending the approach for the deterministic case based on the assessment of expected truthlikeness of different probabilistic theories.

In the second paper, "Truthlikeness for Probabilistic Laws", Alfonso García-Lapeña further explores the issue of probabilistic legisimilitude starting from Rosenkrantz's and Niiniluoto's original contributions. He accepts the Kullback–Leibler divergence as an essentially adequate measure; however, he points to another kind of difficulty with those approaches. According to the author, who builds here on his previous work (García-Lapeña 2021), legisimilitude must take into account two different dimensions: accuracy and "nomicity". Accuracy refers to how close to the true law are the numerical values derived from some probabilistic theory; nomicity has instead to do with how close the "shape" of the theory is to the correct one. For instance, as an approximation to the real trajectory of a planet, a non-elliptical law may be more accurate than an elliptical one; still, the latter may be closer to the truth than the former. The author shows how to apply his dual approach to the case of probabilistic laws, by proposing a measure of probabilistic legisimilitude which balances both accuracy and nomicity considerations.

The third paper takes a more general and abstract look at the problem of probabilistic truth approximation. In "Approaching Deterministic and Probabilistic Truth: a Unified Account", Gustavo Cevolani and Roberto Festa first consider the four possible cases of truth approximation, deriving from combining deterministic or probabilistic theories with a deterministic or probabilistic truth. They then argue that their "basic feature" approach to measuring truthlikeness (Cevolani and Festa 2020) can be extended to cover all the four cases, thus providing a unified account of both deterministic and probabilistic truth approximation. They also show how their approach compares to other accounts in the truthlikeness literature and beyond; in particular, it turns out that, when applied to assessing the distance between probability distributions, their measure is essentially identical to the Brier score (another well-known measure also discussed in several other papers here), but incompatible with the Kullback–Leibler divergence.

In the fourth paper, "Probabilistic Truthlikeness, Content Elements, and Metainductive Probability Optimization", Gerhard Schurz starts from a different classification of possible approaches to deterministic and probabilistic truth approximation. He distinguishes between comparative vs. numeric measures of truthlikeness, between

qualitative vs. quantitative theories, and between deterministic vs. probabilistic truth as the target of approximation. He argues that such distinctions lead to four major cases of truth approximation, to which he applies the account of truthlikeness based on the notion of "content element", originally proposed in Schurz and Weingartner (1987) and further refined in his more recent work. He also shows how to treat the truthlikeness of statements of single case probability by applying the mathematical methods of formal learning theory as developed in his recent work on optimal meta-induction (Schurz 2019).

The fifth paper, by Theo Kuipers, studies both deterministic and probabilistic legisimilitude from the perspective of the theory of inductive probabilities developed in the Carnap-Hintikka tradition. In "Approaching Probabilistic and Deterministic Nomic Truths in an Inductive Probabilistic Way", Kuipers surveys both Carnap's and Hintikka's systems of inductive probability and applies them to modeling truth approximation as convergence to the true probabilistic distribution in a multinomial context (the typical example being random sampling with replacement in an urn with colored balls). He also shows how this approach generalizes and unifies his theory of nomic truth approximation (Kuipers 2000, 2019), here extended to probabilistic approximation to both a deterministic and probabilistic nomic truth. In particular, Kuipers formulates new, probabilistic versions of his original "success theorem", stating (roughly) that higher truthlikeness leads to higher empirical success (in the deterministic case) or higher estimated truth approximation (in the probabilistic case).

The issue of convergence to the true distribution is also the focus of the sixth paper of the collection, "Tracking Probabilistic Truths: A Logic for Statistical Learning", by Alexandru Baltag, Soroush Rafiee Rad, and Sonja Smets. Here the authors study how rational agents can form and revise their beliefs in front of incoming information (e.g., the results of repeated sampling from an urn) and eventually approach the probabilistic truth about the domain (e.g., the unknown statistical distribution of colored balls within the urn). Using techniques from belief revision theory, plausibility logic, and statistical learning theory, they model the agents' beliefs as probability distributions on which a (second-order) plausibility ranking is defined. This setup leads to two kinds of belief revision, one changing the first-order distributions only, the other shrinking the set of possible distributions itself. The authors prove a number of convergence results for learning based on such revision operations, with connections to theories of verisimilitude and classical convergence results in Bayesian reasoning.

