Polarization leakage in epoch of reionization windows

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Polarization Leakage in Epoch of Reionization Windows

Proefschrift

ter verkrijging van de graad van doctor aan de Rijksuniversiteit Groningen
op gezag van de rector magnificus prof. dr. E. Sterken
en volgens besluit van het College voor Promoties.

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# Contents

1 **Introduction** .................................................. 1

1.1 **Observing the EoR signal** ............................... 2

1.1.1 Obstacles in detecting the EoR signal .................. 4

1.2 **An overview of astropolarimetry** ....................... 6

1.2.1 Polarization ellipse ...................................... 7

1.2.2 Poincaré sphere .......................................... 8

1.2.3 Jones calculus ............................................. 10

1.2.4 Stokes parameters ....................................... 10

1.2.5 Mueller calculus .......................................... 11

1.2.6 Polarization leakage ..................................... 12

1.3 **Overview of this thesis** ................................. 13

1.3.1 Motivation .................................................. 13

1.3.2 Structure of the thesis ................................... 14

<table>
<thead>
<tr>
<th>Part One</th>
</tr>
</thead>
</table>

**Introduction** .................................................. 19
# 2 Simulating EoR observations

## 2.1 Mathematical model of a radio interferometer

- **2.1.1 Mueller formalism**
- **2.1.2 Stokes visibilities**

## 2.2 Systematic effects

- **2.2.1 Direction independent effects**
- **2.2.2 Direction dependent effects**

## 2.3 Calibration and imaging

- **2.3.1 AW-projection**

## 2.4 Flux conversion

## 2.5 Rotation measure synthesis

## 2.6 Power spectrum analysis

- **2.6.1 2D power spectrum**
- **2.6.2 3D power spectrum**

## 2.7 Conclusion

# 3 Extragalactic foreground

## 3.1 Pipeline

## 3.2 Direction independent errors

## 3.3 Direction dependent errors

- **3.3.1 Test with a mock sky**
- **3.3.2 3C196 field**
- **3.3.3 Correcting polarization leakage of point sources**

## 3.4 Selfcal errors due to incomplete sky model

## 3.5 Conclusion

# 4 Galactic foreground

## 4.1 Simulation setup

## 4.2 Results

- **4.2.1 RM synthesis**
- **4.2.2 3D Power spectra**

## 4.3 Polarization leakage removal

## 4.4 Conclusion

# Part Two

## 5 Accuracy of the beam model

## 5.1 Introduction

## 5.2 Primary beam model of LOFAR
# Data processing and simulation pipelines

## Observations

## Flagging and averaging

## Calibration

## Imaging

## Simulated observations

## Source flux extraction

## Figures of merit

## Rotation Measure Synthesis

## Direction dependent calibration

# Results

## Observed polarization leakage

## Predicted polarization leakage

## Accuracy of the beam model

## Direction dependent calibration

# Discussion and Conclusions

## Observed polarization leakage in wide fields

## Predicted polarization leakage in wide fields

## Fractional leakage in wide fields

## Discussion

## Conclusion

## Conclusions and outlook

## Conclusions

## Primary beam model of LOFAR

## Extragalactic foreground

## Galactic polarized foreground

## Effects of leakage on EoR window

## Leakage within wider fields

## Removing or avoiding polarization leakage

## Outlook

## Future prospects
One of the main tasks of modern observational cosmology is to make a spatio-temporal map of the whole universe in all its details, thereby constraining the parameters of various cosmological models. The frontier of this exciting research field has been steadily shifting, going back and forth in cosmic time, for the last few decades. First, the frontier was delimited by the $z \sim 1100$ universe, i.e. the recombination epoch (RE), observable through the cosmic microwave background (CMB). There were gaps in the map immediately before and after the RE, which meant that we could understand neither how the temperature fluctuations in the CMB arose, nor how those fluctuations gave rise to the large-scale structures of the universe. Naturally, the next frontiers were set by the boundaries of these gaps giving rise to observational inflationary cosmology to probe the origins of the fluctuations and observational 21-cm cosmology to probe their aftermath.

Both cosmological probes, the 21-cm signal emitted by neutral hydrogen during the epoch of reionization (EoR signal) and the polarization signature on the CMB imprinted by primordial gravitational waves, are heavily impeded by the foregrounds, especially the Galactic diffuse foreground. This thesis is concerned with the former. In this thesis, we study the effects of the Galactic polarized foreground and the systematics of the Low Frequency Array (LOFAR) on the observations of the EoR signal. Here, we first give an overview of observational 21-cm cosmology, and mention the main obstacles in detecting the signal. Because we are mainly concerned with instrumental polarization leakage among the various obstacles, we give a brief overview of astropolarimetry in the context of radio interferometers, and describe the general mechanism of the leakage. Finally, we describe the motivation and structure of this thesis.
1.1 Observing the EoR signal

According to the standard cosmological model, our universe originated around 13.8 billion years ago (Planck Collaboration et al., 2016), and it is expanding and cooling following the laws of general relativity and thermodynamics since then. Hydrogen is a major source for the history of the universe, as all major phases of the information about the universe are imprinted on it.

During the first few thousand years, hydrogen and helium, which were created by primordial nucleosynthesis and constituted almost all baryonic matter at that time, were ionized. The universe was a hot, dense plasma. According to a generic prediction of the inflationary theory, there were small density perturbations in the primordial matter that obey Gaussian statistics with an almost scale-invariant initial power spectrum.

At \( z \sim 1100 \), free electrons and protons combined to produce neutral hydrogen, and the thermalized photons that were previously being scattered from the free electrons were set free. We can still detect these primordial photons; they follow a black body spectrum at microwave frequencies with a temperature of \( \sim 2.7 \) K. This signal is known as ‘Cosmic Microwave Background’ (CMB), and the primordial density fluctuations are imprinted on it.

The era characterized by \( 1100 \gtrsim z \gtrsim 30 \) is called the ‘Dark Ages’ (Madau & Dickinson, 2014; Bromm & Yoshida, 2011) as there were no sources of light during that time. Although the primordial density fluctuations are not very well-understood, their subsequent growth after the epoch of recombination is well-understood. The primordial density fluctuations gave rise to the first structures in the universe, as matter collapsed under gravity in over-dense regions. There is remarkable agreement between the perturbations imprinted on CMB, and the large-scale structure (LSS) of the universe.

Between \( 30 \gtrsim z \gtrsim 10 \), the first structures formed, and this epoch is called the ‘Cosmic Dawn’ (Pritchard & Loeb, 2012). The first structures were ‘population III’ stars and black holes that emitted energy at ultraviolet wavelengths. Even this energy could not light up the universe properly, as they were absorbed by hydrogen.

Between \( 10 \gtrsim z \gtrsim 5 \), bulk of hydrogen were reionized by ultraviolet radiation (Mesinger, 2016, pp. 1–2), and this epoch is called the ‘Epoch of Reionization’ (Furlanetto et al., 2006; Zaroubi, 2013). The boundaries of the EoR is highly uncertain, and they have been determined via indirect methods, specifically by CMB polarization at the high-\( z \) end, and by absorption features in the quasar spectra at the low-\( z \) end. However, the new generation low-frequency, wide-bandwidth, wide-field radio telescopes operating at 100–200 MHz can directly probe the hydrogen during this epoch.

The 21-cm signal emitted by hydrogen during the epoch of reionization is redshifted to the aforementioned frequency range, and this redshifted radiation is called the ‘EoR signal’. Although this signal has not been detected yet, there are several ongoing and planned experiments to detect it using radio telescopes such as Giant Metrewave Radio Telescope (GMRT), Low Frequency Array (LOFAR), Murchison Widefield Array (MWA), Precision Array for Probing the EoR (PAPER), 21-cm Array (21CMA), and the planned Hydrogen Epoch of Reionization Array (HERA) and Square Kilometre Array (SKA).

These telescopes are or will be attempting to detect, and in some cases possibly map the ‘differential brightness temperature’ (\( \delta T_b \)) of the 21-cm signal with respect to CMB. \( \delta T_b \) is
1.1 Observing the EoR signal

Figure 1.1: Evolution of the expected EoR signal created by a semi-numeric simulation 750 Mpc on a side spanning the epochs of reionization and cosmic dawn, from a redshift of 37 to 5. In the upper panel, X-ray luminosity of the primordial galaxies have been assumed to be same as the present-day galaxies, while in the lower panel the primordial galaxies have been assumed to be much more efficient in generating X-rays. We can clearly see the three phases of the signal: at high redshifts the signal is in absorption (red), at the intermediate redshifts it goes into emission (blue), and finally it disappears when reionization is complete. Adapted from Valdés et al. (2013).

related to the radiation intensity $I(\nu)$ measured by the telescope as

$$I(\nu) = \frac{2\nu^2}{c^2} k_B \delta T_b$$

where $I(\nu)$ is calculated from Stokes $I$. The differential brightness temperature is defined as (Mesinger, 2016)

$$\delta T_b(\nu) = \frac{T_S - T_\gamma}{1 + z} \tau_{\nu_0}$$

where $T_S$ is the spin temperature characterized by the relative populations of the two spin states of hydrogen, $T_\gamma$ is the CMB temperature, and $\tau_{\nu_0}$ is the optical depth of hydrogen at the rest frequency $\nu_0$. If $T_S < T_\gamma$, the signal will be in absorption, and if $T_S > T_\gamma$, the signal will be in emission. Expected phase changes of hydrogen during the EoR are shown in Fig. 1.1. At higher redshifts (15 $\sim$ 37), i. e. during the dark ages and the beginning of the cosmic dawn, most of the hydrogen is neutral and the spin temperature, coupled to the cold gas, remains below the CMB temperature resulting in an absorption signal at 21-cm wavelength. During the peak of the reionization process, the signal is in emission, and finally it disappears when hydrogen becomes almost completely ionized at lower redshifts (Mesinger, 2016, pp. 240–280).

The magnitude of the EoR signal is low compared to the foreground and even the thermal noise. Current experiments are thus trying to detect the signal statistically, i. e. detecting the power spectrum or RMS of the signal is the main goal. Most of the current experiments are using interferometers, and interferometers, by definition, only measure the spatial fluctuations. So, in these experiments, not the total power but the spatial fluctuations of $\delta T_b$ as a function
of redshift will be measured. Fractional perturbation to the differential brightness temperature is defined as

\[ \delta_{21}(z) = \frac{\delta T_b(z) - \overline{\delta T_b}}{\overline{\delta T_b}} \]  

which is a zero-mean partially random field. Power spectrum is calculated from the Fourier transform of \( \delta_{21}(z) \), and it gives us the power of the EoR signal as a function of comoving scales, i.e. \( P_{21}(k) \).

### 1.1.1 Obstacles in detecting the EoR signal

The main obstacles in detecting the EoR signal are noise, foregrounds, and systematics. Noise is expressed by the system temperature of a telescope \( T_{sys} \) which includes contributions from the telescope, the receiver system, and the sky. The EoR signal to noise ratio is low for the current generation telescopes, but even greater challenge is posed by the foregrounds.

The radio sky at low frequencies is dominated by diffuse synchrotron (70%) and free-free emission (1%) from our Galaxy and integrated emission from extragalactic sources like radio galaxies and galaxy clusters (27%). Their combined emission is 4–5 orders of magnitude higher than the expected EoR signal. But as interferometers only measure fluctuations, we are affected less by the foregrounds. The ratio of the intensity fluctuations of the foregrounds and the EoR signal on arcmin to degree scales is 2–3 orders of magnitude. The extragalactic foregrounds appear as compact sources in the EoR observations, and can be removed comparatively easily. Removing the diffuse foregrounds is more challenging. There are two main approaches in dealing with diffuse foregrounds in the EoR experiments—one is the ‘foreground removal’ and the other is ‘foreground avoidance’.

To explain these approaches, we have to introduce the concept of ‘EoR window’ in the cylindrically averaged power spectrum (PS). A 3D PS of the EoR observations can be created by Fourier transforming the observed visibilities along the frequency or equivalently the redshift axis. The 3D cube thus produced will give us the power as a function of transverse \( (k_\perp) \) and line-of-sight (LOS, \( k_\parallel \)) wavenumbers. The 3D PS is then averaged in cylindrical bins to produce a 2D spectrum, where the horizontal and vertical axes represent \( k_\perp \) and \( k_\parallel \), respectively. A schematic diagram of such a PS is shown in Fig. 1.2, which is also called the instrumental \( k \)-space, as it is limited by instrumental parameters on all sides.

The low \( k_\perp \) scales are dominated by errors due to limited field of view of the antennae, and the highest \( k_\perp \) scale is determined by the longest baseline of the interferometer. The highest \( k_\parallel \) scale is determined by the spectral resolution of an instrument. Low \( k_\parallel \) scales are in principle limited by cosmic variance, but in practice the lowest LOS scale is determined by the spectral bandwidth. Lower \( k_\parallel \) scales will be dominated by diffuse foreground, because this foreground is mostly contributed by the Galactic synchrotron emission which is very smooth along frequency. However, as we go toward higher \( k_\perp \) scales, the foregrounds leak into higher \( k_\parallel \) scales creating a wedge-shaped region in the Fourier plane. Therefore, along with the low \( k_\parallel \)-modes, all modes within the wedge are most likely to be contaminated by the foregrounds if they are not removed. The region least contaminated by the foregrounds in the instrumental \( k \)-space is called the ‘EoR window’ as the EoR signal is expected to be the dominant component in this window besides thermal noise. Now to come back to the
1.1 Observing the EoR signal

Figure 1.2: A schematic diagram of the EoR window in the Fourier plane created by $k_\perp$ and $k_\parallel$ wavenumbers. This plane is also called the instrumental $k$-space as it is delimited by on all sides by instrumental parameters. From Liu et al. (2014a).

two approaches mentioned earlier, the ‘removal’ approach focuses on removing foregrounds as much as possible, whereas the ‘avoidance’ approach tries to avoid the regions affected by foregrounds in the $k$-space. Removal is always better, but it is not always possible to characterize and remove leakage completely in which case avoidance would be the only option.

There is another potential contaminant of the EoR window. As mentioned earlier, this thesis is mainly concerned with the contamination of the EoR signal by the leakage of polarized emission into total intensity. The mechanism of this will be explained in the next section. The Galactic synchrotron emission is spectrally smooth, but its polarized part might have fluctuations along frequency because of Faraday rotation along the line of sight by the intervening magnetized plasma. A fraction of the spectrally fluctuating polarized diffuse emission will leak into Stokes $I$ because of the polarized primary beam of the observing instrument. The EoR signal is measured in Stokes $I$ and it also fluctuates along frequency, which can be understood from Fig. 1.1—$\delta T_b$ varies significantly along redshift which is equivalent to the frequency axis. Therefore, the leakage might contaminate the signal.

Jelić et al. (2010) showed the potential effects of leakage on the EoR signal by simulating both an EoR signal and a polarized Galactic diffuse emission. Fig. 1.3 (figure 7 of Jelić et al. 2010) shows the spectral structure of a random line of sight though a simulated EoR signal (solid line). The residual leakage (after calibration) of the ‘model D’ of the simulated polarized emission is shown by the dotted line. The model of the polarized emission was created by considering that along the line of sight there are three types of regions: a region where only cosmic-ray electrons are present, a region where only thermal electrons are present, and a region where both types of electrons are mixed together. It was assumed that
Figure 1.3: Spectral structure of a random line of sight through a simulated 21-cm data cube (solid line). The dotted line shows the leakage of simulated polarized Galactic diffuse emission considering 0.15% residual leakage after calibration. The dashed line shows the sum of the 21-cm signal and the leakage. From Jelić et al. (2010).

the resulting residual leakage is 0.15% considering only the geometric projection effects of LOFAR, i.e. 0.15% of the polarized emission is leaked into Stokes $I$. The dashed line shows the sum of the leakage and the EoR signal. If the behavior of leakage is like this in reality, and if this leakage is not removed, we might mistake the polarization leakage to be the EoR signal. This can also be understood in terms of the cylindrical PS. If the leakage indeed has similar spectral structure as the EoR signal, the EoR window shown in Fig. 1.2 will be contaminated by the leakage to some extent.

1.2 An overview of astropolarimetry

Maxwell’s equations give rise to a set of six wave equations for each of the three Cartesian components of the electric ($E$) and magnetic ($B$) fields.\(^1\) If the solution of a sinusoid in a free space is considered, the velocity of the wave is found to be

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

(1.4)

where $\varepsilon_0$ is the permittivity and $\mu_0$ the permeability of free space. Such a wave can be completely described using four parameters:

1. intensity which is proportional to the amplitude of the electric field squared, $E^2$,
2. frequency, $\nu$,
3. direction of propagation, i.e. the Poynting vector $E \times (B/\mu_0)$, and
4. orientation of the electric or magnetic vectors with respect to some reference frame.

Because astronomical observations are largely based on the observations and analyses of electromagnetic waves coming from space, it is not surprising that an independent area of observational astronomy has been dedicated to make time-dependent measurements of each of these four parameters. We know these subject areas as photometry, that measures the intensities, spectrometry, that deals with frequency behaviors, astrometry, that determines the direction of the associated Poynting vectors, and finally polarimetry, that analyzes the

\(^{1}\) $B = \mu_0 H$ where $B$ is the magnetic flux density and $H$ is the magnetic field intensity. We have used $B$ instead of $H$ throughout the thesis.
orientation of the different components of the electric fields. In this thesis, we are interested in astronomical polarimetry or ‘astropolarimetry’ in general, but more specifically ‘radio astropolarimetry’, and even more specifically ‘radio interferometric astropolarimetry’. Much of polarimetry has little to do with astronomy, and thus the bulk of the discussion in this section will be about polarimetry in general, albeit in the context of astronomical waves.\(^2\)

### 1.2.1 Polarization ellipse

The electric and magnetic field vectors in an EM wave are orthogonal, hence equivalent in terms of polarization. So an EM wave can be described using either the electric or the magnetic field vector. But the electric field vector is almost always preferred, because this component interacts most with matter, making it easier to detect with a telescope. Let us consider an EM wave traveling along the \(z\)-axis of a Cartesian frame, where the electric vector is vibrating in the \(xz\)- and \(yz\)-planes. At a certain point along \(z\), the \(x\) and \(y\) components of this vector will take the forms

\[
\begin{align*}
E_x &= E_{x0} \exp[i(2\pi vt + \delta_x)] \\
E_y &= E_{y0} \exp[i(2\pi vt + \delta_y)]
\end{align*}
\]  

(1.5)

where \(E_{x0}\) and \(E_{y0}\) are the amplitudes, and \(\delta_x\) and \(\delta_y\) are the phases. These equations describe the form of the electric field vibration in a plane perpendicular to \(z\), i.e., the \(xy\)-plane, and this is the most interesting plane for describing polarization. If a free electron is placed in this plane, it will be forced by the oscillating electric field to curve out an orbit. The locus of this orbit falls in the family of Lissajous curves.\(^3\) The curve is created by two orthogonal forces with different amplitudes oscillating at the same frequency with a constant phase difference, and it takes the form of, in general, an ellipse, called the *polarization ellipse*. The form of the ellipse, derived from the real parts of Eqns. 1.5, is

\[
\frac{E_x^2}{E_{x0}^2} + \frac{E_y^2}{E_{y0}^2} - \frac{2E_xE_y\phi}{E_{x0}E_{y0}} = \sin^2 \phi
\]  

(1.6)

where \(\phi = \delta_y - \delta_x\) is the phase difference. The equation clearly shows that the form of polarization can be described using only four parameters: \(E_{x0}, E_{y0}, \delta_x\) and \(\delta_y\). These quantities cannot be measured directly; instead, various combinations of them are usually measured, which will be described later. Linear and circular polarizations are two special cases of the elliptical polarization described by this ellipse. For both linear and circular polarizations, the amplitudes of the two components have to be the same, i.e. \(E_{x0} = E_{y0}\). Their difference is determined by the phases—for linear polarization \(\phi = 0\) and for circular polarization \(\phi = \pm \pi/2\). Moreover, the \(\mathbf{E}\)-vector rotates in a clockwise direction if the phase difference is positive, and in an anti-clockwise direction if the phase difference is negative. An instantaneous snapshot of the \(\mathbf{E}\)-vector as it is distributed along the \(z\)-axis follows a helical pattern. To explain the emergence of the polarization ellipse in another way, it can be said that this helix does not rotate but only moves forward along the \(z\)-axis with time, and its point of intersection with the \(xy\)-plane at \(z = 0\) traces out the ellipse.

---

\(^2\)Section 1.1 has been written following two books: Clarke (2010) and Thompson et al. (2001).

\(^3\)First studied by Nathaniel Bowditch in 1815 and later in more detail by Jules-Antoine Lissajous in 1857. For an example of a Lissajous curve see the title sequence of Alfred Hitchcock’s film *Vertigo* (1958).
Chapter 1. Introduction

Figure 1.4: The polarization ellipse. Its major \( a \) and minor \( b \) axes do not coincide with the axes of measurements, i.e. the \( xy \) axes. The major axis is at an angle of \( \chi \) with the \( x \)-axis, and \( \tan \beta \) is the ratio of the major and minor axes. From Clarke (2010).

Another way to describe the polarization ellipse is to use the geometry of the ellipse itself. The axes for measuring the \( x \) and \( y \) components of the \( \mathbf{E} \)-field do not coincide with the major and minor axes of the polarization ellipse, but a relationship between them can be established. If \( a \) and \( b \) are the major and minor axes of the ellipse (as seen in Fig. 1.4), and the \( x \)-axis subtends an angle of \( \chi \) with the major axis, and the ratio of the major and minor axes is \( \tan \beta \), then the relations between the two systems of representing the polarization ellipse would be

\[
\begin{align*}
\tan 2\chi &= \frac{2E_{x0}E_{y0}\cos\phi}{E_{x0}^2 - E_{y0}^2} \\
\sin 2\beta &= \frac{2E_{x0}E_{y0}\sin\phi}{E_{x0}^2 + E_{y0}^2}.
\end{align*}
\] (1.7)

Here, the aforementioned four parameters \( E_{x0}, E_{y0}, \delta_x \) and \( \delta_y \) have been combined to produce a set of two parameters \( \chi \) and \( \beta \) that can completely characterize the polarization ellipse—\( \chi \) is the azimuth of the major axis, and \( \beta \) determines the ellipticity and rotational properties of the ellipse. These terms are physically more meaningful, and a further useful aspect is that they are presented in terms of intensity, which is measurable. The factor \((E_{x0}^2 + E_{y0}^2)\) in Eqns. 1.7 is the total intensity of the EM wave, denoted by \( I \).

1.2.2 Poincaré sphere

The previous section shows that any polarization state can be described by the two double-angles \( 2\chi \) and \( 2\beta \). These angles and the \( I = (E_{x0}^2 + E_{y0}^2) \) can be used to map polarization forms onto the surface of a sphere situated in a polar coordinate system with longitude \( \theta = 2\chi \) where \( 0^\circ \leq \chi \leq 360^\circ \), and latitude \( \phi = 2\beta \) where \(-90^\circ \leq 2\beta \leq +90^\circ \). This sphere is called the Poincaré sphere, where each point \( \mathbf{P}(\theta, \phi) \) corresponds to a unique polarization form.

Different polarizations are represented by different regions of this sphere:

1. The polarizations mapped around the equator, where \( \phi = 0 \), are linear polarizations.
The horizontal linear polarizations are mapped at $\theta = 0^\circ$, and the vertical ones at $\theta = 180^\circ$ at the equator.

2. Those mapped at the poles, where $\theta = 90^\circ$, are circular.

3. The polarizations mapped by all other regions are elliptical. The upper and lower hemispheres correspond to the clockwise and anti-clockwise rotations of the $E$-vector as seen by the observer.

The radius vector representing a particular polarization form is equivalent to the eigenvector of that form. The polarization forms at two opposite ends of a diameter of this sphere have opposite eigenvectors, and are said to be orthogonal; they have perpendicular azimuths but identical ellipticity.

It is interesting to define three new parameters,

$$I_{\cos} = 2E_x E_y \cos \phi = I \cos 2\beta \sin 2\chi$$

$$I_{\sin} = 2E_x E_y \sin \phi = I \sin 2\beta$$

$$I_{\text{diff}} = E_x^2 - E_y^2 = I \cos 2\beta \cos 2\chi.$$  \hspace{1cm} (1.10)

If we consider a Cartesian coordinate system $(I_{\text{diff}}, I_{\cos}, I_{\sin})$, then the polar coordinates of the Poincaré sphere of radius $I$ will be related to these parameters by

$$\theta = \tan 2\chi = \frac{I_{\cos}}{I_{\text{diff}}}$$  \hspace{1cm} (1.11)

$$\phi = \sin 2\beta = \frac{I_{\sin}}{I}.$$  \hspace{1cm} (1.12)
because the radius

\[ I^2 = I_{\text{diff}}^2 + I_{\cos}^2 + I_{\sin}^2. \]  

(1.13)

The concept of polarization ellipse, described in the previous section, is developed by combining two orthogonal waves, that are considered to have a long-term phase coherence so that the form of the ellipse does not change over a long period of time. But if the amplitude and phase of the waves change, which they do in reality, the ellipse will change its form. This transformation can be described using the Poincaré sphere. But this description becomes complicated if the radiation is also attenuated by the transformations. A more general description of the transformations is used in practice.

### 1.2.3 Jones calculus

An electromagnetic wave can be transformed by either the intervening medium between the source and the observer, or the observing instrument. The transformations by various intervening and instrumental processes are most easily described by Jones matrices. The most basic transformation is the rotation of the coordinate frame. The radiation can also be subjected to absorption and phase delays as it passes through a medium or instrument. If an electromagnetic wave \( \mathbf{E} \) is traveling toward the \( z \)-axis of a Cartesian coordinate system designated by \( (x, y, z) \), then the rotation of the \( (x, y) \) frame by an angle \( \gamma \) to a new frame \( (x', y') \), and the absorption and phase delay of the radiation in the new frame can be represented as

\[
\begin{bmatrix}
E_{x'} \\
E_{y'}
\end{bmatrix} =
\begin{bmatrix}
k_{x'} & 0 \\
0 & k_{y'}
\end{bmatrix}
\begin{bmatrix}
e^{-i\Delta_{x'}} & 0 \\
0 & e^{-i\Delta_{y'}}
\end{bmatrix}
\begin{bmatrix}
\cos \gamma & \sin \gamma \\
-\sin \gamma & \cos \gamma
\end{bmatrix}
\begin{bmatrix}
E_x \\
E_y
\end{bmatrix}
\]  

(1.14)

where \( k_{x'}, k_{y'} \) are the amplitude transmission coefficients responsible for absorption, and \( \Delta_{x'}, \Delta_{y'} \) are the phase retardations. The first three matrices on the right-hand side of this equation represent the absorption, phase delay, and coordinate transformations respectively. This formulation is known as Jones calculus, after R. Clark Jones (see Jones, 1941).

In a Jones matrix, the column vectors describing the vertical and horizontal components of a wave is called a Jones vector. Jones vectors describe orthogonal coherent waves that retain a constant phase difference. In reality, astronomical waves do not behave this way. In an electromagnetic radiation coming from the sky, many waves are present simultaneously, and their electric field vector provides a distribution of phases and orientations. In time, some components of the wave are replaced by other components, and their phase coherence is not maintained. If snapshot polarization ellipses of these waves could be recorded, they would show the change of ellipticity and orientations. In reality, we measure the expectation values of the parameters describing the ellipse over the observing time. Stokes parameters, introduced in the next section, are very convenient to represent these values.

### 1.2.4 Stokes parameters

Stokes parameters represent the expectation values of the parameters describing a polarization ellipse over the integration time of an observation. The Stokes parameters are the time averages of \( I_{\text{diff}}, I_{\cos}, I_{\sin} \) and \( I \) introduced above. The four Stokes parameters for an
1.2 An overview of astropolarimetry

integration time of $t$ and averaged over frequency $\nu$, can be written as

$$I = \langle E_{x0}^2 + E_{y0}^2 \rangle_{\nu,t}$$

(1.15)

$$Q = \langle E_{x0}^2 - E_{y0}^2 \rangle_{\nu,t}$$

(1.16)

$$U = \langle 2E_{x0}E_{y0}\cos \phi \rangle_{\nu,t}$$

(1.17)

$$V = \langle 2E_{x0}E_{y0}\sin \phi \rangle_{\nu,t}.$$  

(1.18)

These parameters were first introduced by George Stokes in 1852, and they were introduced to astronomy by Chandrashekhar in 1947.

