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# Evolutionary games played by multi-agent system with different memory capacity

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**Abstract.** The evolution of cooperation is still an enigma. Resolution of cooperative dilemma is a hot topic as a perplexing interdisciplinary project, and has captured wide attention of researchers from many disciplines as a multidisciplinary field. Our main concern is the design of a networked evolutionary game model in which players show difference in memory capability. The idea of different memory capacities has its origin on the pervasive individual heterogeneity of real agents in nature. It is concluded that this proposed multiple memory capacity stimulates cooperation in lattice-structured populations. The networking effect is also investigated via a scale free network which is associated with the heterogeneous populations structure. Interestingly, results suggest that the effectiveness of a heterogeneous network at fostering cooperation is reduced in the presence of individual memory here. A thorough inquiry in the coevolutionary dynamics of individual memory and spatial structure in evolutionary games is planned for the immediate future.

## 1 Introduction

Why and how cooperation among selfish and rational agents can persist in the presence of cheating and the cruel rule of 'survival of the fittest' driven by natural selection, remains a puzzling, fascinating and broad-ranging unsolved question in evolutionary biology [1–6]. Moreover, this interdisciplinary topic has also drawn plenty of attention, interest and research across disciplines, e.g., social sciences, behavioral sciences, psychology, physics, computer science, engineering and so on. Explaining the cooperation evolution is not only an issue of central importance to evolutionary biology but also one of the hot interdisciplinary topics so far, since cooperation is commonplace throughout all levels of the natural world and cooperative behaviors lie at the basis of human societies.

Last few decades have witnessed plenty of studies to be carried out in order to get an idea of what the driving forces behind cooperative behaviors of selfish individuals are. Social dilemma games, such as the Prisoner's dilemma game (PDG), have provided paramount insights into the emergence of cooperation among selfish individuals [7,8]. The PDG has become the paradigm for the evolution of cooperation among egoists. Two players should simultaneously and independently select one of the two decisions: cooperation or defection, and play accordingly with each other. It is regarded as the classical model of how and when concern for the future can lead to cooperation even if all selfish individuals care only their own benefits. The dilemma game promises a defector the highest payoff if encountering a cooperator. Meanwhile, the exploited cooperator is worse than a defector playing with another defector. In line with the principles of Darwin's natural selection, defection will be the dominating strategy of the population.

Since widespread cooperation is crucial for the prosperity of society and is frequently encountered in real-life situations, various mechanisms or solutions aimed at finding under what conditions the cooperation emerges in various games. Prominent examples include repeated interactions [9], direct reciprocity [10,11], indirect reciprocity [12–14], reputation [15], group selection [16], punishment [17–19], teaching ability [20,21], aging [22], emotion [23], population growth [24], phenotypic similarity [25]. Recent years have also witnessed a booming interest in structured population [26–31], and also a large literature has extended the evolutionary games in complex networks from the regular grids to other real-world networks [32,33], and even mobility of players embedded in networks [34,35]. In these studies, interacting strategic agents play games in a specified network, and only closest neighbors interact with one another.

Usually, in the formulation of the cooperative dilemma games only the results generated in the last round are taken into account in deciding the next choice. Studies referring to memory propose that historic memory can be

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implemented by featuring players by a summary of their previous winnings and choices. Various models [36–43], where the effects of full or discounted memory are assessed, have suggested that memory can influence the emergence of cooperation for self-interested agents under suitable conditions.

For instance, the study performed in reference [36] suggests that working memory constrains human cooperation in the PDG, by asking players to play the games either continuously or interrupted after each round by a secondary memory task that constrained the players' working memory capacity. Authors in reference [37] assume that whether a player opts for cooperation or defection in the next round depends on the history of the game and thus on the memory size of the player. Their results suggest that increasing the memory size helps to establish cooperation because traitors can be detected more easily. This is reflected by the vanishing number of traitors present at the end of the simulation runs for longer memories. The literature of [38] introduces a memory-based agent model and investigates the PDG in a heterogeneous Newman-Watts small-world network based on a Genetic Algorithm, focusing on the heterogeneity's role in enhancing the emergence of cooperative behaviors. Moreover, the effects of full and partial memories are assessed in a spatial version of the PDG in reference [39]. They get the conclusion that memory notably stimulates cooperation in the PDG played in ordered lattices, but fails to boost cooperation as the wiring network becomes highly disordered. Further, in the work of reference [42] about the memory-based snowdrift game in networks, the assumption is that by comparing the virtual payoff (by self-questioning) with the actual payoff, each player can get her optimal strategy corresponding to the highest payoff and then record it into her memory. In reference [43], the authors assume that individuals' performance is evaluated in terms of the accumulative payoffs in their memories. Their reach the conclusion that if individuals behave as their successful neighbors, then cooperation can be significantly promoted.

