2021 European Control Conference (ECC)
June 29 - July 2, 2021. Rotterdam, Netherlands

Demand-Side Management in a Micro-Grid with Multiple Retailers: A Coalitional Game Approach

Fernando Genis Mendoza*, Pablo R. Baldivieso-Monasterios, Dario Bauso and George Konstantopoulos

Abstract— **This paper deals with the design and analysis of a novel on-line pricing mechanism based on coalitional game theory. The proposed architecture consists of a micro-grid (MG) where the power demand can be fulfilled by multiple competing energy retailers trying to attract consumers by announcing a price in a hierarchical leader-follower structure. The existence of a Stackelberg equilibrium in such game is shown, leading to a guaranteed consumption value given a price. The coalition formation is then extended to a minimum spanning tree game that affects the rational decision of the players involved. The stability analysis for the resulting coalitions is performed and the steps in the game are presented. Simulations provide a comparison of the profits generated by the proposed scheme against a more traditional single retailer scheme, while simultaneously showing convergence towards steady-state equilibrium.**

I. Introduction

Dynamic pricing schemes in electrical systems constitute a viable way to optimize and shift consumption during peak times without compromising generation and distribution systems [1]. Such schemes can serve as a tool for the retailers to charge more for their services when demand is high, and oppositely, a way to let the consumer know when it is more convenient to utilize such services [2]. An underlying assumption is that both suppliers and consumers are rational and desire to maximize their profits, and in consequence, a price change entails a change in consumption [1], [3]. The advent of the smart grid paradigm has brought changes to the electricity market; where governments and general consumers now seek and switch to better providers and that fulfill custom requirements while being profitable. This has also brought a higher degree of communication between consumers and energy retailers, bringing schemes where these interact and cooperate to optimize their outputs. These aspects of switch-ability and cooperation are the main focus of this study, since the literature about coalitional games applied to smart-grids does not cover scenarios where there are multiple electricity retailers for the end-users to choose.

*Sponsored by Mexico's CONACyT, scholarship number: 440742.
This work is supported by EPSRC under Grants No EP/S001107/1 and EP/S031863/1.

F. Genis Mendoza, P. R. Baldivieso-Monasterios and G. Konstantopoulos are with the Department of Automatic Control and Systems Engineering, The University of Sheffield, Mappin Street, S1 3JD Sheffield, United Kingdom. G Konstantopoulos is now with the Department of Electrical and Computer Engineering, University of Patras, Rion, 26500, Greece. {fgenismendoza1, p.baldivieso, g.konstantopoulos}@sheffield.ac.uk
D. Bauso is with the Jan C. Willems Center for Systems and Control, ENTEG, Fac. Science and Engineering, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands and Dipartimento di Ingegneria, Università di Palermo, viale delle Scienze, Palermo, Italy d.bauso@rug.nl

A. Previous Works and State of the Art

This study builds and extends on the pricing scheme introduced in [3], where consumers and suppliers of electricity maximize their profit in accordance to a Stackelberg-based game with incentive strategies. The concepts involved with these types of games and the Stackelberg equilibrium can be found in [3]–[5] and references therein. The fundamentals about coalitional game theory are studied in detail in [6]. A review of the use of coalitional games along with applications in power networks is carried out in [2], where the types of cooperation applicable to electrical systems are explained, together with game theoretic modeling for agents involved. The formulation of cooperative coalitional games in MGs was first introduced in [7], where the proposed algorithm focuses on reducing power losses and costs by coalitioning neighboring MGs. Similarly, in [8] the coalitions are formed between MGs in a macro station, where their profits are distributed using the Shapley value. A similar approach is used in [9], where a case study is performed. A centralized algorithm where the MGs trade with the macro station is studied in [10]. A study of the case where greedy prosumers do not align with the MG's decision is presented in [5]. The problem formulated in [11] minimizes discomfort, which is modeled as a non-linear function of the power deficit in the MG. In [12] the coalitions are formed between macro stations. A game where local micro-grids cooperate without the participation of a main grid is presented in [13]. In [14] the introduction of auction theory is used to define the pairing of micro-grids. The game proposed in [15] divides the players between consumers and the micro-grid, where the latter makes its payoff function public. The same author also proposes the use of evolutionary game theory in conjunction with coalitions [16]. A bidding system for cooperating prosumers is presented in [17], however constraints on power capacity and losses are ignored.

From the above, it can be seen that the use of coalitional games in the subject of micro-grids is a very recent topic, however, to the best of the authors' knowledge, most of the coalitional games proposed in the literature do not address the end-users as players, and none of them present a scheme in where these can choose from multiple retailers.

We believe that the latter scenario fits the use of coalitional games and has to be studied, since this approach matches the current needs of energy trading platforms, such as [18], [19].

B. Problem Statement and Contributions

We are proposing a dynamic pricing scheme where there are multiple providers of electricity or *retailers*. They are competing to attract the largest number of *consumers* inside a community, in our case represented by a micro-grid.

