Robust Synchronization and Model Reduction of Multi-Agent Systems

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Publication date:
2017

5 CONCLUSIONS

In this thesis we have considered the problems of robust synchronization of networks with uncertain agent dynamics, and the problem of model reduction for networks. In this chapter, we discuss the main contributions and findings presented in Chapters 2–4. Finally, we provide an outlook on some further research opportunities.

5.1 CONTRIBUTIONS

In Chapter 2, we have considered the problem of robust synchronization of undirected and directed multi-agent networks with uncertain agent dynamics. For the agents in the network, the nominal agent dynamics was given by the same linear input-output system. For each agent this nominal dynamics was then perturbed by coprime factor perturbations. We have provided an achievable interval for the values of the $\mathcal{H}_\infty$-norm of the perturbations. We provided methods to construct a dynamic observer-based protocol that achieves consensus robustly. It was shown that in the case of undirected networks with coprime factor perturbed agent dynamics, robust synchronization is equivalent with simultaneous robust stabilization of a single uncertain plant system by $N-1$ controllers, where N is the number of agents in the network. For directed networks, the latter condition implies the former.

For the dynamic protocol proposed in Chapter 2 and for directed network, the supremum of the achievable interval is proportional to the quotient of the smallest real part and the largest modulus of the eigenvalues of the graph Laplacian. If the network is undirected, then the Laplacian eigenvalues are real and we have shown that in this case the supremum of the achievable interval is proportional to the square root of the quotient of the smallest and largest eigenvalues of the Laplacian.

In Chapter 3, we have extended results on clustering based model reduction for leader-follower networks with single integrator agent dynamics to network with arbitrary linear multivariable agent dynamics. The proposed model reduction technique achieve a reduction of the complexity of the network topology by clustering the agents according to an almost equitable partition of the network graph. We have shown that if we reduce the original undirected network according to a specific Petrov-Galerkin projection, then the resulting reduced system can be interpreted as a weighted, directed networked multi-agent system. It was shown that if the network is clustered according to an almost equitable partition, then consensus of the network is preserved. For both the $\mathcal{H}_2$- and the $\mathcal{H}_\infty$-approximation error, we have provided a priori upper bounds. These error bounds are dependent on a auxiliary system which is closely related to the agent dynamics, the Laplacian eigenvalues of the original and reduced network graphs, and the number of cellmates of the leaders. It was shown that the approximation is exact if each leader occupies a cell by itself.

Chapter 3 also briefly considered the case the the agents in the network are clustered according to a general, non-almost equitable partition. We have provided some insight into this problem and have shown how one can obtain upper bounds on both the $\mathcal{H}_2$- and $\mathcal{H}_\infty$-approximation errors by first computing an optimal approximating network for which the chosen partition is almost equitable, and then using the triangle inequality to bound the approximation error.

In Chapter 4, we have introduced a new method for model reduction of networked multi-agent systems where the agent dynamics is given by an arbitrary higher-dimensional symmetric system. This method is based on removing the edges in the network graph that close the cycles. We establish explicit expressions and upper bounds for the $\mathcal{H}_2$-approximation error and give necessary and sufficient conditions for the consensus properties of the original network to be preserved. The approximation error has been expressed in terms of the eigenvectors of the edge Laplacian matrices of the original and reduced network graphs. The presented results are applicable in the case that either a single cycle, or multiple uncorrelated cycles are removed from the graph. We have shown that if the reduced network graph is a star graph, then the obtained expression can be greatly simplified, and the approximation error is independent of the number of agents in the network.

5.2 OUTLOOK

As is common in science, this thesis poses more questions than it can answer. Some interesting topics for future research will now be discussed.

There are few topics in the theory of networked systems that have seen as much interest as the problems of consensus and synchronization. Still, there are opportunities for future research. In the problem of robust synchronization for directed networks especially, finding protocols that allow for non-homogeneous perturbations seems to remain an open problem. This is the case for the protocols presented in this thesis, as well as for existing results on robust synchronization of networks where the agent dynamics are perturbed using additive perturbations.

In this thesis, we have provided upper bounds on the $\mathcal{H}_2$- and $\mathcal{H}_\infty$-approximation errors when clustering the network according to an almost equitable partition of the network graph. In general, graphs may not always have non-trivial almost equitable partitions. We have provided a method to obtain an upper bound on the approximation error in this case. Future research could investigate whether this bound is sharp or could provide different methods for obtaining an upper bound on the modeling error if the network is clustered according to an arbitrary partition.

In the context of cycle-removal based model reduction, this thesis focuses on the case that the reduced network graph is a tree graph. One can use the triangle inequality together with the presented results to obtain upper bounds on the approximation error for general reduced graphs, such as the approximation error when removing an arbitrary cycle. Again, the sharpness of the bounds obtained in this manner could be established after further investigation.

The expressions for the approximation error in the case of cycle-removal require the computation of the eigenvalues and eigenvectors of the Laplacian matrices of the original and reduced networks. For networks with a large number of nodes, this might prove too computationally expensive. Combining the results in this thesis together with results on graph simplification might allow us to compute upper bounds that do not require the computation of (all) the eigenvalues and eigenvectors of both Laplacian matrices. In the case of single integrator agent dynamics, these upper bounds have already been found.

In the case of model reduction by clustering, we have provided upper bounds on the $\mathcal{H}_2$- and $\mathcal{H}_\infty$-approximation error. For model reduction by cycle-removing, only expressions for the $\mathcal{H}_2$-approximation error have been provided. Since the $\mathcal{H}_\infty$-approximation error can be interpreted as a kind of *worst case* error, finding expressions for this error can be a worthwhile venture.

In our chapter on cycle-removal, we have restricted our analysis to networks where the agent dynamics is given by an arbitrary symmetric system. This restriction was made for technical reasons. Future research could extend the results in this thesis to networks with arbitrary linear input-output systems as agent dynamics. Furthermore, we assume the network graph is undirected, but assign a orientation to every edge. If we start with a directed network, are the results still applicable and if this is not the case, can we extend the obtained results to this situation?

Finally, combining the two model reduction techniques that have been investigated in this thesis might lead to low-dimensional, but accurate approximating systems. Finding almost equitable partitions of a general graph is a complex combinatorial problem. First reducing the network to a three graph and only then searching for an almost equitable partition might prove a more feasible approach.