Robust Synchronization and Model Reduction of Multi-Agent Systems

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SUMMARY

This thesis considers two important problems in the theory of networked multi-agent systems: the problem of robust consensus and synchronization and the problem of model reduction of networked systems. We will now give a summary of the problems discussed in this thesis.

A networked multi-agent system consists of a group of subsystems called agents, which are interconnected with each other. The agents are interconnected according to a certain communication topology. This topology is represented by a graph: the network graph. In this graph, the nodes represent the agents and the communication links are represented by the edges.

First, we consider the problem of robust synchronization of networks with uncertain agent dynamics. In this problem, the nominal dynamics of all the agents in the network is given by a common linear input-output system. For each of the agents in the network, this nominal system is then perturbed by a coprime factor perturbation. The goal is to find a communication protocol that robustly synchronizes the network for all such perturbations. This means that for all allowable perturbations, the states of all the agents converge to a common trajectory.

If the network graph is undirected, then robust synchronization is equivalent to simultaneous robust stabilization of a single plant model, closely related to the nominal dynamics, by a given set of stabilizing controllers. This allows us to utilize important results from the theory of robust control, such as the small gain theorem. In the case of a directed network graph, then still it can be shown that robust synchronization can be achieved by solving a simultaneous stabilization problem.

We establish communication protocols that achieve robust synchronization for undirected networks with heterogeneous perturbations and directed networks with homogeneously perturbed agent dynamics. The protocols achieve robust synchronization for a certain achievable interval. For directed networks, the supremum of this interval is proportional

to the ratio of the smallest real part and largest modulus of the nonzero eigenvalues of the Laplacian matrix of the network graph. In the case that the network graph is undirected, the Laplacian eigenvalues are real. It can then be shown that the supremum of the achievable interval is proportional to the square root of the ratio of the smallest and largest nonzero Laplacian eigenvalues.

Next, we investigate two different methods for model reduction of multi-agent systems. We first consider the problem of model reduction by clustering for leader-follower networks. In these networks, a subset of the agents receives an external output. These agents are called the leaders of the network. The other agents are called the followers and can only communicate with other agents in the network. The goal is to accurately approximate these networks by lower dimensional models, while preserving some of the structure of the network.

We establish a model reduction technique for networks with arbitrary higher dimensional agent dynamics that uses a particular class of graph partitions called almost equitable partitions. The agents in the network are clustered according to a almost equitable partition of the network graph, thus reducing the number of agents and in turn the model order of the entire network. If the network is clustered according to an almost equitable partition, then its consensus properties are maintained. We provide upper bounds on the modeling error in both the $\mathcal{H}_2$ and $\mathcal{H}_\infty$-norm. The provided upper bounds depend on the number of cellmates of the leaders, the eigenvalues of the Laplacian matrix, and on an auxiliary model closely related to the agent dynamics.

Since finding almost equitable partitions of a given network graph is computationally expensive, we also investigate the approximation error if the network is clustered according to an arbitrary partition. We first compute a graph that optimal approximates the original network graph, such that the given partition is an almost equitable partition of the reduced graph. Then, we can compute an upper bound using the triangle inequality and the earlier obtained bounds.

Finally, we consider a different approach to model reduction of networks. In contrast to the previous model reduction technique, this technique is not based on reducing the number of nodes in the network, but reducing the number of edges instead. Given a undirected network where the agent dynamics is given by a arbitrary symmetric linear input-output system, we reduce the network by removing all edges from the graph that close the cycles.

We establish necessary and sufficient conditions under which both the original and reduced network reach consensus. We provide explicit expressions for the resulting $\mathcal{H}_2$-approximation error which depend on the signed path vectors describing the removed cycles and the eigenvalues and eigenvectors of the edge Laplacian matrix of both the original and the reduced network.

If the reduced network graph is a star graph, the expressions for the approximation error can be greatly simplified. We provide these reduced expressions and show that in this case, the approximation error is independent of the number of agents in the network. We further show that earlier results on networks with single integrator agent dynamics follow directly from the more general results in this thesis.