However, there are substantive aspects that are not tackled or require deep study along this line, such as the individual heterogeneity on memory capacity. Few prior work has investigated whether merging both memory and individual heterogeneity exhibits positive synergies that lead to increase cooperation even further. A common phenomenon in nature and society is: real agents are not homogeneous but differ in many aspects. This is particulary true in the context of human cooperation where human decision making is probably shaped by a multitude of individual factors. Normally, it is speculated that the memory capability in decision making may be one of the potential factors to affect the behavior of agents. It seems that no combination of memory and individual heterogeneity is implemented in a host of solutions to cooperative dilemma problem in the evolutionary games thus far.

Not only endowing players with a summary of their previous payoffs and strategies, here we also harbor the idea that agents differ in memory capability regarding the historical information of games. For further explanation, we divide the population into four types of agents based on their memory strength: (1) players with unlimited memory. This type of players can collect and remember all the strategy information in previous game rounds that have occurred; (2) players with not strong memory: only half of all the past iterations are remembered; (3) players with weak memory: only the game information in last iteration is available and (4) players without memory. Notably, memoryless players can only update their strategies according to the updating rule based on payoffs, which is also available for other types of players.

In our work, the memory is employed in a way that players will cooperate in next game round, with a probability valued as the fraction of cooperators in their neighborhood in history based on their memory capacity. It is to be stressed that, this assumption of four types is not enough to approximate human reality with high complexity. However, the implied meaning of this work is to broaden the spectrum of studies referring to memory, through designing a framework involving individual heterogeneity and memory simultaneously in this study.

The remainder of this paper is structured as follows. Section 2 is devoted to the description of the model in ample detail. In Section 3 the main results are presented, while in the last section we summarize the results and outline possible real-life implications of our findings.

## 2 Model

Two frequently-used two-agent two-strategy games: the Prisoner's dilemma game (PDG) and the Snowdrift game (SDG), are adopted here. They are both simultaneous two-player games where each player independently and simultaneously decides whether to cooperate or defect. At each point in time, an agent adopts only one of the two strategies. Mutual cooperators each gain the reward $R$, mutual defectors incur the punishment $P$; defectors score the temptation $T$ against cooperators, who score $S$ in such an encounter. For the PDG, payoff matrix should meet the condition of $T > R > P > S$ and the additional constraint $2R > T + S$ for repeated interactions. In line with the Darwin's natural selection rule, defection will be the dominating strategy in the population. Relaxing the inevitability of a social downfall resulted by the PDG is the SDG where $T > R > S > P$.

The following payoff matrices are used to determine the payoffs for the involved strategists. In particular, the payoff obtained by a player using strategy $i$ in an interaction with a player using strategy $j$ is denoted by $m_{ij}$, where $M = (m_{ij})$ is the $2 \times 2$ payoff matrix characterizing the game. Herein, the mentioned payoff matrices of a PDG and a SDG that summarize the feasible payoffs are provided by

$$M_{PDG} = \begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}, \quad M_{SDG} = \begin{pmatrix} b-\frac{c}{2} & b-c \\ b & 0 \end{pmatrix} \quad (1)$$

where $b$ and $c$ ($b > c$) indicate the benefits and costs of cooperation, respectively. And we normalize the cost of cooperation $c$ to 1. The payoff matrices thus only contain one free parameter $b$, but conserve the essence of the employed game.

As stated, the main purpose of this paper is to provide a model that examines individual differences in memory capacity. Unfortunately, the state space increases with the memory size of the players, where theoretical analysis is likely out of reach to the resulted complex scenario. Thus, due to the complexity of the system, our current investigations are limited and only based on extensive numerical simulations.