A finite number of coalitions equal to the number of competing providers can be formed. When consumers choose to be provided by a particular retailer, they join the provider's coalition. It is understood that coalitions between providers cannot be formed since they are quarrels and therefore competing against each other. Another underlying assumption is that both consumers and providers are price-taking rational agents that look forward to increase their profits. Both evaluate the price of energy in their respective profit functions. The ways in which a retailer gains a consumer is by announcing his price for electricity accompanied by an incentive. Every determined period of time, the retailers adjust their prices, which in consequence will cause them to lose or gain consumers, and at the same time, adjust the demand and supply of energy in the MG.

The main contributions of this paper can be summarized as follows:

- We propose a scheme in which there are multiple competing retailers in a MG; represented in a coalitional game framework. To the best of the author's knowledge, this is a novel problem setup in MG literature.
- We perform a stability analysis covering the coalitions formed by our proposed game. We also demonstrate the existence of the equilibrium points in the game, namely the guaranteed existence of a consumption value given a price.
- We illustrate numerically the ways in which the profits of the consumers are improved by comparing it to an scenario where there is a fixed retailer.

This paper is organized as follows, in Section II the models and concepts employed in our study are presented. In Section III the coalitional approach, the game, and a stability analysis for the coalitions and equilibrium points are presented. In Section IV an example comparison between single retailer and multiple retailer scenarios is performed. Section V streamlines conclusions and future directions of this work.

II. System Model Definition and Preliminaries

In this section we introduce the ways in which both retailers and consumers are modeled, we also review concepts from Stackelberg games and graph theory that will be useful for the rest of this study.

A. Sets and Coalitions Definition

To study the coalitional behaviour of our scheme, we recur to a game-theoretic framework. Let us define the universe of players in the MG as $\mathcal{N}$ which contains N players, this is partitioned into two non-overlapping sets: the set of retailers $\mathcal{R} \subset \mathcal{N}$ and the set of consumers $\mathcal{B} \subset \mathcal{N}$; where $\mathcal{R} \cup \mathcal{B} = \mathcal{N}$. For simplicity we have defined that $\mathcal{R} \cap \mathcal{B} = \emptyset$, namely a consumer cannot be a retailer and vice versa. Such sets are composed as $\mathcal{R} := \{r_1, \dots, r_p\}$ and $\mathcal{B} := \{b_1, \dots, b_l\}$, where $p + l = N$. Besides this partition, we seek a pairing between a retailer and a subset of consumers.

Definition 1 (Retailer's coalition): A *coalition* $S_i \subset \mathcal{N}$ is given by assigning k consumers to a single retailer $r_i \in \mathcal{R}$,

$$S_i := \{r_i, b_1, \dots, b_k\}, \forall i \in \mathcal{R}. \tag{1}$$

For the case where a retailer r_i does not succeed to attract any consumer, its coalition is reduced to a singleton, $S_i = \{r_i\}$. In our problem, due to the nature of our market setup, there are underlying assumptions that need to be addressed.

Assumption 1: Each consumer $b_j \in \mathcal{B}$ has to be assigned to one retailer at all times. As a consequence, the union of all coalitions

$$\bigcup_i S_i = \mathcal{N}, \ \forall i \in \mathcal{R}. \tag{2}$$

However, each consumer can decide to have zero consumption, this will be the case according to its profit function, as it will be explained in Section II-B. Coalitions that have more than one retailer or that share consumers are considered infeasible, this is formalized as follows.

Assumption 2: The coalitions comply with the conditions:

1) Two or more retailers cannot be allocated to the same coalition and are considered quarrels:

$$S_i \cap \mathcal{R} \setminus r_i = \emptyset, \ \forall S_i \in \mathcal{N}. \tag{3}$$

2) Coalitions are non-overlapping, a consumer cannot be assigned to more than one coalition:

$$S_i \cap S_j = \emptyset, \ \forall S_i, S_j \in \mathcal{N}. \tag{4}$$

With the above, all coalitions are guaranteed to be feasible.

B. Consumer and Retailer Profit Functions

In our problem setup, both consumers and retailers have the objective of maximizing their profits; which is captured by their profit functions $\Pi(\cdot)$. Such functions represent the remaining amount of money after producing/consuming electricity and after covering the underlying production/consumption fees. For the retailer r_i this is

$$\Pi_i = \Lambda_i x - C(x), \ i \in \mathcal{R} \tag{5}$$

where $C(\cdot)$ is a function which corresponds to the cost of producing x quantity of electricity and Λ_i is the price announced by the retailer which will be applied to its coalition. Similarly, every consumer b_j that has decided to consume from r_i calculates its profit with

$$\Pi_j = U(x) - \Lambda_i x, \ j \in \mathcal{B}, i \in \mathcal{R} \tag{6}$$

where $U(\cdot)$ is the monetary equivalent to the utility from consuming x quantity of electricity. We assume that such utility and costs functions are monotonically increasing, while being concave and convex respectively [1]. The rationale of both players is represented by two maximization problems, the output of such is the price Λ_i for the retailer and a quantity of power consumption $P^d_{b_j}$ for the consumer; these are obtained as