If the fluctuations of the electric field vector have no time coherence, $Q = U = V = 0$ and the wave is said to be completely unpolarized. The values of $Q, U, V$ will change according to the degree of coherence over the integration time. If the degree of coherence is maximum, the wave is completely polarized and in that case

$$I = \sqrt{Q^2 + U^2 + V^2}$$

(1.19)

similar to equation 1.13. Astronomical waves are never completely polarized, and they follow the condition

$$I \geq \sqrt{Q^2 + U^2 + V^2}.$$  

(1.20)

The degree of polarization is measured by

$$d = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}.$$  

(1.21)

The degree of linear polarization

$$p = \frac{\sqrt{Q^2 + U^2}}{I}$$

(1.22)

and the polarization angle

$$\chi = \frac{1}{2} \tan^{-1} \frac{U}{Q}.$$  

(1.23)

The level of linear polarization $P = Q + iU$ and thus $Q = P\cos 2\chi$ and $U = P\sin 2\chi$. The Stokes parameters are usually expressed as a four-element column vector, known as Stokes vector $(I, Q, U, V)^T$.

1.2.5 Mueller calculus

Stokes vector suffers change as the wave interacts with the intervening medium and the instrument. These changes can be expressed by a matrix multiplication of the Stokes vector with a $4 \times 4$ matrix, known as Mueller matrix. The relation between the initial Stokes vector $\{I_i, Q_i, U_i, V_i\}$ and the final Stokes vector $\{I_o, Q_o, U_o, V_o\}$ can be expressed as

$$\begin{bmatrix} I_o \\ Q_o \\ U_o \\ V_o \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{bmatrix} I_i \\ Q_i \\ U_i \\ V_i \end{bmatrix}.$$  

(1.24)
where \(m_{jk}\) are the components of the Mueller matrix. In a radio interferometer, the four correlations of the visibilities are directly related to the four Stokes parameters. Mueller matrices can be constructed from Jones matrices using Kronecker product, which will be shown in Chapter 2.

1.2.6 Polarization leakage

The polarization state of the incoming wave is altered by the observing instrument. One of the most common effects is instrumental polarization, which causes a completely unpolarized wave to appear partly polarized after observation. The instrumental polarization can be demonstrated in terms of Jones matrices by considering two orthogonal feeds of an antenna on the ground plane designated by \(x\) and \(y\). Ideally, each feed of the antenna should be responsive to only one of the orthogonal polarization states. So the signal component \(v_x\) should be detected solely by the \(x\) feed, and the component \(v_y\) by the \(y\) feed. But in reality \(x\) feed also detects a small fraction of \(v_y\). This cross-polarization can be easily understood by considering the polarization ellipse of the wave that would be generated by the antenna, if it was in the transmission mode.

The polarization ellipse of the wave transmitted by an antenna would be similar to that presented in Fig. 1.4. For simplicity, let us assume that the major and minor axes of the antenna polarization ellipse are aligned with the major and minor axes of the polarization ellipse of the incoming wave. Ideally we want the \(x\) feed to detect only the \(v_x\) component, but it will also detect a fraction of the \(v_y\) component. If the signals detected by the \(x\) feed are \(v'_x\) and \(v'_y\), then the ratio \(D_x = v'_y / v'_x\) would give the instrumental polarization caused by the \(x\) feed. Similarly, \(D_y = v'_x / v'_y\) would be the instrumental polarization caused by the \(y\) feed. If we consider the \(\chi\) and \(\beta\) angles for the antenna polarization ellipse, then it can be shown that (Thompson et al., 2001, p. 118)

\[
D_x \simeq \chi_x - i\beta_x
\]

\[
D_y \simeq -\chi_y + i\beta_y
\]

where \(\chi_x, \beta_x\) are measured from the \(x\)-axis, and \(\chi_y, \beta_y\) are measured from the \(y\)-axis. In terms of Jones matrices, the output and input signals are related as

\[
\begin{bmatrix}
  v'_x \\
  v'_y
\end{bmatrix} =
\begin{bmatrix}
  1 & D_x \\
  D_y & 1
\end{bmatrix}
\begin{bmatrix}
  v_x \\
  v_y
\end{bmatrix}
\]

(1.27)

where the \(2 \times 2\) matrix is the leakage Jones matrix \((J_L)\), as it causes leakage from total intensity to polarization and vice versa.

Effects of the instrumental polarization on Stokes parameters can be seen if we take the Kronecker product of the \(2 \times 2\) leakage Jones matrices of two antennae to create a \(4 \times 4\) Mueller matrix, i. e. the leakage Mueller matrix \(M_L\). Two antennae create a baseline in an interferometer, and hence \(M_L\) represents the leakage term for a baseline. In such a representation, Stokes \(I\) will be corrupted by Stokes \(Q, U, V\) because of the \(m_{12}, m_{13}, m_{14}\) components of the Mueller matrix, respectively, as evident from Eq. 1.24. Similarly, Stokes \(Q, U, V\) will be affected by Stokes \(I\) because of the \(m_{21}, m_{31}, m_{41}\) components, respectively.

In radio interferometers, \(J_L\) or \(M_L\) are constituted by the direction independent (DI) gains, and the direction dependent (DD) primary beam. DI effects include cross-talk between the
electronic components of the feeds, and DD effects include the response of the antenna toward different directions in the sky, which can be thought of as DD gains. Polarization leakage is a more general term than instrumental polarization, because instrumental polarization is thought of as the polarization induced by the instrument, whereas leakage can mean intermixing between all the Stokes parameters.

1.3 Overview of this thesis

1.3.1 Motivation

The work presented in this thesis is part of the EoR key science project of the Low Frequency Array (LOFAR), i.e. the LOFAR-EoR project. The simulation pipeline of the LOFAR-EoR project has three main modules: the EoR signal (described in the thesis of Thomas 2009), the foregrounds (described in the thesis of Jelic 2010), and the instrumental response (partly described in the thesis of Lampropoulos 2010). The three modules need to come together to create an end-to-end simulation of the EoR signal taking into account all effects of the intervening medium and the instrument. As the EoR signal is expected to be heavily affected by noise, foregrounds, and systematics, such simulations constitute an important part of all EoR experiments. The effects of the noise and systematics of LOFAR and the unpolarized foregrounds on the expected EoR signal have been described in the thesis of Patil, 2016 (in preparation). In this thesis we focus on the polarized foreground and the leakage of that foreground into Stokes $I$ due to systematics. So the thesis takes into account both the polarized foregrounds and the systematics.

As described above, Jelić et al. (2010) showed that the leakage of polarized Galactic diffuse emission into Stokes $I$ can potentially mimic the EoR signal. However, their result was obtained from simulations, albeit realistic. The spectral structure and the intensity of the EoR signal or the polarized emission were not well known at the time. Now we have a better idea about the intensity and spectral structure of the polarized emission, but the EoR signal, of course, remains unknown to this day. Besides, the systematic processes of LOFAR that give rise to polarization leakage were not explored fully before. Jelić et al. (2010) assumed a generic geometric projection effect to calculate the aforementioned level of leakage. Therefore, the main motivation behind this thesis is to understand the spatial, spectral and scale-dependent behaviors of the diffuse polarized emission, the polarization leakage, and the contamination of the EoR signal by the leakage. We have also explored leakage caused by extragalactic point sources. Moreover, various strategies to remove or avoid the leakage are also discussed in the thesis.

Keeping these motivations in mind, several questions can be asked, that we will try to answer in this thesis. The main questions are:

1. What is the spectral structure of the Galactic diffuse polarized emission observed by LOFAR at the frequencies relevant for EoR observations? The behavior of the emission in the instrumental $k$-space will be dictated by this spectral structure.
2. What is the spatial structure of the polarized emission? More specifically, how does the emission vary between the different observing fields of the LOFAR-EoR project?
3. What is the contribution of the direction independent gains to the polarization leakage? How well can the direction independent leakage be calibrated away by standard LOFAR
calibration software? And what is the effect of incomplete sky models in this calibration?

4. What are the spatial, temporal and spectral structures of the current primary beam model of LOFAR?

5. How accurate is the primary beam model in predicting the leakage of polarized emission into Stokes $I$.

6. How much polarization leakage we should expect in Stokes $I$, based on the observed polarized emission and the model primary beam in the different observing fields of the LOFAR-EoR project? We focus on two fields: ‘3C196’ centered on the radio galaxy 3C 196, and ‘NCP’ centered on the north celestial pole.

7. How much the EoR window of the cylindrical PS is contaminated by the expected polarization leakage in the two fields?

8. How does the leakage vary with distance from the phase center of a field? And how does this variation affect the EoR window in the PS?

9. How does the level of leakage compare with a fiducial EoR signal?

We will come back to these questions in the conclusions of this thesis, in Chapter 7.

1.3.2 Structure of the thesis

This thesis is divided into 3 parts. Part I is adapted from Asad et al. (2015), and it is divided into three chapters, numbered 2, 3 and 4. Part I has an introduction which should be considered as the introduction to all three chapters. However, each chapter has a separate concluding section adapted from the conclusions of Asad et al. (2015).

Chapter 2 revisits the mathematical formalism of a radio interferometer and describes the direction independent effects (DIE) and the LOFAR beam-related direction dependent effects (DDE) within the context of this formalism. Formalisms used for calibration, imaging, flux conversion, rotation measure synthesis and power spectrum analysis are also described briefly. As this formalism is used throughout the thesis, it is worthwhile to include it as a separate chapter.

In Chapter 3, we simulate the LOFAR observations of extragalactic foreground, i.e. point sources, taking into account the full-polarization systematic effects of LOFAR. First, we describe the pipeline and setup of the simulated observations and then present three different results: effect of DI errors and the accuracy of self-calibration in this case, effects of DD-errors and a possible DDE correction strategy, and finally errors due to self-calibration with incomplete sky models.

In Chapter 4, we simulate the LOFAR observations of polarized Galactic foreground in the 3C196 field taking into account the direction dependent systematic effects of LOFAR, and the thermal noise. Here we show the results of rotation measure synthesis and power spectrum analysis, compare the power spectra of the leakage and the expected EoR signal, and also test a potential leakage removal method.

Part II of the thesis is adapted from two papers: Asad et al. (2016a) and Asad et al. (2016b, in preparation). The two papers constitute the two chapters of this part: chapters 5 and 6, respectively.

We calculate the accuracy of the model beam of LOFAR in predicting polarization leakage in Chapter 5. Section 5.2 revisits the nominal model beam of LOFAR and shows the behavior
of the intrinsic cross-polarization ratio of the instrument as a function of the distance from the phase center and also the distance of the observing field from the local zenith. In Section 5.3, we describe the data reduction, calibration and simulation pipelines. Our results are presented in Section 5.4—first, we present the results of the observation and the simulation, then compare them to quantify the accuracy of the beam model, and finally present the results of the DD calibration. The chapter ends with a discussion of our analysis and some concluding remarks.

In Chapter 6, we compare the expected leakages in the 3C196 and NCP fields, and see how the expected leakage varies if the field of view is increased. Section 6.2 presents the PS of the observed polarized emission in the 3C196 and NCP fields with two different fields of view. Section 6.3 presents the calculation of the RMS fractional leakage, by predicting LOFAR observations of a simulated Galactic diffuse polarized emission, for the two fields and for three different fields of view. In both sections, the methods and results are presented in separate subsections. Then, the implications of the calculated fractional leakage in Section 6.3 on the PS presented in Section 6.2 is explored in the discussion section. The chapter ends with some conclusions that include some remarks about our ongoing and future works.

Part III contains the conclusions and future prospects of our work, the bibliography, and the summaries in three languages: English, Dutch and Bengali. In the concluding section of the thesis, in Chapter 7, we revisit the main conclusions of each preceding chapter. Here, we revisit the questions posed in Section 1.3.1, and answer them based on the results of the scientific chapters. The thesis ends with the acknowledgement.
Introduction .................................. 19

2 Simulating EoR observations .............. 23
  2.1 Mathematical model of a radio interferometer
  2.2 Systematic effects
  2.3 Calibration and imaging
  2.4 Flux conversion
  2.5 Rotation measure synthesis
  2.6 Power spectrum analysis
  2.7 Conclusion

3 Extragalactic foreground .................... 39
  3.1 Pipeline
  3.2 Direction independent errors
  3.3 Direction dependent errors
  3.4 Selfcal errors due to incomplete sky model
  3.5 Conclusion

4 Galactic foreground ....................... 53
  4.1 Simulation setup
  4.2 Results
  4.3 Polarization leakage removal
  4.4 Conclusion
Part I is adapted from Asad et al. (2015). The three main sections of the paper constitute the three chapters of this part. The following abstract and introduction are for all three chapters, i.e. Chapter 2, 3 and 4. But each of the three chapters has a separate conclusion adapted from the conclusions of Asad et al. (2015).

Abstract

Detection of the 21-cm signal coming from the epoch of reionization (EoR) is challenging especially because, even after removing the foregrounds, the residual Stokes $I$ maps contain leakage from polarized emission that can mimic the signal. In Part I of the thesis, we discuss the instrumental polarization of LOFAR and present realistic simulations of the leakages between Stokes parameters.

From the LOFAR observations of polarized emission in the 3C196 field, we have quantified the level of polarization leakage caused by the nominal model beam of LOFAR, and compared it with the EoR signal using power spectrum analysis. We found that at 134–166 MHz, within the central 4° of the field the $(Q, U) \rightarrow I$ leakage power is lower than the EoR signal at $k < 0.3$ Mpc$^{-1}$. The leakage was found to be localized around a Faraday depth of 0, and the rms of the leakage as a fraction of the rms of the polarized emission was shown to vary between 0.2–0.3%, both of which could be utilized in the removal of leakage.

Moreover, we could define an ‘EoR window’ in terms of the polarization leakage in the cylindrical power spectrum above the PSF-induced wedge and below $k_{||} \sim 0.5$ Mpc$^{-1}$, and the
window extended up to $k_{\parallel} \sim 1 \text{ Mpc}^{-1}$ at all $k_{\perp}$ when 70% of the leakage had been removed. These LOFAR results show that even a modest polarimetric calibration over a field of view of $\lesssim 4^\circ$ in the future arrays like SKA will ensure that the polarization leakage remains well below the expected EoR signal at the scales of 0.02–1 Mpc$^{-1}$.

**Introduction**

Five phases of the large-scale universe are imprinted on Hydrogen: (i) the primordial phase before redshift $z \sim 1100$—when the universe was a hot, dense plasma—that ended when protons recombined with electrons releasing the photons that we detect today as a $\sim 2.7 \text{ K}$ signal known as the cosmic microwave background (CMB); (ii) the ‘Dark Ages’ ($1100 \gtrsim z \gtrsim 30$) when the baryonic universe contained mostly neutral Hydrogen and freely moving photons; (iii) the ‘Cosmic Dawn’ ($30 \gtrsim z \gtrsim 12$) when the first structures formed; (iv) the ‘Epoch of Reionization’ (EoR; $12 \gtrsim z \gtrsim 6.5$) when high-energy photons emitted by the first sources reionized the Hydrogen in the intergalactic medium; and (v) the current phase ($z \lesssim 6.5$) when almost all Hydrogen in the universe are ionized (e.g. Furlanetto et al., 2006; Mellema et al., 2013; Zaroubi et al., 2012).

The aforementioned highly uncertain boundaries of the EoR have been approximated using indirect probes, e.g. CMB polarization at the high-$z$ end (e.g. Page et al., 2007) and absorption features in quasar spectra at the low-$z$ end (e.g. Fan et al., 2006). However, the new generation low-frequency, wide-bandwidth radio interferometers have the potential to directly detect the 21-cm radiation emitted by neutral Hydrogen during the EoR, redshifted to the wavelengths of around 1.5–3 m (corresponding to 200–100 MHz), as a differential brightness with respect to the CMB. There are several ongoing and planned experiments to detect the EoR signal using radio arrays: Giant Metrewave Radio Telescope (GMRT)$^4$, Low Frequency Array (LOFAR)$^5$, Murchison Widefield Array (MWA)$^6$, Precision Array for Probing the EoR (PAPER)$^7$, 21-cm Array (21CMA)$^8$, and the planned Square Kilometre Array (SKA)$^9$.

In order to detect the EoR, the effect of all other signals, e.g. the Galactic and extragalactic foregrounds, has to be excluded from the observed data; spatial fluctuations of the Galactic foreground can be 2-3 orders of magnitude higher than that of the EoR signal (Bernardi et al., 2009, 2010; Pober et al., 2013) which is around 10 mK within the redshifts 6–10 at 3′ resolution (Patil et al., 2014). However, even after removing the foregrounds with high accuracy the system noise after even hundreds of hours of integration will be an order of magnitude higher than the signal, thereby forcing us to aim for a statistical detection of the signal. One of the methods for detecting the EoR signal statistically entails removing the foregrounds with high accuracy and then measuring the power spectrum of the residual which depends heavily on a proper understanding of the systematic and the random (noise) errors associated with the observing instrument and foreground removal (e.g. Dillon et al., 2015; Liu et al., 2014a,b; Bernardi et al., 2013; Chapman et al., 2013; Vedantham et al., 2014; Morales et al., 2012a; Parsons et al., 2012; Bernardi et al., 2010; Harker et al., 2010; Jelić et al., 2008).

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$^4$http://gmrt.ncra.tifr.res.in/
$^5$http://www.lofar.org/
$^6$http://www.mwatelescope.org/
$^7$http://eor.berkeley.edu/
$^8$http://21cma.bao.ac.cn
$^9$http://www.skatelescope.org/
In Part I of the thesis, we address the systematic errors due to polarized foregrounds associated with the EoR experiment being conducted using LOFAR (the LOFAR-EoR project). After taking out the bright extragalactic foreground, i.e., the resolved point sources, the Galactic foreground can be removed utilizing the fact that the EoR signal has significant correlated structure along the frequency—or equivalently the redshift—axis while the Galactic diffuse foreground is spectrally smooth in Stokes $I$. However, the Faraday rotated polarized Galactic foreground is not always smooth along frequency and hence a leakage of the polarized emission into Stokes $I$ might mimic the EoR signal (e.g., Jelić et al., 2010). Systematic errors can cause this leakage in two different ways: direction independent (DI) and direction dependent (DD). Non-orthogonal or rotated feeds of an antenna of an interferometer can cause $Q$ to leak into $I$ and vice versa while cross-talk between two feeds can cause mixing between all 4 Stokes parameters. As these are DI errors, they can be corrected with high accuracy using traditional self-calibration. However, the DD errors (DDE) caused by the time-frequency-baseline dependent primary beams cannot be corrected so easily. In the latter case, an ellipticity of the beam can cause $I \leftrightarrow Q$ mixing while cross-polarization between two orthogonal components of the beam can mix all Stokes parameters.

Carozzi & Woan (2009) calculated a full polarization Mueller matrix to account for the look-direction dependent polarization aberration inherent in a dipole interferometer due to the fact that a source sees different projections of a dipole at different times. Jelić et al. (2010) used this Mueller matrix to calculate the amount of leakage to be expected over the field of view of LOFAR and found that the leakage should be 0.1–0.7% at 138 MHz within a $5^\circ \times 5^\circ$ patch of sky around the zenith and should increase to 2–20% for an elevation of 45°. If the polarized intensity is $\sim 1$ K, then a 1.5% leakage would give a polarized emission of $\sim 15$ mK in Stokes $I$ which is comparable to the EoR signal. Moore et al. (2013) simulated the sky with randomly generated Faraday rotated, polarized point sources and found that the power of $Q \rightarrow I$ leakage due to the model beam of PAPER that has a FWHM of around 45° at 150 MHz is of the order of thousands of [mK]$^2$ which is several orders of magnitude higher than the expected EoR signal power. Their result turned out to be pessimistic because of their choice of the model; in reality, point sources are much more weakly polarized at low frequencies (Bernardi et al., 2013).

Here, we predict the level of polarization leakage to be expected in the 3C196 window of the LOFAR-EoR experiment using reasonable models of the field and the model beam of LOFAR produced by Hamaker (2011) using an electromagnetic simulation of the ASTRON Antenna Group$^{10}$, and also test some leakage-correction strategies. This part is organized as follows.

Chapter 2 revisits the mathematical formalism of a radio interferometer and describes the DI errors and the LOFAR beam-related DD errors within the context of this formalism. Formalisms used for calibration, imaging, flux conversion, RM synthesis and power spectrum analysis are also described briefly.

In Chapter 3 we describe the pipeline and setup of the simulations of extragalactic point sources and present three different results: effect of DI errors and the accuracy of self-calibration in this case, effect of DD-errors and a possible DDE correction strategy, and finally errors due to self-calibration with incomplete sky models.

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$^{10}$M. J. Arts; http://www.astron.nl
Pipeline, setup and results of the simulation of Galactic foreground are presented in Chapter 4, where we show the results of rotation measure synthesis and power spectrum analysis, compare the power spectra of the leakage and the expected EoR signal, and test a potential leakage removal method.
2. Simulating EoR observations

Adapted from Section 2 of Asad et al. (2015).

2.1 Mathematical model of a radio interferometer

Here, we give an outline of the mathematical model of a radio interferometer and refer the readers to Hamaker et al. (1996); Smirnov (2011) for a detail description.

Consider a quasi-monochromatic electromagnetic wave propagating through space from a single point source. Using the Cartesian coordinate system $xyz$ where the signal propagates along $z$ direction, the signal, at a specific point in time ($t$) and space, can be described by the complex vector $\mathbf{E}(x,y,t)$ and transformations (e.g. contaminations) of this signal along its path can be represented by $2 \times 2$ Jones matrices. Assuming all such transformations to be linear, a cumulative Jones matrix ($\mathbf{J}$) can be constructed from the products of the matrices. The signal detected by our telescope will be the intrinsic signal multiplied by this cumulative matrix, mathematically\(^1\) $\mathbf{E}' = \mathbf{J} \mathbf{E}$.

The electric field represented by this vector hits an antenna of our interferometer that has two feeds, each one sensitive to a specific polarization state of the vector in case of a perpendicularly incident electric field. Let us assume that the $p$ and $q$ feeds are sensitive to the $x$ and $y$ polarization states of the signal respectively. The feeds convert the respective electric

\(^1\)In this paper vectors are represented by calligraphy, matrices by bold and scalars by normal typefaces.
fields into voltages and this conversion can be expressed as yet another Jones matrix yielding

\[ \mathbf{V} = \mathbf{J}' \mathbf{E}' \Rightarrow \begin{pmatrix} v_p \\ v_q \end{pmatrix} = \mathbf{J}' \begin{pmatrix} e_x \\ e_y \end{pmatrix}. \] (2.1)

Let us denote this antenna as \( a \) and assume that there is another antenna in our interferometer denoted by \( b \). Voltages from each antenna are fed to a correlator that cross-correlates them to create 4 pairwise correlations that can be written as a \( 2 \times 2 \) matrix, known as the visibility matrix,

\[ \mathbf{V}_{ab} = \langle \mathbf{V}_a \mathbf{V}_b^H \rangle = \begin{pmatrix} \langle v_{ap} v_{bp}^* \rangle & \langle v_{ap} v_{bq}^* \rangle \\ \langle v_{aq} v_{bp}^* \rangle & \langle v_{aq} v_{bq}^* \rangle \end{pmatrix} \] (2.2)

which is related to the electric field correlations according to Eq. 2.1, i.e.

\[ \mathbf{V}_{ab} = \mathbf{J}_a \mathbf{J}_b^H. \] (2.3)

Here * denotes a complex conjugate, \( H \) the conjugate transpose or Hermitian conjugate and \( \langle \rangle \) the time averages. Polarized waves are best described by Stokes parameters and their relation with the correlations of the electric field components, for a linear experiment, can be written as (Hamaker et al., 1996)

\[ \begin{pmatrix} \langle e_x e_x^* \rangle & \langle e_x e_y^* \rangle \\ \langle e_y e_x^* \rangle & \langle e_y e_y^* \rangle \end{pmatrix} = \begin{pmatrix} I + Q & U + iV \\ U - iV & I - Q \end{pmatrix} \equiv \mathbf{B} \] (2.4)

where \( \mathbf{B} \) is the brightness matrix. Therefore, Eq. 2.3 becomes

\[ \mathbf{V}_{ab} = \mathbf{J}_a \mathbf{B} \mathbf{J}_b^H \] (2.5)

which contains all effects along the signal path in the form of Jones matrices. The effect fundamental to all interferometers is the phase difference between the measured voltages \( \mathbf{V}_a \) and \( \mathbf{V}_b \). To account for the phase delays in Eq. 2.5, consider the interferometer to be situated in a Cartesian coordinate system represented by \( u, v, w \) and the antenna \( a \) to be located at the coordinates \( \mathcal{U}_a = (u_a, v_a, w_a) \). The phase delay between the baselines \( a \) and \( b \) then becomes

\[ K_{ab} = e^{-2\pi i (u_ab l + v_ab m + w_ab (n-1))} \] (2.6)

where \( \mathcal{U}_{ab} = \mathcal{U}_a - \mathcal{U}_b \); \( l, m \) are the cosines of the right ascension and declination of the source respectively; and \( n = \sqrt{1 - l^2 - m^2} \). If we take out the phase delay scalar matrices \( K \)-Jones from \( \mathbf{J} \) for both antennae and express them as a single scalar associated with the baseline, then Eq. 2.5 becomes

\[ \mathbf{V}_{ab} = \mathbf{J}_a \mathbf{B} K_{ab} \mathbf{J}_b^H = \mathbf{J}_a \mathbf{X}_{ab} \mathbf{J}_b^H \] (2.7)

where \( \mathbf{X}_{ab} = \mathbf{B} K_{ab} \) is called the coherency matrix as it represents the spatial coherence function (Clark, 1999) of the electric field for this particular baseline.

If, instead of a single source, we have a continuum of sources, the visibility matrix has to be written as an integration over all directions within the field of view and the cumulative Jones matrix has to be separated into two different matrices, one representing the direction
2.1 Mathematical model of a radio interferometer

independent effects (DIE, G-Jones) and another the direction dependent effects (DDE, E-Jones),

\[
V_{ab} = G_a \left[ \int \int E_a B K_{ab} E_b^H dldm \frac{1}{n} \right] G_b^H.
\] (2.8)

This is the standard equation to describe the mathematical model of a radio interferometer that, from now on, we will refer to as the measurement equation.

### 2.1.1 Mueller formalism

For understanding the effects of systematic errors on the images produced from the visibilities, it helps to write this equation in terms of baseline-based Mueller matrices \((M)\) instead of antenna-based Jones matrices \((J)\) remembering the relation between the two (Hamaker et al., 1996),

\[
M_{ab} = S^{-1} (J_a \otimes J_b^H) S
\] (2.9)

where the coordinate transformation matrix,

\[
S = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & 1 & -i \\ 1 & -1 & 0 & 0 \end{bmatrix}
\] (2.10)

and \(\otimes\) denotes the Kronecker product. To do so, instead of taking the matrix product of \(v_a\) and \(v_b\) like in Eq. 2.2, we have to take their Kronecker product to get the voltage correlation vector \(\gamma_{ab} = (V_{pp} V_{pq} V_{qp} V_{qq})^T\) where \(T\) represents transpose. Then Eq. 2.8 becomes

\[
\gamma_{ab} = G_{ab} \int \int E_{ab} S \mathcal{J} K_{ab} dldm \frac{1}{n}
\] (2.11)

where \(G_{ab} = G_a \otimes G_b^H\), \(E_{ab} = E_a \otimes E_b^H\) and brightness vector \(\mathcal{J} = (I Q U V)^T\).