Then, we situate the investigated population on a graph where agents occupy nodes and edges represent game connections, i.e. the other agents with whom an agent can interact. The population of size $N$ thus situates at the nodes of the underlying network, and each individual connects with all the closest neighbors to whom it is linked. And, two quintessential network, i.e. a Lattice network and a Barabási-Albert (BA) scale-free network [32], are employed as paradigmatic examples to explore the potential effects of heterogeneity in the number of edges on evolution dynamics. Throughout the interaction phase, each agent adopts which strategy ($C$ or $D$) to perform during each game round, then plays the game with her neighbors-the chosen strategy being the same with all of them-and collects the final payoff. Choosing independent strategies with different neighbors may be more realistic in the presence of individual heterogeneity, and may be more interesting. However, considering the larger difficulty of realizing this assumption and the only research focus on memory capacity here, we thus only consider the case of taking same strategy with all neighbors in this work. Besides, the payoff for each player depends only on the outcome of the previous round, and thus remains unaffected by the memory capacity.

Afterwards, synchronous strategy updating follows. As stated, the memory capability of each agent is a crucial observable of the system. Whether a player opts for cooperation or defection in the next round depends on (but not fully) the history of the game and thus on the memory size of the player. A player with memory can keep track of all the strategies adopted by her neighbors in previous game rounds within her memory capability. It is plausible that a strong memory capacity likes a search engine with formidable power, helping individuals to remember history effectively. Players will cooperate in next game round, with a probability equivalent to the fraction of cooperators in her neighborhood in history based on their memory. However, taking into account history information is optional, and not indispensable, in the decision making process performed by players. The reason is that the information stored in memory as an external resource, can also be ignored or not used under some decision-making situations. For example, players incline to obey the strategy updating rule (described in the following) for the renewal of strategies.

Inspired by the above fact, we employ a parameter $\delta$ (where $0 \leq \delta \leq 1$) as the probability of memory information will be recurred to help the focal agents make decision for next game round. The choice of $\delta$ thus simulates the weight of memory, storing the strategy information already occurred in previous rounds, in strategy updating. As $\delta$ increases, the effects of memory gradually increase, and players have less chance to follow the appointed strategy updating rule (provide in the following). Thus, the limit case $\delta = 1$ corresponds to completely rely on the information stored by memory, whereas smaller values of $\delta$ reduce the effects of memory; the choice of $\delta = 0$ corresponds to the ahistoric model where agents act like a memoryless one and update their strategy in line with the given update rule.

As described, the population consists of four types of players: (1) players with unlimited memory. They can cooperate with a probability equated with the fraction of cooperators in their neighborhood in history, or abide by the given strategy update rules in deciding next move; (2) players with not strong memory. Only strategy information in half of all the past iterations is recorded; (3) players with weak memory. Players can only remember each opponent's last action and (4) players without memory. Since no game information is recorded, memoryless players can only renovate their strategies based on the payoffs gained in last game round, through randomly selecting a game partner. This is regarded as a common framework of strategy updating rule, and the details of the adopted strategy updating rule in this study are mentioned as follows.

Thus, a player using strategy $x$ and receiving payoff $\pi_x$ in last interaction randomly chooses another player from her neighbors; if that player happens to have employed the alternative strategy $y$ and accumulated payoff $\pi_y$, then the focal agent will adopt $y$ with a probability $u_{x\to y}$ that reflects the payoff difference $\pi_y - \pi_x$. This probability is given by the following function

$$u_{x\to y} = \frac{\pi_y - \pi_x}{R}, \tag{2}$$

where $R$ ensures the proper normalization and is given by the maximum possible difference between $\pi_x$ and $\pi_y$ at any given instance of the game.

## 3 Simulation results and discussion

In this section the displayed results rely on simulations carried out for constant populations size $N = 2500$ and fixed average degree $\bar{k} = 4$. The initial strategies ($C$ and $D$) of the population are randomly distributed. At the start of simulation there is a random distribution of the four types of agents with different memories. The simulation results are obtained by averaging over the last 1000 generations of the entire $10^4$ generations. Moreover, each data point averages over 100 realizations of both the networks and the initial conditions. The dynamics of a system is investigated as follows by simulation while the using probability of memory $\delta$ and the temptation to defect $b$ vary.