$$\Lambda_i = \underset{\lambda \in [\underline{\lambda}, \bar{\lambda}]}{\arg\max} \quad \lambda \cdot \Big(\sum_{b_j \in S_i} P^d_{b_j} - P^{loss}_i \Big) - C(P^g_i), \tag{7}$$

$$P^d_{b_j} = \underset{\zeta \in [\underline{\zeta}, \bar{\zeta}]}{\arg\max} \quad U(\zeta) - \Lambda_i \zeta, \tag{8}$$

where λ and ζ are the maximization argument variables for the retailer's price and power used by each consumer respectively, P_i^g is the power generated by the retailer and P_i^{loss} equals to the power losses incurred by the individual retailer in the MG transmission lines. In essence, the retailers try to maximize their profits by announcing a price which in turn is formulated as a function from the expected demand, and the consumers consume as much as possible with the given price to also maximize their profits. Such tension between leader (retailer) and followers (consumers) is captured by the maximization problems (7)-(8). When both leader and followers select their optimal outputs, it is said that the game is at an equilibrium.

The consumer is expected to evaluate all the prices Λ_i announced by all retailers r_i in the MG. This will determine its decision on picking a retailer, namely what coalition to join. By adding consumers to its coalition, the retailer also has to take into account the constraint on its own power capabilities:

$$|\sum_{b_j \in S_i} P_{b_j}^d + P_i^{loss}| \leq |P_i^g|. \tag{9}$$

C. Network Systems Review

Each provider knows the cost of connection for every consumer in the MG; these can be delineated by a graph where the weight of each edge represents a cost. These non-physical cost edges vary from consumer to consumer depending on various factors like the physical position relative to each supplier, plausible power losses or fees from regulatory agencies. The topology of each cost network is represented by a connected, undirected and weighted graph $\mathcal{G}_i(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of nodes (vertices); in it we find a retailer and a number of consumers $\mathcal{V} = \{r_i, b_j, \ldots, b_k\}$. The set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of unordered pairs $\{i, j\}$ where the value (weight) of each one represents a cost. The out-degree δ_k of a node refers to the number of edges that connect to a certain node k. The *minimum spanning tree* (MST) of a graph refers to the subset of edges that connect all the nodes in the network, with the minimum total edge weight. In order to get from a supplier to a consumer, a certain *path* can be followed. This path is described by the succession of edges in the network that can be used to reach a consumer. The absence or presence of certain consumers in the provider's network can affect the path and result on higher or lower costs by adding the edge weights.

III. Coalitional Game with Multiple Retailers

In this section we formally present the rules of the game and the ways in which the coalitions are formed. We also perform a stability analysis in the coalitional games sense.

A coalitional game is defined by the tuple $\langle \mathcal{N}, v \rangle$, where $v : 2^{\mathcal{N}} \to \mathbb{R}$ is a function that assigns a value to every coalition $S_i \subset \mathcal{N}$. In our case this is the savings that the consumers in a coalition achieve by choosing the same retailer. To define the value function $v(S_i)$ for any coalition $S_i \subset \mathcal{N}$ the costs inferred by each consumer relative to all available retailers has to be defined.

A. Cost Definition and MST Problem

Given a coalition S_i, each retailer r_i has knowledge on how much it costs to provide electricity to a consumer b_j in the retailer's coalition, i.e. a connection fee to be paid by the consumer. We refer to it as *direct connection* cost and can be denoted as $c(\{(r_i, b_j)\})$ which is represented by the weight of edge (r_i, b_j) in a network. Such cost $c(\{(r_i, b_j)\})$ is equal to the cost for having $b_j \in \mathcal{B}$ as the only client of $r_i \in \mathcal{R}$, namely, for $S_i = \{r_i, b_j\}$, $c(S_i) = c(\{r_i, b_j\})$. The retailer also allocates different *aggregate connection* costs that are enabled depending on the consumers in the coalition, namely the cost for connecting consumer b_j if b_i is already in the coalition; denoted by $c(\{(b_i, b_j)\})$ and represented in a network as edges connecting consumers. The existence of the latter is subject to various exogenous factors such as geographical location, high costs, etc. The paths and edges are defined in a way that yields higher savings for the consumers when more consumers join the coalition. However, a consumer has to be able to join whatever retailer it wants in accordance to its individual objectives. In the present manuscript we will refer to this kind of network as *retailer's cost network*. We are now ready to present the definition below.

Definition 2 (Cost of a coalition): Given a coalition S_i with an associated graph $\mathcal{G}_i$, the value of its cost function $c(S_i)$ is given by the MST of $\mathcal{G}_i$.

