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*Published in:*  
Journal of Public Economic Theory

*DOI:*  
[10.1111/jpet.12171](https://doi.org/10.1111/jpet.12171)

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*Document Version*  
Publisher's PDF, also known as Version of record

*Publication date:*  
2017

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*

Heijnen, P., & Schoonbeek, L. (2017). Signaling in a Rent-Seeking Contest with One-Sided Asymmetric Information. *Journal of Public Economic Theory*, 19(2), 548-564. <https://doi.org/10.1111/jpet.12171>

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# SIGNALING IN A RENT-SEEKING CONTEST WITH ONE-SIDED ASYMMETRIC INFORMATION

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## Abstract

We consider a two-player rent-seeking Tullock contest where one player has private information about his valuation of the prize, which can be high or low. This player can send a costly signal to his opponent, i.e., he can commit to reduce the prize either by some absolute amount of money or proportionally, conditional on winning it. We show that both kinds of signaling imply completely opposite results for separating equilibria, both in terms of conditions for existence and the type of player who sends the costly signal.

## 1. Introduction

In practice, we encounter many instances where two players compete for a single prize. In such cases, we observe now and again that a player commits to give up part of the prize conditional on winning it. For example, take two rival contractors who lobby officials to obtain a building contract. It then may happen that one of them commits to organize an (excessively) expensive feast to celebrate the completion of the project, if the contract will be awarded to him.<sup>1</sup> Next, consider the take-over contest for the Dutch bank ABN-AMRO in 2007 between the British bank Barclays and a consortium of the Royal Bank of Scotland, Fortis, and Santander. Barclays committed to invest 20 million Euros in the Amsterdam Business School of the University of Amsterdam conditional on winning the contest. Or, in litigation, we sometimes see that a celebrity commits to give

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<sup>1</sup> A committee of the Dutch parliament, investigating possible bribing of government officials by contractors, observes that such feasts occurred in the Dutch construction sector (Tweede Kamer 2003).

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We thank the associate editor and two anonymous referees for very helpful suggestions that improved the paper substantially. We also thank Allard van der Made and seminar participants at the University of Nottingham, the 4th World Congress of the Game Theory Society in Istanbul, the European Public Choice Society Meeting in Zurich, and EEA 2014 in Toulouse for useful discussions.

Received April 29, 2013; Accepted July 11, 2015.

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*Journal of Public Economic Theory*, 19 (2), 2017, pp. 548–564.

away part of his indemnity (e.g., as a donation to charity) if he wins the lawsuit from the defendant.<sup>2</sup> Finally, consider two politicians running for election. Then, one of them might commit to reward specific individuals or organizations, regardless of their merit, in case he is elected.<sup>3,4</sup>

We argue that sacrificing part of the prize conditional on winning can be rationalized as a strategic choice in a signaling game, used in order to induce less aggressive behavior of the opponent. We investigate this possibility in the context of the two-player rent-seeking contest of Tullock (1980). The game has many applications, e.g., in terms of lobbying, rent-seeking, litigation, and political campaigning (Nitzan 1994; Lockard and Tullock 2001; Congleton, Hillman, and Konrad 2008; Konrad 2009). In the standard contest, the players have complete information about their valuations of the prize. However, we examine the case with asymmetric information, where one player has private information regarding his valuation, which can be either high or low with given probabilities. The valuation by the other player is publicly known. In this setup, we introduce the option of costly signaling.

We analyze the following three-stage game. In stage 1, the privately informed player (referred to as the informed player) learns his type, i.e., his true valuation of the prize. The opponent (the uninformed player) only knows the distribution. In stage 2, the informed player can send an observable costly signal to the uninformed player by committing to give up part of the prize conditional on winning. In stage 3, the players play the actual contest. We consider two simple variants of signaling in stage 2. In the first variant, the informed player can commit to reduce his prize by some absolute amount of money (e.g., 100,000 euros) conditional on winning. We call this absolute signaling. In the second variant, the informed player can commit to lower his prize proportionally to some fraction (e.g., 90%) of its initial value contingent on winning. We call this proportional signaling. For both variants of signaling, we derive the necessary and sufficient condition for the existence of separating equilibria. In such equilibria, the informed player strategically reveals his type to his opponent through his decision in stage 2 (Fudenberg and Tirole 1991).

Signaling has a direct negative effect on the payoff of the player who sends the signal, since it reduces the size of his prize contingent on winning. In addition, signaling has an indirect effect on this player's payoff because it influences the efforts subsequently chosen by the players in the contest. Importantly, the indirect effect is positive if it entails a reduction of the effort of the opponent. The interplay of the direct and indirect effect determines the conditions for the existence of separating equilibria. We find that the game with absolute signaling has separating equilibria if and only if the informed player is relatively strong (in a sense to be specified below). In such equilibria, the high type informed player is willing to send a costly absolute

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<sup>2</sup> This type of behavior is especially prevalent in U.K. libel cases, where celebrities sue newspapers and magazines over the publication of allegedly defamatory stories. For instance, in 2007, Kate Winslet sued *Grazia*, a weekly women's magazine, for publishing a story that claimed she had sought medical help for weight loss. She promised that the indemnity would be donated to charities, which combat eating disorders. After winning the court case, she indeed donated the money.

<sup>3</sup> For example, take any country in which nepotism and rent-seeking are prevalent. A presidential candidate, who is open about his intention to appoint clearly incompetent family members as government officials, is reducing the rent from office.

<sup>4</sup> In the examples above, the relevant player makes a public announcement and he or she does not renege on this announcement for fear of tarnishing his or her reputation. In that sense, the player commits to giving up part of the prize conditional on winning it.

signal. On the contrary, under proportional signaling, there are separating equilibria if and only if the informed player is relatively weak. In that case, the low type informed player sends the costly proportional signal. We further demonstrate that in each variant of signaling, the costly signal sent is unique if we apply the intuitive criterion (Cho and Kreps 1987). Finally, we consider the welfare implications of signaling. Numerical analysis shows that the welfare effect of signaling is positive if, given the prize valuations of the informed player, the probability that this player is a high type is small enough.

Interestingly, the two kinds of signaling technology thus lead to completely opposite results, both in terms of the condition for the existence of separating equilibria and the type of player who sends the costly signal. In order to understand this intuitively, note that it is optimal for the informed player if the equilibrium effort of his opponent is as small as possible. In case the informed player is relatively strong (relatively weak), the effort of the uninformed player is smallest if she believes that the informed player is a high type (low type). Consider now the case with absolute signaling. If the informed player is relatively strong, then the high type is interested in revealing his type. It turns out that he is indeed willing to send a signal that is so costly that it will not be mimicked by the low type. On the other hand, if the informed player is relatively weak, then the low type would like to signal his type. However, now there is not enough leeway to find a costly signal that is both profitable for the low type and would not be mimicked by the high type. Next, examine the case with proportional signaling. If the informed player is relatively strong, then again the high type is interested in revealing his type. However, separation is not possible, since now the low type will always mimic the high type. In order to understand this, note that sending a given proportional signal is less costly in absolute terms for the low type than for the high type. Next, if the informed player is relatively weak, then the low type would like to signal his type. Now there is enough leeway to find a signal for the low type that will not be mimicked by the high type (since doing that would be too costly for the high type in absolute terms). Finally, examine the signaling technology used in the separating equilibria. Suppose, again, that the informed player is relatively strong, and take a separating equilibrium with absolute signaling for this case. The absolute signal sent by the high type in this equilibrium can formally be written as the product of the high value of the prize times some fraction. Hence, the high type is indifferent between sending the absolute signal and the proportional signal based on this fraction. Yet, there is no separating equilibrium with proportional signaling based on this fraction. The reason is that the low type would then have an incentive to mimic the high type. Note that, for the low type, sending this proportional signal is in absolute terms less costly than sending the absolute signal of the equilibrium with absolute signaling (in fact, it appears that there is no separating equilibrium with proportional signaling for any fraction in this case). In a similar way, we can explain that a separating equilibrium must be based on proportional signaling if the informed player is relatively weak.