The seventh paper is "Propositional and Credal Accuracy in an Indeterministic World" by Graham Oddie. The author frames the issue of truth approximation within a general theory of questions: how accurate is a possible answer to a relevant question? i.e., how close is it to its complete and correct answer? Two kinds of accuracy are distinguished, propositional and credal, depending on whether answers are construed as propositions or as epistemic probability distributions. Oddie further distinguishes between precise and imprecise answers and between determinate and indeterminate questions, depending on whether the answer (respectively, the question's target) is construed as a singleton possibility or instead as a set of possibilities (e.g., an interval of numerical values, or a nomic truth in Kuipers' sense). His approach thus covers both deterministic and probabilistic truth-approximation; moreover, it sheds light on the relations between the research program on truthlikeness (usually based on propo-

sitional accuracy) and that on epistemic utility theory or so-called accuracy first epistemology (based on credal accuracy, see, e.g., Pettigrew 2016).

The tricky relation between accuracy and truthlikeness (i.e., between credal and propositional accuracy in Oddie's sense) has been the object of some very recent work (Oddie 2019, Schoenfield 2020, McCutcheon 2021) and remains an open problem. It is also the focus of Igor Douven in the eighth paper of the collection, "Scoring, Truthlikeness and Value". Here, Douven departs from standard literature on so-called scoring rules (i.e., measures of the closeness between probabilistic—e.g., meteorological—forecasts) by assessing competing measures by taking into account verisimilitude. In a nutshell, if the relevant hypotheses are ordered with respect to their closeness to the true one, an adequate scoring rule should favor forecasts that assign higher probabilities to hypotheses which are close to the truth, all else being equal. Douven argues for this thesis both on theoretical grounds and with a "quasi-empirical" approach employing computer simulations in assessing the performance of different scoring rules, following the idea (by Murphy 1993) that forecasts should be evaluated also as guides to action, i.e., in terms of the quality of the decisions taken on their basis.

In the ninth paper, "Probabilistic Truth Approximation and Fixed Points", David Atkinson and Jeanne Peijnenburg tackle our central issue by discussing mathematical methods employed in genetics to compute the probabilities of some traits in biological populations of interest. In this context, the method of fixed points allows one to approximate with increasing accuracy the probability that, say, an individual has some trait knowing that his ancestors had the trait with some given estimated probability. The authors argue that this is a typical case of probabilistic truth approximation when the relevant events form a (inheritance) chain in which each event is made more probable by its immediate predecessor. They also show how this scenario differs, despite a superficial similarity, from another kind of probabilistic truth approximation—corresponding to the Bayesian case of the "washing out of the priors"—which is instead relevant for sequences of independent events (similar to those studied in Carnapian inductive logic, e.g., in Kuipers' paper).

Finally, in the tenth paper of the Collection, Leander Vignero and Sylvia Wenmackers revive the Popperian account to the analysis of scientific theories by the three central concepts mentioned in their title: "Degree of Riskiness, Falsifiability, and Truthlikeness: A Neo-Popperian Account Applicable to Probabilistic Theories". In the first part of the paper, relying on a classical forecasting example, they reason about the notions of riskiness (or boldness) and of falsifiability of competing scientific hypotheses making more or less vague predictions about some magnitude of interest. In the second part, they link such discussion to the issue of verisimilitude of probabilistic theories. They start by discussing and criticizing the definitions of truthlikeness given in Cevolani and Schurz (2017), which are limited to deterministic theories, and proceed in developing new definitions of both the truthlikeness and approximate truth of probabilistic hypotheses; they also analyze the relations between such definitions and the notions of riskiness, informativeness, and falsifiability discussed in the first part.