Conceptually, the MST contains all the nodes and the minimum possible total edge weight, that is, the smallest possible sum of all weights. With the cost of a coalition, its value function can be obtained by turning the consumer's cost game into a *costs-saving* game. The value function of any S_i is expressed as:

$$v(S_i) = \begin{cases} \sum_{j \in S_i} c\left(\{(r_i, b_j)\}\right) - c\left(S_i\right) & \text{if } S_i \text{ is feasible} \\ 0 & \text{if } r_{i_1}, r_{i_2} \in S_i \\ 0 & \text{if } r_k \notin S_i \ \forall k \in \mathcal{R} \end{cases}, \tag{10}$$

where the value equals the sum of savings of all individual consumers. Having defined the ways in which the savings are obtained through $v(\cdot)$, the following straightforward assumption is formulated.

Assumption 3: Given a coalition S_i of a retailer r_i and two or more consumers b_j, the following condition holds:

$$v(\{r_i, b_j\}) \leq v(S_i), \ \forall j \in S_i. \tag{11}$$

The purpose of the costs-saving game is to use it as a tool to incentivize consumers for joining a retailer's coalition, by aiding to increase the consumers' profits with such savings.

B. Savings Imputation via Shapley Value

Given a coalition S_i, the savings produced by the consumers of retailer r_i have to be distributed fairly. We recur to the Shapley value to do so since it is a well known and standard solution [4]. A few concepts have to be introduced, such as the marginal value which determines how valuable a player can be when joining a coalition. Assuming that the consumers enter in a certain sequence σ to an already defined S_i (e.g. $\sigma = \{r_i, b_6, b_8, ..., b_k\}$), where the ordering number of a buyer b_j is given by $\sigma^{-1}(b_j)$ (in the example $\sigma^{-1}(b_8) = 3$), the set of predecessors of consumer b_j is defined as

$$\rho^{\sigma}_{b_j} := \{b_l \in S_i \mid \sigma^{-1}(b_l) < \sigma^{-1}(b_j)\}. \quad (12)$$

From this, the marginal value for b_j given an arbitrary sequence σ can be defined as

$$m^{\sigma}_{b_j}(v) = v(\rho^{\sigma}_{b_j} \cup \{b_j\}) - v(\rho^{\sigma}_{b_j}), \quad (13)$$

the marginal value for each sequence can be stored in vector form as

$$m^{\sigma}(v) = \{m^{\sigma}_{b_j}(v), b_j \in S_i\}. \quad (14)$$

Finally, the Shapley value is then calculated as the average of the marginal vector over all permutations of sequences, namely

$$\Phi(v) = \frac{1}{k!}\sum_{\sigma} m^{\sigma}(v), \quad (15)$$

where k is the total number of consumers in the coalition. The resulting vector outputs the corresponding portion of savings imputed to each consumer $b_j \in S_i$.

C. Price, Consumption and Coalition Formulations

The coalitional game is introduced to the cost functions by including directly into the cost of the retailer the term $v(S_i)$, which is equal to the consumer savings that will be rewarded back via imputation. The consumers also include the individual base payment $-c(\{r_i, b_j\})$ since it has to be paid regardless of the prospective savings. The retailers also announce a *potential subsidy* $\kappa_{r_i}\delta^{r_i}_{b_j}$ to the consumer. Where $\delta^{r_i}_{b_j}$ is the degree of the node represented by b_j in the cost network relative to r_i. The scalar κ_{r_i} is a positive constant to adjust the potential subsidy proportionally to the number of connections in the cost network. The subsidy term is an equivalent to the Bahnzaf power index employed in cooperative games which dictates how pivotal is a players' presence in a coalition [20]. The potential subsidy is announced by the retailer to the consumer as means to incentivize the latter to join the retailer's coalition, since in MST games the players with more connections in a network potentially hold more value [4]. We have defined such functions as

$$C(P^g_i, S_i) = \alpha_{r_i}\cdot(\Lambda_i P^g_i)^2 + v(S_i), \ \forall i \in \mathcal{R} \quad (16)$$

$$U(P^d_{b_j}) = \alpha_{b_j}\cdot(P^d_{b_j})^{\frac{1}{6}} + \kappa_{r_i}\delta^{r_i}_{b_j} - c(\{r_i, b_j\}), \quad \forall r_i \in \mathcal{R}, b_j \in \mathcal{B} \quad (17)$$

where, due to the respective quadratic and radical terms, it is clear that the functions are monotonically increasing and convex and concave respectively. The constant α_{r_i} is a scalar associated to the costs of operating/generating the power corresponding to retailer r_i. Analogously, α_{b_j} represents each consumer's interest in consuming power. The inclusion of (16)-(17) to the profit functions (7)-(8), yields the maximization problems:

$$\Lambda_i = \underset{\lambda\in[\underline{\lambda},\bar{\lambda}]}{\arg\max}\ \lambda\cdot\Big(\sum_{b_j\in S_i} P^d_{b_j} - P^{loss}_i\Big) + \sum_{j\in S_i} c(\{(r_i, b_j)\}) - \alpha_{r_i}\cdot(\lambda P^g_i)^2 - v(S_i), \quad (18)$$

$$P^d_{b_j} = \underset{\zeta\in[\underline{\zeta},\bar{\zeta}]}{\arg\max}\ \alpha_{b_j}\cdot(\zeta)^{\frac{1}{6}} + \kappa_{r_i}\delta^{r_i}_{b_j} - c(\{r_i, b_j\}) - \Lambda_i\zeta, \quad (19)$$

for each $r_i \in \mathcal{R}$ and $b_j \in \mathcal{B}$ respectively.