Returning to the examples above, we can rationalize the behavior observed there in terms of a separating equilibrium. Focusing on the contractor example, suppose that the private information is about the profits of the project (e.g., because of privately known costs). By committing to organize an excessively expensive feast to celebrate the completion of the project, a relatively strong informed contractor can signal to the other contractor that he can afford to waste some absolute amount of money after winning the building contract, which may induce the other one to compete less fiercely. Note that this is an example of absolute signaling. On the contrary, we would have proportional

signaling if the contractor could commit to donate a fraction of its profits, provided that these are observable and verifiable.<sup>5</sup>

Our study is related to recent work by Denter Morgan, and Sisak (2011). These authors examine a setup similar to ours. However, in their model, the informed player can build up effort prior to playing the actual contest. The costs thus incurred have no direct effect on the outcome of the contest, but can only aim to signal the player's type. The authors show that their model has separating equilibria—wherein the player with the high valuation of the prize builds up effort prior to the contest—if and only if the informed player is relatively strong. This coincides with our conclusion for the case of absolute signaling. Notice that in Denter *et al.* (2011), the costs of signaling are sunk before the actual contest is played. This implies that if the informed player signals that his type is high, the effort level subsequently chosen by the players does not depend on the exact size of this informative signal. On the contrary, in our model, the exact magnitude of the signal has an effect on the size of the efforts subsequently chosen.

Two papers have examined signaling in contests with two-sided private information. Katsenos (2010) explores the same kind of signaling as Denter *et al.* (2011), whereas Münster (2009) investigates a model in which a contest is played twice. In his model, the effort level chosen in the first contest may serve as a signal of the player's type. The conditions for the existence of separating equilibria in these two studies are different from ours, i.e., there is a separating equilibrium if and only if the probability of the high valuation of the prize is small enough (the corresponding thresholds are different in the two studies).<sup>6,7</sup>

The paper is organized as follows. Section 2 introduces the game. Section 3 gives preliminary results. Section 4 presents our results on separating equilibria. Section 5 concludes. Proofs can be found in the Appendix.<sup>8</sup>

## 2. The Game

We consider a game where risk-neutral players 1 and 2 compete for a single prize by exerting nonrefundable effort. Player 1 has a high valuation of the prize,  $V = V_H$ , with

<sup>5</sup> The commitment to give up part of the prize conditional on winning may have other effects that could play a role. For example, the commitment of Barclays to invest in the University of Amsterdam might be an attempt to gather political support for its proposal, which should enter as an effort to increase the probability of winning. Similarly, if a celebrity, like Kate Winslet, promises to give part of the indemnity to charity, this might show that she is not interested in the indemnity itself but rather in defending her image. In turn, this might deter future attacks on her image. However, we cannot rule out the signaling motive considered by us. In order to analyze its implications and to obtain clear-cut results, we study the signaling motive without confounding the analysis with the other possible effects.

<sup>6</sup> A number of papers have examined one-sided and two-sided private information in Tullock contests without the option of signaling; see, for example, Hurley and Shogren (1998a, 1998b), Malueg and Yates (2004), Schoonbeek and Winkel (2006), Fey (2008), Ryvkin (2010), and Wasser (2013).

<sup>7</sup> There is a large literature on signaling in contests other than the rent-seeking contest of Tullock (1980). For example, Gill (2008) studies a two-stage patent contest between two firms where each firm has private information about its intermediate research results. Gill studies the incentive of the leader firm to disclose its results to deter the rival's exit. Gordon (2011) analyzes a patent contest with two firms that must each complete two phases of research. Each firm knows privately whether it has completed the first phase. Gordon investigates each firm's incentive to disclose this information impacting its rival's R&D investment. A large number of industrial organization studies examine information disclosure not related to patent races. For example, Jansen (2008) examines information acquisition and disclosure on a common demand intercept by oligopolistic firms.

<sup>8</sup> The Appendix includes the proofs for the case with absolute signaling. The proofs for the case with proportional signaling are similar and available from the authors upon request.

probability  $p$ , or a low valuation,  $V = V_L$ , with probability  $1 - p$ , where  $V_H > V_L > 0$  and  $0 < p < 1$  are given. In stage 1, player 1 learns the realization of his type, which is drawn by nature. Player 2 does not observe the type of player 1, she only knows the distribution of the types. Without loss of generality, the valuation of the prize by player 2 is normalized to 1, which is known to both players.

In stage 2, player 1 has the option to send a (costly) signal to player 2. For this stage, we consider two variants. In the first variant, referred to as absolute signaling, type  $i \in \{L, H\}$  of player 1 can make a statement of the following kind: "I commit to lower my prize by the absolute amount  $a_i \geq 0$ , conditional on winning it." The size of the absolute signal  $a_i$  is determined endogenously. After observing the (arbitrary) signal  $a \geq 0$ , player 2 can update her belief of player 1's type. She now believes that  $V = V_H$  with probability  $\lambda(a) \in [0, 1]$ . In the second variant, referred to as proportional signaling, type  $i \in \{L, H\}$  of player 1 can make a statement of the following kind: "I commit to lower my prize proportionally to a fraction  $r_i \in [0, 1]$  of its original value, contingent on winning it."<sup>9</sup> The size of the proportional signal  $r_i$  is determined endogenously. After observing the (arbitrary) signal  $r \in [0, 1]$ , player 2 believes that  $V = V_H$  with probability  $\lambda(r) \in [0, 1]$ .

In stage 3, the players play a Tullock (1980) contest in which they simultaneously choose their effort levels. Let  $e_{1L} \geq 0$  denote the effort of player 1 if his type is low,  $e_{1H} \geq 0$  if his type is high, and  $e_2 \geq 0$  the effort of player 2. If player 1 is of type  $i \in \{L, H\}$ , the probability that he wins the prize is given by  $e_{1i}/(e_{1i} + e_2)$ . If both players exert zero effort, each one wins the prize with probability  $\frac{1}{2}$ .

Disregarding the situation where both players exert zero effort (because this will not happen in equilibrium), the (expected) payoff in stage 3 of player 1 of type  $i$  in the variant with absolute signaling equals<sup>10</sup>

$$\pi_{1i} = \frac{e_{1i}}{e_{1i} + e_2} (V_i - a_i) - e_{1i}, \quad i \in \{L, H\}. \tag{1}$$

The (expected) payoff of player 2 reads

$$\pi_2 = \lambda(a) \frac{e_2}{e_{1H} + e_2} + (1 - \lambda(a)) \frac{e_2}{e_{1L} + e_2} - e_2. \tag{2}$$

In the variant with proportional signaling, we must replace  $V_i - a_i$  by  $r_i V_i$  in (1) and  $\lambda(a)$  by  $\lambda(r)$  in (2).<sup>11</sup> Each player will maximize *ex ante* expected payoff. We solve the game by backward induction to find separating equilibria.