Before concluding, let us take stock on the progress made and on the prospects for future research on the topic of probabilistic truth approximation.

First, we believe that the ten papers in the Collection offer an informative and quite comprehensive overview of the kinds of problems arising in this field and of the different approaches one can take to tackle them. The first four papers (by Niiniluoto, García-Lapeña, Cevolani and Festa, and Schurz) are firmly in the tradition of post-Popperian theories of truthlikeness and aim at extending existing accounts of truthlikeness to deal with probabilistic theories and approximation to a probabilistic truth. The two papers by Kuipers and Baltag, Rad, and Smets also deal with the latter problem but focusing on probabilistic convergence and learning as based on incoming empirical evidence. The next two papers by Oddie and Douven discuss the differences and relations between two notions of truth approximation, respectively based on truthlikeness and on accuracy. Finally, the papers by Atkinson and Peijnenburg, and by Vignero and Wenmackers, connect the issue of truth approximation with related approaches and problems, including fixed points methods, Bayesian updating, and Popper's ideas on risky prediction and falsifiability.

In turn, this multiplicity of approaches is reflected in the quite wide array of analytical methods employed in the ten papers. Indeed, these significantly enrich the traditional toolbox of philosophers of science and formal epistemologists, essentially based on deductive and inductive logic and probability and decision theory. Many contributions apply methods taken from relevant scientific disciplines, including the mathematics of distance and entropy measures, formal and statistical learning theory, theories of belief revision, population genetics, and other ones. We see this richness as instrumental both in bringing forward increasingly powerful approaches to old and new methodological and philosophical problems, and in connecting different fields by highlighting the pervasiveness of some central issues and comparing the different solutions proposed to tackle them.

In this connection, some recurring themes in the ten papers of the collection suggests themselves as central problems that invite for future research. We briefly mention them in turn. (i) One is the issue of whether, and to what extent, the analysis of probabilistic truth approximation can be "reduced" to theories of deterministic truthlikeness. Many contributors more or less explicitly touch on this problem, arguing for more or less positive answers; however, more work seems to be needed to fully clarify the relations between deterministic and probabilistic truth approximation. (ii) The answer to the above question may depend, at least partly, on the discussion of another important distinction, that between the logical and the epistemic problem of truthlikeness. While not all the authors deal with both problems, their results together suggest that the epistemic problem—assessing (deterministic or probabilistic) truthlikeness on the basis of available evidence—is both crucial and may require new or separate discussion when the focus is on a probabilistic context. (iii) Another interesting, more technical, problem has to do with the different measures (or relations, in the comparative case) one can employ to rigorously quantify (estimated) closeness to the truth and related notions like accuracy, approximate truth, and convergence to the truth. Since there is a plethora of such measures, and some of them are not even ordinally equivalent to each other (e.g. quadratic and entropy-based measures), this raises the problem of so-called measure sensitivity: the solutions to central methodological problems can depend in a crucial way on the specific measure adopted for its analysis (Fitelson 1999, Brössel 2013, Cevolani 2017). For this reason, finding a

common ground to discuss the limits and merits of competing measures is likely an important goal of future research. (iv). Finally, and on a more general level, a more systematic discussion seems needed of the different conceptions of "truth approximation" employed in the literature. These include at least truthlikeness as (estimated) closeness to the whole truth (either deterministic or not), truth tracking as probabilistic convergence to the true state of affairs, accuracy as closeness of credal states to the probabilistic or deterministic truth, and forms of learning in different scenarios, from Bayesian updating to belief revision, merging and opinion dynamics (cf., e.g., Cevolani 2014).

Future research will hopefully address these, and many other specific issues; we hope that the papers in this Topical Collection are instrumental in setting the stage for further developments, and in making a step forward on the route of approaching truth.