### 2.1.2 Stokes visibilities

In order to describe the relation between Stokes parameters and voltage correlations in Fourier space, let us define

\[
V^{(ab)}_Z = J_a Z K_{ab} J_b^H
\] (2.12)

where \(V^{(ab)}_Z = V_t, V_Q, V_U, V_V\) is a Stokes visibility and \(Z = I, Q, U, V\) is a Stokes parameter. Comparing equations 2.12, 2.7 and 2.2, and remembering the definition of the coherency and brightness matrices, we can establish the relation between Stokes visibilities and the voltage
correlations as \( \text{(Sault et al., 1996; Bunn, 2007)} \),

\[
\begin{align*}
V_l &= \frac{1}{2} (V_{pp} + V_{qq}) \\
V_Q &= \frac{1}{2} (V_{pp} - V_{qq}) \\
V_U &= \frac{1}{2} (V_{pq} + V_{qp}) \\
V_V &= \frac{1}{2i} (V_{pq} - V_{qp}).
\end{align*}
\]  

(2.13a) (2.13b) (2.13c) (2.13d)

2.2 Systematic effects

In this section, we will discuss the effects of the systematic errors \( (G \text{ and } E \text{ Jones}) \) on the Stokes visibilities and the Stokes parameters for the case of LOFAR, although the aforementioned formalism is universal. LOFAR is a phased array covering the frequency range from 10–240 MHz. LOFAR stations consist of two types of antennae—LBA (low band antenna; 10–90 MHz) and HBA (high band antenna; 110–240 MHz). We use the HBA stations in our simulations and a schematic diagram of a typical 24-tile LOFAR HBA core (situated within the central 3.5 km) station is shown in Fig. 2.1. In this case, 16 dipoles are combined to create a tile and 24 tiles are combined to create a station (for details see van Haarlem et al., 2013).

2.2.1 Direction independent effects

To simplify calculations, while discussing DIEs, we will ignore the DDEs by assuming the \( E\)-Jones terms of Eq. 2.11 to be identity matrices. Consequently, the Mueller-matrix form of the measurement equation (Eq. 2.11) becomes,

\[
\begin{align*}
\mathcal{V}_{ab} &= \mathbf{G}_{ab} \int \int l, m S.\mathcal{I} K_{ab} \frac{dldm}{n} = \mathbf{G}_{ab} \int \int l, m \mathcal{S}_{\hat{V}Z} \frac{dldm}{n}
\end{align*}
\]  

(2.14)

where \( \mathcal{S}_{\hat{V}Z} = \mathcal{S} K_{ab} \) represents the Stokes visibilities without any systematic errors. The DIEs, denoted here by \( \mathbf{G}_{ab} \), are caused by errors in the electronic gains of the antennae (gain errors) and non-orthogonal and/or rotated feeds (feed errors). Gain and feed errors, for antenna \( a \), can be modelled by the Jones matrices,

\[
\begin{align*}
\mathbf{G}_g^a &= \begin{pmatrix} g_{ap} & 0 \\ 0 & g_{aq} \end{pmatrix} \quad \text{and} \quad \mathbf{G}_f^a = \begin{pmatrix} 1 & \varepsilon_{ap} \\ -\varepsilon_{aq} & 1 \end{pmatrix}
\end{align*}
\]  

(2.15)

where \( g_{ap} \) is the gain error of the feed \( p \) of the antenna \( a \) and \( \varepsilon_{ap} \) is the spurious sensitivity of the \( p \) feed to the \( y \) polarization. The Jones matrix for all DIEs, i.e. \( G\)-Jones of Eq. 2.8, then becomes \( \mathbf{G}_a = \mathbf{G}_g^a \mathbf{G}_f^a \). Gain and feed errors affect different Stokes visibilities (Eq. 2.13) in different ways which can be illustrated by taking into consideration how the Stokes visibilities observed by an instrument with DIEs differ from that of an error-free ideal instrument. Let’s assume that both \( \mathbf{G}^g \) and \( \mathbf{G}^f \) of the ideal instrument are identity matrices and for a realistic instrument gains and feeds are in error by,

\[
\begin{align*}
\Delta \mathbf{G}_g^a &= \begin{pmatrix} \Delta g_{ap} & 0 \\ 0 & \Delta g_{aq} \end{pmatrix} \quad \text{and} \quad \Delta \mathbf{G}_f^a = \begin{pmatrix} 0 & \varepsilon_{ap} \\ -\varepsilon_{aq} & 0 \end{pmatrix}
\end{align*}
\]  

(2.16)
2.2 Systematic effects

Figure 2.1: Schematic diagram of a 24-tile LOFAR HBA station. A tile is made of 16 dual polarization dipoles. Dipoles see almost the whole sky (FWHM $\sim 90^\circ$), while the FWHM of a tile beam is $\sim 20^\circ$ and that of a station beam is only $\sim 4^\circ$. There is a 15 cm gap between the tiles which is not shown here.
Then, seven error parameters (hereafter DI-error parameters) can be defined following Sault et al. (1996, equations 36-42) as,

\[
\delta_s = (\Delta g_{ap} + \Delta g_{aq}) + (\Delta g_{bp} + \Delta g_{bq})
\]

\[
\delta_{I,Q} = (\Delta g_{ap} - \Delta g_{aq}) + (\Delta g_{bp} - \Delta g_{bq})
\]

\[
\delta_{U,V} = (\Delta g_{ap} - \Delta g_{aq}) - (\Delta g_{bp} - \Delta g_{bq})
\]

\[
\delta_{Q,U} = (\varepsilon_{ap} + \varepsilon_{aq}) + (\varepsilon_{bp} + \varepsilon_{bq})
\]

\[
\delta_{I,U} = (\varepsilon_{ap} - \varepsilon_{aq}) + (\varepsilon_{bp} - \varepsilon_{bq})
\]

\[
\delta_{I,V} = (\varepsilon_{ap} + \varepsilon_{aq}) - (\varepsilon_{bp} + \varepsilon_{bq})
\]

\[
\delta_{Q,V} = (\varepsilon_{ap} - \varepsilon_{aq}) - (\varepsilon_{bp} - \varepsilon_{bq})
\]

where the subscript \( I,Q \) stands for mixing between Stokes \( I \) and \( Q \). Now, if the difference between the ideal Stokes visibilities and the Stokes visibilities affected by these errors is

\[
\Delta V = V_{\text{ideal}} - V_{ab}
\]

then by assuming errors to be very small it can be shown that (see Sault et al., 1996, appendix B),

\[
\Delta V = \frac{1}{2} \begin{pmatrix}
\delta_s & \delta_{I,Q} & \delta_{I,U} & -i\delta_{I,V} \\
\delta_{I,Q} & \delta_s & \delta_{Q,U} & -i\delta_{Q,V} \\
\delta_{I,U} & -i\delta_{Q,U} & \delta_s & i\delta_{U,V} \\
-i\delta_{I,V} & \delta_{Q,V} & -i\delta_{U,V} & \delta_s
\end{pmatrix} \cdot V_{\text{ideal}}
\]

(2.18)

Here, the \( 4 \times 4 \) matrix is the instrumental Mueller matrix for the DIEs (hereafter DI-Mueller) and it determines the full Stokes response of an instrument without any direction dependent errors. It can be seen from the equation that a completely unpolarized source \((Q,U,V = 0)\) will appear to have non-zero Stokes \( Q, U \) and \( V \) in an interferometric observation because of the DIEs \( \delta_{I,Q}, \delta_{I,U} \) and \( \delta_{I,V} \) respectively and these same errors will cause leakage into Stokes \( I \) from Stokes \( Q, U \) and \( V \) respectively. The DIE-parameters can be used to determine calibration errors if, instead of comparing the ideal and the actual gains, we compare the input and the solved gains (Sault et al., 1996).

### 2.2.2 Direction dependent effects

Direction dependent errors in a radio interferometer are caused mainly by the Earth’s ionosphere and the primary beams—i.e. the radiation patterns—of the antennae. Here, we restrict ourself only to the LOFAR beam errors. The beam we use for the bowtie dipoles has been modelled by an analytic expression whose coefficients are determined by fitting to a numerically simulated beam raster generated by the ASTRON Antenna Group (Hamaker, 2011; hereafter H11). Here, we will give a brief overview of this model; for further details we refer the readers to H11.

From basic symmetry considerations a generic expression for a dual dipole antenna \( E \)-Jones matrix has been derived by H11 which, for azimuth \( \phi \) and zenith angle \( \theta \equiv (\pi/2−\text{elevation}) \) can be written as,

\[
E_x(\theta, \phi) = \sum_{k' = 0}^{N} R(k', \phi) P_k(\theta)
\]

(2.19)
where the azimuth dependent rotation matrix

\[
R(k', \phi) = \begin{pmatrix}
\cos((-1)^{k'}(2k' + 1)\phi) & -\sin((-1)^{k'}(2k' + 1)\phi) \\
\sin((-1)^{k'}(2k' + 1)\phi) & \cos((-1)^{k'}(2k' + 1)\phi)
\end{pmatrix}
\]

(2.20)

and the zenith angle and frequency (ν) dependent projection matrix that contains the detailed geometry of the dipoles and the ground plane is

\[
P_{k'}(\theta, \nu) = \begin{pmatrix}
p_{\theta,k'}(\theta, \nu) & 0 \\
0 & -p_{\phi,k'}(\theta, \nu)
\end{pmatrix}
\]

(2.21)

and \(k' = 0\) gives the ‘ideal’ beam, whereas the higher order terms represent the differences between the ideal and the more realistic beams. Each element of the projection matrix \(p(\theta, \nu)\), for each harmonic \(k'\), is calculated as \(\tilde{\theta}[C]\nu\) where \(\tilde{\theta}\) is a row vector \((\theta^0 \theta^1 \ldots \theta^N_\theta)\), \(\nu\) is a column vector \((\nu^0 \nu^1 \ldots \nu^N_\nu)^T\), \([C]\) is a 2D matrix of dimensions \((N_\theta + 1) \times (N_\nu + 1)\) that contains the complex coefficients determined by fitting to an electromagnetic simulation, and \(N_\theta = N_\nu = 4\).

In Eq. 2.19, \(E_e\) has been expressed in a topocentric (azimuth-zenith angle) coordinate, but in reality the source is carried around through the beam by the apparent rotation of the sky during an observation. To account for this effect, the position of the source is transformed from equatorial celestial coordinate system to the topocentric system. For polarized sources, there is an additional factor— the relative rotation between the equatorial and the topocentric grids at the position of the source that causes the beam to rotate with the parallactic angle, known as the parallactic rotation which has been incorporated in the dipole beam model as a separate Jones matrix. Hereafter, by \(E_e\) we will refer to an element beam where all these effects have been taken into account.

In an element beam Jones matrix the diagonal terms determine the primary beam of the element and the off-diagonal terms the level of cross-polarization. Errors related to antenna pointing, beamwidth and beam ellipticity are all included in the diagonal terms. For a dipole of size \(D \sim 1.25 \text{ m}\) the FWHM at 150 MHz becomes \(\lambda/D \sim 90^\circ\) and the shape of the diagonal terms of the matrix is similar to an Airy pattern. The polarization response of a LOFAR station is completely determined by \(E_e\). Therefore, it would be interesting to analyse the beam Mueller matrix corresponding to an interferometer constructed by two such elements before entering into the discussion of the tile and the station beams.

In a two-element interferometer, the component at the first row and first column of the Mueller matrix (hereafter \(M_{11}\)) represents the Stokes \(I\) response of the interferometer to a completely unpolarized point source of unity flux and \(M_{12}\) gives the corresponding Stokes \(Q\) response. Examples of Stokes \(I\) and \(Q\) responses of a LOFAR LBA dipole can be seen in Fig. 3.8 and 3.9 of Bregman (2012, hereafter B12) respectively. From the figures we see that Stokes \(I\) response is almost circular with amplitudes decreasing from the centre toward the edges until the first null. Stokes \(Q\) response, on the other hand, has a cloverleaf pattern with 2-fold symmetry corresponding to the physical structure of the dual dipole. The cross-polarization over a beam is conventionally measured by the ratios \(Q(\theta)/I(\theta), U(\theta)/I(\theta)\) and \(V(\theta)/I(\theta)\). Comparing Fig. 3.9 and 3.8, B12 finds that \(Q/I\) is lowest at the centre and increases quadratically with \(\theta\) and reaches a value of 0.5 at the FWHM. It implies that an unpolarized source situated at FWHM of a dipole beam will become 50% polarized in the observed data due to instrumental polarization.
Figure 2.2: (a) Direction dependent Mueller matrix representing the polarization response of the baseline 0-1 (127 m) of LOFAR at 150 MHz over the 3C196 field (20° × 20°) at the time when the centre of the field culminates. (b) Spatio-temporal profiles as a percentage of total intensity—i.e. first row, first column ($M_{11}$) of the matrix representing Stokes $I$—for leakages from (1) $I$ to linear polarization ($P$), i.e. $\sqrt{M_{21}^2 + M_{31}^2}$; (2) linear to $I$, i.e. $\sqrt{M_{21}^2 + M_{31}^2}$; (3) $I$ to circular, $M_{14}$ and (4) circular to $I$, $M_{41}$. Here, $\Delta \theta$ represents distance from the phase centre. See section 2.2.2 for details.
Figure 2.3: (a) Gaussian fit to the azimuthally averaged Stokes $I$ response of the 0-1 baseline of LOFAR at 150 MHz over the 3C196 field when the field culminates ($M_{11}$ component of Fig. 2.2a). (b) FWHM of the Stokes $I$ beam at different frequencies (solid); the $\alpha \lambda / D$ curve (dashed) is overplotted. (c) A single line through the centre of the Mueller term responsible for linear polarization leakage (see caption of Fig. 2.2) at different frequencies. The leakage is shown as a percentage of Stokes $I$ flux density.
The beams of the 16 dipoles (Ee) in a tile are combined in an analogue way to form the tile beam which is narrower (∼ 20°, Fig. 2.1) and the beams of all the tiles in a station are digitally combined to form the station beam which has the smallest width (∼ 4°). Assuming the tile beams (Et) have been created by phasing the constituting dipole beams, the beam of the station a can be written as (Yatawatta, 2009)

$$E_a(\theta, \phi) = w^H \cdot v(k) \odot E_t(\theta, \phi)$$  \hspace{1cm} (2.22)

where \( \odot \) denotes the Hadamard product, \( k \) is the wave vector, \( v(k) \) is the steering vector, i.e. the delay an incoming wavefront experiences depending on the position \( (r_i) \) of the observing tile in a station that can be expressed as

$$v(k) = \begin{pmatrix}
e^{-jkr_0} \\ e^{-jkr_1} \\ \vdots \\ e^{-jkr_{N-1}}
\end{pmatrix}$$  \hspace{1cm} (2.23)

for \( N \) number of tiles and \( w \) is the weight vector that contains the complex weights associated with each tile. Station beams cut only a small portion of the element beam and get a polarization response depending on which part of the element beam it is tracing. The sidelobes of the station beam cut yet another part of the element beam and accordingly acquire a different polarization response. Station beams that are formed to track a source in the sky follow a trace in azimuth and elevation over the polarized element beam. Hereafter, by beam we will refer to the beam of a single station, \( E_a \).

We could, in principle, derive a direction dependent equivalent of Eq. 2.18 using \( E_a \) as the only systematic error and ignoring the DIEs, but it will be much more complicated in this case. So, instead, we numerically calculate the baseline-dependent Mueller matrices (e.g. \( E_{ab} \)) from the constituent station beams (\( E_a \) and \( E_b \)) following the formalism of section 2.1.1. Such a Mueller matrix for baseline 0-1 (a 127 m baseline formed by the two sub-stations of the central core stations, CS001HBA0 and CS001HBA1) at 150 MHz, at the time when the centre of the target field (20° × 20°) culminates has been shown in Fig. 2.2a. The components of the matrix have been normalized with respect to the Mueller matrix at the phase centre resulting in a differential Mueller matrix; hereafter, by differential beam or nominal beam we will refer to this form of the Mueller matrix. Let’s denote this matrix by \( M_{01} \), where the superscript represents the station numbers.

\( M_{01} \) can be thought of as a direction dependent equivalent of the DI-Mueller (Eq. 2.18), hence we can call it the DD-Mueller. By comparing these two matrices, we can see that \( M_{21} \) component of the DD-Mueller will cause Stokes I to leak into Stokes Q. The off-diagonal terms of \( M_{01} \) show the spatial variation of the instrumental polarization—it is lowest at the phase centre and increases toward the edges until the first null and then, after a gap, we get further polarization at the location of the first sidelobe. In addition to the spatial variation, all components of the instrumental Mueller matrix also vary with zenith angle, or equivalently with hour angle, of the source during an observation. To show the dependence on the directions and sidereal time simultaneously, i.e. spatio-temporal dependence, we calculated \( M_{01} \) for all hour angles. In Fig. 2.2b, we show spatio-temporal profiles of various leakages as a percentage of total intensity. Leakage from linear polarization to total intensity, i.e. \( \sqrt{M_{12}^2 + M_{13}^2}/M_{11} \times 100 \), at different distances from the phase centre (x axis) and at
different hour angles (y axis) during an eight-hour observation is shown in the top panel. The second panel shows fractional leakage from Stokes $I$ to linear polarization and the third and fourth panels show fractional $I \rightarrow V$ and $V \rightarrow I$ leakages respectively. These figures show the variation of the leakages along a single line through the centre of the field at every hour angle during a night-long observation.

From the spatio-temporal profiles, we see that leakage increases with both distance from the phase centre and zenith angle. During the beginning and the end of the observation zenith angle is very high and the beam is extremely attenuated which results in a very high percentage of leakage. Leakages vary across the FoV mainly due to polarization aberrations caused by geometric projection of the antenna on the plane perpendicular to the line of sight (see section 5.3 and Fig. 2 of Carozzi & Woan 2009). The projection changes as a function of direction and zenith angle because of both the coordinate rotation and parallactic rotation that were introduced in the beam model (as discussed before). We see that at high zenith angle the leakages change more rapidly, but these effects can be considered constant within ten minutes (B12) which is a useful assumption for primary beam correction.

Besides direction and elevation, the width and shape of the beam also vary with frequency. Fig. 2.3a shows a Gaussian fit to the azimuthally averaged station beam ($M_{11}$) that gives us an FWHM of $3.8^\circ$ at 150 MHz. Fig. 2.3b shows the beamwidths obtained by Gaussian fitting as a function of frequency and we can see that the curve closely follows the $\alpha \lambda / D$ relation where $\lambda$ and $D$ denote wavelength and station size respectively (for an analogous fitting, see Fig. 21 of van Haarlem et al. 2013). Leakages also vary with frequency, albeit not in a very prominent way; as evident from Fig. 2.3c, within approximately ten degrees leakage changes very slowly with frequency. Therefore, if we have multi-frequency data, the leakages can be removed by utilizing their spectral smoothness.

Ideally, the beam should be exactly same for all elements and, consequently, for all baselines, for traditional calibration to work efficiently, but making them slightly different in configuration could be advantageous in another way. In case of LOFAR, although all dipoles are rotated into the same position, station configurations are rotated with respect to one another to minimize blind angle effects and to average out the effect of grating lobes (B12).

### 2.3 Calibration and imaging

In DI-calibration, it is assumed that all baselines of an array observe the Fourier transform of a common sky which is only true if DDEs are taken to be identical across all antennae. Consequently, $E_a$ of Eq. 2.8 becomes a function of just $l, m$ and the common sky observed by all baselines becomes $B_c = E B H$, i.e. the true sky attenuated by the beam. Then, Eq. 2.8 can be written as

$$V_{ab} = G_a X_{ab}^c G_h^H$$

(2.24)

where $X_{ab}^c$ is the element by element 2D Fourier transform of $B_c$. The most widely used DI-calibration method, self-calibration or selfcal works with this form of the measurement equation. The first step of selfcal is to create a model of the observed sky and to ‘predict’ the corresponding visibilities, $V_{ab}^{\text{mod}}$ that an interferometer would produce. Then, the values
of \( G \) terms that minimize \( V_{\text{mod}}^{ab} - V_{ab} \) are determined. \( G \) terms can be calculated to a very high accuracy, because an array provides over-determined information as \( N(N-1) \) complex visibilities are available for computing only \( 2N-2 \) error parameters, \( N \) being the number of antennae.

The inferred values (\( \tilde{G} \)) are applied to the observed visibilities to yield the corrected visibilities as

\[
V_{\text{corr}}^{ab} = \tilde{G}^{-1}_a V_{ab} \tilde{G}^{-H}_b.
\]

(2.25)

Inverse Fourier transform of the weighted and gridded visibilities produce a ‘dirty’ image, which is the true sky convolved with the PSF. To recover the true sky as closely as possible, the PSF is deconvolved from the dirty image iteratively producing a ‘clean’ image. As the primary beam has not been corrected for, this clean image is actually the true sky attenuated by the primary beam (\( B_c \)). If the primary beam is assumed to be same for all antennae and at all times, the true brightness distribution \( B \) can be extracted from \( B_c \) by just multiplying it with the inverse of \( E \). Traditionally, this is what has been done for dish instruments with small FoV. But in case of wide FoV instruments, e.g. LOFAR, time-frequency-baseline variations of the instrumental Mueller matrices (\( M \), Fig. 2.2) cannot be ignored and one way of dealing with this is AW-projection (Tasse et al., 2013).

### 2.3.1 AW-projection

The problem of imaging can be expressed in Mueller formalism as \( V = \mathcal{A} I + \varepsilon \) where \( V \) is the total set of visibilities, \( I \) is the set of Stokes images to be estimated, \( \varepsilon \) is the noise, \( \mathcal{A} = W I \mathcal{F} M \) ignoring the ionospheric effects, \( W \) is the set of visibility weights, \( \mathcal{F} \) is the sampling function, \( \mathcal{F} \) is the Fourier transform kernel, and \( M \) is the Mueller matrix corresponding to the primary beam. Each of these parameters is a multi-dimensional matrix (for explanation see Tasse et al., 2013). AW-projection, as implemented in AWImager, calculates \( \hat{I} \), an estimate of \( I \), iteratively as,

\[
\hat{I}^{n+1} = \hat{I}^n + \Phi \mathcal{A}^H (V - \mathcal{A} \hat{I}^n)
\]

(2.26)

where \( \Phi \) is a non-linear operator that estimates the deconvolved sky from the residual dirty image \( \mathcal{A}^H (V - \mathcal{A} \hat{I}^n) \). Here the construction of the residual dirty image constitutes the major cycle and the deconvolution the minor cycle. Note that \( \mathcal{A} \hat{I}^n \) is the forward Fourier transform taking into account all instrumental effects and this has to be done accurately for the solutions to converge; during prediction of visibilities using AWImager, only this step is performed. On the other hand, during minor cycle only an approximation of \( (\mathcal{A}^H \mathcal{A})^{-1} \) is calculated and applied on the residual. A-projection, as described in Bhatnagar et al. (2008), is a fast way for applying \( \mathcal{A} \) or \( \mathcal{A}^H \). In AWImager, the element beam (\( E_e \)) and the array factor (\( w^H v(k) \) of Eq. 2.22) of LOFAR have been taken out of the \( M \) matrix of the A-term and they are applied separately.

### 2.4 Flux conversion

For easier comparison with the predicted level of the EoR signal we convert fluxes to intensities and express them as temperature. If \( F_{Jy} \) is the flux of a radio source in Jy, then the
2.5 Rotation measure synthesis

The rotation of the plane of polarization (\(\chi\)) of a linearly polarized signal while propagating through a magnetized plasma is called Faraday rotation which, for a single Faraday screen along the LOS, can be written mathematically as \(\chi = \chi_0 + \Phi \lambda^2\) where \(\chi_0\) is the intrinsic polarization angle and Faraday depth, \(\Phi = 0.81 \int_{\text{source}}^{\text{observer}} n_e B_{||} dI\) \(^{28}\) where \(n_e\) is the density of electrons and \(B_{||}\) is the magnetic field component along the LOS. Note that rotation measure (RM) is defined as \(d\chi/d\lambda^2\) and hence for a single phase screen along the LOS it is equivalent to Faraday depth. Polarized surface brightness per unit Faraday depth, \(F(\Phi)\) can be obtained from the polarized surface brightness per unit squared-wavelength, \(P(\lambda^2)\) using the technique of RM-synthesis (Brentjens & de Bruyn, 2005); mathematically, \(F(\Phi) = R(\Phi) \ast \int_{-\infty}^{\infty} P(\lambda^2) e^{-2i\Phi \lambda^2} d\lambda^2\) \(^{29}\) where \(R(\Phi)\) is the Fourier transform of the wavelength sampling function, known as ‘rotation measure spread function’ (RMSF) and \(\ast\) denotes convolution.

The polarized brightness, \(\mathcal{P} = Q + iU^2\) is a complex valued function and, hence, \(F(\Phi)\) is also complex. However, a Faraday dispersion function for real valued Stokes I, \(F_I(\Phi)\) can also be calculated assuming its imaginary parts to be zero in all spectral bands (e.g. Geil et al., 2011). As the Fourier transform of a real function is always Hermitian, \(F^*_I(\Phi) = F_I(-\Phi)\). The same can be done for Stokes V. In section 4, we will present some of our results in terms of \(F(\Phi), F_I(\Phi)\) and \(F_V(\Phi)\).

2.6 Power spectrum analysis

The power spectrum (hereafter PS) of an image is the measure of the variance per unit angular wavenumber \((k = 2\pi/\theta)\). As the first detections of the EoR signal will be statistical, and its PS is the most widely used statistic (e.g. Bowman et al., 2006; Harker et al., 2010; Moore et al., 2013; Patil et al., 2014; Chapman et al., 2016), most of our analysis will be done through PS. We present three types of PS: 2D, 3D cylindrical and 3D spherical, and in all of them

\[ T_K = \frac{\lambda^2 F_J}{2k_B \Omega_E} 10^{-26} \] \(^\text{(2.27)}\)

where \(k_B\) is the Boltzmann constant and \(\Omega_E = \pi \theta^2 / (4 \ln 2)\) is the beam solid angle, \(\theta\) being the FWHM of the Gaussian restored PSF calculated during imaging.

\(^{26}\)In this paper \(\mathcal{P}\) always refers to \(Q + iU^2\), while \(P\) is always \(|Q + iU|\), and note that the 2D and 3D power spectra, denoted by \(P_{2D}\) and \(P_{3D}\) respectively, are not related to \(\mathcal{P}\) or \(P\).
the wavenumbers are converted to the unit of comoving Mpc$^{-1}$ at the redshift corresponding to the observing frequency. PS can be calculated from the weighted visibilities directly. As the imaging process puts weights on the visibilities and calculates the resulting PSF, we have measured the PS from the Fourier transform (hereafter FT) of the images remembering that the squared complex modulus of a FT yields the PS of a signal.

### 2.6.1 2D power spectrum

Assume that ˘$I_{uv}$ is the 2D FT of the image $I_{lm}$ where $u, v$ represent the spatial frequencies corresponding to the angular scales $l, m$. The minimum and maximum spatial frequencies of ˘$I_{uv}$ are determined by $1/(N, \theta_{\text{pix}})$ and $1/(2 \theta_{\text{pix}})$ respectively where $\theta_{\text{pix}}$ is the angular size of the pixels in $I_{lm}$ and $N, \theta_{\text{pix}}$ are the total number of pixels in $l$ and $m$ directions respectively. We cut the portion of ˘$I_{uv}$ delimited by the minimum and maximum physical baselines and calculate the 2D PS as $P_{2D}(u, v) = |˘I_{uv}|^2$.

To produce 1D angular PS, we divide $P_{2D}$ in several concentric circular bins and calculate the average power at every bin. Finally, we plot the average power in the bins as a function of comoving transverse wavenumbers corresponding to the bins defined as (Morales et al., 2012b, equations 2-3),

$$k_\perp = \frac{2\pi U_\perp}{D_e(z)}$$

(2.30)

where $U_\perp = \sqrt{u^2 + v^2}$ in units of wavelengths, transverse comoving distance at redshift $z$, $D_e(z) = \int_0^z dz'/E(z')$, and dimensionless Hubble parameter, $E(z) = (\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda)^{1/2}$, $\Omega_m$, $\Omega_k$ and $\Omega_\Lambda$ being the matter density, curvature and cosmological constant parameters respectively. Thus, we obtain $k_\perp$ in units of Mpc$^{-1}$ and $P_{2D}(k_\perp)$ in units of K$^2$ Mpc$^2$. Note that the minimum and maximum values of $k_\perp$ are determined by $U_{\perp, \text{min}}$ and $U_{\perp, \text{max}}$ respectively, as shown in Vedantham et al. (2012, equations 13-14).