### 3. Preliminary Results

In this section, we present a number of results that are useful in Section 4. The preliminary results increase the brevity of the proofs, but also hint at which form the separating equilibria will take. First, consider a standard two-player Tullock contest (with complete information) where the prize of player 1 is  $V$ , the prize of player 2 is unity, and the probability that player 1 wins is  $e_1/(e_1 + e_2)$  if  $e_1 + e_2 > 0$  and  $\frac{1}{2}$  otherwise, with  $e_i$  the effort of player  $i = 1, 2$ . Then, it is known that the (Nash) equilibrium efforts of players

<sup>9</sup> We assume in both variants of signaling that the commitment made by player 1 is credible. In the case of proportional signaling, this means the valuation of the prize by player 1 must be observable after stage 3 in order to be sure that this player fulfills his commitment.

<sup>10</sup> It is immediate that in equilibrium we have  $a_i \leq V_i$ ,  $i \in \{L, H\}$ .

<sup>11</sup> For notational simplicity, we use the same symbols  $\lambda(\cdot)$ ,  $e_{1i}$ ,  $e_2$ ,  $\pi_{1i}$ , and  $\pi_2$  in both variants of signaling,  $i \in \{L, H\}$ . It will be clear from the context whether we have either  $\lambda(a)$  or  $\lambda(r)$ , and so on.

1 and 2 are given by, respectively,  $V^2/(V + 1)^2$  and  $V/(V + 1)^2$ , while the corresponding equilibrium payoff of player 1 equals  $V^3/(V + 1)^2$  (Konrad 2009, p. 45). For later use, we also notice that the optimal payoff of player 1 if his opponent plays  $e_2 > 0$  can be written as

$$\max_{e_1} \frac{e_1}{e_1 + e_2} V - e_1 = \begin{cases} (\sqrt{V} - \sqrt{e_2})^2 & \text{if } e_2 \leq V, \\ 0 & \text{if } e_2 > V. \end{cases} \tag{3}$$

Second, take stage 3 of the game of Section 2. In this stage, the belief  $\lambda \in [0, 1]$  of player 2 that she plays against a high type player 1 is given. We present the following lemma.

LEMMA 1: *Consider stage 3 of the game where player 1 has private information about his type. The following holds for  $\lambda \in [0, 1]$ .*

- (i) *Let  $V_L V_H < 1$ . Then, the equilibrium effort of player 2 is strictly increasing in  $\lambda$ .*
- (ii) *Let  $V_L V_H > 1$ . Then, the equilibrium effort of player 2 is strictly decreasing in  $\lambda$ .*

We say that player 1 is relatively weak if  $V_L V_H < 1$ , i.e., if the geometric mean of the valuations of the prize of the two possible types of player 1 is smaller than the valuation of the prize by player 2 (cf. Denter *et al.* 2011). Similarly, player 1 is called relatively strong if  $V_L V_H > 1$ . Lemma 1 shows that the equilibrium effort of player 2 is strictly increasing (decreasing) in  $\lambda$  if player 1 is relatively weak (strong).<sup>12</sup> The lemma hints at which type of player 1 might benefit from signaling and why. A necessary condition for a player to send a costly signal is that he should be willing to reveal his type in case he would be able to do that without any cost. Lemma 1 implies that if  $V_L V_H > 1$ , this is true for the high type. The high type is willing to reveal his type because it will reduce the effort level of player 2. If the resulting increase in the high type’s payoff is large enough, then there might be room for a separating equilibrium to exist in the case with costly signaling. If  $V_L V_H < 1$ , a similar argument holds for the low type.

Finally, we show that the game of Section 2 satisfies a single-crossing property under both variants of signaling. This is convenient for our analysis in Section 4. First, consider the game with absolute signaling. Given signal  $a \in [0, V_i)$  and effort  $e_2$ , let  $e_{1i}(a, e_2)$  be the optimal effort of player 1 of type  $i \in \{L, H\}$ . Using payoff (1), and ignoring the boundary condition of nonnegative efforts, we find the best response function of type  $i$  of player 1, i.e.,  $e_{1i}(a, e_2) = \sqrt{e_2(V_i - a)} - e_2$ .<sup>13</sup> Using this, we have

$$\frac{d\pi_{1i}}{da} = -\frac{e_{1i}(a, e_2)}{e_{1i}(a, e_2) + e_2},$$

$$\frac{d\pi_{1i}}{de_2} = -\frac{e_{1i}(a, e_2)}{(e_{1i}(a, e_2) + e_2)^2} (V_i - a).$$

<sup>12</sup> Katsenos (2010, lemma 1) gives a related result for the case with two-sided private information.

<sup>13</sup> The best response function of player 2 reads  $e_2(e_1) = \sqrt{e_1} - e_1$ . This function is independent of  $a$  and the belief of player 2 regarding the type of player 1. Further, if player 2 believes that player 1 has type  $i \in \{L, H\}$ , then in equilibrium the efforts satisfy  $e_1 = V_i e_2$ . The intersection of this line with the best response of player 2 determines the equilibrium efforts. Thus, by revealing  $V_i$ , player 1 chooses one of the two points on player 2’s best response function.



Hence, the slope of the indifference curve of type  $i$  in the  $(a, e_2)$  space is given by

$$\frac{de_2}{da} = -\frac{d\pi_{1i}/da}{d\pi_{1i}/de_2} = -\sqrt{\frac{e_2}{V_i - a}}. \tag{4}$$

As a result, we obtain the single-crossing property that the indifference curve of the high type is less steep than the indifference curve of the low type. Both indifference curves are strictly decreasing in  $a$  (at an increasing rate).

In a similar way, we obtain for the game with proportional signaling that the slope of the indifference curve of player 1 of type  $i$  in the  $(r, e_2)$  space equals

$$\frac{de_2}{dr} = \sqrt{\frac{e_2 V_i}{r}} \tag{5}$$

for  $r \in (0, 1]$ . In this case, we obtain the single-crossing property that the indifference curve of the high type is steeper than the indifference curve of the low type. Both indifference curves are strictly increasing in  $r$  (at a decreasing rate).

### 4. Costly Signaling and Separating Equilibria

We now examine separating equilibria of the model of Section 2. Having observed the signaling decision of player 1 in stage 2, player 2 knows that either  $V = V_L$  or  $V = V_H$ . In turn, the corresponding equilibrium effort levels are realized in stage 3. We consider the two variants of signaling in separate subsections.

#### 4.1. Absolute Signaling

We first take the variant with absolute signaling. Let  $a_L^* \in [0, V_L]$  and  $a_H^* \in [0, V_H]$  be the equilibrium signals of both types of player 1 in stage 2, with  $a_L^* \neq a_H^*$ . We then have  $\lambda(a_L^*) = 0$  and  $\lambda(a_H^*) = 1$ . The equilibrium payoffs of the two types are denoted by  $\pi_{1L}^*$  and  $\pi_{1H}^*$ . Using the first paragraph of Section 3 yields the corresponding efforts:  $e_{1i}^*(a_i^*) = (V_i - a_i^*)^2 / (V_i - a_i^* + 1)^2$  and  $e_{2i}^*(a_i^*) = (V_i - a_i^*) / (V_i - a_i^* + 1)^2$ , with  $i \in \{L, H\}$ . It is useful to introduce  $e_2(a, \lambda(a))$ , i.e., the optimal effort of player 2 if she has observed the (possibly out-of-equilibrium) signal  $a$ , and consequently believes that player 1 is a high type with probability  $\lambda(a)$ . Note that  $e_{2i}^*(a_i^*) = e_2(a_i^*, \lambda(a_i^*))$ .

We have the following lemma.

LEMMA 2: *Consider the game with absolute signaling.*

- (i) *Let  $V_L V_H < 1$ . Then, in any separating equilibrium, we must have  $a_H^* = 0$ .*
- (ii) *Let  $V_L V_H > 1$ . Then, in any separating equilibrium, we must have  $a_L^* = 0$ .*

Lemma 2 implies that in case player 1 is relatively weak (strong), the low type (high type) must send the costly positive absolute signal in a separating equilibrium.