The process of consumer b_j selecting a coalition S_i is done by evaluating all prices $\Lambda_i \ \forall i \in \mathcal{R}$, base payments $c(\{r_i, b_j\}) \ \forall i \in \mathcal{R}$ and the value of the potential subsidies $\kappa_{r_i}\delta^{r_i}_{b_j}$ announced by all the retailers into its profit function as in (19) and taking the one yielding the largest profit:

$$S_i \leftarrow S_i \cup \{b_j\} \iff i = \arg\max_i \big\{\Pi_{b_j}\big(\Lambda_i, \kappa_i\delta^{r_i}_{b_j}, c(\{r_i, b_j\})\big), \ i\in\mathcal{R}\big\}, \quad (20)$$

the consumption is then optimized in (19) by selecting its own quantity of power $P^d_{b_j}$. In the case where the constraint (9) does not hold for S_i, retailer r_i will have to reject the consumers that generate the least profit in its coalition until the constraint is fulfilled, leaving the rejected consumers to re-evaluate (20) without the former selected retailer. The actual savings value (subsidy) per consumer in the coalition is obtained and added back by calculating the Shapley value $\Phi(v)$ as explained in Section III-B.

D. Stability of the Coalitional Game

From the game proposed in Section III-A and Assumption 2, we conclude that the grand coalition value $v(\mathcal{N}) = 0$ since by definition, it would include uncooperative retailers. This, in consequence, renders conventional cooperative game stability analysis [4], [17] unusable. We recur to the notion of stable partitions first introduced in [21], more specifically $\mathbb{D}_{hp}$-stability, where the *defection function* $\mathbb{D}(S_i)$ is defined in a way that it outputs collections of players that can leave S_i to form *homogeneous partitions*. A coalition is $\mathbb{D}$-stable when no group of players is interested in leaving it.

Definition 3 ($\mathbb{D}_{hp}$ stability): A coalition $S_i = \{r_i, b_j, \ldots, b_k\}$ is $\mathbb{D}_{hp}$-stable if the following is satisfied:

1) given a collection $\{P_{i_1}, \ldots, P_{i_L}\}$ resulting from an arbitrary partition of S_i, such that $\cup^L_{j=1} P_{i_j} = S_i$:

$$v(S_i) \geq \sum_{j=1}^{L} v(P_{i_j}), \ \forall i \in \mathcal{R}, \quad (21)$$

2) given the coalitions S_i in the subset $\mathcal{T} \subseteq \{1, \ldots, K\}$, where $i \in \mathcal{T}$ and $K \leq p$:

$$\sum_{i\in\mathcal{T}} v(S_i) \geq v\Big(\bigcup_{i\in\mathcal{T}} S_i\Big), \quad (22)$$

Theorem 1: Given the coalitional game with multiple energy retailers $\langle\mathcal{N}, v\rangle$. The coalitions S_i formed by such are $\mathbb{D}_{hp}$ stable.

Proof: For a fixed $i \in \mathcal{R}$ and associated coalition $S_i \subset \mathcal{N}$, consider a partition $\{P_{i_1}, \ldots, P_{i_L}\}$. It follows that $r_i \in P_{i_j}$ for some $i_j \in \{1, \ldots, L\}$ which implies $v(P_{i_j}) \geq 0$, and $v(P_{i_k}) = 0$ for the rest. The cost associated with P_{i_j} satisfies $c(S_i) \geq c(P_{i_j})$ since the MST of P_{i_j} is contained in the one corresponding to S_i. Condition (21) follows from

$$v(S_i) - v(P_{i_j}) \geq c(S_i) - c(P_{i_j})$$

since $c(\{r_i, b_{i_k}\}) > 0$ for all $i_k \in S_i \setminus P_{i_j}$. For the second condition, it can be inferred from (3)-(4) that $v(\cup_{i\in\mathcal{T}} S_i) = 0$. From the value formulation for a coalition (10), it is true that $v(S_i) \geq 0$. From the above, it is trivial that (22) holds. ■

TABLE I

PARAMETERS FOR RETAILERS AND CONSUMERS.

