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Robust Synchronization and Model Reduction of Multi-Agent Systems

Jongsma, Hidde-Jan

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ROBUST SYNCHRONIZATION
AND MODEL REDUCTION
OF MULTI-AGENT SYSTEMS

HIDDE-JAN JONGSMA



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 groningen

The research for this doctoral dissertation has been carried out at the Faculty of Mathematics and Natural Sciences, University of Groningen, The Netherlands, within the Johann Bernoulli Institute for Mathematics and Computer Science.



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Hidde-Jan Jongsma

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