Let us now consider the situation where  $V_L V_H > 1$ . Using Lemma 2, it follows that in a separating equilibrium we must have  $a_L^* = 0$  and  $a_H^* = a' \in (0, V_H]$ . In order to determine the range of possible values of  $a'$ , notice that the high type must be willing to send  $a'$  if this convinces player 2 that his type is high instead of low, i.e., using (3), we must have

$$\left(\sqrt{V_H - a'} - \sqrt{e_2(a', 1)}\right)^2 \geq \left(\sqrt{V_H} - \sqrt{e_2(0, 0)}\right)^2, \tag{6}$$



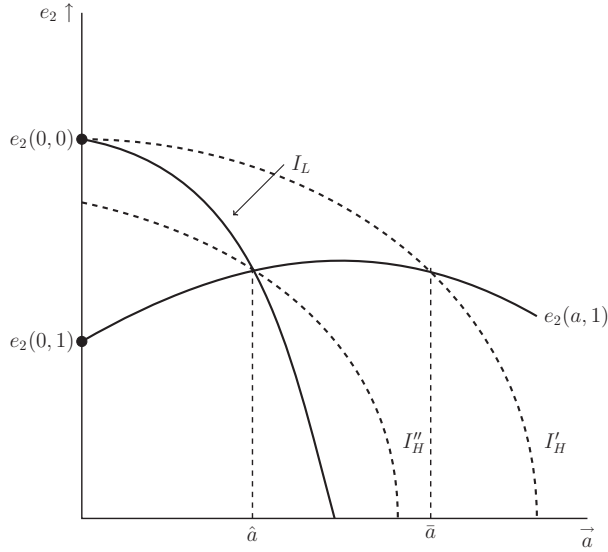


Figure 1: Range of possible signals of a high type player 1 in separating equilibria of the game with absolute signaling when  $V_L V_H > 1$  (cf. Lemma 3).

where  $e_2(a', 1) = (V_H - a') / (V_H - a' + 1)^2$  and  $e_2(0, 0) = V_L / (V_L + 1)^2$ . In addition, the low type should not be willing to send the signal  $a'$ , which confirms his opponent that his type is low rather than high. Hence, if  $a'$  is such that

$$e_2(a', 1) \leq V_L - a', \tag{7}$$

the following condition must be fulfilled:

$$\left(\sqrt{V_L} - \sqrt{e_2(0, 0)}\right)^2 \geq \left(\sqrt{V_L - a'} - \sqrt{e_2(a', 1)}\right)^2. \tag{8}$$

If (7) holds, the term between brackets on the right-hand side (RHS) of (8) is nonnegative. In case (7) is not fulfilled, it is immediate that the low type is not willing to send the signal  $a'$  because then his payoff would be zero (use (3)) rather than  $V_L^3 / (V_L + 1)^2$  (i.e., the left-hand side of (8)). In sum, there exists a separating equilibrium if either (6)–(8) hold simultaneously, or if (6) is fulfilled but (7) is not. We present the following lemma.

**LEMMA 3:** *Let  $V_L V_H > 1$ . Then, in any separating equilibrium of the game with absolute signaling, we must have  $a_H^* = a' \in [\hat{a}, \bar{a}]$ . Here,  $\hat{a} = \hat{a}(V_L, V_H) \in (0, V_L)$  and  $\bar{a} = \bar{a}(V_L, V_H) \in (\hat{a}, V_H)$  are the unique values such that, respectively, (8) and (6) hold with equality.*

Lemma 3 can be illustrated by using Figure 1, which has  $a$  and  $e_2$  along the axes (details are in the proof). In a separating equilibrium, the low type sets  $a_L^* = 0$  and the corresponding effort of player 2 is  $e_{2L}^*(0) = e_2(0, 0)$ . The indifference curves of the low type and high type through the point  $(0, e_2(0, 0))$  are denoted by, respectively,  $I_L$  and  $I_H'$ . Better pairs  $(a, e_2)$  are below the indifference curves. The point of intersection of  $I_H'$  and  $e_2(a, 1)$  defines  $\bar{a}$ . The point of intersection of  $I_L$  and  $e_2(a, 1)$  defines  $\hat{a}$ . The indifference curve of the high type through  $(\hat{a}, e_2(\hat{a}, 1))$  is denoted by  $I_H''$ . Due to the single-crossing property, we have  $\bar{a} > \hat{a}$ , while the high type prefers  $(\hat{a}, e_2(\hat{a}, 1))$  to

$(0, e_2(0, 0))$ . Lemma 3 follows easily since the high type is willing to signal his type if and only if  $a \leq \bar{a}$ , while the low type is not willing to mimic the high type if and only if  $a \geq \hat{a}$ .

Next, we present our first proposition, showing the existence of separating equilibria.

**PROPOSITION 1:** *Let  $V_L V_H > 1$ . Then, for each  $a' \in [\hat{a}, \bar{a}]$ , the game with absolute signaling has separating equilibria with (i) signals  $a_L^* = 0$  and  $a_H^* = a'$ , and (ii) out-of-equilibrium beliefs  $\lambda(a) = 0$  if  $a \in (0, a')$  and  $\lambda(a) \in [0, 1]$  if  $a > a'$ . Here,  $\hat{a}$  and  $\bar{a}$  are defined in Lemma 3.*

Sending the signal  $a'$  has a direct negative effect on the payoff of the high type, i.e., the player can only win  $V_H - a'$  rather than  $V_H$ . Yet, it also implies that his opponent chooses an effort level  $e_2(a', 1)$  instead of  $V_L/(V_L + 1)^2$ . In equilibrium, the former effort level is smaller than the latter, which induces a favorable indirect effect that outweighs the negative direct effect. The out-of-equilibrium belief is arbitrary for  $a > a'$ . Hence, for each  $a' \in [\hat{a}, \bar{a}]$ , we have a continuum of separating equilibria characterized by different  $\lambda(a) \in [0, 1]$  if  $a > a'$ .

We present the following corollary of Proposition 1.

**COROLLARY 1:** *Let  $V_L V_H > 1$ . Then, the game with absolute signaling has separating equilibria satisfying the intuitive criterion (IC). The corresponding signals are uniquely given by  $a_L^* = 0$  and  $a_H^* = \hat{a}$ , where  $\hat{a}$  is defined in Lemma 3, while the out-of-equilibrium beliefs follow from Proposition 1.*

Hence, if player 1 is relatively strong, then in a separating equilibrium satisfying the IC, the signal sent by the high type is equal to the smallest possible, i.e., least costly, value of the interval of Lemma 3.

Finally, we turn to the case where player 1 is relatively weak or  $V_L V_H = 1$ . We then have the following result.

**PROPOSITION 2:** *Let  $V_L V_H \leq 1$ . Then, the game with absolute signaling has no separating equilibrium.*

If  $V_L V_H = 1$ , then the two types of player 1 exert the same effort regardless of the belief of player 2. It is immediate that there is no separating equilibrium in this case. If  $V_L V_H < 1$ , then Lemma 2 shows that we must have  $a_H^* = 0$  in a candidate-separating equilibrium. This implies that the low type player should be willing to send a positive absolute signal in order to distinguish himself from the high type. However, the single-crossing property implies that the low type prefers to mimic the high type, which breaks the candidate equilibrium. Hence, there is no separating equilibrium in this case (see the proof for details).

Combining Propositions 1 and 2, we see that in the model with absolute signaling, we only have separating equilibria if player 1 is relatively strong. Furthermore, in such equilibria, the high type sends the costly positive absolute signal.