Retailer	α_{r_i}	κ_{r_i}	$\underline{\lambda}_i$	$\overline{\lambda}_i$	P_i^g
r_1	1e-4 $\$^{\frac{1}{2}}$	65 \$	0.01 \$/W	4 \$/W	30 kW
r_2	7e-5 $\$^{\frac{1}{2}}$	64 \$	0.01 \$/W	2 \$/W	30 kW
r_3	5e-5 $\$^{\frac{1}{2}}$	63 \$	0.01 \$/W	3.5 \$/W	30 kW

Consumer	α_{b_j}	$P_{b_j}^{d_{rated}}$	$\underline{\zeta}_{b_j}$	$\overline{\zeta}_{b_j}$
b_1	1800 W^6	3 kW	0 kW	6 kW
b_2	150 W^6	3.5 kW	0 kW	7 kW
b_3	140 W^6	2.8 kW	0 kW	5.6 kW
b_4	100 W^6	4 kW	0 kW	8 kW
b_5	1600 W^6	1.5 kW	0 kW	3 kW

E. Stackelberg Equilibrium

Having fully formulated the maximization problems involved in our proposed game, we present the following result that guarantees the existence of a Stackelberg equilibrium.

Theorem 2: There exists an equilibrium point $(\Lambda_i^*, P_{b_j}^{d*}) \in \mathbb{R}^2, \forall r_i, b_j \in S_i$ for the Stackelberg game (18)-(19).

Proof: Let r_i and b_j be in the same coalition, where they maximize (18) and (19) respectively. The maximum of both profits is obtained by taking the derivative of both functions and equaling to zero:

$$\frac{\partial \Pi_{r_i}}{\partial \Lambda_i} = P_{b_j}^d - P_i^{loss} - 2\alpha_{r_i} {P_i^g}^2 \Lambda_i = 0, \tag{23}$$

$$\frac{\partial \Pi_{b_j}}{\partial P_{b_j}^d} = \frac{\alpha_{b_j}}{6} (P_{b_j}^d)^{-\frac{5}{6}} - \Lambda_i = 0, \tag{24}$$

following the procedure to obtain the equilibrium point in a two-player Stackelberg game [3], [4]. From (24) an expression for the consumer's demand as a function of price is obtained

$$P_{b_j}^d = \frac{1}{6\sqrt[5]{6}} \left(\frac{\Lambda_i}{\alpha_{b_j}}\right)^{-\frac{6}{5}}, \tag{25}$$

substituting (25) in (23), yields the following expression:

$$\frac{1}{6\sqrt[5]{6}} \frac{\Lambda_i}{\alpha_{b_j}}^{-\frac{6}{5}} - 2\alpha_{r_i} {P_i^g}^2 \Lambda_i - P_i^{loss} = 0, \tag{26}$$

substituting parameter values in (26) and solving for Λ_i, equals a real positive value which corresponds to Λ_i^*. Substituting it into (25) results in $P_{b_j}^{d*}$, thus yielding both equilibrium values. ∎

IV. SIMULATIONS

To demonstrate the ways in which retailers calculate new prices according to demand as well as how consumers react to a change of prices in the MG, we have formulated two scenarios. The first consists of a MG that contains five consumers $\mathcal{B} = \{b_1, b_2, b_3, b_4, b_5\}$ that are supplied with power by only one retailer $\mathcal{R} = \{r_1\}$. The second consists of the same consumers supplied by two additional retailers $\mathcal{R} = \{r_1, r_2, r_3\}$. The parameters for all players are listed in Table I, the cost networks for the different retailers are defined as in Fig. 1. We show the response along ten time periods, where at the end of each, the game is played.

From Fig. 2, it can be seen that the rationality of the players has been captured, namely that the consumers tend

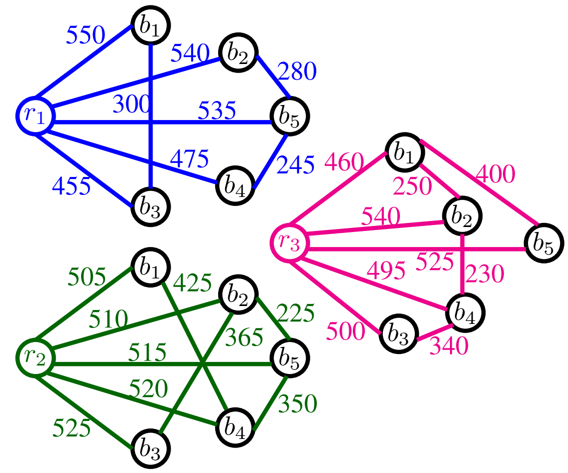

Fig. 1. Cost networks defined by each retailer for the same set of consumers.

to consume more (less) given a lower (higher) price, and that the retailers tend to lower (raise) their price when the consumption is low (high). This is more evident in the single retailer scenario, where is also demonstrated that the game eventually converges to a Stackelberg equilibrium [3], [4]. Also Fig. 2 shows that there are instances where the game yields zero consumption to certain coalitions, leaving retailers without consumers for a period of time due to their decisions. The individual consumptions are captured in Fig. 3, it is clear that all the consumers are able to consume more in the multiple retailer scenario; where the consumption converges above the rated values for all consumers, even with those that do not prioritize consumption as much (lower α_{b_j}). Nonetheless, the advantage of the multiple retailer scheme is evident from Fig. 4; where by comparing the total sum of profits of the consumers in the problem, the one in the multiple retailer case is significantly larger. The decision made by each consumer on what coalition to join in the second scenario is delineated by the plots in Fig. 5, where the consumers do not stay fixated with one retailer in their effort to take the one that gains them the largest profit.