We begin the report of our numerical results by plotting the final fraction of cooperators in the population when the system evolves to the steady state. Figure 1

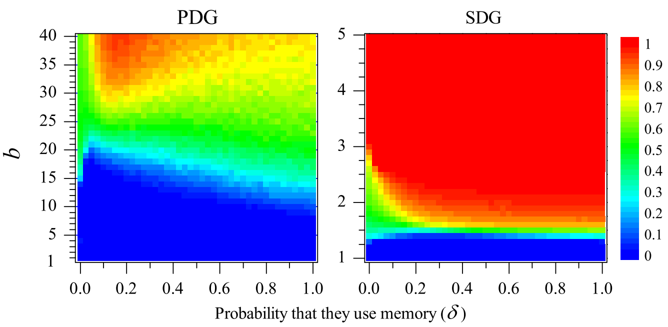


**Fig. 1.** The simulation results of average cooperation level of the whole population situating in Lattice networks, as a function of $b$ and $\delta$. The employed parameters are set by $N = 2500$ and $\bar{k} = 4$. The color bar encodes the average cooperation level of the population. The population is composed by the four types of agents: (1) players with unlimited memory; (2) players with not strong memory. Only strategy information in half of all the past iterations is recorded; (3) players with weak memory. Players can only remember each opponent's last action and (4) players without memory.

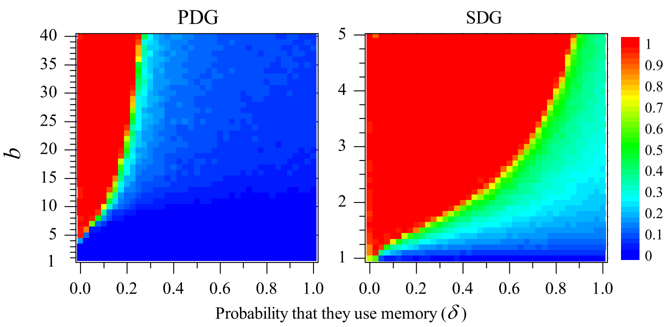


**Fig. 2.** The simulation results of average cooperation level of the whole population embedded in BASF networks, in dependence on $b$ and $\delta$. All results are obtained by setting $N = 2500$ and $\bar{k} = 4$. The population is composed by the four types of agents: (1) players with unlimited memory; (2) players with not strong memory. Only strategy information in half of all the past iterations is recorded; (3) players with weak memory. Players can only remember each opponent's last action and (4) players without memory.

summarizes the results of cooperation levels in Lattice networks with average degree $\bar{k} = 4$. Evolutionary results for PDG (left plot) on Lattice graphs follow a trend that: for small $b$, larger probability of using memory in deciding next action, can almost better foster the survival and maintenance of cooperators. However, for large $b$, an interesting point here is that, the variation of $\delta$ in proper range can best promote the domination of cooperation. It is evident that, cooperators survive at $b > 15$ when $\delta = 0$, while this number first increases (e.g., see $\delta = 0.025$) indicating that cooperators get suppressed by the proposed memory mechanism. And then the related $b$ decreases to below 15 and approaches 10 finally, which yield exclusive dominance of cooperators even in highly unfavorable conditions (lower $b$ herein).

The non-monotonic trend of cooperation levels with increasing $b$ and $\delta$ indicate that the influence of differentiated memory capacity on evolutionary dynamics is complicated since many factors may be involved simultaneously. It is supposed that each node (except the memoryless ones) becomes aware of the opposite player's strategy behavior by this memory mechanism. When using memory to adjust their behavior, players will choose cooperation in next game round, with a probability valued as the fraction of cooperators in their neighborhood in history based on their memory. In this sense, the information provided by this mechanism is that this proposed memory shows some similarity with the strategy like 'TIT FOR TAT' (TFT) [44]. And the dominant strategy between the two strategies in the population will spread more easily by the aid of memory mechanism than the minority one. Meanwhile, the strategy updating rule which almost coexists simultaneously with the memory mechanism will foster the evolution of strategy with higher payoffs. Considering the above, the amount of cooperation levels in the steady state is dependent on the combined effect of the two strategy updating ways in this work. In the strict cooperative dilemma of PDG, it is not easy to gain a straightforward conclusion about the role of memory played in influencing the evolutionary fate. However, when the cooperation dilemma turns milder (SDG in the following), the results seem to transfer more explicit intuitive understanding: keeping memory of past is positive and partly relying on memory in decision making is already enough.