## 4.2. Proportional Signaling

Proceeding, we study proportional signaling and denote the signals of the low type and high type player 1 in a separating equilibrium by, respectively,  $r_L^*$  and  $r_H^*$ , with  $r_L^* \neq r_H^*$ . Let  $\pi_{1L}^*$  and  $\pi_{1H}^*$  again be the corresponding payoffs of these two types. Note that

$\lambda(r_L^*) = 0$  and  $\lambda(r_H^*) = 1$ . Let  $e_2(r, \lambda(r))$  be the optimal effort of player 2 if she has observed signal  $r$  and believes that player 1 is a high type with probability  $\lambda(r)$ .

We have the following lemma.

LEMMA 4: *Consider the game with proportional signaling.*

- (i) *Let  $V_L V_H < 1$ . Then, in any separating equilibrium, we must have  $r_H^* = 1$ .*
- (ii) *Let  $V_L V_H > 1$ . Then, in any separating equilibrium, we must have  $r_L^* = 1$ .*

The lemma shows, similar to Lemma 2, that in case player 1 is relatively weak (strong), the low type (high type) must send the costly signal in a separating equilibrium.

Let us now take  $V_L V_H < 1$ . Lemma 4 implies that in a separating equilibrium we must have  $r_L^* = r' \in [0, 1)$  and  $r_H^* = 1$ . In order to find the range of possible values of  $r'$ , we notice that the incentive constraint for the high type reads

$$\left(\sqrt{V_H} - \sqrt{e_2(1, 1)}\right)^2 \geq \left(\sqrt{r V_H} - \sqrt{e_2(r, 0)}\right)^2, \tag{9}$$

where  $e_2(1, 1) = V_H / (V_H + 1)^2$  and  $e_2(r, 0) = r V_L / (r V_L + 1)^2$ . If  $V_L \geq V_H / (V_H + 1)^2$ , the incentive constraint for the low type is

$$\left(\sqrt{V_L} - \sqrt{e_2(r, 0)}\right)^2 \geq \left(\sqrt{V_L} - \sqrt{e_2(1, 1)}\right)^2. \tag{10}$$

In case  $V_L < V_H / (V_H + 1)^2$ , the low type is always willing to send the signal  $r'$ , since he then obtains a positive payoff and zero otherwise. We have the following result.

LEMMA 5: *Let  $V_L V_H < 1$ . Then, in any separating equilibrium of the game with proportional signaling, we must have  $r_L^* = r' \in [\hat{r}, \bar{r}]$ . Here,  $\hat{r} = \hat{r}(V_L, V_H) \in (0, 1)$  is the unique value such that (10) holds with equality in case  $V_L > V_H / (V_H + 1)^2$  while  $\hat{r} = \hat{r}(V_L, V_H) = 0$  otherwise, and  $\bar{r} = \bar{r}(V_L, V_H) \in (\hat{r}, 1)$  is the unique value such that (9) holds with equality.*

Lemma 5 is the counterpart of Lemma 3. It can be illustrated with Figure 2 (here, we focus on the case  $V_L > V_H / (V_H + 1)^2$ ; see the proof for further details). In a separating equilibrium, the high type signals  $r_H^* = 1$  and, in turn, player 2 sets  $e_{2H}^* = e_2(1, 1)$ . The indifference curves of the low type and high type through  $(1, e_2(1, 1))$  are, respectively,  $I'_L$  and  $I_H$ . Better pairs  $(r, e_2)$  are below the indifference curves. The points of intersection of  $e_2(r, 0)$  with  $I'_L$  and  $I_H$  define, respectively,  $\hat{r}$  and  $\bar{r}$ . The indifference curve of the low type through  $(\bar{r}, e_2(\bar{r}, 0))$  is  $I''_L$ . The single-crossing property implies that both  $\bar{r} > \hat{r}$  and that the low type prefers  $(\bar{r}, e_2(\bar{r}, 0))$  to  $(1, e_2(1, 1))$ . For the low type, it is profitable to signal his type if and only if  $r \geq \hat{r}$ . The high type has no incentive to mimic the low type if and only if  $r \leq \bar{r}$ .

We present the following result as the counterpart of Proposition 1.

PROPOSITION 3: *Let  $V_L V_H < 1$ . Then, for each  $r' \in [\hat{r}, \bar{r}]$ , the game with proportional signaling has separating equilibria with (i) signals  $r_L^* = r'$  and  $r_H^* = 1$ , and (ii) out-of-equilibrium beliefs  $\lambda(r) \in [0, 1]$  if  $r \in [0, r')$  and  $\lambda(r) = 1$  if  $r \in (r', 1)$ . Here,  $\hat{r}$  and  $\bar{r}$  are defined in Lemma 5.*

The commitment to lower the prize conditional on winning means that the low type can only win  $r V_L$  rather than  $V_L$ , which directly and negatively impacts his payoff. However, it also induces player 2 to set an effort level  $r V_L / (r V_L + 1)^2$  instead of the larger

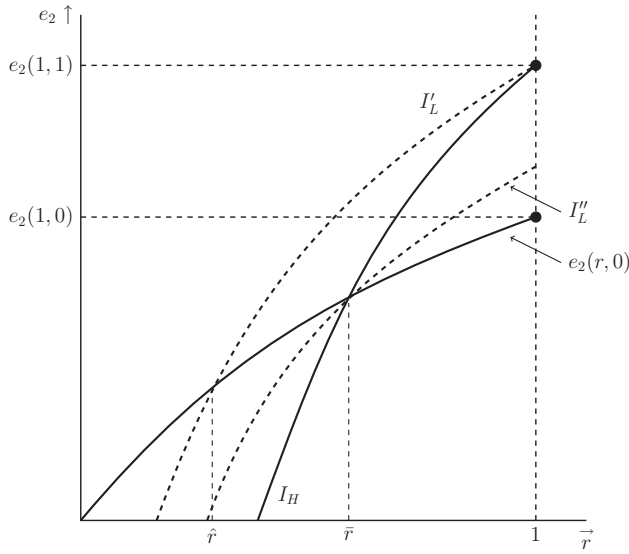


Figure 2: Range of possible signals of a low type player 1 in separating equilibria of the game with proportional signaling when  $V_L V_H < 1$  and  $V_L > V_L / (V_H + 1)^2$  (cf. Lemma 5).

$V_H / (V_H + 1)^2$ , which is always beneficial for the low type. In equilibrium, this indirect positive effect is dominating. Note that in case  $\hat{r} = r' = 0$ , the out-of-equilibrium belief reduces to  $\lambda(r) = 1$  for  $r \in (0, 1)$ . Further,  $\lambda(r)$  is arbitrary for  $r > r'$ .

The next corollary shows that the low type signals the least costly proportional signal  $\bar{r}$  in a separating equilibrium if we invoke the IC.

**COROLLARY 2:** *Let  $V_L V_H < 1$ . Then, the game with proportional signaling has separating equilibria satisfying the IC. The corresponding signals are uniquely given by  $r_L^* = \bar{r}$  and  $r_H^* = 1$ , where  $\bar{r}$  is defined in Lemma 5, while the out-of-equilibrium beliefs follow from Proposition 3.*

We conclude with a result showing that the game with proportional signaling has no separating equilibrium if player 1 is relatively strong or if  $V_L V_H = 1$ .

**PROPOSITION 4:** *Let  $V_L V_H \geq 1$ . Then, the game with proportional signaling has no separating equilibrium.*

Proposition 4 is the counterpart of Proposition 2 and can be discussed in a similar way (cf. Section 4.1). We leave the details to the reader.