V. CONCLUSIONS AND FUTURE DIRECTIONS

We have proposed an on-line pricing scheme that encompasses the concepts of a hierarchical structure with the Stackelberg game and coalitional games where there are competing players. The definitions and steps for the game have been established and the stability of the game has been demonstrated. A comparison between single and multiple retailer scenarios has been shown numerically, demonstrating the advantages of the latter from an economic point of view. Future directions of this work dwell in the analysis of the computational burdens that our setup entails, the implementation of the present scheme integrated with the physical MG dynamics, together with a stability analysis, the inclusion of disconnection penalties, quality of the service, among others.

REFERENCES

[1] T. Namerikawa, N. Okubo, R. Sato, Y. Okawa, and M. Ono, "Real-Time Pricing Mechanism for Electricity Market with Built-In Incentive

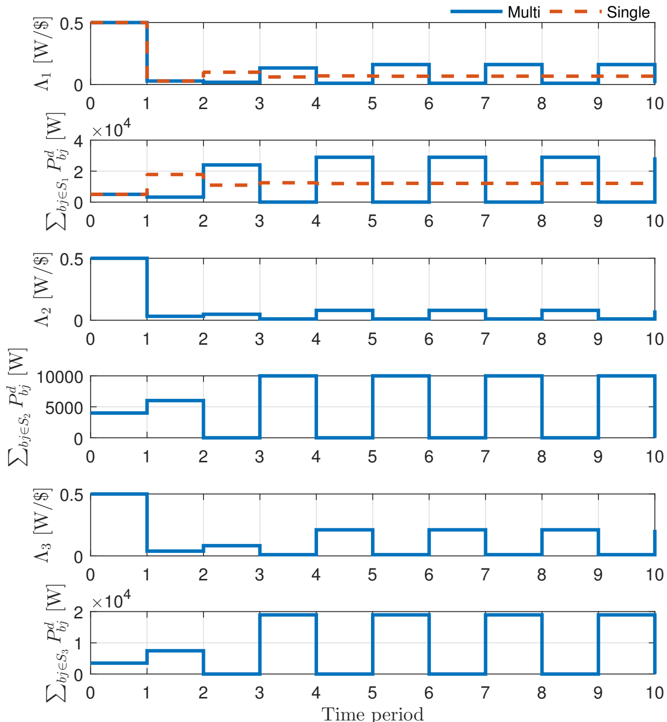

Fig. 2. Price and coalition consumption responses over time.

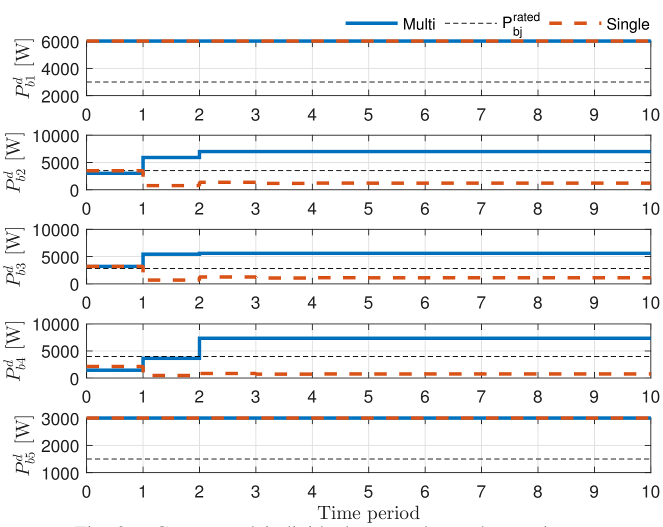

Fig. 3. Consumers' individual power demand over time.

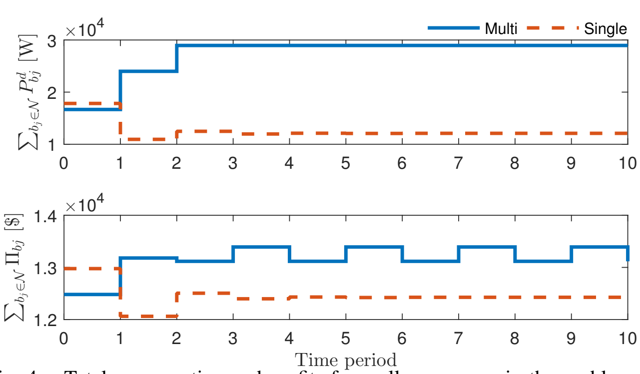

Fig. 4. Total consumption and profits from all consumers in the problem.