Then we shift our attention to the results in the case of SDG when other parameters are unaltered with that in PDG. It is obvious that the red region for a majority of parameter choices the system corresponds to a full $C$ state. In the regions where full cooperation state is reached, almost all selfish players adopt cooperation when the system evolves to a steady state. In this situation, cooperative behaviors survive and prevail in a selfish population even if the temptation to defect is large (i.e. low $b$). Cooperators get dominance in SDG in the Lattice network when parameters fulfill the conditions of $b = 3$ at $\delta = 0$, while this number decreases to $b = 1.5$ or so when $\delta > 0.3$, as shown in Figure 1. Compared with the result when $\delta = 0$, $\delta > 0$ reflects the monotonous and positive role played by the memory capacity in solving the perplexing puzzles of cooperation in snowdrift games. Different with the case of PDG, the monotonic trends occur here in the framework of SDG.

Since BASF networks provide a more realistic model of the individual interaction features discovered in many nature or social networks. Herein the evolution dynamics are also performed in this most-often used heterogeneous network to check the influences of population structures. Figure 2 manifests examples of the behavior of the average cooperation level in a system evolving in BASF networks with the average degree $\bar{k} = 4$. Plenty of the research have stated that heterogeneous networks can better inspire cooperation than homogeneous ones even in strongly defection-prone environments. However, this phenomenon does not occur in our study. Memory drives the system to a low cooperation level state, even in mild cooperation situations denoted by relatively large $b$, as suggested in Figure 2.

Different with outcomes in the Lattice network, larger adoption probability of memory significantly inhibits the evolution fate of cooperators. We have checked that, increasing $\delta$ leads the system to lower cooperation levels, and smaller red regions signifying domination of cooperation narrow when players make their strategy decision according to the history information. The speed and range of this shrinkage is closely associated with the parameter $\delta$ and the employed games. Thus, Figure 2 evidences the nonnegligible dependence of our results on the overall population structure. Previous studies have emphasized that spatially constrained populations can sustain sizeable levels of cooperation of players located in heterogeneous networks. The highly heterogeneous state of population structure is crucial for the fortified facilitative effect on cooperation, since it incubates cluster of cooperators in a BASF network. Thus, clusters uphold and inspire the spreading of cooperative strategy around the large-degree nodes, according to the strategy update rules based on payoffs. Conversely, defectors sitting on large-degree nodes are incapable of reaping lasting benefits from clustering, simply because they become exceedingly weak if all the neighbors of the defecting cluster become defectors themselves.

However, once history information acts as the determining factor in strategy updating, the spreading of strategy (especially the cooperation) adopted by large-degree nodes will potentially be blocked off. Lower faction of cooperators induces the cooperators located at large-degree nodes inevitably change their strategy from cooperation to defection. Under this situation, the constructive influences of heterogeneous interaction numbers on promoting cooperation decrease or even disappear, and the propagation of defection is hard to avoid. Thereby, the clear indication from the observed results presented in Figure 2 is a strong correlation between the population structure and cooperation enhancement.

To investigate thoroughly the mechanism responsible for the emergence of cooperation in Figures 1 and 2, as comparisons we resort here to the evolutionary results when only three kinds of memory capacity are employed in the population: (1) players with unlimited memory; (2) players with not strong memory: only half of all the past iterations are remembered and (3) players without memory. All the other model rules (e.g., the employed network, the strategy updating rule, the simulation process) are kept the same with the four-type-agent model considered above, and results are summarized by Figure 3. The results transfer a clear information: the main conclusions stated above robustly and qualitatively remain the same within this three-type-agent model.

While as discussed in previous paragraphs, the suppressed cooperation levels in scale-free networks may be closely related with the memory rule here. Previous work has argued that, the feasibility of the $C$-strategy to spread increases with the degree of the node where the $C$ is located. In scale-free structured populations, the majority of agents own few interactions, whereas a few are highly linked individuals. Therefore, most of the largely linked individuals (say hubs) interact with low-degree agents, which provide favorable conditions for a $C$ located on a hub to spread to other nodes. However, the evolutionary fate of the large-degree $C$s changes in our study here where using memory indicates imitating the most-often used strategy in history. More precisely, players will cooperate in next game round, with a probability equivalent to the fraction of cooperators in their neighborhood in history based on their memory capacity. In this case, the large-degree $C$ in scale-free networks can not effectively spread her strategy to the neighbors when memory plays significant role in decision making. Incredible though it may seem at first glance, the gained results give us clear understanding about the role of large-degree nodes in facilitating cooperation.