Summarizing, under proportional signaling, we only have separating equilibria if player 1 is relatively weak. In those equilibria, the low type sends the costly proportional signal.

### 4.3. Absolute versus Proportional Signaling

Roughly summarizing Sections 4.1 and 4.2, we have obtained the following. When player 1 is relatively strong, the high type can rely on absolute signaling to signal his type. Conversely, when player 1 is relatively weak, the low type can use proportional signaling. It is obvious why the relative strength of player 1 matters, since it determines who benefits

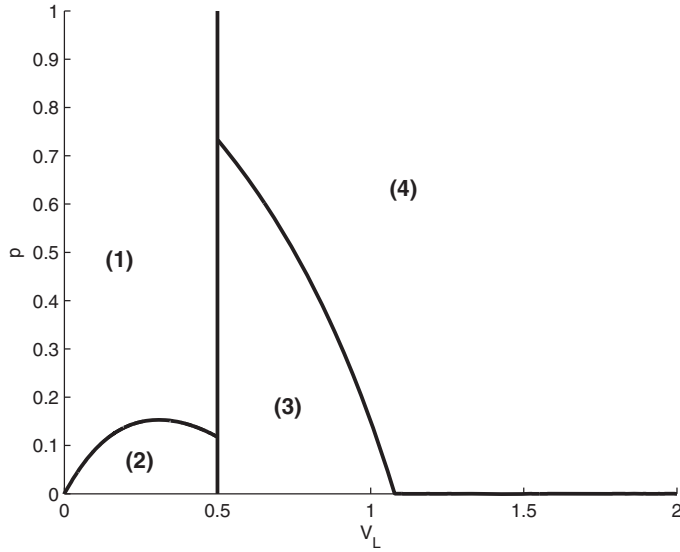


Figure 3: Welfare effects of signaling, with  $V_H = 2$ . Signaling decreases welfare in (1) and (4), and increases welfare in (2) and (3).

from revealing its type (cf. Lemma 1). However, the need for two different signaling technologies is more subtle, and is largely determined by the incentive to deviate from the equilibrium strategy. For example, consider the case  $V_L V_H > 1$ . In the separating equilibria of Proposition 1, the high type sacrifices  $a' \in [\hat{a}, \bar{a}]$ . Clearly, the high type is indifferent between sacrificing the absolute amount  $a'$  or reducing his prize to a fraction  $r'' = 1 - a'/V_H$  of the original valuation. However, it follows that  $V_L - a' < r'' V_L$ , and consequently the low type has a stronger incentive to deviate under proportional signaling than under absolute signaling. It turns out that for any  $r''$ , it is optimal for the low type to mimic the high type, and therefore these equilibria do not exist (cf. Proposition 4).

#### 4.4. Welfare Implications

In the contest literature, different definitions of welfare are used. Traditionally, welfare *loss* is defined as the sum of the efforts. This approach is justified only if all players have the same valuation of the prize. However, if valuations differ, then it is inefficient for the player with the lowest valuation to win the contest. Therefore, we include the prize of the winning player in the welfare measure, i.e., welfare is the sum of *ex ante* expected payoffs. We are interested in this welfare measure in the case where signaling is possible compared to the case where signaling is not possible.

When  $V_L V_H \neq 1$ , there are three factors affecting welfare in the equilibrium with signaling *vis-à-vis* the equilibrium without signaling: (i) the direct cost of signaling (i.e., the reduction of the prize contingent on winning), (ii) the probability that the player with the highest (or lowest) valuation wins the prize, and (iii) the size of the aggregate efforts. Apart from the direct cost of signaling, the effects of these factors are ambiguous and, because of the values of the parameters, we are not able to analytically derive an expression for which the welfare effect of signaling is positive. Based on numerical simulations, Figure 3 shows (for  $V_H = 2$ ) the combinations of  $p$  and  $V_L$  where signaling

improves welfare (assuming that the low type signals with  $r = \bar{r}(V_L, V_H)$  when  $V_L V_H < 1$ , and the high type signals with  $a = \hat{a}(V_L, V_H)$  when  $V_L V_H > 1$ ). The line  $V_L = \frac{1}{2}$  separates the area with relative signaling (1 and 2) from the area with absolute signaling (3 and 4). In areas (2) and (3), the use of signaling increases welfare. In areas (1) and (4), the opposite happens. We see that signaling is welfare-improving if, given  $V_L$  and  $V_H$ , the probability that player 1 is a high type is small enough. Calculations done by us (not reported here) indicate that this increase in welfare is mainly caused by smaller aggregate efforts.

### 5. Conclusion

This study has shown that committing to give away a part of the prize conditional on being successful can always be used as a signal, unlike the unconditional signal in Denter *et al.* (2011), where only a relatively strong player will signal his type. Our results show that information disclosure does not depend on the strength of the player; it only determines which form the signaling might take.

One of the limitations of our model is the fact that it only allows for one-sided asymmetric information. In applications, it can happen that both players are uncertain about the valuation of the other player. The existing literature on signaling in contests with two-sided asymmetric information (Münster 2009; Katsenos 2010) has not considered the type of signaling studied by us. This seems a promising avenue for future research.

### Appendix: Proofs

*Proof of Lemma 1:* To begin with, take  $\lambda \in (0, 1)$ . We then know from Schoonbeek and Winkel (2006, p. 125) that there are two possible cases: (i) both types of player 1 and player 2 exert a positive equilibrium effort, or (ii) the high type player 1 and player 2 exert a positive equilibrium effort, while the low type player 1 exerts zero effort. They show that case (i) occurs if and only if  $V_L V_H > \lambda(\sqrt{V_L V_H} - V_L)$ . This condition is certainly fulfilled if  $V_L V_H > 1$ . First, examine case (i). Then, the following first-order conditions must hold:

$$\frac{e_2 V_L}{(e_{1L} + e_2)^2} = 1, \tag{A.1}$$

$$\frac{e_2 V_H}{(e_{1H} + e_2)^2} = 1, \tag{A.2}$$

$$\lambda \frac{e_{1H}}{(e_{1H} + e_2)^2} + (1 - \lambda) \frac{e_{1L}}{(e_{1L} + e_2)^2} = 1. \tag{A.3}$$

Using this, we find

$$e_2 \equiv e_2(\lambda) = V_L \left( \frac{\alpha(\lambda)}{\beta(\lambda)} \right)^2, \tag{A.4}$$

where  $\alpha(\lambda) \equiv \frac{\lambda}{\sqrt{V_L V_H}} + \frac{1-\lambda}{V_L} > 0$  and  $\beta(\lambda) \equiv 1 + \frac{\lambda}{V_H} + \frac{1-\lambda}{V_L} > 0$ . Hence,  $de_2(\lambda)/d\lambda > 0$  is equivalent to

$$\beta(\lambda) \frac{d\alpha(\lambda)}{d\lambda} - \alpha(\lambda) \frac{d\beta(\lambda)}{d\lambda} = \frac{1}{\sqrt{V_L V_H}} - \frac{1}{V_L} + \frac{1}{V_L \sqrt{V_L V_H}} - \frac{1}{V_L V_H} > 0, \tag{A.5}$$

which can be rewritten as  $(1 - \sqrt{V_L V_H})(\sqrt{V_L V_H} - V_L) > 0$ . Since  $V_H > V_L$ , we have  $\sqrt{V_L V_H} > V_L$ . Thus, in case (i), we see that  $e_2(\lambda)$  is strictly increasing in  $\lambda$  if  $V_L V_H < 1$ , respectively, strictly decreasing in  $\lambda$  if  $V_L V_H > 1$ . Next, take case (ii). Then, (A.2), (A.3) and  $e_{1L} = 0$  imply that  $e_2 \equiv e_2(\lambda) = V_H \lambda^2 / (V_H + \lambda)^2$ . It follows that  $e_2(\lambda)$  is strictly increasing in  $\lambda$  in this case. Note that (A.1) holds with a smaller than or equal sign now. Further, case (ii) cannot occur if  $V_L V_H > 1$ .