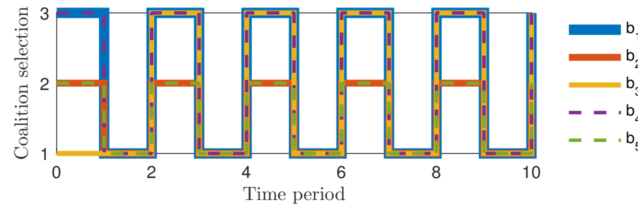

Fig. 5. Consumers' coalition selection over time.

for Participation," *IEEE Trans Smart Grid*, vol. 6, no. 6, pp. 2714–2724, 2015.

[2] A. Loni and F.-A. Parand, "A survey of game theory approach in smart grid with emphasis on cooperative games," in *2017 IEEE Int Conf Smart Grid and Smart Cities*. IEEE, jul 2017, pp. 237–242.

[3] F. Genis Mendoza, D. Bauso, and G. Konstantopoulos, "Online Pricing via Stackelberg and Incentive Games in a Micro-Grid," in *2019 18th European Control Conference*. IEEE, jun 2019, pp. 3520–3525.

[4] D. Bauso, *Game Theory with Engineering Applications*. SIAM's Advances in Design and Control, 2016.

[5] L. Han, T. Morstyn, and M. McCulloch, "Incentivizing Prosumer Coalitions With Energy Management Using Cooperative Game Theory," *IEEE Trans. Power Syst*, vol. 34, no. 1, pp. 303–313, jan 2019.

[6] W. Saad, Z. Han, M. Debbah, A. Hjørungnes, and T. Başar, "Coalitional game theory for communication networks," *IEEE Signal Processing Magazine*, vol. 26, no. 5, pp. 77–97, 2009.

[7] W. Saad, Z. Han, and H. V. Poor, "Coalitional Game Theory for Cooperative Micro-Grid Distribution Networks," in *2011 IEEE Int Conf Commun Workshops*. IEEE, jun 2011.

[8] C. Wei, Z. M. Fadlullah, N. Kato, and A. Takeuchi, "GT-CFS: A Game Theoretic Coalition Formulation Strategy for Reducing Power Loss in Micro Grids," *IEEE Trans. Parallel Distrib Syst*, vol. 25, no. 9, pp. 2307–2317, sep 2014.

[9] R. Pilling, S. Chang, and P. Luh, "Shapley Value-Based Payment Calculation for Energy Exchange between Micro- and Utility Grids," *Games*, vol. 8, no. 4, p. 45, oct 2017.

[10] J. Ni and Q. Ai, "Economic power transaction using coalitional game strategy in micro-grids," *IET Gener. Transm. Distrib.*, vol. 10, no. 1, pp. 10–18, jan 2016.

[11] M. Yasir, M. Purvis, M. Purvis, and B. T. R. Savarimuthu, "Dynamic coalition formation in energy micro-grids," in *Lecture Notes in Computer Science*, 2015. [Online]. Available: http://link.springer.com/10.1007/978-3-319-25524-8_10

[12] Y. Xu, E. Hou, and N. Ansari, "Economic coalition strategies for cost reductions in microgrids distribution networks," in *2014 IEEE Global Commun Conf*. IEEE, dec 2014, pp. 2703–2708.

[13] A. Chis and V. Koivunen, "Coalitional game-based cost optimization of energy portfolio in smart grid communities," *IEEE Trans. Smart Grid*, vol. 10, no. 2, pp. 1960–1970, mar 2019.

[14] J. Mei, C. Chen, J. Wang, and J. L. Kirtley, "Coalitional game theory based local power exchange algorithm for networked microgrids," *Applied Energy*, vol. 239, pp. 133–141, apr 2019.

[15] A. Mondal and S. Misra, "Dynamic coalition formation in a smart grid: A game theoretic approach," in *2013 IEEE Int Conf Commun Workshops*. IEEE, jun 2013, pp. 1067–1071.

[16] S. Misra and A. Mondal, "DCoE: Game-theoretic dynamic coalition extension with micro-grid failure in smart grid," in *11th IEEE Int Conf Adv Netw Telecommun Syst, 2017*. IEEE, 2018.

[17] P. Chakraborty, E. Baeyens, K. Poolla, P. P. Khargonekar, and P. Varaiya, "Sharing Storage in a Smart Grid: A Coalitional Game Approach," *IEEE Trans. Smart Grid*, vol. 10, no. 4, pp. 4379–4390, jul 2019.

[18] L. Stoker, "Good Energy formally launches peer-to-peer renewables trading service Selectricity," 2016. [Online]. Available: https://www.current-news.co.uk/news/good-energy-formally-launches-peer-to-peer-renewables-trading-service-5133

[19] Good Energy LTD, "Selectricity - the power of choice," 2018. [Online]. Available: https://www.goodenergy.co.uk/selectricity/

[20] E. Algaba, S. Béal, E. Rémila, and P. Solal, "Harsanyi power solutions for cooperative games on voting structures," *Int J Gen Syst*, vol. 48, no. 6, pp. 575–602, 2019.

[21] K. R. Apt and T. Radzik, "Stable partitions in coalitional games," 2006. [Online]. Available: http://arxiv.org/abs/cs/0605132