References

Brössel, P. (2013). The Problem of Measure Sensitivity Redux. *Philosophy of Science*, 80(3), 378–397

Cevolani, G. (2014). Truth Approximation, Belief Merging, and Peer Disagreement. *Synthese*, 191(11), 2383–2401

Cevolani, G. (2017). "Truthlikeness and the Problem of Measure Sensitivity.". *EPSA15 Selected Papers* (pp. 257–271). Cham: Springer

Cevolani, G., & Festa, R. (2020). A Partial Consequence Account of Truthlikeness. *Synthese*, 197, 1627–1646

Cevolani, G., & Schurz, G. (2017). Probability, Approximate Truth, and Truthlikeness: More Ways out of the Preface Paradox. *Australasian Journal of Philosophy*, 95(2), 209–225

Cohen, L. J. (1980). What has Science to do with Truth? *Synthese*, 45, 489–510

Fitelson, B. (1999). "The Plurality of Bayesian Measures of Confirmation and the Problem of Measure Sensitivity." *Philosophy of Science*, S362–78

García-Lapeña, A. (2021). "Truthlikeness for Quantitative Deterministic Laws". *The British Journal for the Philosophy of Science*. Forthcoming

Kuipers, T. (Ed.). (1987). *What is Closer-to-the-truth?* Amsterdam: Rodopi

Kuipers, T. (2000). *From Instrumentalism to Constructive Realism*. Dordrecht: Kluwer A.P

Kuipers, T. (2019). *Nomic Truth Approximation Revisited*. Cham: Springer

McCutcheon, R. G. (2021). "A Note on Verisimilitude and Accuracy". *British Journal for the Philosophy of Science*. Forthcoming

Miller, D. (1974). Popper's Qualitative Theory of Verisimilitude. *The British Journal for the Philosophy of Science*, 25(2), 166–177

Murphy, A. H. (1993). What is a Good Forecast? An Essay on the Nature of Goodness in Weather Forecasting. *Weather Forecasting*, 8, 281–293

Niiniluoto, I. (1987). *Truthlikeness*. Dordrecht: D. Reidel

Niiniluoto, I. (1998). Verisimilitude: The Third Period. *The British Journal for the Philosophy of Science*, 49, 1–29

Niiniluoto, I. (1999). *Critical Scientific Realism*. Oxford: Oxford University Press

Niiniluoto, I. (2018). *Truth-Seeking by Abduction*. New York, NY: Springer Berlin Heidelberg

Oddie, G. (1986). *Likeness to Truth*. Dordrecht: Reidel

Oddie, G. (2016). "Truthlikeness", *The Stanford Encyclopedia of Philosophy* (Winter 2016 Edition), Edward N. Zalta (ed.), https://plato.stanford.edu/archives/win2016/entries/truthlikeness/

Oddie, G. (2019). What Accuracy could not be. *British Journal for the Philosophy of Science*, 70, 551–580

Pettigrew, R. (2016). *Accuracy and the Laws of Credence*. Oxford: Oxford University Press

Popper, K. (1963). *Conjectures and Refutations*. London: Routledge and Kegan Paul

Rosenkrantz, R. (1980). Measuring Truthlikeness. *Synthese*, 45, 463–488

Schoenfield, M. (2020). "Accuracy and Verisimilitude: The Good, the Bad and the Ugly". *British Journal for the Philosophy of Science*. Forthcoming
Schurz, G., & Weingartner, P. (1987). "Verisimilitude Defined by Relevant Consequence-Elements". In T. A. Kuipers (Ed.), *What is closer-to-the-truth?* (pp. 47–78). Amsterdam: Rodopi
Schurz, G. (2019). *Hume's Problem Solved: The Optimality of Meta-Induction*. Cambridge: MIT Press
Tichý, P. (1976). Verisimilitude Redefined. *The British Journal for the Philosophy of Science*, 27, 25–42
Zwart, S. D. (2001). *Refined Verisimilitude*. Dordrecht: Springer Netherlands