Finally, examine the limit cases with  $\lambda = 0$  or  $\lambda = 1$ , where the type of player 1 is known. Note that in case (i), we have  $\lim_{\lambda \downarrow 0} e_2(\lambda) = V_L / (V_L + 1)^2$  (case (ii) is not relevant now). Further, in both cases (i) and (ii), we have  $\lim_{\lambda \uparrow 1} e_2(\lambda) = V_H / (V_H + 1)^2$ . The lemma follows easily. ■

*Proof of Lemma 2:* Consider a candidate separating equilibrium with  $a_L^* \in [0, V_L]$  and  $a_H^* \in [0, V_H]$ . We write  $\pi_{1i}(a, e_2(a, \lambda(a)))$  for the payoff of player 1 of type  $i \in \{L, H\}$  if he deviates to signal  $a \neq a_i^*$  and plays his best response to  $e_2(a, \lambda(a))$ .

– Part (i): Let  $V_L V_H < 1$ . We demonstrate that  $a_H^* = 0$ . Suppose, to the contrary, that  $a_H^* \in (0, V_H^*]$ . Consider a deviation of the high type to  $a \in [0, a_H^*)$ . Using (i) of Lemma 1, we know that  $e_2(a, \lambda(a)) < e_2(a, 1)$  for  $\lambda(a) \in [0, 1)$ . Using this and the fact that  $(V_H - a)^3 / (V_H - a + 1)^2$  is strictly decreasing in  $a \in [0, a_H^*)$ , we find that  $\pi_{1H}(a, e_2(a, \lambda(a))) > \pi_{1H}(a, e_2(a, 1)) > \pi_{1H}^*$  for any  $\lambda(a) \in [0, 1)$ . As a result,  $\pi_{1H}(a, e_2(a, \lambda(a))) > \pi_{1H}^*$  for any  $\lambda(a) \in [0, 1]$ , and the high type will deviate to  $a$ , giving a contradiction. Thus,  $a_H^* = 0$ .

– Part (ii): Let  $V_L V_H > 1$ . We show that  $a_L^* = 0$ . Suppose, to the contrary, that  $a_L^* \in (0, V_L^*]$ . Consider a deviation of the low type to  $a \in [0, a_L^*)$ . Using (ii) of Lemma 1, we have  $e_2(a, \lambda(a)) < e_2(a, 0)$  for  $\lambda(a) \in (0, 1]$ . From this and the fact that  $(V_L - a)^3 / (V_L - a + 1)^2$  is strictly decreasing in  $a \in [0, a_L^*)$ , we find  $\pi_{1L}(a, e_2(a, \lambda(a))) > \pi_{1L}^*$  for any  $\lambda(a) \in [0, 1]$ . Hence, the low type deviates to  $a$ , a contradiction. So,  $a_L^* = 0$ . ■

*Proof of Lemma 3:* Let  $V_L V_H > 1$ . Lemma 2 shows that in a candidate-separating equilibrium the low type player 1 sets  $a_L^* = 0$ . The corresponding effort of player 2 is  $e_{2L}^*(0) = e_2(0, 0) = V_L / (V_L + 1)^2$ . In Figure 1, the indifference curve,  $I_L$ , of the low type through the point  $(0, e_2(0, 0))$  is given by the pairs  $(a, e_2)$  such that  $(\sqrt{V_L - a} - \sqrt{e_2})^2 = (\sqrt{V_L} - \sqrt{e_2(0, 0)})^2$  (the associated payoff level is  $V_L^3 / (V_L + 1)^2$ ). The indifference curve,  $I_H'$ , of the high type through this point is given by  $(\sqrt{V_H - a} - \sqrt{e_2})^2 = (\sqrt{V_H} - \sqrt{e_2(0, 0)})^2$ . Better pairs of  $(a, e_2)$  are below the indifference curves. Also, note that  $e_2(a, 1)$  is strictly increasing in  $a$  if  $a < V_H - 1$  and strictly decreasing if  $a > V_H - 1$ . Further,  $e_2(0, 1) = V_H / (V_H + 1)^2 < e_2(0, 0)$ , where the inequality follows from  $V_L V_H > 1$ .

Given  $V_L$  and  $V_H$ , we demonstrate that there exists a unique point of intersection of  $e_2(a, 1)$  and  $I_H'$ . In order to do that, we show that there exists a unique value of  $a$  such that  $(\sqrt{V_H - a} - \sqrt{e_2(a, 1)})^2 = (\sqrt{V_H} - \sqrt{e_2(0, 0)})^2$ , or equivalently  $f(a) = \sqrt{V_H} - \sqrt{V_L} / (V_L + 1)$ , with  $f(a) \equiv (V_H - a)^{3/2} / (V_H - a + 1)$ . We have the following: (i)  $\sqrt{V_H} - \sqrt{V_L} / (V_L + 1) > 0$ , (ii)  $f(0) > \sqrt{V_H} - \sqrt{V_L} / (V_L + 1)$  since  $V_L V_H > 1$ , (iii)  $f(V_H) = 0$ , and (iv)  $f'(a) < 0$  for  $a \in [0, V_H]$ . The result follows directly. We denote the value of  $a$  corresponding to the point of intersection by  $\bar{a} = \bar{a}(V_L, V_H) < V_H$ .

Due to the single-crossing property discussed in Section 3, it follows now that there also exists a unique point of intersection of  $e_2(a, 1)$  and  $I_L$  denoted by  $\hat{a} = \hat{a}(V_L, V_H)$ , with  $\bar{a} > \hat{a}$ . We have  $\hat{a} < V_L$ , since  $I_L$  intersects the horizontal axis to the left of  $a = V_L$ . Since  $\hat{a}$  exists, it follows that  $e_2(\hat{a}, 1) \leq V_L - \hat{a}$ . The indifference curve of the high type through  $(\hat{a}, e_2(\hat{a}, 1))$  is denoted by  $I_H''$ . Due to the single-crossing property, the high type must prefer  $(\hat{a}, e_2(\hat{a}, 1))$  to  $(0, e_2(0, 0))$ .



The high type is willing to signal his type if and only if  $a \leq \bar{a}$ . The low type is not willing to mimic the high type if and only if  $a \geq \hat{a}$ . ■

*Proof of Proposition 1:* Let  $V_L V_H > 1$ . Consider a candidate-separating equilibrium with  $a_L^*$  and  $a_H^*$ . We know from Lemmas 2 and 3 that we must have  $a_L^* = 0$  and  $a_H^* = a' \in [\hat{a}, \bar{a}]$ . By construction, the low type will not deviate to  $a_H^*$ , while the high type will not deviate to  $a_L^*$ .

Take an arbitrary  $a' \in [\hat{a}, \bar{a}]$ . We define the corresponding out-of-equilibrium belief by  $\lambda(a) = 0$  if  $a \in (0, a')$ , and  $\lambda(a) \in [0, 1]$  if  $a > a'$ .