### 2.6.2 3D power spectrum

Assume that ˘$I_{uv\eta}$ is the 3D FT of the image $I_{lm\eta}$ where $\eta$ represents the LOS spatial frequency corresponding to the LOS distance signified by the frequency $v$ (see Morales et al., 2012b, Fig. 2). After taking only the portion of the cube that represents real baseline distribution as before, the 3D PS can be calculated as $P_{3D}(u, v, \eta) = |˘I_{uv\eta}|^2$. Two types of binned PS can be calculated from this PS-cube: cylindrical, $P_{3D}(k_\perp, k_\parallel)$, and spherical, $P_{3D}(k)$.

In the cylindrical case, averaging is done in concentric cylindrical bins centred on the centre of the cube. Hence, $P_{3D}(k_\perp, k_\parallel)$ is the average power of all $uv\eta$ cells within a logarithmic cylindrical bin around $k_\perp, k_\parallel$ where the comoving LOS wavenumber,

$$k_\parallel = \frac{2\pi H_0 E(z) v_{21}}{c(1+z)^2},$$

(2.31)

$v_{21}$ being the rest frequency of 21-cm radiation emitted by HI, and $k_\perp$ is the same as defined by Eq. 2.30. The minimum and maximum values of $k_\parallel$ are given by $\eta_{\text{min}} = 1/B$ and $\eta_{\text{max}} = 1/\Delta v$ respectively where $B$ is the bandwidth and $\Delta v$ is the frequency resolution.
provided by the instrument. From the minimum and maximum values of $k_\perp$ and $k_\parallel$, it is evident that the boundaries of the $k$-space are defined by the instrumental parameters (see e.g. Vedantham et al., 2012, Fig. 4). Instead of showing the raw power we plot the quantity $\Delta^2(k_\perp,k_\parallel) = k_\perp^2 k_\parallel P_3D(k_\perp,k_\parallel)/(2\pi)^2$ in our 2D figures which has the dimensions of temperature squared.

For constructing the spherical 3D PS, we divide the PS-cube in concentric spherical annuli around the centre of the cube and average the power in every annulus. Consequently, we get a 1D PS as a function of $k = \sqrt{k_\perp^2 + k_\parallel^2}$. Here, we plot the quantity $\Delta^2(k) = k^3 P_3D(k)/(2\pi^2)$ that has the same dimensions as $\Delta^2(k_\perp,k_\parallel)$.

## 2.7 Conclusion

We have revisited the measurement equation of a radio interferometer and modelled the direction independent (DI) and direction dependent (DD) errors as $2 \times 2$ Jones matrices and the corresponding $4 \times 4$ Mueller matrices have been used to show the polarization properties of the instrument. The full polarization DD-Mueller matrix (Fig. 2.2) describing the time-frequency-direction dependent behaviour (e.g. see Fig. 2.2 and 2.3) of the response of a baseline of LOFAR, created by two stations (Fig. 2.1), has been presented to be a DD equivalent of the DI-Mueller matrix of Eq. 2.18.

We have found that the polarization leakage predicted by the model beam increases with distance from the phase center of the field, and also with distance of the field from the local zenith. The components of the Mueller matrix responsible for leakage were found to be 3–4 orders of magnitude lower than the Stokes I beam. The FWHM and spatial structure of the beam model change smoothly with frequency. The LOFAR beam model has an FWHM of $3.776^\circ$ at 150 MHz and decreases with increasing frequency.

The primary beam model and the mathematical formalism described in this chapter will be used throughout the thesis.
3. Extragalactic foreground

Adapted from Section 3 of Asad et al. (2015).

To show the effects of direction independent errors on calibration, we simulate the observations of a mock sky with point sources. In case of the direction dependent errors, we first simulate a mock sky to show the trend of the effects, and then proceed to simulate the realistic sky to quantify the effects expected in the LOFAR-EoR observations. We did not include any additive noise in the simulations described in this section. Below we describe the general pipeline of the simulations followed by the set-ups and results of the specific simulations.

3.1 Pipeline

A block diagram of the pipeline for simulating extragalactic point sources is shown in Fig. 3.1. We start from a given model of the sky (described in the specific sections) and predict the visibilities that LOFAR would produce in the presence of certain DI and DD (beam) errors. Simulations with the two systematic errors are done separately, although some steps are common to both of them.

DI errors are introduced in accordance with the formulation described in section 2.2.1. After prediction, the visibilities corrupted by the DIEs are self-calibrated using the same sky model that was used to predict. Then, the gains determined by selfcal are compared with the input gains to calculate the error parameters defined by equations 2.17a-g. Additionally, the solved gains are applied to the model visibilities to produce corrected visibilities. All
Figure 3.1: Block diagram of the pipeline of the simulations of extragalactic foreground. Blocks with solid and dotted borders represent simulations with DD and DI errors respectively; blocks with dashed borders represent steps performed for both simulations, but separately. Arrows with dashed line-styles have been used to avoid intersection between arrows. FT, IFT and SC stand for Fourier transform, inverse Fourier transform and self-calibration respectively.
3.1 Pipeline

Table 3.1: Observational setup for simulations of extragalactic sources:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of LOFAR HBA stations used, ( N )</td>
<td>59</td>
</tr>
<tr>
<td>Number of baselines, ( N(N - 1)/2 )</td>
<td>1711</td>
</tr>
<tr>
<td>Number of spectral subbands</td>
<td>1</td>
</tr>
<tr>
<td>Number of channels in the subband</td>
<td>1</td>
</tr>
<tr>
<td>Central frequency of the channel</td>
<td>150 MHz</td>
</tr>
<tr>
<td>Width of the channel, i.e. frequency resolution</td>
<td>0.19 MHz</td>
</tr>
<tr>
<td>Total observation time</td>
<td>8 h</td>
</tr>
<tr>
<td>Integration time, i.e. time resolution</td>
<td>10 s</td>
</tr>
<tr>
<td>Number of timeslots</td>
<td>2874</td>
</tr>
<tr>
<td>Number of visibilities</td>
<td>5090520</td>
</tr>
<tr>
<td>Baseline cut (( u_{\min} \sim u_{\max} )) for imaging</td>
<td>0.06 – 20 km</td>
</tr>
<tr>
<td>Baseline cut for PS estimation</td>
<td>0.06 – 1 km</td>
</tr>
<tr>
<td>Angular resolution (PSF) of the images, ( \alpha \lambda / u_{\max} )</td>
<td>( \sim 0.34 ) arcmin</td>
</tr>
<tr>
<td>Physical width of the HBA stations, ( D )</td>
<td>30 m</td>
</tr>
<tr>
<td>FWHM of station primary beams, ( \alpha \lambda / D )</td>
<td>( \sim 3.78 ) deg</td>
</tr>
<tr>
<td>Field of view, ( \pi(FWHM/2)^2 )</td>
<td>11.2 deg²</td>
</tr>
</tbody>
</table>

processes up to this point are performed using the standard LOFAR calibration and simulation software, Black Board Selfcal (BBS; Pandey et al. 2009). We image both the corrupted and the corrected visibilities using CASA and produce 2D PS from the images through the procedure described in section 2.6.1.

DD errors are introduced by multiplying every point source in the model with the relevant station beam at the position of the source at every timeslot. Fourier transform of the beam attenuated sky yields the visibilities corrupted by DDEs. We carry out two different simulations with these dataset: one to measure effects of DD errors, and another to quantify the errors in calibration due to incomplete calibration sky model. The latter could be done meaningfully without introducing systematic errors at all, but we did it this way to make it more realistic.

To quantify the effects of DD errors, first, we correct the corrupted visibilities for the beam at the phase centre which, in reality, normalizes the DDEs with respect to the phase centre so that only the differential nominal beam effects remain (this step is not shown in Fig. 3.1). Then, we image both the corrupted and uncorrupted (ideal) visibilities and produce 2D PS from the images. Furthermore, we extract the fluxes and positions of the brightest point sources in the corrupted and uncorrected images using PyBDSM\(^1\) and compare them. Finally, we correct the visibilities for the differential beam and produce images from them using AWImager. Fluxes of the beam-corrected images are compared with the uncorrected fluxes to quantify the quality of the correction.

To determine calibration errors due to an incomplete sky model, we calibrate the corrupted visibilities using different incomplete sky models. As the same DDEs are included during both prediction and calibration, the remaining errors will be only due to the incompleteness of the models. The deviation of the different corrected visibilities from the corrupted visibilities is demonstrated through PS.

\(^1\)http://tinyurl.com/PyBDSM-doc
Figure 3.2: Left: Fractional error on the 7 DIE parameters defined in Eq. 2.17 for a single baseline as a percentage of the input rms DI-errors. The \( \Delta g \) and \( \Delta \varepsilon \) used to calculate these parameters are the differences between the components of the input Jones matrix and the Jones matrix calculated by self-calibration. Right: Square-root of the fractional residual power spectra, which is equivalent to the rms of the image, for different rms DI-errors and Stokes parameters. See section 3.2 for details.
3.2 Direction independent errors

To show the effects of DI errors and test their correction strategy, we ignore the DDEs and introduce DIEs for every station and timeslot as G-Jones matrices. Both gain ($g$) and feed ($\varepsilon$) error terms of $G$ are modelled as complex numbers that are random at every time-step drawn from a Gaussian distribution with zero mean and a certain standard deviation (rms). Then, we create a sky model containing 25 sources of 5 Jy Stokes $I$ flux ($Q, U, V = 0$) in a $5 \times 5$ uniform grid of $1^\circ$ separation, predict the DIE-corrupted visibilities for all baselines of LOFAR and perform all the other steps described in the previous section and shown in Fig. 3.1 (see the blocks with dotted and dashed borders). The rms of the introduced errors is the same for every term of the G-Jones of every station and we repeat this experiment thrice for three different rms DI-errors: $10^{-3}$, 0.01 and 0.1. Note that, as the calibration was done with a perfect sky model, the errors will be due only to the calibration process itself.

We analyse the results using two parameters: fractional rms selfcal error ($\delta_f$) and square-root of the residual power spectrum ($\sqrt{P(k)}$) which, in effect, gives the rms of the images at different spatial frequencies. To determine $\delta_f$, we calculate $\Delta G^g$ and $\Delta G^f$ (see Eq. 2.16) by differencing the model gains and the solved gains for two stations, and then, calculate the DIE-parameters ($\delta$) for the baseline created by those stations (equations 2.17). We did not plug in the values of $\Delta G^g$ and $\Delta G^f$ directly in Eq. 2.16 to calculate $\delta_f$, but created an error DI-Mueller matrix from the $\Delta G$ matrices of two stations following Eq. 2.9 and extracted the 7 relevant parameters from it. $\delta_f$ for a given $\delta$ is the rms of the $\delta$ as a percentage of the input rms DI-error. The seven $\delta_f$ are plotted as a function of the input rms DI-errors on the left panel of Fig. 3.2. We see that fractional selfcal errors increase linearly with rms DI-errors, and for an rms DI-error of $10^{-3}$, which is not unrealistic, the error on these parameters is less than 0.002%.

For calculating residual $P(k)$, we subtract the corrected Stokes images from the corrupted ones and measure the PS of the residuals. As we did not subtract any source from the corrected visibilities, if the calibration error is low the difference between the corrected and the corrupted visibilities should also be low. $P_r(k)$ is the PS of a residual image as a percentage of the PS of a corrupted image. $\sqrt{\langle P_r(k) \rangle}$ of the different Stokes images for the three simulations are plotted on the right panel of Fig. 3.2 which clearly shows that the calibration errors propagated to the PS are negligible as expected in the absence of additive noise. For an rms DI-error of $10^{-3}$, errors on $\sqrt{P(k)}$ or, equivalently, on the rms of the image is less than 0.005%. Furthermore, by comparing the Stokes $I, Q + iU$ and $V$ power spectra for an rms DI-error of 0.01, we see that the errors on different Stokes parameters are the same, as expected. This simulation shows that self-calibration can correct for the DI-errors to a very high accuracy if we have a sufficiently accurate model of the sky.

3.3 Direction dependent errors

To show the effects of DD errors on point sources and to test one of their correction strategies, we ignore the DIEs, introduce DDEs as station beams and carry out the steps outlined in Fig. 3.1 (see the blocks with solid and dashed borders). As mentioned before, we implemented two different simulations with the DDE-corrupted dataset; the purpose of the first one is
Table 3.2: Sky models used for the different simulations of extragalactic foreground with DDEs.

<table>
<thead>
<tr>
<th>Field</th>
<th>Phase centre (Equatorial J2000)</th>
<th>FoV (deg)</th>
<th>Number of sources</th>
<th>Max (Jy)</th>
<th>Min (Jy)</th>
<th>Total(^a) (Jy)</th>
<th>Spectral index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mock 8(^b)</td>
<td>8(^h)13(^m)36(^s), 48(^\circ)13(^\prime)0(^\prime)</td>
<td>10</td>
<td>225</td>
<td>100</td>
<td>0.4</td>
<td>189.6</td>
<td>-0.75</td>
</tr>
<tr>
<td>3C196(^b)</td>
<td>8(^h)13(^m)36(^s), 48(^\circ)13(^\prime)0(^\prime)</td>
<td>10</td>
<td>4567</td>
<td>83</td>
<td>0.027</td>
<td>796.64</td>
<td>-0.75</td>
</tr>
</tbody>
</table>

\(^a\) All flux densities shown here are at 150 MHz.

\(^b\) Sources taken from the \textbf{F}aint \textbf{I}mages of the \textbf{R}adio \textbf{S}ky at \textbf{T}wenty-cm (FIRST) survey, produced by NRAO VLA at 1365 and 1435 MHz and 5\(^\prime\) resolution; noise \(\sim 0.15\) mJy.

to show the effects of DD-errors on the Stokes parameters and this has been done for two different sky models, some information about which are listed in table 3.2.

### 3.3.1 Test with a mock sky

To show the general trend of the effects of DD-errors, we make a mock sky model comprising 225 unpolarized point sources arranged in a 15 \times 15 uniform grid of 0.66\(^\circ\) separation centred on the position of 3C196\(^2\) and simulate an 8-hour, 150-MHz observation of LOFAR, taking into account the beams described in section 2.2.2. The source at the centre of the grid is given a flux density of 100 Jy, while each of the other sources have a flux density of 0.4 Jy. The central source has been made exceptionally bright (analogous to the 3C196 field) to be able to check the consequence of calibrating an otherwise dim sky with a very bright point source which will be described in section 3.4.

As the sources were completely unpolarized, the Stokes \(Q, U, V\) images created from this dataset contain only the flux leaked from Stokes \(I\), i.e. instrumentally polarized sources. These sources are shown on the middle and right panels of Fig. 3.3. Each bubble in the plots represent an instrumentally polarized point source and the size and colour of the bubble represent the flux of the source as a percentage of its Stokes \(I\) flux. The figures show that leakages to both linear and circular polarizations increase as we go out from the centre of the field. As for the levels of leakage, within the central 4 degrees, i.e. within the first null of the primary beam at 150 MHz, linear polarization leakage \((I \rightarrow P)\) is around 0.5%, and circular polarization leakage \((I \rightarrow V)\) is less than 0.003%. Instrumental polarization of the central bright source (not shown in the figure) is very low, because before imaging the visibilities corrupted by the DDEs were corrected for the element beam \((E_e\) of Eq. 2.19) at the phase centre, thereby making the leakage terms very close to zero at that point. In physical terms this means that the projection of the beams on the sky had been made perfectly orthogonal at the phase centre. What is left after this centre-correction is the effect of the differential beam (e.g. Fig. 2.2a). There is an anomaly in the south-east corner of the middle and the right panels of Fig. 3.3 which can be attributed to the errors in extracting fluxes of very dim sources situated near the null of the primary beam.

These results are consistent with the beam model described in section 2.2.2. For example, we can understand both the trend and the level of linear leakage seen in Fig. 3.3 by comparing it to the \(M_{21}\) and \(M_{31}\) components of the instrumental Mueller matrix shown Fig. 2.2a, or to

\(^2\) A quasar situated at \(z \sim 0.871\) with a flux density of 74.3 Jy at 174 MHz.
Figure 3.3: *Left:* Distribution of 103 sources from the 225 sources arranged in a $15 \times 15$ uniform grid within a $10^\circ$ field of view. Flux (bubble size) and position (colour) errors due to calibration with only the prominent central source are shown. *Middle:* Same distribution with corresponding fluxes leaked from Stokes I to linear polarization as a percentage of Stokes I flux (size and colour). *Right:* Same as the middle figure except that it is for the leakages to circular polarization which is much lower.
Figure 3.4: *Left:* Distribution of the brightest 33 sources ($I > 100$ mJy) in the 3C196 field with their corresponding Stokes I fluxes (colour) and flux errors (bubble size) due to calibration with different number of sources (numbers in the legend) in the sky model. The percentages in the legend refer to the minimum and maximum flux errors. Note that after calibration with 1000 sources errors for most of the sources decrease. *Right:* Same distribution with the corresponding linear polarization leakages as a percentage of Stokes $I$ flux. Both colour and size of the bubbles represent fractional leakage.
Figure 3.5: \(a\): Square-root of the power spectra of the \(Q,U,V\) leakages as a percentage of the Stokes \(I\) power spectrum within the central 10 degrees of the 3C196 field. \(b\): Residual (after subtracting calibrated data from the uncalibrated ones) power spectra of Stokes \(I\) as a percentage of the uncalibrated Stokes \(I\) PS of the same field. The different cases are for calibration with different number of sources in the sky model. These residuals correspond to calibration errors due to incomplete sky model.
the spatio-temporal profiles of the leakages shown in Fig. 2.2b. We expect to see leakage at this level also in the realistic simulations and this expectation will be put to the test in the next section where we describe the simulation of one of the LOFAR-EoR target fields.

3.3.2 3C196 field

The 3C196 field (centred on the bright quasar, 3C196; Bernardi et al. 2010) is well-suited for EoR observations because the presence of a bright and almost unresolved source at its centre allows very accurate direction independent calibration, and it is situated in one of the colder regions of the Galactic halo. To make an unpolarized sky model for simulating this field, we extract Stokes $I$ fluxes and positions of the sources brighter than 25 mJy within a radius of 5° around 3C196 from the FIRST survey catalogue (see table 3.2) and extrapolate the fluxes to that of 150 MHz using a spectral index of -0.75 which is typical for the radio sources at these frequencies. The eponymous source, 3C196, has been taken out of this model, and a 4-component improved model of the source made from LOFAR data by V. N. Pandey has been inserted in its place.

Linear leakage of the brightest 33 sources (Stokes $I > 100$ mJy) is shown on the right panel of Fig. 3.4. Both colour and size of the bubbles in the figure represent the percentage of leakage. Extraction of fluxes and positions of the sources in this case is not as precise as that of the gridded sky model as here sources are much more closely spaced; thus some errors in this scatter plot originate from the source extraction process. Nevertheless, the figure, as a whole, is quite informative; we see that linear leakage can be as high as 4%, but for most of the sources it is less than 2% and for the sources very close to the phase centre only less than a percent leak, as expected. The sources with the highest leakages (the three reddest bubbles) are very dim in Stokes $I$ which can be seen by comparing these three bubbles with the corresponding bubbles on the left panel where the Stokes $I$ fluxes are shown as colour of the bubbles. These leakages might not be real, but a consequence of errors in the source extraction process. The leakage from 3C196 itself is very low and hence is not shown here.

The overall level of the leakage can be better understood from the fractional (as a percentage of Stokes $I$ PS) power spectra of Stokes $Q, U, V$ shown in Fig. 3.5a. The PS of $Q/I$ and $U/I$ tell us that the rms of the linear leakage is $0.05 \sim 0.06\%$ of the rms of the Stokes $I$ image. On the other hand, rms of circular leakage is almost 4 orders of magnitude lower. Leakage from linear polarization to Stokes $I$, which is relevant for the EoR experiments, will be similar to the $I \rightarrow P$ leakage shown in this simulation, as evident from a comparison of the first and second panels from the top of Fig. 2.2b. However, compact radio sources are usually unpolarized or very weakly polarized and hence the leakage from polarized point sources into Stokes $I$ is very low and even that leakage can be removed by direction dependent calibration (e.g. SAGECal; Kazemi et al. 2011; Kazemi & Yatawatta 2013) and/or AW-projection (Tasse et al., 2013).

3.3.3 Correcting polarization leakage of point sources

There are several strategies for correcting beam-related DDEs which are classified broadly into two categories: image-plane and Fourier-plane corrections. Here, we test one of the Fourier-plane strategies called AW-projection (see section 2.3.1), a particular version of which is implemented by AWImager for the LOFAR AW-terms.
3.4 Selfcal errors due to incomplete sky model

Incomplete sky models can lead to many problems in directionally independent self-calibrated data, among them generation of spurious source components, removal of real source components and the generation of ghost sources, a spurious source whose flux is proportional to the flux of an unmodelled source (Grobler et al., 2014). We try to quantify the calibration errors
due to incomplete sky models with different numbers of sources in the models for the 3C196 field. Note that, as we included the beam during both prediction and calibration, its effect was taken out and we were left with only selfcal errors. The calibration is performed using BBS which is based on the matrix formalism described in section 2.

We make 5 different calibration sky models that contain roughly 10, 15, 30, 60 and 75% of the total flux of the field; the models have 1, 5, 50, 400 and 1000 sources respectively. After self-calibrating the field with each one of these models, we calculate the difference of fluxes of the sources between the calibrated and uncalibrated data, and also create corresponding residual PS. The left panels of Fig. 3.4 show the flux errors on the sources that contribute to the largest errors in the field; the filled bubbles represent the errors after calibrating with only 10% of the total flux, while the unfilled bubbles with red borders are for the case when 75% flux is modelled. According to the figure, errors go down significantly after improving the sky models.

In Fig. 3.5b, we show the PS of the Stokes $I$ residual after subtracting the calibrated images from the uncalibrated ones as a fraction of uncalibrated Stokes $I$ PS. As there is an exceptionally bright source (the second brightest source is only 7.7 Jy) at the centre of the 3C196 field, rms of the residual is already low (1% of the rms of the original image) after calibrating with only 3C196 which contains 10% of the total flux of the field. Errors go down significantly when we include 15% of the total flux by adding another 4 sources in the model, but after that there is no rapid improvement.

3.5 Conclusion

This chapter presents a first step in the analysis of the systematic errors of a radio interferometer, with a focus on polarization leakage, by simulating the LOFAR observations of both compact and diffuse emission in the presence of direction independent and direction dependent errors which are treated separately.

We have simulated an observation with DI-errors by assuming them to be random at every timestep and the rms of the random numbers drawn from a Gaussian distribution with zero mean is dubbed the ‘rms DI-error’. We find that self-calibration can solve for these errors to an extremely high accuracy if the sky model is perfect as, in that case, the information provided by an interferometer will be highly redundant. For an rms DI-error of $10^{-3}$, the selfcal error is less than 0.002% and the corresponding error in the rms of the resulting residual image is less than 0.005% (Fig. 3.2).

The only DD-error that we have simulated is the differential (normalized with respect to the phase centre) station beam of LOFAR. We simulated the LOFAR observations of extragalactic unpolarized point sources in the 3C196 observing field of the LOFAR-EoR experiment including the DD-errors, and estimated the flux and position errors due to self-calibration with incomplete sky models, and the percentage of $I \rightarrow (Q, U)$ leakage of the brightest sources (see Fig. 3.4). We see that the errors go down significantly as the sky model is improved. However, calibrating with only unpolarized sources has its limitations, e.g. the unitary ambiguity (Wijnholds et al., 2012; Carozzi, 2014). There is no plan for using polarized sources in calibrating LOFAR EoR data until now, as there are very few intrinsically polarized
point sources in the data, and the polarized emission is dominated by diffuse emission (e.g. see Yatawatta et al., 2013; Jelić et al., 2014).

We test a possible strategy of correcting the DD errors from point sources using AWImager with an unrealistic, exaggerated sky model and see that AWImager can remove up to 80% of the leakage from Stokes $Q, U$, but a more elaborate testing of this algorithm with realistic sky models has to be done to reach any final conclusion.
4. Galactic foreground

Adapted from Section 4 of Asad et al. (2015).

So far we have considered leakages from unpolarized point sources into Stokes $Q, U, V$ only, but, as mentioned before, our interest lies in the opposite case, i.e. leakage from polarization to total intensity. Compact radio sources are very weakly polarized and most of the point sources seen in polarization maps can be attributed to instrumental polarization and leakage. As at frequencies of tens to hundreds of MHz the polarized sky is dominated by Galactic diffuse synchrotron emission, we take real data of the 3C196 field observed by LOFAR, and create the simulated dataset using it as a sky model following the pipeline described in the next section. In these simulations, except for the one represented by Fig. 4.3h, our Stokes $I$ data contain only the noise leaked from Stokes $Q, U$. We do not add realistic noise to Stokes $I$ until the final test because that would make the quantification of the intrinsic instrumental polarization over the complete $k$-space difficult, as the expected level of leakage is lower than the system noise. However, as a final test we add system noise to check the efficiency of a leakage removal technique.

4.1 Simulation setup

The general pipeline of the simulation of Galactic foreground is almost same as that of the extragalactic foreground (boxes with solid borders in Fig. 3.1), but there are two major differences: here we simulate datasets for 161 spectral bands instead of just one, and examine the leakages from Stokes $Q, U$ to $I, V$ rather than that from $I$ to $Q, U, V$. The former enables us to examine the frequency behaviour of the leakages through rotation measure synthesis
Table 4.1: Setup of the simulation of Galactic foreground.

<table>
<thead>
<tr>
<th>Baseline used for simulated observation</th>
<th>up to 3 $k\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of spectral subbands</td>
<td>161</td>
</tr>
<tr>
<td>Number of channels in each subband</td>
<td>1</td>
</tr>
<tr>
<td>Width of the channels / frequency resolution, $\delta\nu$</td>
<td>0.19 MHz</td>
</tr>
<tr>
<td>Central frequency of the observing band</td>
<td>150 MHz</td>
</tr>
<tr>
<td>Total bandwidth, $\Delta\nu$</td>
<td>32 MHz</td>
</tr>
<tr>
<td>Total observation time</td>
<td>8 hr</td>
</tr>
<tr>
<td>Integration time / time resolution</td>
<td>10 s</td>
</tr>
<tr>
<td>Baseline cut for imaging and PS estimation</td>
<td>30 – 800 $\lambda$</td>
</tr>
<tr>
<td>Angular resolution (PSF) of the images</td>
<td>4.3 arcmin</td>
</tr>
<tr>
<td>Number of pixels in the images</td>
<td>$480 \times 480$</td>
</tr>
<tr>
<td>Size of each pixel</td>
<td>0.5 arcmin</td>
</tr>
<tr>
<td>Maximum detectable Faraday depth, $\sqrt{3}/\delta\lambda^2$</td>
<td>160 rad/m$^2$</td>
</tr>
<tr>
<td>Largest resolvable structure in Faraday depth, $\pi/\lambda_{\min}^2$</td>
<td>0.96 rad/m$^2$</td>
</tr>
<tr>
<td>Resolution in Faraday depth space, $2\sqrt{3}/\Delta\lambda^2$</td>
<td>1 rad/m$^2$</td>
</tr>
<tr>
<td>Minimum and maximum $k_{\perp}$ [Mpc$^{-1}$]</td>
<td>0.02 – 0.53</td>
</tr>
<tr>
<td>Minimum and maximum $k_{\parallel}$ [Mpc$^{-1}$]</td>
<td>0.011 – 1.85</td>
</tr>
</tbody>
</table>

and 3D power spectrum analysis, and the latter provides us with a realistic estimate of the amount of leakage into Stokes $I$ to be expected in the current LOFAR-EoR observations of the 3C196 field.

To make the 3C196 polarization sky model, we took Stokes $Q, U$ images for 161 subbands spanning 32 MHz centred at 150 MHz that were produced from a single-night (8 hr) LOFAR observation using the standard LOFAR calibration and imaging pipeline (e.g. see Yatawatta et al., 2013; Jelić et al., 2014). During the reduction process, DI-errors were removed and the data was also corrected for the element beam at the phase centre, thereby removing most of the instrumentally polarized point sources. We removed the remaining point sources by just masking them with noise so that diffuse emission dominates the image. The most significant systematic errors that still remains in these images are the ionospheric Faraday rotation and the differential beam. A dataset with ionospheric correction implemented is not necessary for our case as we are not concerned with analysis of the real data here, but only with the fraction of leakage; thus, any reasonable input model would serve our purpose. Also note that we are applying the ‘model’ differential beam to an image that already has the ‘true’ differential beam in it. This cannot be avoided as direction dependent calibration or differential beam correction are yet to be done in this observing window, but DD-correction would not bring any dramatic change in the final results that we want to produce, as the polarization maps are dominated by diffuse emission and differential beam is only an 1% effect.