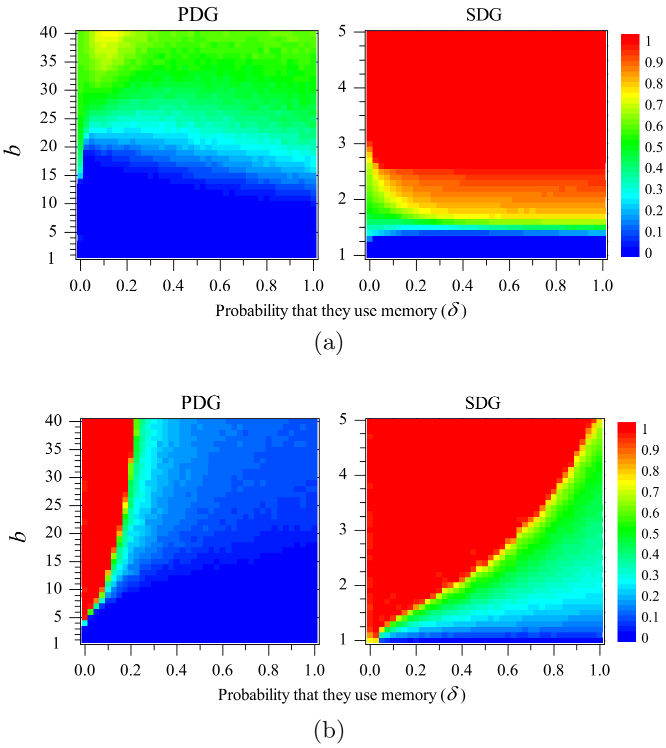


**Fig. 3.** (a) Fraction of cooperators in the population located in Lattice networks. (b) Fraction of cooperators in the population located in BASF networks. All results are obtained by setting $N = 2500$ and $\bar{k} = 4$. The population is composed by the three types of agents: (1) players with unlimited memory; (2) players with not strong memory. Only strategy information in half of all the past iterations is recorded and (3) players without memory.

## 4 Conclusion

The archetypical tensions that generate social dilemmas exist in many important issues of real social society: resource depletion, pollution, and climate change. This work, inspired by social reality, aims at seeking and establishing an evolutionary framework capable of modeling individual heterogeneity in terms of memory capacity.

Following this, the game is held among a population consisted with four types of agents in terms of their memory strength: (1) players with unlimited memory, who can collect and remember all the strategy information in previous game rounds that have occurred; (2) players with not strong memory, who can only remember the information from half of all the past iterations; (3) players with weak memory, who can only store the game information in last iteration and (4) players without memory, who can only update their strategies according to the given updating rule, which is also available for other types of players. There may be multiple ways of exploiting the information stored in memory, here the memory is implemented in a way that players will cooperate (or defect) in next game round, with a probability equivalent to the fraction of cooperators (or defectors) in their neighborhood in history based on their memory capacity.

Besides, we compare our results in two typical networks, drawing conclusions as to the relevance of the proposed dynamics to the population structure. Results suggest that the heterogeneous memory capacity proposed here boosts cooperation in the context of Prisoner's dilemma game and Snowdrift game when the Lattice network is employed. Intriguingly, in the scale-free graphs used to model heterogeneous populations, the diversity of memory capability leads to a reduced cooperative level of population, compared with the outcomes generated in the memoryless population. Therefore, our understanding can be enhanced by the investigations of how memory and population structure affect the evolutionary dynamics taking place among its nodes.

Our work may provide some novel hints to resolve the cooperative dilemma and foster the evolution of cooperation through pouring attention to individual memory capacity. It is planned to study in a near future the effect that more types of memory sizes of the player population has on the evolutionary fate. Further, the evolution and variation of memory capacity which serves as a fixed character in the present study when game proceeds will be necessary. We believe that the above mentioned subjects are worth studying, at least as a promising extension of the current model.

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All authors contributed equally to the paper.

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