We show that the low type has no incentive to deviate. We have two possible cases. First, suppose that  $a' \leq V_L$ . Take a deviation to  $a \in (0, a')$ . Using  $\lambda(a) = 0$  and the fact that  $(V_L - a)^3 / (V_L - a + 1)^2$  is strictly decreasing in  $a \in (0, a')$ , we see that the low type will not choose such a deviation. Next, examine a deviation to  $a > a'$ . We know that the low type is not willing to sacrifice  $a > \hat{a}$  in order to (falsely) convince player 2 that he is a high type. Using this,  $\lambda(a) \in [0, 1]$ , and part (ii) of Lemma 1, it follows that such a deviation is not profitable. Second, suppose that  $V_L < a'$ . Using the same argument as above, we see that he will not deviate to  $a \in (0, V_L)$ . Clearly, it is not profitable to deviate to  $a \geq V_L$ , with  $a \neq a'$ , for any belief.

Next, examine possible deviations of the high type. First, we show that the high type will not deviate to  $a \in [0, \hat{a})$ . Recall that  $\hat{a}$  is defined such that

$$\sqrt{V_L - \hat{a}} - \frac{\sqrt{V_H - \hat{a}}}{V_H - \hat{a} + 1} = \sqrt{V_L} - \frac{\sqrt{V_L}}{V_L + 1}. \tag{A.6}$$

Note that  $\sqrt{x} - \frac{\sqrt{x}}{x+1}$  is strictly increasing in  $x > 0$ . Using this, we see that (A.6) yields

$$\sqrt{V_L - \hat{a}} - \frac{\sqrt{V_H - \hat{a}}}{V_H - \hat{a} + 1} \geq \sqrt{V_L - a} - \frac{\sqrt{V_L - a}}{V_L - a + 1}, \tag{A.7}$$

or

$$\sqrt{V_L - \hat{a}} - \sqrt{V_L - a} \geq \frac{\sqrt{V_H - \hat{a}}}{V_H - \hat{a} + 1} - \frac{\sqrt{V_L - a}}{V_L - a + 1}. \tag{A.8}$$

Further, note that

$$\sqrt{V_H - \hat{a}} - \sqrt{V_L - \hat{a}} \geq \sqrt{V_H - a} - \sqrt{V_L - a}, \tag{A.9}$$

since the RHS is increasing in  $a \in [0, \hat{a})$ . Using this, (A.8) implies

$$\sqrt{V_H - \hat{a}} - \sqrt{V_H - a} \geq \frac{\sqrt{V_H - \hat{a}}}{V_H - \hat{a} + 1} - \frac{\sqrt{V_L - a}}{V_L - a + 1}, \tag{A.10}$$

or

$$\sqrt{V_H - \hat{a}} - \frac{\sqrt{V_H - \hat{a}}}{V_H - \hat{a} + 1} \geq \sqrt{V_H - a} - \frac{\sqrt{V_L - a}}{V_L - a + 1}. \tag{A.11}$$

Using  $\lambda(a) = 0$ , it follows that a deviation to  $a \in [0, \hat{a})$  is not profitable.

Proceeding, examine a deviation to  $a \in [\hat{a}, a')$ . Using  $\lambda(a) = 0$  and the fact that  $(V_H - a)^3 / (V_H - a + 1)^2$  is strictly decreasing in  $a \in [\hat{a}, a')$ , the high type will not choose such a deviation. Next, consider a deviation to  $a \in (a', V_H]$ . Since  $(V_H - a)^3 / (V_H - a + 1)^2$  is strictly decreasing in  $a \in (a', V_H]$ , and using  $\lambda(a) \in [0, 1]$  and part (ii) of Lemma 1, we see that the high type will not make such a deviation. Finally, it is immediate that the high type will not deviate to  $a > V_H$  given  $\lambda(a) \in [0, 1]$ . ■

*Proof of Corollary 1:* Let  $V_L V_H > 1$ . Consider a candidate-separating equilibrium with  $a_L^*$  and  $a_H^*$ . Using Lemmas 2 and 3, we know that we must have  $a_L^* = 0$  and  $a_H^* = a' \in [\hat{a}, \bar{a}]$ . We write  $\pi_{1i}(a, e_2(a, \lambda(a)))$  for the payoff of player 1 of type  $i \in \{L, H\}$  if he sends a deviation signal  $a \neq a_i^*$  and plays his best response to  $e_2(a, \lambda(a))$ .

We show that  $a' = \hat{a}$ . Suppose, to the contrary, that  $a' \in (\hat{a}, \bar{a}]$ . We demonstrate that this candidate equilibrium can be eliminated by using the IC. The IC means that if there exists a deviation  $a$  such that either

$$\pi_{1L}^* > \pi_{1L}(a, e_2(a, \lambda)) \quad \text{for all } \lambda(a) \in [0, 1] \tag{A.12}$$

or

$$\pi_{1H}^* > \pi_{1H}(a, e_2(a, \lambda)) \quad \text{for all } \lambda(a) \in [0, 1], \tag{A.13}$$

but not both, then  $\lambda(a) = 1$  (if (A.12) is true), or  $\lambda(a) = 0$  (if (A.13) is true). Because it is most tempting to deviate when the out-of-equilibrium beliefs are equal to unity, (A.12) and (A.13) reduce to, respectively,

$$\pi_{1L}^* > \pi_{1L}(a, e_2(a, 1)) \tag{A.14}$$

and

$$\pi_{1H}^* > \pi_{1H}(a, e_2(a, 1)). \tag{A.15}$$

Using  $\pi_{1L}^* = \pi_{1L}(\hat{a}, e_2(\hat{a}, 1))$ , we find that (A.14) holds if and only if  $a > \hat{a}$ . Further, (A.15) holds if and only if  $a > a'$ . Since we suppose that  $a' \in (\hat{a}, \bar{a}]$ , it now follows from the IC that  $\lambda(a) = 1$  for all  $a \in (\hat{a}, a')$ . Consequently, the high type will deviate to a lower value of  $a$ , breaking the candidate equilibrium. Thus,  $a' = \hat{a}$  in any equilibrium that satisfies the IC.

Applying Proposition 1, we now obtain a separating equilibrium with  $a_L^* = 0$ ,  $a_H^* = \hat{a}$ , and  $\lambda(a) = 0$  if  $a \in (0, \hat{a})$  and  $\lambda(a) \in [0, 1]$  if  $a > \hat{a}$ . Because  $a_H^* = \hat{a}$ , we see that both (A.14) and (A.15) hold for  $a > \hat{a}$ , and the IC puts no additional restrictions on  $\lambda(a)$  for these values of  $a$ . ■

*Proof of Proposition 2:* Let  $V_L V_H = 1$ . Then, there is no separating equilibrium because the efforts of the low type and high type would be the same.

Next, let  $V_L V_H < 1$ . Using Lemma 2, we then know that in a separating equilibrium (if any), we must have  $a_L^* \in (0, V_L]$  and  $a_H^* = 0$ . Consider a candidate-separating equilibrium with  $a_L^* = a_0$  and make a graph with  $a$  and  $e_2$  along the horizontal and vertical axes, respectively (similar to Figure 1). Draw the indifference curve,  $I_H$ , of the high type through the point  $(0, e_2(0, 1))$  associated with the candidate equilibrium. Since in equilibrium, the high type should not be tempted to mimic the low type, the point  $(a_0, e_2(a_0, 0))$  must be located above  $I_H$ . Let  $I'_L$  be the indifference curve of the low type through  $(a_0, e_2(a_0, 0))$ , and  $I''_L$  be this player's indifference curve through  $(0, e_2(0, 1))$ . Using the single-crossing property discussed in Section 3, it follows that the low type prefers  $I''_L$  to  $I'_L$ . In particular, the low type prefers  $(0, e_2(0, 1))$  to  $(a_0, e_2(a_0, 0))$ , which breaks the candidate equilibrium. ■

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