Stokes $I$ and $V$ in the model images were put to zero so that after applying the beam, they contain only leakages from $Q, U$. The following steps were performed to produce the final results.

1. DDE-corrupted visibilities at different frequencies are simulated using\texttt{AWImager}, as a prediction using\texttt{BBS} would currently take too much time. Here\texttt{AWImager}, in effect, carries out the forward transform of the major cycle and stops.
4.1 Simulation setup

Figure 4.1: Faraday dispersion images of Galactic diffuse polarized emission within the central 4 degrees of the 3C196 field (top) and their leakages to Stokes $I$ (middle) and $V$ (bottom) caused by the LOFAR differential beams at the Faraday depths of 0 (left), +1 (middle) and +2 (right) rad/m$^2$. The diameters of the inner and the outer circles are 2$^\circ$ and 3.8$^\circ$ (FWHM of LOFAR station beam at 150 MHz) respectively. The images have 480 × 480 pixels of 0.5$'$ with a PSF of 3$'$. 
2. Images from the simulated visibilities are produced using CASA. Different parameters of the input model images and the final CASA images were kept the same; for details see table 4.1.

3. We make 4 image-cubes by combining the images for 4 Stokes parameters and also convert the fluxes in Jy to intensities in temperature following Eq. 2.27.

4. To analyse Faraday structure of the leakage of polarized emission, RM-synthesis is performed on the cubes according to the formalism of section 2.5 resulting in 3 ‘dirty’ (without deconvolving the RMSF) RM-cubes: $F(\Phi), F_I(\Phi)$ and $F_V(\Phi)$.

5. Cylindrical and spherical 3D power spectra for all Stokes parameters are calculated from the image-cubes according to the formalism described in section 2.6.2.

4.2 Results

We first show the results of RM-synthesis of both polarization and leakage, but with a focus on the leakages, and then present the 3D power spectra produced from the image-cubes.

4.2.1 RM synthesis

The polarization RM-cube, $F(\Phi)$ basically represents the real data that we took as our input model, as leakage from $Q$ to $U$ and vice versa will be very small compared to their brightness. The maximum intensity in the cube is $\sim 5$ K which is seen at $\Phi = +1.2$ rad/m$^2$ and the brightest structures are located within the Faraday depths of -1.5 and +5.0. Three slices of the $F(\Phi)$ cube at $\Phi=0,1,2$ rad/m$^2$ are shown in the top 3 panels of Fig. 4.1 and the diffuse Galactic emission is prominent in all of them, but increases toward $\Phi = +1$ rad/m$^2$. For a detailed analysis of polarized emission in the 3C196 window seen by LOFAR, we refer the reader to Jelić et al. (in preparation). The corresponding slices of $F_I(\Phi)$ leakage are shown in the middle panels of the figure and here we see that the highest leakages appear at $\Phi = 0$ and their peak is $\sim 10$ mK. As differential beams vary slowly with frequency (e.g. see Fig. 2.3), the leakages caused by them are a smooth function of frequency, thereby making them localized around $\Phi = 0$ in RM space. This property can be utilized to correct the effects of leakage, but performing a realistic leakage removal is beyond the scope of this paper (e.g. see Geil et al., 2011). However, in section 4.3, we will show the results of a correction that does not take the differential beam into account. Another aspect of these images can be seen by focusing on the central 2 degrees; for example, although $F(\Phi = 0)$ is highest in the central part, the corresponding $F_I(\Phi = 0)$ leakage is still much lower than that of the outer region (between the inner and the outer circle). This is expected as leakage terms of the beam Mueller matrix increase toward the outskirts.

The 3 bottom panels of Fig. 4.1 show the same Faraday dispersion images for leakages into Stokes $V$ and they are much lower than the corresponding leakages into Stokes $I$—so much lower that they are dominated by leaked noise. This can be understood in terms of the differential beam of Fig. 2.2— the $M_{42}$ and $M_{43}$ components of this matrix are responsible for leaking $Q,U$ to $V$ and we see that they are 2-3 orders of magnitude lower than the components responsible for leakage into Stokes $I$, i.e. $M_{12}$ and $M_{13}$.

\footnote{1The RM-synthesis code of Michiel Brentjens was used for this purpose.}
Figure 4.2: Four lines of sight along Faraday depth (Faraday spectrum) for 4 bright pixels in the Faraday dispersion images $F(\Phi)$ (top), leakage into $F_I(\Phi)$ (middle) and leakage into $F_V(\Phi)$ (bottom). The pixels were chosen according to their intensity in $F_I(\Phi)$ and their RA and DEC are shown in the legend.
Chapter 4. Galactic foreground

Behaviour of the leakage in Faraday space can be seen more clearly in Fig. 4.2, where we show 4 lines of sight along Faraday depth (Faraday spectrum) for 4 bright pixels in $F(\Phi)$ (top), $F_I(\Phi)$ (middle) and $F_V(\Phi)$ (bottom). The bright pixels were chosen in $F_I(\Phi)$ and then the corresponding pixels were found in $F(\Phi)$ and $F_V(\Phi)$. The fact that instrumental polarization and leakage appears at $\Phi=0$ in a Faraday spectrum, convolved with the RMSF, is evident from the middle panel of the figure. It is not so evident in $F_V(\Phi)$ due to the dominance of leaked noise.

4.2.2 3D Power spectra

A 3D power spectrum analysis of the DDE-corrupted image-cubes would be most interesting, as this would allow us to calculate the amount of polarized Galactic foreground leaked into a possible ‘EoR window’ (a region in 3D Fourier space where the EoR signal is taken to be least contaminated) of LOFAR; for an example of an EoR window, see Fig. 1 of Dillon et al. (2014) that was made using the instrumental parameters of MWA. In the top panels of Fig. 4.3, we show the 3D cylindrical power spectra of the beam-corrupted polarized emission ($P$), its leakages into Stokes $I$ and $V$, and the ratio between the power spectra of the $I$-leakage and that of the 21-cm differential brightness temperature $\delta T_b$ of the fiducial model of Mesinger et al. (2011). The plots show the power that lies within a given $k_\perp,k_\parallel$ bin in units of $[\text{mK}]^2$ or as a ratio.

The $P$ spectrum (panel a) exhibits the same characteristics that one would expect based on the behaviour of the polarized emission in RM-space (described in the previous section). As in RM-space the brightest polarized emission were found near $\Phi=0$ rad/m$^2$, so here the power is high at low $k_\parallel (<0.1 \text{ Mpc}^{-1})$. Some additional power is seen in a wedge-shaped region at $k_\parallel > 0.1$ and $k_\perp > 0.1$ which can be attributed to the frequential unsmoothing of the intrinsically smooth polarized foreground by the frequency-varying PSF, and the extra power at high $k_\perp$,$k_\parallel$ is due to noise. At $k_\perp < 0.04$ power is very low, as expected, and it reaches its maximum at around $0.3 \text{ Mpc}^{-1}$. The maximum power is around $4.4 \times 10^5 \text{ [mK]}^2$ which is found at the highest $k_\perp,k_\parallel$.

The $I$-leakage spectrum (c) looks very similar to the $P$ spectrum, and the leakage power reaches up to $\sim 5.5 \text{ [mK]}^2$. At $k_\perp < 0.1$ and $k_\parallel > 0.1$ leakage power is 2–3 orders of magnitude lower than the maximum. In order to see if $I$ is just a scaled down version of $P$, we calculate the ratio $\sqrt{I/P}$ as a percentage of $P$, shown in panel e, which gives an estimate of the percentage of rms leakage at different $k_\perp$ and $k_\parallel$. Evidently, at $k_\parallel < 0.06$ leakage rms is 0.2%–0.3% of the polarization rms, and can go as high as 0.4% at high $k_\parallel$ where noise leaked from $P$ to $I$ dominates.

The $V$-leakage spectrum is shown in panel b, and its level is much lower, the peak being around $0.06 \text{ [mK]}^2$. The region at high $k_\parallel$ and high $k_\perp$ is dominated by noise, as signal-to-noise ratio is lower for longer baselines. By comparing this spectrum with that of $P$, it can be seen that the rms of the $V$-leakage at low $k_\parallel$ is only $\sim 0.003\%$ of the rms of the polarized emission which means that the uncorrected $V$ leakage is negligible compared to the current noise levels in the EoR experiments within a FoV of $4^\circ$.

As noted in the previous section, leakage is lower near the centre of the field (see Fig. 4.1). In order to quantify the associated decrease in power, we calculated the power spectra of $I$, $P$ and $\sqrt{I/P}$ within the inner $3^\circ$ of the field. We found that, in this case, maximum leakage
Figure 4.3: Top: Cylindrically averaged 3D power spectra (PS) of the polarized emission ($\mathcal{P}$) within the central 4° of the 3C196 field (a) and its leakages into Stokes $V$ (b) and $I$ (c). Panel d shows the ratio between panel c and the corresponding PS of the 21-cm differential brightness temperature $\Delta^2_{21}$ for the fiducial model of Mesinger et al. (2011) at $z = 9$. The contours are drawn where this ratio is 1 for both the normal leakage and the leakage reduced by 70%. Bottom: (e) Square-root of the ratio between the panels c and a expressed as a percentage. The $g$ and $f$ panels represent the same quantity as that of the c and e panels respectively, but for the case when 4 foreground components were removed from the leakage into Stokes $I$. Panel $h$ shows the same percentage as the $e$ and $f$ panels, but when 60 mK noise was added to the $I$-leakage before foreground removal. The subscript $R$ stands for GMCA residual.
into $I$ at low $k_\parallel$ is $\sim 4.9 \text{[mK]}^2$ and $\sqrt{I/P} \approx 0.2\%$ at $k_\parallel < 0.06$ which is lower than the level of leakage within the inner 4°.

To see how $P \rightarrow I$ leakage affects the EoR signal, we took the 3D spherical power spectrum of the fiducial model of 21-cm differential brightness temperature $\delta T_b$ at $z = 9$ from Mesinger et al. (2011) and calculated the corresponding cylindrical power spectrum as $\Delta^2_{21}(k_\perp, k_\parallel) = k^2_\perp k_\parallel \Delta^2_{21}(k)/[2(k^2_\perp + k^2_\parallel)^{3/2}]$. The spherical PS is plotted in Fig. 4.4 along with the spectra of $P$, $I$ and $V$, and the figure clearly shows that the EoR signal power is higher than the $I$-leakage at $k < 0.3 \text{ Mpc}^{-1}$, and can be 2 orders of magnitude higher at the lowest scales. The ratio between $\Delta^2_{21}(k_\perp, k_\parallel)$ and $\Delta^2_I(k_\perp, k_\parallel)$ is plotted in Fig. 4.3d where the contours are drawn at $\Delta^2_I/\Delta^2_{21} = 1$ for the normal case and the case when 70% of the leakage had been removed. Evidently, there is an ‘EoR window’ above the PSF-induced wedge and below $k_\parallel \sim 0.5 \text{ Mpc}^{-1}$, and the window extends up to $k_\parallel \sim 1 \text{ Mpc}^{-1}$ when 70% leakage is removed.

### 4.3 Polarization leakage removal

As Jelić et al. (2010, section 7.2) have discussed at length, a leakage of the polarized foreground into total intensity will be a major obstacle in detecting the EoR signal if (1) the level of leakage is comparable to the intensity of the EoR signal, and/or (2) frequency spectrum of the leakage mimics that of the signal. Fortunately, in the 3C196 field the latter is not the case, as we have seen that there is no significant polarization at high Faraday depths, or, equivalently, at high $k_\parallel$. However, the power of leakage could be comparable to that of the signal at high $k_\parallel$ and hence leakage needs to be removed with sufficient accuracy to extend the EoR window.

There are many methods for removing foregrounds from Stokes $I$; some assume spectral smoothness of the foreground and try to fit it out using polynomials, while others do not assume anything and hence are called ‘blind’ or non-parametric methods (for a list see Chapman et al., 2016). The best way to remove the leakage contribution of the foreground is, of course, to use the time-frequency-baseline dependent Mueller matrices during calibration and/or imaging to produce beam- and leakage-corrected images. Another potential way is to correct them in the Faraday dispersion images, i.e. correcting the $F_I(\Phi)$ using information from $F(\Phi)$ as demonstrated by Geil et al. (2011, see section 6.2 and Fig. 6). For leakages as smooth as in the field of 3C196, simply filtering the $F_I(\Phi)$ for $\Phi \sim 0$ could be another potential solution. However, testing these methods is beyond the scope of this paper, and here we use a non-parametric foreground removal method, called GMCA (generalized morphological component analysis; Bobin et al. 2007, 2008a,b), that has been shown to be able to remove foregrounds from simulated LOFAR-EoR data with high accuracy (Chapman et al., 2013).

If a signal is represented as $X = AS + N$ where $S$ is the foreground to be extracted, $N$ is noise and $A$ is the mixing matrix, then GMCA tries to calculate a mixing matrix for which $S$ is sparsest (have the least number of non-zero wavelet coefficients) in the wavelet domain. For details of the algorithm we refer the readers to Chapman et al. (2013). We run GMCA on the Stokes $I$-leakage cube to extract and subtract 4 components of the leaked foreground, as this number has been shown to yield good results (Chapman et al., 2016), and produce 3D cylindrical power spectrum from the residual cube which is shown in Fig. 4.3g. It clearly
shows that the power of the smooth foregrounds at low $k_\parallel (< 0.06)$ has been reduced by almost two orders of magnitude by GMCA; compare it with the input $I$-leakage spectra of panel c that is plotted on the same scale. On the other hand, everything above $k_\parallel = 0.1$ has been kept completely untouched due to low SNR—where S is the foreground and N is the noise including the cosmic signal—as GMCA cannot produce reliable model for the foregrounds when the SNR is low. In panel f, we plot the ratio of the GMCA residual PS and the polarization PS which shows that after GMCA subtraction, rms residual leakage at $k_\parallel < 0.1$ is around 0.1% of the polarized intensity. However, the EoR signal could also be removed along with the foreground in this case as there was no noise in Stokes $I$ except for a very low level of noise leaked from $Q, U$.

To see how additive noise affects the removal of leakage, we add 60 mK (rms) noise, which should be reached after 600 hours of integration using LOFAR (Chapman et al., 2016), to the Stokes $I$ leakage maps at all frequencies. The noise was added to the visibilities and a new image cube was produced from the noisy visibilities. We run GMCA on the noisy $I$-leakage cube and produce a 3D cylindrical PS from the residual and take the square-root of the ratio of this PS with respect to the $I$-leakage which is shown as a percentage in Fig. 4.3h. We see that almost no leakage has been removed in this case, not even in the relatively high SNR region at low $k_\perp$. Therefore, we conclude that in case of such levels of noise, either a different strategy should be taken to remove foreground-leakage, or the leakage dominated region (where the leakage is more than the EoR signal) should be avoided to some extent (see Chapman et al. (2016) for a discussion on the relative merits of foreground removal and avoidance.)

Figure 4.4: Spherically averaged 3D power spectra of the polarized emission within the central 4° of the 3C196 field (top solid), its leakages into Stokes $I$ (middle solid) and $V$ (bottom solid) caused by the LOFAR model beam, and the PS of the 21-cm differential brightness temperature $\Delta^2_{21}$ at $z = 9$ (solid with circles) for the fiducial model of Mesinger et al. (2011).
4.4 Conclusion

To predict the level of polarization leakage in the Stokes I images of the 3C196 field, we took the real LOFAR observations of Galactic diffuse polarized emission in this field and created an unreal sky model where $I = V = 0$ to quantify the leakages from $Q, U$ to $I, V$ caused by the DD errors. An RM-synthesis of the DDE-corrupted $\mathcal{D}$ image cubes showed that in this particular field polarization peaks within the Faraday depths ($\Phi$) of -1 and +5 rad/m$^2$. From the effective Stokes I Faraday dispersion images we saw that polarization leakage is localized around $\Phi = 0$ (Fig. 4.2), as DD-errors do not have any rapid variation along frequency. Maximum leakage was found to be around 15 mK which could be comparable to the EoR signal (Fig. 4.1).

To understand the level of leakage contaminant in the ‘EoR window’ of the instrumental $k$-space, we calculated the cylindrically and the spherically averaged 3D power spectra (PS) of $I, P, V$ cubes. The $P$ spectrum shows characteristic smooth polarized foregrounds at low $k_\parallel$ (Fig. 4.3) and the $I$-leakage spectrum looks very similar to this. From the power ratio, $\sqrt{I/P}$ we showed that the percentage of rms leakage over the $k_{\perp}, k_{\parallel}$ space varies by a factor of 2 and ranges from 0.2% to 0.4%. We compared the $I$-leakage with the 3D PS of the expected 21-cm differential brightness temperature at $z = 9$ simulated by Mesinger et al. (2011) and saw that the region above the PSF-induced wedge and below $k_\parallel \sim 0.5$ Mpc$^{-1}$ is dominated by the cosmic signal (Fig. 4.3d) and hence defines a potential ‘EoR window’, and the window expands substantially after removing 70% of the leakage.

As the $I$-leakage do not mimic the EoR signal in this case, we tried to remove it using GMCA which is being used to remove diffuse foreground from the LOFAR-EoR data. From the 3D PS of the residual left after the removal of foreground leakage components by GMCA, we saw that (Fig. 4.3f,g) at $k_\parallel < 0.1$, i.e. in the high SNR regime, GMCA could reduce the leakage by up to two orders of magnitude while the region above that scale was left completely untouched. For a more realistic analysis, we added 60 mK noise to the Stokes I leakage maps, reran GMCA on it and saw that (Fig. 4.3h) in this case almost no leakage was removed, not even in the relatively high SNR region.

Antennas for the future arrays like SKA, that have EoR detection as one of the main scientific objectives, are being designed in such a way that their polarimetric performance is good enough to be able to minimize the effects of polarization leakage (de Lera Acedo; private communication). A recently proposed figure of merit for quantifying the polarimetric performance is the intrinsic cross-polarization ratio (IXR) which, in Mueller formalism, can be directly related to the instrumental polarization (Carozzi & Woan, 2011, eq. 23). Our LOFAR results show an instrumental polarization of around 0.3% (Fig. 4.3e; ignoring $V \rightarrow I$ leakage) within the FWHM of the nominal station beams, i.e. within a FoV of $\sim 4^\circ$. This corresponds to an IXRM (Mueller IXR) of 25 dB, or equivalently an IXRJ (Jones IXR; see eq. 25 of Carozzi & Woan 2011) of 56 dB, and if the leakage can be reduced by 70%, IXRM will improve to 35 dB.

Therefore, we can say that if SKA has a minimum IXRM of 25 dB within the central $\sim 4^\circ$ of its nominal station beams, then even a modest polarimetric calibration ($\sim 70\%$ leakage removal) will ensure that the polarization leakage remains well below the expected EoR signal at the scales of 0.02–1 Mpc$^{-1}$. However, if the IXRM is lower within a FoV of $4^\circ$, more leakage needs to be removed to reach the same level as before in relation to the EoR signal in
the power spectra, e.g. if the $\text{IXR}_M$ is 20 dB, 91% leakage has to be removed, and if it is 15 dB, 97% has to be removed.

The major conclusions of this chapter are the following.

1. Two properties of the polarization leakage can be utilized for its removal in this specific case: it appears around a Faraday depth of 0 rad/m$^2$ in RM-space and the overall variation of the rms of the fractional leakage in the instrumental $k$-space is less than a factor of 2.

2. In the cylindrically averaged 3D power spectra, a clear ‘EoR window’ can be defined in terms of polarization leakage above the wedge and below $k_\parallel \sim 0.5 \, \text{Mpc}^{-1}$. Within this window, the EoR signal dominates the polarization leakage and the window takes up the whole $k$-space at $k_\parallel < 1$ after removing 70% of the leakage.

3. A DDE-blind foreground removal method like GMCA is not ideal for removing leakage of diffuse polarized emission, as the level of leakage is lower than the current noise level in the LOFAR observations.
5 Accuracy of the beam model ........ 67
5.1 Introduction
5.2 Primary beam model of LOFAR
5.3 Data processing and simulation pipelines
5.4 Results
5.5 Discussion and Conclusions

6 Polarization leakage in wide fields .... 93
6.1 Introduction
6.2 Power spectra of polarized emission
6.3 Fractional leakage in wide fields
6.4 Discussion
6.5 Conclusion
5. Accuracy of the beam model

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Abstract

Leakage of diffuse polarized emission into Stokes I caused by the polarized primary beam of the instrument might mimic the spectral structure of the 21-cm signal coming from the epoch of reionization (EoR) making their separation difficult. Therefore, understanding polarimetric performance of the antenna is crucial for a successful detection of the EoR signal. Here, we have calculated the accuracy of the nominal model beam of LOFAR in predicting the leakage from Stokes I to Q, U by comparing them with the corresponding leakage of compact sources actually observed in the 3C295 field. We have found that the model beam has errors of \( \leq 10\% \) on the predicted levels of leakage of \( \sim 1\% \) within the field of view, i.e. if the leakage is taken out perfectly using this model the leakage will reduce to \( 10^{-3} \) of the Stokes I flux. If similar levels of accuracy can be obtained in removing leakage from Stokes Q, U to I, we can say, based on the results of our previous paper, that the removal of this leakage using this beam model would ensure that the leakage is well below the expected EoR signal in almost the whole instrumental k-space of the cylindrical power spectrum. We have also shown here that direction dependent calibration can remove instrumentally polarized compact sources, given an unpolarized sky model, very close to the local noise level.
Chapter 5. Accuracy of the beam model

5.1 Introduction

One of the fundamental obstacles in statistically detecting the 21-cm signal coming from the epoch of reionization (EoR) is the leakage of polarized signal into total intensity caused by the time-frequency-baseline-direction dependent primary beams of the telescope. The Galactic diffuse foreground, the most dominant contaminant of the EoR signal after the extragalactic compact sources (e.g. Bernardi et al., 2009, 2010; Patil et al., 2014), is expected to be separated from the signal by utilizing the fact that the foreground is spectrally smooth and the signal is not (Jelić et al., 2008; Datta et al., 2010b; Harker et al., 2010; Trott et al., 2012; Morales et al., 2012a; Bernardi et al., 2013; Pober et al., 2013; Chapman et al., 2013; Dillon et al., 2015; Thyagarajan et al., 2015b). However, the Faraday-rotated polarized foreground is also not smooth along frequency, and its leakage into total intensity might mimic the frequency structure of the EoR signal making the separation of the two difficult (Pen et al., 2009; Jelić et al., 2010; Moore et al., 2013; Asad et al., 2015, hereafter A15). Moreover, chromaticity of the beam—characterized, e.g., by the first derivative of the beam as a function of frequency—can cause the spectrally smooth diffuse foreground to show fluctuations along frequency (Mozdzen et al., 2016). The EoR detection experiments with GMRT\(^1\) (Pen et al., 2009), LOFAR\(^2\) (A15, Jelić et al. 2015), PAPER\(^3\) (Kohn et al., 2016), MWA\(^4\) (Sutinjo et al., 2015), HERA\(^5\) (Neben et al., 2016), SKA\(^6\) (de Lera Acedo et al., 2015) will be affected by ‘polarization leakage’ to various degrees depending on the directional gain properties and the fields of view of the instruments.

A number of papers dealing with the direction dependent (DD) gains, i.e. the primary beam, of low frequency radio telescopes have been published recently that shows the relevance of polarimetric analysis in the detection of the EoR signal. Pober et al. (2016) demonstrated the effects of the Stokes \(I\) primary beam of MWA that can leak power from the foreground wedge into the EoR window, claiming that the foreground in even the sidelobes of the primary beam needs to be modeled and removed for a successful detection of the EoR signal as the farther the source is from the phase center the worse the leakage of power from the wedge. Polarized power will be leaked from the wedge in a similar way albeit to a much lower amplitude. Efforts are underway to better understand the MWA beam and calculate the accuracy of the beam model. Sutinjo et al. (2015) found that with the ‘Full Embedded Element Pattern’ model, the beam can be 2–5% different from reality. With the improved model they found that a \(I \rightarrow Q\) leakage of a few per cent (with outliers up to 10%) is achievable which is higher than that of the LOFAR case, as the field of view of an MWA tile is significantly higher than that of a LOFAR station. Some preliminary results of the ‘intrinsic cross-polarization ratio’ (IXR; Carozzi & Woan, 2011) of MWA tiles (Sutinjo & Hall, 2013) have been published. Foster et al. (2015, fig. 1) show an example of the variation of IXR\(_J\) (IXR calculated in terms of Jones matrices; see Carozzi & Woan (2011) for more details) of a simple all sky dipole element at 130 MHz. The beam model exhibits an IXR\(_J\) of 70 dB toward the zenith with a low-IXR\(_J\) structure along the 45 degree line between the orthogonal receptors, and based on this model they have used a polarization leakage of up to

\(^1\)http://gmrt.ncra.tifr.res.in/
\(^2\)http://www.lofar.org/
\(^3\)http://eor.berkeley.edu/
\(^4\)http://www.mwatelescope.org/
\(^5\)http://reionization.org/
\(^6\)http://www.skatelescope.org/
-30 dB in their simulations of pulsar times of arrival. A stringent limit on the accuracy of the model beamwidth of a wide-field transit radio telescope has been set by Shaw et al. (2015). By simulating the CHIME\(^7\) observations of the foreground-contaminated 21-cm signal in the presence of instrumental errors, they have found that in order to recover unbiased power spectra, the model beamwidth of each element should be known to an accuracy of at least 0.1% within each minute. Compared to beamwidth, beamshape errors would be even more difficult to model and hence would be a more problematic source of bias in the power spectra. In Chapter 4, we predicted the polarization leakage from Stokes \(Q, U\) to \(I\) to be expected in the ‘EoR window’ of the cylindrically averaged power spectra (PS) using the LOFAR observations of the 3C196 field. The prediction was based on the nominal model beam of LOFAR produced by Hamaker (2011) using an electromagnetic simulation of the ASTRON Antenna Group\(^8\) (Schaaf & Nijboer, 2007). We found that within a field of view (FoV) of 3 degrees the rms of the leakage as a fraction of the rms of the polarized emission varies between 0.2 to 0.3 per cent, and the leakage is lower than the EoR signal at \(k < 0.3\) Mpc\(^{-1}\). We thus concluded that even a modest polarimetric calibration over the FoV would ensure that the polarization leakage remains well below the expected EoR signal at scales of 0.02–1 Mpc\(^{-1}\). The accuracy of this prediction depends mainly on the accuracy of the model beam.

In this chapter, we have used LOFAR observations of the compact sources in the 3C295 field to quantify the accuracy of the nominal model beam of LOFAR (Hamaker, 2011), as this field is less contaminated by polarized diffuse emission than the 3C196 field. In addition to quantifying the accuracy of the beam, we demonstrate the efficiency of a DD calibration method in removing instrumentally polarized compact sources. This chapter is organized as follows. Section 2 revisits the nominal model beam of LOFAR and shows the behavior of the intrinsic cross-polarization ratio of the instrument as a function of distance from the phase center and also distance of the observing field from the zenith. In section 3, we describe the data reduction, calibration and simulation pipelines. Our results are presented in section 4—first, we present the results of the observation and the simulation, then compare them to quantify the accuracy of the beam model, and finally present the results of the DD calibration. The chapter ends with a discussion of our analysis and some concluding remarks.

5.2 Primary beam model of LOFAR

The primary beams of LOFAR HBA stations are modeled in three steps: an analytic expression is used for the dipole beams whose coefficients are calculated by fitting to a beam raster generated by electromagnetic simulation, the 16 dipole beams are phased in an analog way to create the tile beams, and the tile beam patterns are multiplied together with the respective weights and phases using an ‘array factor’ to create the station beams.\(^9\) As a low-frequency aperture array, LOFAR does not have any moving parts and hence the beams of two orthogonal feeds are projected non-orthogonally away from zenith while tracking a moving source giving rise to mutual coupling between the beams. The projection-induced mutual coupling is the

\(^7\)http://chime.phas.ubc.ca
\(^8\)M. J. Arts; http://www.astron.nl
\(^9\)The simulation procedure is described in detail in Hamaker (2011) and its key points are also mentioned in Asad et al. (2015, section 2.2.2). Also note that the software package that creates the directional response of the antenna elements is included in the standard LOFAR calibration software BBS (Pandey et al., 2009), publicly available at https://svn.astron.nl/LOFAR/trunk/CEP/Calibration.
Figure 5.1: Simulated model of the primary beam Jones matrix of the LOFAR station CS001HBA0 at 150 MHz for a zenith pointing. The color-bar is shown in decibel units. The left and right panels show the \((xx + yy)/2\) and \((xx - yy)/2\) components respectively.

Figure 5.2: Spectral structure of the primary beam model of LOFAR calculated by taking a slice in frequency space at different zenith angles at an azimuth of 180°. The color-bar is shown in decibel units. The left and right panels show the \((xx + yy)/2\) and \((xx - yy)/2\) components respectively.
Figure 5.3: IXR$_M$ of a typical LOFAR baseline within the central 8.3° × 8.3° of the 3C295 field at eight different instances during an 8-hr synthesis. The EM-simulated LOFAR model beam has been used to calculate the parameter. The panels correspond to different hour angles, mentioned in the top-left corners. IXR$_M$ is lowest at ±0.6 h when the field is very close to its culmination point or equivalently in the zenith.
Chapter 5. Accuracy of the beam model

Principal contributor to polarization leakage and its removal completely depends on how well we can model the beam.

Although projection effects are worse at lower elevation, it is possible to remove the effects of the primary beam to high accuracy toward at least one direction, the phase center, at all elevations using direction independent (DI) calibration (Sault et al., 1996) and a model of the dipole beam (e.g. see Chapter 2). After calculating the electronic gains of a station via DI calibration and correcting the data for the dipole beam at the phase center, only the effect of the array factor and the errors in correcting the dipole beam effects remain, which can be thought of as a differential beam with respect to the phase center. An example of such a ‘differential’

\footnote{In this chapter, whenever we talk about the LOFAR primary beam or station beam, it should be understood to be the ‘differential’ primary beam, i.e. the beam where each component of the Jones or Mueller matrix has been normalized with respect to the phase center.}

station beam at 150 MHz is shown in Fig. 5.1 where the field of view, the nulls and the sidelobes are clearly visible. The left panel of the figure shows the sum of the diagonal terms of the beam Jones matrix, i.e. \( \left( xx + yy \right) / 2 \), and the right panel shows their difference, i.e. \( \left( xx - yy \right) / 2 \). If we divide the difference by the sum, we obtain the fraction of Stokes I flux that leaks into Stokes Q. A more intuitive way to calculate this leakage is to use Mueller matrices instead of Jones matrices, and we calculate the leakage in terms of Mueller matrices below and follow the Mueller formalism throughout the chapter. Fig. 5.2 shows the spectral structure of the sum (left panel) and the difference (right panel) of the diagonal terms of the beam Jones matrix and they demonstrate that the position of the sidelobes changes smoothly as a function of frequency. We will demonstrate the accuracy of this beam in predicting the polarization leakage. We will call this leakage the ‘off-axis’ leakage as opposed to the ‘on-axis’ leakage at the phase center. Off-axis leakage increases as we go away from the phase center and the zenith or, in case the observing field never reaches the local zenith, the culmination point of the field.

A fundamental figure of merit (FoM) to evaluate the polarization performance of a polarimeter is the intrinsic cross-polarization ratio (IXR) introduced by Carozzi & Woan (2011). The ‘intrinsic’ in IXR signifies that the parameter is independent of the choice of coordinate systems. IXR is related to the invertibility of a DD Jones matrix. The Jones matrices calculated by calibration are inverted and multiplied with the data to give the ‘corrected’ data, and hence the intrinsic invertibility of a Jones matrix put a fundamental limit to the extent to which a data can be corrected. For Stokes polarimeters, IXR can be easily converted to a Mueller IXR, or \( \text{IXR}_M \) which, in turn, is directly related to the fractional polarization leakage (fraction of Stokes I signal leaked into Stokes Q, U, V and vice versa) caused by the beam, mathematically

\[
\text{IXR}_M = 10 \times \log_{10} \left[ \sqrt{\frac{M_{10}^2 + M_{20}^2 + M_{30}^2}{|M_{00}|}} \right] \text{ dB} \quad (5.1)
\]

where \( M \) is a 4 × 4 Mueller matrix corresponding to the outer product of the DD Jones matrices of two elements that make up a baseline of an array, and by the subscript ‘10’ in \( M_{10} \) we mean the second row and first column of the matrix (for explanation see section 2.2.1 of A15). \( M_{10}, M_{20}, \) and \( M_{30} \) give leakages from \( I \) to \( Q, U, \) and \( V \) respectively, and \( M_{00} \) gives the Stokes I beam. For an example of a Mueller matrix that completely characterizes the beam of a baseline see fig. 2 of A15. Note that \( \text{IXR}_M \) is usually taken to be the opposite of this...
Figure 5.4: Azimuthal profiles of $\text{IXR}_M$ at eight different instances of time during an eight hour observation of the 3C295 field. The texts correspond to the hour angle of the field during the observation which clearly shows that the leakage is lowest at $\pm 0.6$ h. The solid and dashed lines correspond to the negative and positive hour angles respectively.

The values are usually expressed as a positive integer and in dB units. However, here we express the dB values as negative integers so that they correspond to the increment of leakage with distance from the phase center more intuitively.

$\text{IXR}_M$ distributions within the central $8.3^\circ \times 8.3^\circ$ of the 3C295 field, one of the secondary observing windows of the LOFAR-EoR project, at eight different instances of time during an 8 hour synthesis are shown in Fig. 5.3 for example; the observation time increases as we go from left to right panels on the top, and then from right to left panels on the bottom. We see that $\text{IXR}_M$ increases as we go away from the phase center of the field and also from the culmination point, and this increment directly corresponds to an increase in leakage. There is a reversal of orientation of the elliptical shape of the spatial distribution of the $\text{IXR}_M$ which is due to the reversal of the orientation of the projected dipole beams. For a more quantitative understanding we show the azimuthal profiles of $\text{IXR}_M$ at the same eight instances of time in Fig. 5.4. We see the same trend as Fig. 5.3 here: an increase of $\text{IXR}_M$ as we go away from the center and the zenith. $\text{IXR}_M$ or equivalently the leakage is lowest near the zenith, $3.4 \sim 4.5$ hours after the beginning of the synthesis.

All plots in this section have been calculated from the model beam of LOFAR (Hamaker, 2011), the same beam that was used to predict the amount of leakage from linearly polarized diffuse Galactic emission into total intensity (A15). Our main aim in this chapter is to demonstrate the accuracy of this model within the FoV of a typical LOFAR HBA (high band antenna, 110–200 MHz) station. After finding the accuracy, we will be able to constrain our previous prediction more robustly and the need for improvement of the model. The accuracy has been demonstrated below by comparing the leakage actually seen in an 8-hr synthesis data of the 3C295 field and the leakage predicted by the model beam that we have introduced.
### Table 5.1: Observational parameters of the 3C295 synthesis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation ID</td>
<td>L104068</td>
</tr>
<tr>
<td>Start time [UTC]</td>
<td>22 Mar, 2013, 21:41:05</td>
</tr>
<tr>
<td>Phase center, RA</td>
<td>14h11m20.656</td>
</tr>
<tr>
<td>Phase center, DEC</td>
<td>+52°12'9''</td>
</tr>
<tr>
<td>Frequency range</td>
<td>115–189 MHz</td>
</tr>
<tr>
<td>Spectral resolution</td>
<td>3.2 kHz</td>
</tr>
<tr>
<td>Observing time</td>
<td>8h</td>
</tr>
<tr>
<td>Integration time</td>
<td>2s</td>
</tr>
</tbody>
</table>

#### 5.3 Data processing and simulation pipelines

We have used real and simulated LOFAR observations of the 3C295 field. This field was chosen because the compact sources in Stokes $Q, U$ leaked from Stokes $I$ are less contaminated by diffuse polarized emission compared to the 3C196 field. Observed data were processed using the standard LOFAR software pipeline, and the simulated observations were produced using the simulation pipeline presented in A15. In this section, we briefly describe the observational setup and data processing steps. Next, the process of simulating the desired observations, taking into account the systematic effects of LOFAR, by implementing our previous pipeline is described.

#### 5.3.1 Observations

The 3C295 field was observed multiple times by LOFAR. Here we have used an 8-hour synthesis observation taken in March 2013. An overview of the observational parameters is presented in Table 5.1, and an instantaneous uv-coverage of the inner 3 km of the configuration used for this observation is shown in Fig. 5.5.

For this observation, the phased array was set up in the HBA DUAL INNER configuration consisting of 48 core stations (CS) and 14 remote stations (RS). As the RS have 48 tiles in contrast to the 24-tile CS (van Haarlem et al., 2013, fig. 4), half of the tiles of the RS were turned off to make them equivalent to the CS. The observations spanned the frequency range from 115 to 189 MHz, that was divided into 380 sub-bands, each of width $\sim$ 195 kHz. Each subband was further divided into 64 channels. All four correlations of the voltages between the orthogonal pairs of dipoles were recorded and the data were integrated every 2s at the correlator. Data were taken only during the night and the syntheses were symmetric around the time of culmination of the fields.

#### 5.3.2 Flagging and averaging

The acquired data were first processed using the AOFlagger (Offringa et al., 2010, 2012) to remove terrestrial radio frequency interference (RFI). Within the frequency range from
5.3 Data processing and simulation pipelines

Figure 5.5: The instantaneous uv-coverage of the LOFAR configuration. Only the Dutch stations have been shown here. The coverage within the inner 3km, relevant for our experiment, is shown on the inset. The \( u \) and \( v \) distances are shown in frequency independent physical units, i.e. in km.

115–177 MHz, on average only 1% data were flagged due to RFI. However, above 177 MHz more than 40% of the data were flagged due to interfering signals from Digital Audio Broadcasting. After flagging, the data were averaged in time and frequency to reduce their volume for further processing. Every 10s of the data were averaged, and the inner 60 channels of every sub-band were averaged to produce a single channel of width 183 kHz. The four edge channels were excluded from the averaging process to remove edge effects from the polyphase filter. Averaging usually results in bandwidth and time smearing, but we are not affected by them as only the short baselines were considered in this study which are less prone to smearing effects.

5.3.3 Calibration

After flagging and averaging, we performed calibration in two steps: direction independent (DI) and direction dependent (DD). DI calibration was performed using the Black Board Selfcal (BBS) package (Pandey et al., 2009). We used a sky model consisting of only 3C295, the central source that dominates the visibilities on all baselines. The model (Scaife & Heald, 2012) had two components with a total Stokes \( I \) flux of 97.76 ± 2 Jy at 150 MHz and it also sets our broad band spectral model. Note that the model "lead to unacceptably high flux scale uncertainty at frequencies below 70 MHz," but we would not be affected by this as we restrict ourselves within the frequency range of 134–166 MHz. BBS calculates the four complex components of the DI Jones matrices for each station taking into account the changing location of 3C295 within the primary beam of the dipole elements, and the variation of parallactic angle which minimizes the instrumental polarization in the vicinity of the phase center of the field. The gains are then applied on the model visibilities which in turn are
Table 5.2: Imaging parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline cut</td>
<td>30 – 1000 $\lambda$</td>
</tr>
<tr>
<td>Weighting</td>
<td>Natural</td>
</tr>
<tr>
<td>Angular resolution (PSF)</td>
<td>3.44 arcmin</td>
</tr>
<tr>
<td>Frequency range</td>
<td>134–166 MHz</td>
</tr>
<tr>
<td>Spectral resolution</td>
<td>1.9 MHz</td>
</tr>
<tr>
<td>Synthesis time</td>
<td>8h</td>
</tr>
<tr>
<td>Time resolution</td>
<td>10s</td>
</tr>
<tr>
<td>Number of pixels in the image</td>
<td>$1024 \times 1024$</td>
</tr>
<tr>
<td>Size of each pixel</td>
<td>0.5 arcmin</td>
</tr>
</tbody>
</table>

...subtracted from the observed visibilities to remove the 3C295 source with its DI gains. In addition, BBS removes the clock and short-timescale ionospheric phase errors, and sets the frequency-dependent intensity and astrometric reference frame for the field (Pandey et al., 2009; Yatawatta, 2012; Jelić et al., 2014).

DI-calibration works well only for the sources on or very close to the phase center. However, there are another 10 sources brighter than 0.75 Jy within the central 8° of the 3C295 field, and we removed them with their corresponding DD-gains using SAGECAL, a DD-calibration package (Kazemi et al., 2011; Kazemi & Yatawatta, 2013). SAGECAL calculates complex Jones matrices for every station toward the directions of the 10 sources. Although SAGECAL does not have any information about the time-frequency varying polarized primary beams of the stations, it should be able to reproduce their effects through the DD-gains. In principle, all significant DD effects should be absorbed in the DD-gains, among them also the position-dependent ionospheric delays. Each direction is associated with one source, and the solution interval is 10 minutes for each direction which is sufficient to remove the sources down to the confusion noise, resulting from the background unresolved radio sources, on the short baselines. Removing these 10 sources does not affect the other sources in the field since no gain solutions are applied to the data in DD calibration; they are only applied to the model and subtracted from the data. Note that the data has not been corrected for ionospheric Faraday rotation, that depolarizes the signal depending on the level of total electron content in the ionosphere. However, it will not affect our experiment as the variation of the ionospheric Faraday rotation is usually comparatively small within 8 hours (e. g. see fig. 2 of Jelić et al., 2015) and it affects only the intrinsically polarized sources which are excluded while calculating the accuracy of the beam model.

### 5.3.4 Imaging

Two different sets of images were produced, one from only the DI-calibrated data and the other from both DI- and DD-calibrated data. Imaging was performed using the standard LOFAR-EoR imaging software, excon (Yatawatta, 2014, [http://exconimager.sf.net](http://exconimager.sf.net)). Baselines only up to 1 k$\lambda$ were used and, although higher resolution images would produce even better results, an angular resolution of 3.44 arcmin is both sufficient for our purposes and computationally less expensive. The visibilities were weighted naturally in all cases. We took 160 subbands within the frequency range of 134–166 MHz centered around 150 MHz to conduct our simulation and analysis described below. We use the same parameters to
5.3 Data processing and simulation pipelines

create images from both the real observations and the simulated observations. The imaging parameters are listed in Table 5.2.

### 5.3.5 Simulated observations

For simulating the LOFAR observations of the 3C295 field, we use the pipeline described in our previous paper, A15. Here we briefly outline the steps of the simulation specific to this experiment.

First step in simulating an observation is to create a realistic sky model from observed data that we want to compare with. We have taken the Stokes $I$ images for 160 frequency channels and created a sky model using buildsky that uses the available frequency information to calculate the spectral index of each source. The aim is to predict the leakage from Stokes $I$ to $Q, U$, hence we do not want to include any polarization in our sky model. We used a flux cut of 100 mJy to remain well above the local noise around the sources (≈ 5 mJy/beam). Our model consisted of 140 point sources, mainly within the first null of the primary beam, several of which were constructed by more than one components. Note that, we have created a sky model from an image that was not corrected for the primary beam. Therefore, there is a systematic decrease in flux away from the phase center until the first null, and then there are some more sources on the sidelobes of the beam. The attenuation caused by the primary beam does not pose any difficulty in quantifying the fractional leakages, as the attenuation effect drops out in the ratio of different Stokes parameters, the parameter we are interested in. Also note that in the case of calibrating real data, sky models are usually constructed from very high resolution images, but for our purpose such precision is not required as, again, we are only interested in the fraction of Stokes $I$ flux leaked into the other Stokes parameters, and not in the absolute Stokes $I$ flux.

We calculated visibilities from the sky model using the same baselines as that of the observation at all frequency channels and taking into account the station-time-frequency dependent model primary beams of the instrument. This was done, in effect, by multiplying the fluxes of the individual sources with the beams at the corresponding positions, times and frequencies and Fourier transforming them using BBS. Therefore, although the sky was completely unpolarized, the predicted visibilities had non-zero values in all four visibility correlations due to instrumental polarization. The parameters of this simulated observation are listed in Table 5.3. Note that, the set-up of the instrument was the same for the simulation.

<table>
<thead>
<tr>
<th>Baselines used</th>
<th>$30 - 1000 \lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase center, RA</td>
<td>$14^h11^m20.6^s$</td>
</tr>
<tr>
<td>Phase center, DEC</td>
<td>$+52^\circ12'9''$</td>
</tr>
<tr>
<td>Frequency range</td>
<td>134–166 MHz</td>
</tr>
<tr>
<td>Spectral resolution</td>
<td>1.9 MHz</td>
</tr>
<tr>
<td>Synthesis time</td>
<td>8h</td>
</tr>
<tr>
<td>Time resolution</td>
<td>10s</td>
</tr>
<tr>
<td>Minimum flux in the sky model</td>
<td>100 mJy</td>
</tr>
<tr>
<td>Number of sources</td>
<td>140</td>
</tr>
</tbody>
</table>
as that of the observation.

The simulated visibilities were inverted to produce four images corresponding to the four Stokes parameters. Same imaging parameters were used in this case as in the case of imaging from the observed data; the key parameters are listed in Table 5.2. The standard imaging software CASA was used for all imaging. Although we created images for all Stokes parameters, here we will use only Stokes $I$, $Q$ and $U$ images for our analysis, as the SNR of $I \rightarrow V$ leakage is too low to be useful for a comparison between the observation and the simulation. We used the standard definition of Stokes visibilities to calculate the Stokes parameters from the visibility correlations, as given by the equations (13)a–d of A15, and the linear polarization $P$ was calculated as $Q + iU$. We also created an average of the images of all frequency channels to get an increased SNR that will facilitate the extraction of source fluxes.

5.3.6 Source flux extraction

Once we have all the images, the next step was to calculate the fluxes of the sources that we want to compare. The quantity we use in this case is the ‘peak flux’, as it is straightforward to determine and sufficient for illustrating the difference between the observation and the simulation. To extract the peak fluxes, first, we created small non-overlapping circular apertures around the point sources of interest in the averaged observed $P$ image—as the source must be present in the observed $P$ image for us to be able to compare it with the simulation—where the sources are clearly visible due to high SNR. The sizes of the apertures depended on the structure of the sources, some of which had double lobes, but their radii were never more than 3.3 arcmin in an 8.3 degree image of 0.5 arcmin pixel size. The apertures thus produced from the observed $P$ image were used in all images, and the maximum flux within the apertures were extracted in each case. Following this method, we produced eight different lists of the sources with their peak flux corresponding to the Stokes $I$, $Q$, $U$ and the linear polarization $P$ images of both the observation and the simulation. The minimum threshold set during flux extraction depended on the SNR of individual images and the numbers will be mentioned when we describe the results of the data analysis and simulation. The observation also has diffuse polarization, but, as mentioned before, in the 3C295 field this emission is small compared to the 3C196 field, motivating the use of this field rather than the latter; the diffuse emission in the real data set a lower limit on the accuracy of the measurements in the data.

It should be noted that we do not include the effects of the total intensity of the diffuse foregrounds in our simulations. One could argue that we would be affected by the total intensity of the diffuse emission here, if the emission was sufficiently bright. But, we have seen that this is not the case. In fact, we could not detect any diffuse emission in Stokes $I$ even after removing the brightest compact sources. More sources have to be subtracted before we can start looking for diffuse emission in total intensity. However, the case is very different in polarization. Polarized diffuse emission can be comparable to both the instrumentally and intrinsically polarized compact sources. But, again, we have seen that in the 3C295 field that is not the case. In fact, less contamination from diffuse polarized emission was the very reason we chose this field for this experiment. For example, note in Fig. 5.6a that very few compact sources are seen through diffuse polarized emission, and even the sources that are, are much brighter than the diffuse emission around them. More details about this figure are
5.3 Data processing and simulation pipelines

5.3.7 Figures of merit

The figures of merit used in this chapter are mainly the fractional linear polarization leakages, equivalent to the degrees of polarization. However, we also calculate the leakage for Stokes $Q, U$ separately. The following three parameters are most frequently used:

$$m_P = \frac{|Q + iU|}{I} \times 100$$  \hspace{1cm} (5.2)

$$m_Q = \frac{Q}{I} \times 100$$  \hspace{1cm} (5.3)

$$m_U = \frac{U}{I} \times 100.$$  \hspace{1cm} (5.4)

From now on, $m_P, m_Q$ and $m_U$ will refer to the the observed data, and for the simulated data we will use $m'_P, m'_Q$ and $m'_U$, the ratios of the corresponding simulated Stokes parameters. The ratio parameters $m_P/m'_P, m_Q/m'_Q, m_U/m'_U$ and the difference parameters $m_P - m'_P, m_Q - m'_Q$ and $m_U - m'_U$ are the figures of merit we are most interested in. $m_P$ of the sources should follow a Rice distribution as they are essentially the degrees of polarization (for a review see Trippe, 2014).

Note that these parameters are different from the IXR$_M$ introduced in section 2. $m_P$ is most closely related to the IXR$_M$ and, as Stokes $V$ leakage is $\sim 3$ orders of magnitude lower than the linear polarization leakage (e. g. see figs. 6 and 11 of A15), the value of $m_P$ should be comparable to IXR$_M$ in magnitude. However, we would like to point out that IXR$_M$ is calculated directly from the model of the beam, whereas $m_P$ is calculated either from the observed data or from the data created by applying the model beam on the sky and, also, in this case the data is averaged over 8 hours within which time the sky moves in the beam. In case of instrumental polarization, $m_Q$ is determined by $M_{10}$, $m_U$ by $M_{20}$ and $m_P$ by a combination of them.

5.3.8 Rotation Measure Synthesis

A good way to distinguish between the intrinsic and the instrumental polarization is rotation measure (RM) synthesis (Brentjens & de Bruyn, 2005). A linearly polarized wave can undergo Faraday rotation, the rotation of its polarization angle ($\chi$), during its journey from the source to the observer if there are magnetized plasma in between. This wavelength-dependent rotation is quantified by rotation measure, defined as $d\chi / d\lambda^2$, which is equivalent to Faraday depth

$$\Phi = 0.81 \int_{\text{source}}^{\text{observer}} n_e B_{\parallel} dl$$  \hspace{1cm} (5.5)

if the intervening magneto-ionized medium is assumed to be a single screen; here $n_e$ is the density of electrons and $B_{\parallel}$ is the magnetic field component along the line-of-sight component.
$dl$. $\Phi$ and $\lambda^2$ are a Fourier conjugate pair, and this Fourier relationship is the basis of RM-synthesis, a per-pixel one dimensional Fourier transform along $\lambda^2$ for a multi-frequency data. If a source is intrinsically polarized, Faraday rotation will introduce spectral structures in the broadband signature of the source. The more it fluctuates along $\lambda^2$ the higher Faraday depth it will appear at, a basic consequence of the Fourier relationship. On the other hand if the source is not intrinsically polarized, the only broadband signatures that it will have in its polarization is that of the ‘differential’ beam, which is very smooth along frequency (as shown in Fig. 5.2), and the ionosphere. Due to the spectral smoothness of the beam, and due to the fact that we do not apply an ionospheric RM correction, instrumental polarization will produce a strong signal at $\Phi = 0 \text{ rad m}^{-2}$.

We have performed RM-synthesis—using the code written by Michiel Brentjens\textsuperscript{11}—in our analysis mainly to distinguish between the intrinsic and instrumental polarization, which is necessary if we want to compare the leakage predicted by the model beam with the instrumental part of the linear polarization seen in the observed data.

The image obtained after RM-synthesis is usually called Faraday dispersion function $F(\Phi)$, which is just the polarized surface brightness per unit Faraday depth. We have not cleaned $F(\Phi)$ in our analysis, which means in our case $F(\Phi)$ is actually the polarized surface brightness convolved with the rotation measure spread function (RMSF), the equivalent of power spread function (PSF) in imaging. However, to clearly determine the fluxes and degrees of polarization of the instrumentally polarized sources, we have subtracted the RMSF from the Faraday depth profiles, $F(\Phi)$ as a function of $\Phi$, for the sources that could be confused with sidelobes. Sometimes a source can appear at a higher Faraday depth even if it is not intrinsically polarized due to the sidelobes of the RMSF. However, sidelobes are usually symmetric whereas real RM structures are not. By subtracting the RMSF we could eliminate the possibility of false detection of intrinsic polarization.

### 5.3.9 Direction dependent calibration

Both modeling and calibration have been or are being tested for removing polarization leakage in Fourier space. In the former case, leakages are predicted using a model primary beam of the instrument and then deconvolved from visibilities which is essentially similar to primary beam correction in Fourier space. One such method, called AW-projection (Tasse et al., 2013), was tested in A15, and it was found to be able to remove up to 80% leakage. The latter method solves for the leakages instead of modeling them by minimizing a leakage-free data set, simulated from a sky model, with the observed data toward different directions; the solutions thus produced are then applied on the observed visibilities to remove leakage. \textsc{sagecal} is being used as the standard tool for direction dependent calibration and source removal from Stokes $I$ in the LOFAR-EoR key project. It can also be used to remove point sources from polarized data (Jelić et al., 2015). Like self-calibration, \textsc{sagecal} tries to solve for gains to match the model visibilities with the observed ones, but \textsc{sagecal} does it for all given directions instead of just one. If the solutions are good, the corrected data after multiplying the inverse of the gains with the observed visibilities should correspond very closely to the model visibilities. If the model visibilities are calculated without taking into account the primary beam, without any leakage from Stokes $I$ to $Q, U$, \textsc{sagecal} should be

\textsuperscript{11}\url{https://github.com/brentjens/rm-synthesis}
able to blindly incorporate the beam and the corresponding leakage terms in its gain solutions. SAGECAL’s performance in this regard has not been tested yet, and here we perform one such test. We will show how well it can remove instrumentally polarized point sources by incorporating beam-leakage terms into the gain solutions. For this we have run DD calibration on the DI calibrated data set and then used RM synthesis to see if SAGECAL removed the sources at all Faraday depths.

5.4 Results

We will describe the polarization leakage found in the observed data and the simulated observations separately and then go on to compare them to demonstrate the accuracy of the model beam.

5.4.1 Observed polarization leakage

We extracted the peak fluxes of 138 sources from the frequency-averaged $P$ image of the 3C295 field. The faintest source in our list had a flux of 1 mJy which is $6.5\sigma$ above the noise level in the averaged image. Most of these sources are instrumentally polarized and we could find their Stokes $I$ counterparts from which they were leaked. After finding Stokes $I, Q, U$ fluxes of all the sources, we could calculate $m_P, m_Q$ and $m_U$ of the sources. The degrees of linear polarization $m_P$ are shown by the bubble sizes in Fig. 5.6a, that ranges from 0.15% to 4%. The trend of increasing $m_P$ as we go out from the phase center is also clearly visible suggesting the effect is a systematic one, and principally caused by the primary beam of the instrument; compare this with the increase of leakage as a function of distance from the phase center seen in Fig. 5.3, 5.4.

We then proceed to create the RM profiles, i.e. $F(\Phi)$ as a function of Faraday depth, for all sources detected in $P$. The average and standard deviation of the fluxes of all the sources at each Faraday depth is shown in Fig. 5.7a. We can already see from this figure that the fluxes of most of the sources peak at around a single Faraday depth, and that peak is always around $\Phi = 0$. The 16 sources that show peaks at higher Faraday depths along with the 0-peak were isolated. Among them, only 6 could be identified as intrinsically polarized, as described in section 5.4.1. In case of the other 10 sources, either their peaks were caused by the sidelobe of the RMSF, or the SNRs of the peaks were too low to be considered as a detection. We did not take these 16 sources into account while comparing the predicted and observed leakages to calculate the accuracy of the beam model.

Intrinsically polarized point sources

We have found 6 intrinsically polarized compact sources in the 3C295 field. The RM profiles of these sources are plotted in Fig. 5.8. The ‘dirty’ RM profiles (convolved with the RMSF) are shown in green, and the red line shows only higher RM peaks as it was created by subtracting the product of the RMSF and the 0-peak from the ‘dirty’ profile. Only three sources show more than 1% polarization and the minimum degree of polarization is only

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12 A special concern in this regard is the unitary ambiguity that might cause the beam-incorporated solutions to appear differently rotated than the actual beam (Yatawatta, 2012).
(a) Observed polarized emission \(|P + iQ|\) in the 3C295 field after averaging 149 frequency channels. Most of these point sources are leaked from Stokes \(I\) due to instrumental polarization. Only six sources among them were found to be intrinsically polarized as shown in Fig. 5.8. The size of the bubbles indicate the amount of leakage as a percentage of Stokes \(I\) flux, i.e. \(m_P\) of our figures of merit.

(b) Predicted polarization leakage for the point sources in the sky model created from observation. Sources outside the FoV are not visible because the model beam is much more attenuated than the real beam outside its FWHM. The size of the bubbles represent the fractional leakages as a percentage of the Stokes \(I\) flux.

Figure 5.6
(a) RM profiles of the polarized point sources in the 3C295 field. The average (solid line) and standard deviation (shaded region) of the fluxes of almost 100 compact sources have been plotted here at each Faraday depth. What one can understand from the plot is that almost all the sources have peak flux at around $\Phi = 0 \text{ rad m}^{-2}$. Only 16 sources have higher-$\Phi$ peaks and even in that case most of the peaks were due to the sidelobe of the RMSF.

(b) RM profiles of the simulated polarized point sources in the 3C295 field. The average (solid line) and standard deviation (shaded region) of the fluxes of almost 100 compact sources have been plotted here at each Faraday depth. In contrast to Fig. 5.7a, here all sources have their peak flux at around $\Phi = 0 \text{ rad m}^{-2}$.

Figure 5.7
Chapter 5. Accuracy of the beam model

0.17%. A comparison of the green and red curves in Fig. 5.8 shows that for most of the sources intrinsic polarization is either comparable to or more than the polarized flux seen at around $\Phi = 0$. It should be noted that the Faraday depth and the polarization fraction are affected by the ionospheric Faraday rotation and the depth and beam depolarizations. As we did not correct for these effects, the measured values (shown on the top right corners of each panel in Fig. 5.8) cannot be considered to be the true values. But knowing the true values is not necessary for calculating the accuracy of the beam model; knowing whether the sources are intrinsically polarized or not is enough.

Based on previous observations, Bernardi et al. (2013) stated that "one would expect to have one polarized source every four square degrees with an average polarization fraction of a few percent" between 1.4 GHz and 350 MHz. In their 2400 deg$^2$ survey performed using MWA at 189 MHz with an angular resolution of 15.6 arcmin and a noise level of 15 mJy beam$^{-1}$, they found only one polarized point source that shows a 320 mJy peak at $\text{RM} \sim +34.7$ rad m$^{-2}$ and a polarization fraction of $\sim 1.8\%$. On the other hand, in our 10 deg$^2$ LOFAR image averaged over 134 to 166 MHz with an angular resolution of 3.44 arcmin and a noise level of 0.15 mJy beam$^{-1}$, we have found 6 intrinsically polarized point sources. This discrepancy is mainly due to the different sensitivities of the two observations. A polarized source in the MWA observation had to have a flux of at least 75 mJy (5$\sigma$ above their noise level) to be considered a detection, whereas in our case even the brightest intrinsically polarized point source have a flux of only $\sim 7$ mJy. Bernardi et al. (2013) did not find any polarization in the 137 point sources brighter than 4 Jy, and concluded that if any of them were polarized, the polarization fraction would be less than $\sim 2\%$. Our result is in general agreement with this conclusion, as we see that even for fainter sources—all our point sources except one are fainter than 2.5 Jy in Stokes $I$—the polarization fraction is not more than $\sim 1.3\%$.

5.4.2 Predicted polarization leakage

We have identified 95 instrumentally polarized sources in the frequency-averaged image of the visibilities predicted using the unpolarized sky model created from observation. Sources appear in Stokes $Q$ and $U$ because of the primary beam induced leakage and their degrees of polarization $m'_p$ are shown as bubbles in Fig. 5.6b. Only the sources within the first null of the primary beam are shown here, as the current software for simulating visibilities can reproduce the effects of the ‘real’ primary beam well only within the FoV. The polarization leakage from outside the FoV only comes in via the sidelobes and is a very small effect and because the EoR analysis is limited to the FoV, this is the only region of interest in terms of polarization leakage. Leakages from Stokes $I$ into polarization increases as a function of distance from the phase center and they range from 0.05 to 1.22 per cent as shown by the sizes of the bubbles. This is consistent with the fractional leakages observed in the 3C196 field which is expected as the two fields roughly have similar declinations. The average and standard deviation of the fluxes of all the sources are plotted at each Faraday depth in Fig. 5.7b and the contrast with Fig. 5.7a is clearly visible. In the previous figure, intrinsically polarized sources were found at higher Faraday depths due to the rotation of their polarization angle by intervening magneto-ionic medium, but in the latter figure there are no such sources as here the polarization is caused only by the spectrally smooth primary beam. The sources that do appear at slightly higher Faraday depths than 0 rad m$^{-2}$ in the latter figure do so only
Figure 5.8: RM profiles of the 6 intrinsically polarized point sources in the 3C295 field (in green). The red lines show the profiles after the product of the RMSF and the peak flux at $\Phi = 0$ has been subtracted from the green lines. The dashed line shows the 10$\sigma$ level. The positions of the sources in RA, DEC, and their Faraday depths and degrees of polarization are shown in each panel. Here, the resolution (FWHM) in Faraday depth is 1 rad m$^{-2}$. 
due to the sidelobe of the RMSF.

5.4.3 Accuracy of the beam model

As a first step toward understanding the accuracy of the primary beam model, we have compared the degrees of polarization of the observed and predicted polarized sources, i.e. $m_P$ and $m'_P$, by taking their ratio. The parameter $m_P/m'_P$ for the sources found in both the observed and the predicted images is plotted in Fig. 5.9. Both the size and color of the bubbles correspond to the ratios of the observed and predicted leakages. The most general trend in Fig. 5.6a, 5.6b and 5.9 is that the observed leakage is almost always more than the one induced by the model beam which seems to show that the model beam is under-predicting the leakage, but one should note that the observed data has noise and diffuse emission that contribute to the estimation of source fluxes. The observed leakage is seen to be 0.75 to 4.61 times higher than the predicted leakage, but for most of the sources the ratio is less than 2. The overestimation of the degrees of polarization in the observed data could be due to the well-known bias in the presence of noise (Simmons & Stewart, 1985), and the diffuse emission faintly visible in Fig. 5.6a.

A more natural way to calculate the accuracy of the beam would be to compare the leakages into Stokes $Q$ and $U$ separately and take the difference between the observed and predicted leakages, i.e. $m_Q - m'_Q$ and $m_U - m'_U$, instead of their ratios. A bar chart of these difference parameters are plotted together in Fig. 6.9. As individual Stokes parameters follow Gaussian noise statistics, their difference should also be Gaussian, and although here we rescale the Gaussian by taking the difference between the ratios of Stokes parameters and although the diffuse foreground might not follow Gaussian noise, the distribution still approximately follows a Gaussian. Both $m_Q - m'_Q$ and $m_U - m'_U$ follow approximately a Gaussian with means close to zero (0.02 for $Q$ and -0.03 for $U$) and a standard deviation $\sigma$ of 0.3. Therefore, we can say with a 68% certainty that the leakage predicted by the model beam of LOFAR will be 30% different from the actual leakage. If the actual leakage is $\sim 1\%$, the model beam might predict the leakage to be around 0.7% – 1.3%.

We have calculated the uncertainty in the prediction of the beam model induced polarization leakage, but there are uncertainties in that uncertainty arising from the errors in extracting fluxes of the sources. To show these uncertainties we plot the FoM $m_Q - m'_Q$ and $m_U - m'_U$ for the sources as a function of their Stokes $I$ fluxes in Fig. 5.11, and as a function of their distances from the phase center in Fig. 5.12. In the former plot, as the flux of the source decreases thereby decreasing the local SNR of the source and enhancing the effect of the Gaussian noise, the random scatter of the aforementioned FoM increases. Therefore, this trend can be attributed to the Gaussian noise in the image that leaves its imprint on the extracted fluxes. As sources are attenuated as we go away from the phase center due to the azimuthally decreasing primary beam, we should expect an increase in the scatter of the FoM as we go outward from the phase center, and this is exactly what we see in the latter figure. Hence, this incremental trend of the FoM as a function of distance from the center should not be attributed to a systematic bias in the model of the beam, but again to the imprint of the image noise on the extracted fluxes.

As, here, we are mainly limited by the image noise and the errors in extracting fluxes of faint sources, one would expect the uncertainty in the calculation of the accuracy of the beam
Figure 5.9: Ratios of the observed and predicted leakages $m_p/m'_p$ represented by the size of the bubbles. It seems that for a few source model is a factor of 3–4 off from the reality, but as discussed in the text, the scenario is not that pessimistic for most of the sources, and this anomaly could be attributed to the bias and diffuse emission in the observed data. The background image is that of the simulated frequency-averaged linearly polarized image, and the color in the bubbles correspond to the polarized flux in mJy.
Figure 5.10: Difference between the fractional observed and predicted leakages into Stokes $Q$ (blue) and $U$ (red). Both of them follow approximately a Gaussian with means close to zero and a standard deviation of 0.3. The dashed lines show the Gaussian fits to the bar chart.

Figure 5.11: Difference of observed and predicted leakages as a function of the corresponding Stokes $I$ fluxes of the sources. More scatter at the dimmer end indicates errors related to extracting flux of dim sources.
model to go down if a higher flux density cut is used. And we see exactly this trend. We have taken only the 18 sources brighter than 600 mJy in Stokes $I$ and made bar charts similar to that of Fig. 6.9 and found that the standard deviation indeed improves significantly—although the mean of $m_Q - m'_Q$ remained 0.02%, its standard deviation improved to 0.1%. On the other hand, for the 26 sources brighter than 500 mJy, the $\sigma$ was found to be 0.2% showing that, due to the effect of the image noise, $\sigma$ increases as we include more fainter sources. The contribution of flux extraction error in the calculation of the accuracy of the model beam can also be seen clearly by comparing Fig. 6.9 and 5.11—the sources for which the difference between the observation and prediction is more than 0.5% are the ones with low flux and high scatter, and if we discard these sources the bar chart becomes narrower and exhibits a lower standard deviation for both Stokes $Q$ and $U$. Note that the standard deviation in these figures is contributed solely by the observed images as there was no additive noise in our simulation. The 18 brightest sources provide a clean model that is precise enough to predict leakage more accurately over the FWHM of the primary beam. Therefore, we can now say that the errors are $\leq 10\%$ on the predicted levels of leakage of $\sim 1\%$ typically in 68% of the cases, i. e. in these cases polarization leakage after calibration with the nominal LOFAR beam should be $\leq 10^{-3}$ of Stokes $I$ within the FoV.

### 5.4.4 Direction dependent calibration

We solved for DD gains toward 10 clusters using SAGECAL (Kazemi et al., 2011; Kazemi & Yatawatta, 2013). Instead of solving toward the direction of every source in the sky model, SAGECAL groups the sources into different clusters and solves for the gains toward the center of each cluster (Yatawatta et al., 2013). However, in our case, each cluster had only one source, as solving for only the brightest sources is sufficient for our demonstration purpose.
Figure 5.13: RM profiles of the 10 sources used in DD calibration both before (red) and after (blue) the calibration. For most of the sources, more than 80% flux could be subtracted using this calibration. RA and DEC of the sources are given on the top left corner of every panel for ease of comparison. The texts on the top right corners show percentages of flux subtracted, and the residual levels with respect to the image noise (in brackets).
As the sky model was completely unpolarized, SAGECAL should subtract polarized flux at all Faraday depths irrespective of instrumental or intrinsic polarization. RM profiles of the 10 sources were created both before and after SAGECAL and they are shown together in Fig. 5.13 in red and blue respectively. The figure shows that the sources with high SNR, i.e., half of the sources, were subtracted to more than 80%, and the brightest two sources were subtracted to $\geq 90\%$. Local noise level in these images was on average 0.2 mJy, and the brighter sources were removed sufficiently close to the noise level. The brighter the source, the better it was removed; the residual of the 20 mJy source was only $2.4 \sigma$ above the local noise. Residuals after subtracting all the sources are mentioned on the top right corner of each panel, and we see that most residuals are $< 5\sigma$.

## 5.5 Discussion and Conclusions

We have calculated the accuracy of the nominal model beam of LOFAR—created from the EM simulations of the ASTRON antenna group (Hamaker, 2011)—by comparing the leakages predicted by the model beam with that of the observation of the 3C295 field. Fig. 5.1 shows the model beam of a typical station (left panel), and the mismatch between the beams of the two dipoles (right panel), and Fig. 5.2 shows that the position of the sidelobe of the beam varies smoothly along frequency. Although the mismatch of the feed-beams already shows the extent of the polarization leakage, we have quantified the polarimetric performance of the beam using the $\text{IXR}_M$, the Mueller matrix version of the intrinsic cross-polarization ratio, a standard figure of merit for measuring the polarimetric performance of low-frequency arrays (see, e.g., de Lera Acedo et al., 2015). Fig. 5.3 and 5.4 show that the polarimetric performance of low-frequency aperture arrays like LOFAR is best near the phase center of the field and when the field is close to its culmination point. However, narrowing the field of view or filtering out the observations close to horizon result in reduced sensitivity and a balance between data filtering and calibration and modeling of the systematic errors needs to be maintained. In A15, we showed that taking data only within the central 3 degrees decreases the effect of polarization leakage. Here, from Fig. 5.4, we see the significant improvement of polarimetric performance close to the zenith, and further work is needed to establish a balance between the calibration and/or modeling of the DD systematic effects and the avoidance of the systematics dominated observation. Note that we did not use the $\text{IXR}_M$ directly while calculating the accuracy of the model beam, but the figures of merit we used for this purpose is very closely related to $\text{IXR}_M$, as explained in section 5.3.7.

The prediction of polarization leakage in the ‘EoR window’ of the cylindrical PS can be made more robust in the context of LOFAR based on the calculations of this chapter. A15 found that even without any leakage correction the simulated EoR signal is higher than the rms of the leakage in a significant portion of the cylindrical PS, and this EoR window extends to almost the whole instrumental $k$-space of LOFAR if 70% of the leakage could be removed. In this chapter, by comparing the leakages from Stokes $I$ to $Q, U$, we have found that the prediction of the beam, in 68% of the cases, will have an error of $\leq 10\%$, i.e., if the predicted leakage is 1%, the actual leakage might be between 0.9% to 1.1%. Therefore, if the differential beam effects are taken out perfectly using the nominal model beam of LOFAR, the errors in the correction will be $\leq 10\%$, i.e., the residual leakage in Stokes $Q, U$ will be $10^{-3}$ of Stokes $I$ flux.
We could calculate the accuracy of the beam model only up to the first null; accuracy of the sidelobes of the model could not be calculated for two interconnected reasons. First, the beam model under-predicts leakage on the sidelobes to some extent which can be seen by comparing the observed (Fig. 5.6a) and the simulated (Fig. 5.6b) images. In the former figure, some sources can be seen on the sidelobes, whereas in the latter all sources are within the FoV (note that the FoV would also change with frequency). Of course, the accuracy of the model beam could still be calculated, if we could quantify the under-prediction, and that’s where the second reason comes in. The Stokes I fluxes of the sources in the sidelobes were already very low as they were attenuated by the primary beam, and when we predicted leakage from these "faint" sources, the resulting leakage was even lower. So, we could not find compact sources bright enough to give rise to a detectable polarization leakage, even after the under-prediction of the beam, that would make the calculation of the accuracy possible at these distances from the phase center. Due to this limitation, we claim our measurement of the accuracy of the beam model to be reliable only within the FoV. However, a future paper (in preparation) will take into account both the leakage and the accuracy of the beam model farther away from the phase center, as they are crucial for EoR experiments.

The result of this experiment obtained using the $I \rightarrow Q,U$ leakages should hold true even for the $Q,U \rightarrow I$ leakages, as their relationship is symmetric for both the on-axis (Sault et al., 1996) and off-axis (e.g. see fig. 2b of A15) beams. Therefore, we can say that the beam model used to predict the $Q,U \rightarrow I$ leakage in A15 had a 10% error, and if the leakage could be removed, this error would be one of the constituents of the residual. However, we do not know how well this subtraction can be performed given that the leakage is even below the noise level, let alone the total intensity of the diffuse foregrounds. One should be careful about the uniqueness of each field in terms of both the projection effects of the beam and the diffuse polarization structure. For example, the diffuse polarized emission in the 3C295 field is very different in both amplitude and spatial and Faraday structure from that of the 3C196 field, but the projection of the beam toward these fields are not that different as they are situated at similar declinations.

We used DD-calibration to remove leakages of compact sources from Stokes $I$ to $Q,U$ and found that for sources with sufficiently high SNR, more than 80% of the flux could be removed and the residuals were generally very close to the local noise level. More work is needed to see how this blind correction of leakage compare with the correction using model beam. A good way to compare the modeling and DD-calibration approaches would be to test the effectiveness of AW-projection (using, e.g., AWIMAGER) and SAGECAL in removing linear polarization leakage. However, both AWIMAGER and SAGECAL can remove only the leakages of compact sources from Stokes $I$ to $Q,U$, whereas for the EoR project we are interested in the leakages of diffuse emission from Stokes $Q,U$ to $I$. 
Abstract

Leakage of polarized Galactic diffuse emission into total intensity can potentially mimic the 21-cm signal coming from the epoch of reionization (EoR), as both of them might have fluctuating spectral structure. Although we are sensitive to the EoR signal only in small fields of view, chromatic sidelobes from further away can contaminate the inner region. Here, we explore the effects of leakage into the ‘EoR window’ of the cylindrically averaged power spectra (PS) within wide fields of view using both observation and simulation of the 3C196 and NCP fields, two observing fields of the LOFAR-EoR project. We present the polarization PS of two one-night observations of the two fields and find that the ‘NCP’ has higher fluctuations along frequency, and consequently exhibits more power at high-$k_{\parallel}$ that could potentially leak to Stokes $I$. Subsequently, we simulate LOFAR observations of a model Galactic diffuse polarized emission to assess what fraction of polarized power leaks into Stokes $I$ because of the primary beam toward different directions. We find that the fractional leakage over the instrumental $k$-space is 0.35% in the 3C196 field, and 0.27% in the NCP field, and it does not change significantly within the three fields of view: 15°, 9° and 4° in diameter. Based on the observed PS and simulated fractional leakage, we show that a similar level of leakage into Stokes $I$ is expected in the 3C196 and NCP fields.
Chapter 6. Polarization leakage in wide fields

6.1 Introduction

Polarization leakage is one of the least explored effects that can potentially contaminate the 21-cm signal coming from the epoch of reionization (EoR). A fraction of the polarized emission (Stokes $Q$, $U$) always leaks into total intensity (Stokes $I$) due to instrumental effects, specifically a mismatch of the primary beams of the two feeds of an antenna. The EoR signal is expected to be detected statistically by current telescopes such as GMRT (Paciga et al., 2011), LOFAR (van Haarlem et al., 2013), MWA (Tingay et al., 2013) and PAPER (Parsons et al., 2010), and future telescopes such as HERA (DeBoer et al., 2016) and SKA (Koopmans et al., 2015). For a successful detection, the foregrounds contaminating the signal need to be removed one by one. First, bright point sources are removed. Then, the total intensity of the Galactic diffuse synchrotron emission is removed by utilizing the fact that this emission is spectrally smooth, whereas the EoR signal is not (Jelić et al., 2008; Datta et al., 2010b; Harker et al., 2010; Trott et al., 2012; Morales et al., 2012a; Bernardi et al., 2013; Pober et al., 2013; Chapman et al., 2013; Dillon et al., 2015; Thyagarajan et al., 2015b). Removing polarization leakage comes at the very end if at all necessary, because the leakage level is usually lower than noise in these observations (Asad et al., 2015; Kohn et al., 2016), although potentially still above the EoR signal.

Jelić et al. (2010) showed that if the polarization angle of the Galactic diffuse polarized emission is differentially Faraday-rotated by the magnetized plasma in the interstellar medium, then the emission that reaches us can have significant spectral fluctuation. If this is indeed the case, and if this high-rotation measure polarized emission is leaked into Stokes $I$, the leakage might mimic the EoR signal, which is expected to have similar fluctuations as a function of frequency.

There are two main approaches toward detecting the EoR signal—foreground ‘avoidance’ and ‘removal’ (Chapman et al., 2016). In the former approach, the region of the cylindrically averaged power spectra (PS) most contaminated by foregrounds and noise is avoided. The region least contaminated by the foregrounds and systematics is called the ‘EoR window’ and this is the only region where the EoR signal is looked for. In the latter approach, foregrounds are subtracted from the data employing various strategies.

The levels of foreground and system noise are expected to be much higher than the polarization leakage in Stokes $I$. For example, in the LOFAR-EoR observations, an excess noise is detected, which is higher than the expected level of leakage, and this noise is not contributed by leakage (Patil et al. 2016, in preparation). In case of PAPER, Kohn et al. (2016) found no evidence of polarization leakage in the EoR window with their current sensitivity. Although polarization leakage is not one of the main concerns of the current EoR experiments, previously it was thought to pose a greater problem (private communication, de Bruyn). For example, based on the experience of WSRT\textsuperscript{1}, a higher level of polarization leakage was expected than what was found by Asad et al. (2015) in case of LOFAR. The diffuse polarized emission was found to be rich in Faraday structures (Jelić et al., 2015), but the instrumental polarization of LOFAR was much lower than WSRT, resulting in a lower leakage in the LOFAR observations. Although the level of leakage is low, it is worth exploring because it will be relevant for the more sensitive experiments in the future. Once we reach

the sensitivity limit where leakage becomes relevant, we have to decide whether to avoid or remove leakage. If the leakage toward certain directions is found to be spectrally structured, however, avoidance would not be a proper strategy.

Asad et al. (2015) showed that in the 3C196 field, one of the observing fields of the LOFAR-EoR project, leakage from observed polarized emission into Stokes $I$ is contained within a wedge-shaped region at the high-$k_\perp$ (transverse wavenumber), low-$k_\parallel$ (line of sight wavenumber) corner of the cylindrical PS, and there is a wide region at the opposite corner of the PS relatively free from leakage-contamination. This prediction of leakage was performed using the model beam of LOFAR. Asad et al. (2016) showed that this beam model has an accuracy of $\sim 10\%$ within the first null of the primary beam. These two papers enabled us to properly assess the level of leakage that could potentially make EoR detection harder if not mitigated properly.

In this chapter we take our previous analyses one step further. Here we present cylindrical PS of both the 3C196 and NCP fields and compare them to each other. Another worry about polarization leakage is that the leakage is expected to increase with distance from the phase center—wider fields are expected to suffer from more leakage. Although in case of LOFAR, we are sensitive to the EoR signal only within small fields of view (FoV), chromatic sidelobes of the diffuse and compact emission and the polarization leakage from further away might corrupt the inner regions to some extent. We therefore present simulations of leakage for different FoV ranging from $4^\circ \times 4^\circ$ to $15^\circ \times 15^\circ$, and compare their PS. Once we know the fractional leakage, we can predict the level of leakage based on the observed polarized emission in different fields.

This chapter is organized as follows: Section 6.2 presents the PS of the observed polarized emission in the 3C196 and NCP fields with two different FoV. Section 6.3 presents the calculation of the fractional leakage, by predicting LOFAR observations of a simulated Galactic diffuse polarized emission, for the two fields and for three different FoV. In both sections, the methods and results are presented in separate subsections. The implications of the calculated fractional leakage in Section 6.3 on the PS, presented in Section 6.2, are explored in the discussion section. The chapter ends with some conclusions and remarks about our ongoing and future works.

### 6.2 Power spectra of polarized emission

This section presents cylindrically averaged power spectra of the diffuse polarized emission in the 3C196 and NCP observing fields. A power spectrum of the polarized emission $P = Q + iU$ in the 3C196 field was shown in Asad et al. (2015, hereafter A15), but here the Stokes $Q$ and $U$ power spectra are also presented. Moreover, the power spectrum presented by A15 was created from the observed emission convolved with the beam, whereas here we create power spectra from the observed emission itself. In the previous case, the power spectrum was, in effect, convolved with the beam twice, as the emission was not deconvolved to correct for the beam before convolving it again with the beam. This double-convolution was not that relevant because in that test, our aim was to calculate the fractional polarization leakage. We found that the ratio of the polarized emission and the leakage is almost constant over the instrumental $k$-space in the power spectrum. It is expected to be constant, because leakage is caused by the convolution of the visibilities with a polarized primary beam that remains
almost same for all baselines, and thus for all $k$-scales. Therefore, if we know the power spectrum of the polarized emission and the leakage ratio for a certain field, we can predict the level of leakage in the $k$-space and potentially correct for this bias in the EoR power spectrum, in a similar fashion as correction for the noise bias. The power spectra of polarization in the 3C196 and NCP fields presented in this chapter provide an idea of how much the ‘EoR window’ can be corrupted toward different pointing directions.

6.2.1 Method

The power spectra have been created using the same pipeline as used in Part I of this thesis. Here we briefly outline the main steps relevant for this experiment. We took polarization images (Stokes $Q$ and $U$) of 50 spectral subbands corresponding to about 10 MHz of bandwidth from 150 to 160 MHz for both the NCP and 3C196 fields. The images are created from observed visibilities using natural weighting and an angular resolution of 0.5 arcmin within an area of $10^\circ \times 10^\circ$. Two sets of image cubes are produced from these images, one containing the inner 4 degrees, and the other the inner 9 degrees in diameter. These two fields of view (FoV) were chosen because it will make the comparison of these PS with the PS of the experiment described in the next section more convenient. The image cubes are then Fourier transformed, and the squared absolute values of the transformed cubes are cylindrically averaged resulting in a 2D cylindrical PS as a function of comoving transverse ($k_\perp$) and LOS ($k_\parallel$) wavenumbers. Finally, spherical PS as a function of wavenumbers $k$ are produced by averaging the same Fourier cubes in spherical shells.

<table>
<thead>
<tr>
<th>Table 6.1: Parameters related to the power spectrum estimation:</th>
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<tbody>
<tr>
<td>Observation ID</td>
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<tr>
<td>Phase center ($\alpha$, $\delta$)</td>
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<td>Observation date</td>
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<td>Observing time</td>
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<tr>
<td>Baseline range</td>
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<td>Frequency range</td>
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<td>Spectral subbands</td>
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<tr>
<td>Bandwidth</td>
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<td>Spectral resolution</td>
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<td>Integration time</td>
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<td>Pixel scale</td>
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Here an additional parameter, the *wedge*, has been indicated in the cylindrical PS. The total intensity of diffuse foregrounds is expected to be smooth along frequency, and hence should appear only at low $k_\parallel$. However, the frequency-dependent synthesized beam leaks power from low $k_\parallel$ to high $k_\parallel$ as one goes to higher $k_\perp$ wavenumbers, spreading the foreground in a wedge-shaped region on the bottom-right corner of the PS. The spread increases at higher $k_\perp$ because longer baselines have higher fringe rates making them more susceptible to spectral
6.2 Power spectra of polarized emission

distortions. In an optimistic scenario, the wedge should spread no further than the region delimited by the FoV set by the primary beam of the instrument. In fact, the line delimiting the foreground wedge is usually calculated from the primary FoV as (Morales et al., 2012a; Liu et al., 2014a; Dillon et al., 2014)

\[
k_{\parallel} = \left[ \sin \theta_{\text{FoV}} \frac{H_0 D_c(z) E(z)}{c(1+z)} \right] k_{\perp}
\]

(6.1)

where \(H_0\) is the Hubble parameter at redshift \(z = 0\), \(c\) is the speed of light, \(E(z)\) is the dimensionless Hubble parameter defined in Chapter 2, Section 2.6.1, \(D_c(z) \equiv \int_0^z dz'/E(z')\), and \(\theta_{\text{FoV}}\) is the angular radius of the field of view delineated by the first null of the primary beam. This wedge can be called the ‘primary wedge’ as its boundary is fixed by the width of the primary beam. In a pessimistic scenario, foreground might leak beyond the primary wedge, but even then it should not leak beyond the ‘horizon wedge’, the wedge delimited by the angular distance to the horizon. The region above the wedge is considered to be the ‘EoR window’ as it is expected to be least contaminated by foreground and systematics. Although the concept of the wedge has originally been developed for analyzing the Stokes \(I\) PS (Datta et al., 2010a; Vedantham et al., 2012; Parsons et al., 2012), it can also be used for polarization PS, which we intend to do here.

6.2.2 Results

The cylindrical PS of the 3C196 and NCP observations within an area of \(4^\circ \times 4^\circ\) are shown in Fig. 6.1, and the corresponding spherical PS are shown in Fig. 6.2. The corresponding cylindrical and spherical PS within an area of \(9^\circ \times 9^\circ\) are shown in Fig. 6.3 and 6.4 respectively. In each case, the PS of \(Q\), \(U\), and \(P = Q + iU\) have been presented. In the cylindrical PS, the lower blue and upper green dashed lines correspond to the boundaries of the ‘primary’ and ‘horizon’ wedges respectively.

The bottom panels of Fig. 6.1 show the PS of the polarized emission in the 3C196 field. The diffuse polarized emission is clearly contained within a wedge-shaped region, but the wedge extends beyond the limit set by the primary beam. Significant power is seen between the primary and horizon wedges, i.e. between the blue and green dashed lines. The region at \(k_{\parallel} \geq 0.2\) and \(k_{\perp} \leq 0.15\) is relatively free of foreground and noise. The power of the polarized emission can be seen more clearly from the corresponding spherical PS shown on the right panel of Fig. 6.2. The polarized emission is of the order of \(10^5\) in Stokes \(Q\) and \(U\), but goes down by almost a factor of 5 in \(|Q + iU|\), because the polarization vector rotates by 180° many times across the averaged frequency band.

The top panels of Fig. 6.1 show the PS of the polarized emission in the NCP field. The diffuse polarized emission is almost an order of magnitude lower here compared to the 3C196 field, which is seen by comparing the low \(k_{\parallel}\) power in Stokes \(Q, U\) in the top and bottom panels. Another difference is that there is significant power at high \(k_{\parallel}\) and low \(k_{\perp}\) which will leak into the EoR window of the Stokes \(I\) PS. The power at high \(k_{\parallel}\) is caused by differential Faraday rotation along of the intrinsic polarized signal by the intervening magnetized plasma along the LOS. The difference between the two fields, thus, shows that one can expect considerable difference in the level of Faraday rotation toward different directions in the sky and hence considerable differences in polarization leakage.
Chapter 6. Polarization leakage in wide fields

Figure 6.1: Cylindrically averaged power spectra of the observed polarized emission in the inner $4^\circ \times 4^\circ$ of the NCP (top row) and 3C196 (bottom row) fields within the frequency range of 150–160 MHz. The columns from left to right correspond to the PS of respectively $Q$, $U$, and $Q + iU$ in units of $[\text{mK}]^2$. The lower blue and upper green dashed lines correspond to the boundaries of the primary and horizon wedges respectively.

Figure 6.2: Spherically averaged PS of the polarized emission in the inner $4^\circ \times 4^\circ$ of the NCP (left) and 3C196 (right) fields within the frequency range of 150–160 MHz. $Q$, $U$, and $Q + iU$ spectra are plotted in red, blue, and green respectively.
Figure 6.3: Cylindrically averaged power spectra of the observed polarized emission in the inner $9^\circ \times 9^\circ$ of the NCP (top row) and 3C196 (bottom row) fields within the frequency range of 150–160 MHz. The columns from left to right correspond to the PS of respectively $Q$, $U$, and $Q + iU$ in units of $[\text{mK}]^2$. The lower blue and upper green dashed lines correspond to the boundaries of the primary and horizon wedges respectively.

Figure 6.4: Spherically averaged PS of the polarized emission in the inner $9^\circ \times 9^\circ$ of the NCP (left) and 3C196 (right) fields within the frequency range of 150–160 MHz. $Q$, $U$, and $Q + iU$ spectra are plotted in red, blue, and green respectively.
The power of polarized emission decreases in both the 3C196 and NCP fields, if we create PS from $9^\circ \times 9^\circ$, instead of $4^\circ \times 4^\circ$, which can be seen by comparing Fig. 6.1 with Fig. 6.3. This is primarily because of the fact that the region outside $4^\circ$ is dominated by noise, and averaging signals with noise within the larger area gives a lower power. Averaging signals with anti-correlated position angles could be another potential reason, as this effect could be higher within the larger FoV.

These PS show that the ‘EoR window’ could be more prone to leakage-contamination in the NCP field, if the level of fractional leakage in this field is comparable to the 3C196 field. If, on the other hand, the fractional leakage is lower in the NCP field, a smaller fraction of the high level of power at high $k_\parallel$ would leak into Stokes $I$, making this field more or less similar to the 3C196 field. We calculate the fractional leakage caused by the LOFAR beam model in the two fields in the next section. The implications of the level of fractional leakage on the ‘EoR window’ will be described in the discussion section.

6.3 Fractional leakage in wide fields

In Chapter 4, it was found that the ratio between the power spectra of the polarized emission within $3 \times 3$ degrees and its leakage into Stokes $I$ varies very little over the instrumental $k$-space. However, that ratio was calculated using a noisy polarization observation. To understand this ratio better, and to see how it changes if the field of view is increased, we now perform a different experiment where a model of polarized emission is used instead of an observation.

6.3.1 Method

A model of the diffuse Galactic polarized emission is created using the foreground simulations discussed by Jelić et al. (2010). The model, that will be described in Section 6.3.2, contains diffuse foreground within $15^\circ \times 15^\circ$ and 150–160 MHz, and has a pixel scale of 18.75 arcmin, sufficient to analyze the short baselines that we consider here. A coarse grid of $50 \times 50$ pixels is used because simulating visibilities for more pixels is computationally very expensive. We note however that for the purposes of calculating polarization leakage, oversampling of the PSF is not necessary. The polarization model has both Stokes $Q$ and $U$ emission. For each Stokes parameter three image cubes are created from the original model cube for three different observing areas: $15^\circ \times 15^\circ$, $9^\circ \times 9^\circ$, and $4^\circ \times 4^\circ$. Hence, the three cubes contain $50^2$, $28^2$, and $16^2$ pixels, respectively, in the spatial domain.

To produce point source sky models, each pixel in the Stokes $Q, U$ cubes is considered a point source with $I = V = 0$ and $Q, U$ taken from the values of the pixels. The three sky models for the three different fields of view contain 2500, 784, and 256 polarized point sources. The visibilities corresponding to these sky models are simulated toward two phase centers, one centered on the NCP, and the other on 3C 196. All parameters for the six simulated observations, for three areas centered on two different directions, were kept same except for the total observing time. Because the NCP always remains above the horizon as seen from the LOFAR site, it can be observed for considerably more time than the 3C196 field, which in general is only visible during the shorter spring and summer nights.\(^2\)

\(^2\)EoR observations are carried out during the night when the ionospheric phase fluctuations are more benign.
To reconstruct the effects of the actual observations as precisely as possible, we have simulated the NCP observations for more time than 3C196. LOFAR-EoR visibilities are usually integrated every 2 seconds, and the 2 s visibilities are again averaged down to every 10 s. If, to remain as close to reality as possible, we wanted to simulate observations every 10 s for all LOFAR baselines, we would have to predict 442 million visibilities for 13 hours and for 50 spectral subbands, which is computationally very expensive. Instead, we have predicted visibilities for every 120 s (as mentioned in Table 6.2), and for only the core baselines of LOFAR—the maximum baseline is 3 km long. In the end, baselines longer than 180 $\lambda$ will not be needed, because the model has a resolution of only 18.75 arcmin, and any baseline longer than 180 $\lambda$ would resolve the diffuse emission into point sources. Also note that time-smearing on these short baselines is negligible.

Full-Stokes visibilities are simulated in four main steps. First, the sky model is Fourier transformed to produce four correlations of the visibilities for every baseline, frequency channel, and timeslot. Then, the visibilities are convolved with the Fourier transform of the primary beam corresponding to the specified baseline, frequency, and timeslot, and toward the specified directions. Third, the convolved visibilities are corrected for the polarized primary beam toward the phase center only via an inverse Mueller matrix multiplication. After this correction, all effects of the beam toward the phase center are removed, and only the ‘differential’ effects, i.e., those of the wide-field beam with respect to the phase center, remain. Finally, Stokes visibilities are calculated from the four visibility correlations following Eqn. 2.13. The first three steps are performed using the standard LOFAR calibration and simulation software BBS (Pandey et al., 2009).

The sources in the sky model does not have any Stokes $I$ flux, but Stokes $Q, U$ emission is leaked into Stokes $I, V$ resulting in non-zero values of the latter. The aim of this experiment is to measure the fraction of power leaked from $Q$ and $U$ to $I$ because of the polarized primary beam, and in this respect it is similar to the experiment presented in Chapter 4. But there are two major differences. First, in Chapter 4, fractional leakage was calculated from real observations, and here we calculate it from simulated observations to avoid the effect of noise and the effect that in real observations Stokes $I$ will also leak in to Stokes $Q$ and $U$ and hence needs to be corrected for. Second, the visibilities were predicted using AWIMAGER in Chapter 4, whereas here BBS is used. In the previous case, a gridded sky map was directly Fourier transformed to produce visibilities and then convolved with the beam. Here, direction-dependent Jones matrices corresponding to the model beam are applied to each source in the sky model. Although AWIMAGER is much faster, BBS is used for the current experiment because currently AWIMAGER cannot produce visibilities from maps wider than the width of the main lobe of the primary beam.

The results of this experiment are presented in terms of power spectra. Some power spectra of polarized emission and leakage have been presented in Chapter 4, and Section 6.2.2 of this chapter. These PS were calculated from observed images, whereas here we calculate PS directly from the visibilities. One of the motivations behind the current approach is that the model used here has very low resolution and covers very few pixels. Another motivation is that, it would be easier to characterize instrumental effects in visibility space, as we will not need to worry about imaging artifacts. The procedure of creating PS from visibilities and the figures of merit used to present the final results are described in section 6.3.3.
Table 6.2: Parameters related to the simulated observations:

<table>
<thead>
<tr>
<th></th>
<th>NCP</th>
<th>3C196</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase center ((\alpha, \delta))</td>
<td>0°, 90°</td>
<td>123.4°, 48.2°</td>
</tr>
<tr>
<td>Observing time</td>
<td>13 hours</td>
<td>8 hours</td>
</tr>
<tr>
<td>Baseline range</td>
<td>30–180 (\lambda)</td>
<td></td>
</tr>
<tr>
<td>Frequency range</td>
<td>150–160 MHz</td>
<td></td>
</tr>
<tr>
<td>Spectral subbands</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Bandwidth</td>
<td>10 MHz</td>
<td></td>
</tr>
<tr>
<td>Spectral resolution</td>
<td>0.195 MHz</td>
<td></td>
</tr>
<tr>
<td>Integration time</td>
<td>120 s</td>
<td></td>
</tr>
<tr>
<td>Pixel scale</td>
<td>18.75 arcmin</td>
<td></td>
</tr>
</tbody>
</table>

6.3.2 Diffuse polarized foreground model

We use the foreground simulations of Jelić et al. (2010), in which the 3D emission coefficients of the Galactic synchrotron and free-free emission are obtained from the cosmic-ray and thermal electron densities, and the Galactic magnetic field. It is assumed that both cosmic-ray and thermal electrons are mixed in a region of 1 kpc in depth along the LOS. The synchrotron emission produced by the cosmic-ray electrons are depolarized due to differential Faraday rotation. This is the ‘Model B’ of Jelić et al. (2010) and we refer the reader to their paper for more details.

We have created 3D cubes of dimension 50\(^3\) for Stokes \(Q\) and \(U\) models within a field of 15° × 15° and a frequency range of 150–160 MHz. An example slice of the model is shown in Fig. 6.5. Note that level of polarized foreground in this simulation is lower than the typical observed polarized emission, as shown in Fig. 6.1. However, the main result of this exercise will not be affected by this unrealistic choice of simulated polarized emission, because we are only interested in the leakage ratio. The top panels show Stokes \(Q\) and \(U\) emission at 150 MHz in mK units. The power spectra of \(P = Q + iU\) of this slice is shown on the bottom-left panel. The high value of power at small \(k\)-scales is caused by the large-scale diffuse emission. The frequency profile of an example pixel of this cube is shown in the bottom-right panel of the figure. The figure shows that the polarized emission fluctuates along frequency, caused by differential Faraday rotation of synchrotron emission by the mixed thermal and cosmic-ray electrons in the intervening medium.

Jelić et al. (2010) noted that if a fraction of this polarized emission is leaked into total intensity, the leakage might mimic the EoR signal. However it has been found that in the 3C196 field, the observed diffuse polarized emission does not fluctuate too much along frequency, the level of differential Faraday rotation was found to be small (Jelić et al., 2015; Asad et al., 2015). However, in Section 6.2.2, we have shown that the situation is different in the NCP field. Although the diffuse polarized emission in the NCP field is lower than that of the 3C196 field, the former has more fluctuation along frequency, i.e. it has higher power at high \(k_{||}\)-scales. It would be useful to examine the effect of leakage on the EoR window in the worst case scenario, i.e. when the fluctuation of the emission along frequency
6.3 Fractional leakage in wide fields

Figure 6.5: Top: Galactic diffuse foreground model of Stokes $Q$ and $U$ at 155 MHz created from the ‘model B’ of Jelić et al. (2010). Bottom left: The Power spectrum of the above model. Bottom right: Frequency spectra of the Stokes $Q$ and $U$ emission of the model for a single pixel.

is high. Therefore, even though the choice of model is not important for the measurement of fractional leakage—as leakage is caused solely by $uv$-plane effects—, we have used a spectrally fluctuating model. Our findings related to both the fractional leakage and the corruption of the EoR window by the spectrally varying polarized emission is described below.

6.3.3 Power spectrum from visibilities

We have used Stokes visibilities, defined in Section 2.1.2 of Chapter 1, to create cylindrically averaged PS. Consider a Stokes visibility $V_2(b, t, \nu)$ for the baseline of length $b = \sqrt{u^2 + v^2}$ at the time $t$ and frequency $\nu$, where $Z = I, Q, U, V$ denotes different Stokes parameters. In synthesis observations, the position of a baseline with respect to the astronomical source changes as the Earth rotates, producing many more baselines than is possible with a snapshot
We have presented
where
where
The visibility in each voxel of this cube can be written as

Chapter 6. Polarization leakage in wide fields

To produce PS, the uv-plane within the baseline range \( b_{\text{min}} - b_{\text{max}} \) is gridded\(^3\) in \( N_b \times N_b \) pixels. The width and height of each pixel is \( 16\lambda \), corresponding to the diameter of a LOFAR core HBA-station. To correct for the \( w \)-terms, the visibilities were also \( w \)-projected using 32 \( w \)-planes. A visibility cube is produced taking the gridded visibilities for all spectral subbands. The visibility in each voxel of this cube can be written as \( V_Z(b_n, \nu) \), where \( b_n \) refers to the \( n \)-th pixel, \( n \) going from 0 to \( (N_b - 1)^2 \).

Each pixel of the visibility grid is Fourier transformed along the frequency axis using the one-dimensional FFT algorithm. The Fourier conjugate of frequency is ‘delay’ (\( \tau \)) and the transform can be written mathematically as

\[
\tilde{V}_Z(b_n, \tau) = \frac{1}{N_\nu} \sum_{\nu=0}^{N_\nu-1} V_Z(b_n, \nu) \exp \left[ -2\pi i \frac{\nu \tau}{N_\nu} \right] \tag{6.2}
\]

where \( N_\nu \) is the number of frequency channels. The squared absolute values of \( \tilde{V}_Z(b_n, \tau) \) give the 3D PS, i.e. \( P_{3D}(b_n, \tau) = |\tilde{V}_Z(b_n, \tau)|^2 \).

To create cylindrically averaged PS, the uv-plane of \( P_{3D} \) is divided into \( N \) annuli and the powers within an annulus are averaged for each delay. The averaged power at a certain delay and a certain uv-annulus \( b_N \)

\[
P_{2D}(b_n, \tau) = \frac{1}{N_p} \sum_{b=b_N}^{b=b_{N+1}} P_{3D}(b, \tau) \tag{6.3}
\]

where \( N_p \) is the number of pixels within the annulus \( b_N \).

Delay \( \tau \) is related to \( k_\parallel \), and the baseline length for a particular annulus \( b_N \) is related to \( k_\perp \). We have presented \( P_{2D}(b_n, \tau) \) in terms of \( k_\perp \) and \( k_\parallel \), i.e. as \( P_Z \equiv P_{2D}(k_\perp, k_\parallel) \). The relation between baseline length and \( k_\perp \) is given in Section 1.6 of this thesis, and the relation between time delay and \( k_\parallel \) can be found in Thyagarajan et al. (2015a, equation 6). Dimensionless power spectrum is calculated as \( \Delta^2(k_\perp, k_\parallel) = k_\perp^2 k_\parallel P_Z(k_\perp, k_\parallel) / (2\pi)^2 \).

We are interested in Stokes \( Q, U \) and their leakage into Stokes \( I \). The power of Stokes \( I, Q, U \) visibilities are \( P_I, P_Q, P_U \), respectively, and the power of the linear polarization \( Q + iU \) is denoted as \( P_P \). Then, the fractional leakage from \( Q, U \) to \( I \)

\[
L_I = \sqrt{\frac{P_I}{P_P}} \times 100. \tag{6.4}
\]

\( L_I \) represents the leakage into Stokes \( I \) as a percentage of linear polarization as a function of \( k_\perp \) and \( k_\parallel \). Both the 2D spectrum and histogram of \( L_I \) for different fields of view and toward different directions are presented below.

\(^3\)Visibility gridding is done using the excon imager.
6.3 Fractional leakage in wide fields

6.3.4 Results

Cylindrically averaged PS of the model polarized emission observed by LOFAR and its leakage into total intensity are presented in Fig. 6.6 and 6.7. Fig. 6.6 shows the spectra for the 3C196 field, and Fig. 6.7 for the NCP field. In both figures, the columns represent $P_P$, $P_I$, and $L_I$ spectra from left to right, respectively. And the rows represent the spectra for field areas of $15^\circ \times 15^\circ$, $9^\circ \times 9^\circ$ and $4^\circ \times 4^\circ$ from top to bottom, respectively.

Polarization power

PS of linear polarization ($P_P$) show the characteristics of the sky model in both fields. In the left panels of Fig. 6.6 and 6.7, most polarized power are found within a wedge shaped region at high-$k_\perp$, but the the wedge is located at higher $k_\parallel$ scales as compared to the observed emission of Fig. 6.1. High $k_\parallel$ modes are affected by diffuse emission, because the emission had significant spectral fluctuations, as seen in the bottom-right panel of Fig. 6.5. A comparison between Fig. 6.6 and 6.7 shows that more polarization power is predicted in the NCP than in the 3C196 field. The simulated sky model is same for both fields, but the visibilities have been convolved with two different beams, and also the fields have been observed for different durations, 3C196 for 8 hours and NCP for 13 hours. The difference in polarization power could be caused by these two dissimilarities.

The difference between the polarization beams toward the 3C196 and NCP fields can be seen in the top two rows of Fig. 6.8. The figures show some components of the beam Mueller matrix, the outer product of the beam Jones matrices of two stations constituting a baseline. For an example of a complete beam Mueller matrix see Fig. 2.2 of Part I of this thesis. $M_{22}$ and $M_{33}$ components of the matrix represent the Stokes $Q$ and $U$ beams, respectively. Fig. 6.8 shows the polarization beam $\sqrt{M_{22}^2 + M_{33}^2}$, for a FoV of $15^\circ \times 15^\circ$, and all beams are normalized with respect to the Mueller matrix at the phase center. We see that the shape of the polarization beam is different in the NCP field compared to the 3C196 field—the former has a 2-fold symmetry, whereas the latter is either circular or elliptical depending on the hour angle. Different panels in the figures show beams at different hour angles, and we can see the rotation of the beam with the apparent rotation of the sky. The 3C196 field rises, reaches very close to the zenith, and then sets during the observation, but the NCP field rotates around the north celestial pole with the rotation of the Earth without rising or setting. Exactly why the beam is so different in the two fields, and how this difference gives rise to a difference in power in the PS will be explored in our future work.

Another interesting aspect of the PS is that they show the extent of contamination of the ‘EoR window’ by the leakage of a spectrally fluctuating polarized emission. Looking at the middle columns of Fig. 6.6 and 6.7, one can clearly see that there is significant foreground power at the high-$k_\parallel$, low-$k_\perp$ corner of the PS, which is expected to be a clean window for statistically detecting the EoR signal. Comparing the simulated leakages in the two fields, we can see that the contamination of the EoR window is higher in ‘NCP’ than in ‘3C196’. However, in real observations, this level of spectral structure has not been found in all fields. For example, we refer to the PS made from real observations shown in Fig. 6.1 and 6.3. Between the observed emission in the two fields, ‘3C196’ shows limited spectral fluctuation and almost no leakage in the EoR window, whereas ‘NCP’ suffers from considerable level of leakage in the EoR window due to its high-rotation measure polarized emission.
Figure 6.6: Power spectra created from the visibilities corresponding to the model of Fig. 5.6b toward the 3C196 field. The three rows represent three fields of view centered around 3C196: from top to bottom 15\(^2\), 9\(^2\) and 4\(^2\) deg\(^2\) respectively. The three columns represent Stokes \(P = |Q + iU|\), \(I\), and \(\sqrt{I/P}\) from left to right respectively. Note that the structure in \(k_\parallel\) direction corresponds to the 'wave' seen in Fig. 6.5 (bottom-right panel).
Figure 6.7: Power spectra created from the visibilities corresponding to the model of Fig. 5.6b toward the NCP field. The three rows represent three fields of view centered around NCP: from top to bottom $15^2$, $9^2$ and $4^2$ deg$^2$ respectively. The three columns represent Stokes $P = |Q + iU|$, $I$, and $\sqrt{I/P}$ from left to right respectively. Note that the structure in $k_\parallel$ direction corresponds to the ‘wave’ seen in Fig. 6.5 (bottom-right panel).
Chapter 6. Polarization leakage in wide fields

Figure 6.8: *Top two rows:* LOFAR primary beam model for linear polarization for a single baseline at 150 MHz within $15^\circ \times 15^\circ$, i.e. $\sqrt{M_{22}^2 + M_{33}^2}$ where the subscripts represent the components of the beam Mueller matrix (e.g. Fig. 2.2). The top and bottom panels show the beams for the 3C196 and NCP fields, respectively. All beams have same FoV, and are normalized with respect to the Mueller matrix at the phase center. *Bottom two rows:* Leakage from linear polarization to Stokes $I$ as a fraction of Stokes $I$ for the 3C196 (top) and NCP (bottom) fields, i.e. $\sqrt{M_{12}^2 + M_{13}^2} / M_{11}$. All plots are shown in dB, and the numbers on the top-left corners show the hour angles at which the beams are measured.
Table 6.3: Statistics of $L_I$ (%) for different fields of view in the 3C196 field.

<table>
<thead>
<tr>
<th>Fields of view</th>
<th>15°</th>
<th>9°</th>
<th>4°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.62</td>
<td>0.61</td>
<td>0.59</td>
</tr>
<tr>
<td>Median</td>
<td>0.36</td>
<td>0.36</td>
<td>0.34</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.39</td>
<td>1.12</td>
<td>1.47</td>
</tr>
<tr>
<td>Peak</td>
<td>0.18</td>
<td>0.15</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 6.4: Statistics of $L_I$ (%) for different fields of view in the NCP field.

<table>
<thead>
<tr>
<th>Fields of view</th>
<th>15°</th>
<th>9°</th>
<th>4°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.63</td>
<td>0.63</td>
<td>0.49</td>
</tr>
<tr>
<td>Median</td>
<td>0.26</td>
<td>0.26</td>
<td>0.29</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.76</td>
<td>1.67</td>
<td>0.94</td>
</tr>
<tr>
<td>Peak</td>
<td>0.09</td>
<td>0.09</td>
<td>0.12</td>
</tr>
</tbody>
</table>

**Leakage power and RMS**

The PS of the leakage of polarized emission into Stokes $I$ ($P_I$) are shown in the middle panels of Fig. 6.6 (3C196 field) and 6.7 (NCP field). The figures show that $P_I$ is almost a scaled down version of $P_P$ for all fields of view and toward both observing fields. The square-root of the ratio between the two PS $L_I$ are shown in the right-hand panels of both figures as a percentage. Because $L_I$ is the square root of power or variance, it represents the RMS of the fractional leakage as a function of $k_\perp$ and $k_\parallel$.

In both 3C196 and NCP fields, $L_I$ varies little over the instrumental $k$-space. Whatever variation is present is mostly due to the sample variance. The difference among the three fields of view $15^\circ \times 15^\circ$, $9^\circ \times 9^\circ$ and $4^\circ \times 4^\circ$ is not significant, although the fraction seems to be higher within $4^\circ \times 4^\circ$ compared to the other fields of view. There is significant difference between the $L_I$ PS of the 3C196 and NCP fields. It is considerably lower in the NCP field.

The comparison between the two observing fields and the three FoV would be easier if we look at the histogram of the ratio $L_I$ for each of the voxels in the 3D box before averaging cylindrically or spherically, shown in Fig. 6.9. The top and bottom rows show the histograms for the 3C196 and NCP fields, respectively. The three columns show the histograms for the three FoV. The mean, median, and position of the peak of $L_I$ are shown by blue, green and red dashed lines, respectively. These values are also presented in Table 6.3 and 6.4. The histograms have several general characteristics: they all have a long tail, their medians are close to the positions of the peaks, and the means are greater than the medians due to the tails. Therefore, median would be a better estimator for $L_I$.

The median of the fractional leakage is higher in the 3C196 field compared to the NCP, which seems paradoxical given that the beam model shows more leakage in the NCP field, as evident from the bottom two panels of 6.8. Leakage is lower near the zenith and increases as one moves away from the zenith. As the 3C196 field comes very close to the zenith during the mid-point of the observation, the leakage decreases substantially. But the NCP field rotates
Figure 6.9: Histograms of the leakage RMS ($L_{\text{rms}}$) over the instrumental $k$-space over the NCP (bottom row) and 3C196 (top row) fields. The three columns correspond to the fields of view of $15^2$, $9^2$ and $4^2$ deg$^2$ from left to right respectively. The mean and median of the fractional leakage are shown on the inset in each panel.

around a fixed elevation resulting in a constant leakage pattern throughout the observing time. Therefore, from the beam plots 3C196 seems to exhibit more leakage than NCP. However, in the power spectra, the leakages at all hour angles are averaged, and we think Stokes $Q$ and $U$ are averaged down more in the NCP than in the 3C196 field resulting in a lower fractional leakage in the former field.

The median of $L_I$ has a very weak dependence on the fields of view; it remains almost same in the three FoV. This also seems paradoxical, because the fractional leakage calculated directly from the model beam increases as a function of distance from the phase center (Asad et al., 2015, fig. 2b). However, as we go away from the phase center, the fractional leakage increases, but the power of the polarized emission decreases. This can be clearly seen by comparing the top two panels with the bottom two in Fig. 6.8. The top panels show the polarization beams that attenuate the polarized emission, and the bottom panels show the leakage patterns that determine the level of leakage. It might be the case that these two effects
cancel each other out so that leakage does not vary significantly with the change of fields of view. Another reason could be that the polarization vector are averaged down more in a larger area, thereby diminishing the effect of leakage in wider FoV. The exact mechanism of these attenuations, and their mathematical interpretation will be explored in a future work.

We take the medians of $L_I$ to be the best estimator of the fractional leakage. As the variation of this value is negligible between the three FoV, as seen in Table 6.3 and 6.4, we take the average of the three medians in each field to be the fractional leakage. The resulting fractional leakages are:

$$L_{I}^{3C196} = 0.35\%$$  \hspace{1cm} (6.5)
$$L_{I}^{NCP} = 0.27\%.$$  \hspace{1cm} (6.6)

## 6.4 Discussion

The observed polarized emission is lower in amplitude but has larger fluctuation along frequency in the NCP field, compared to the 3C196 field. The power of the polarized emission ($P_P$) in both fields has been shown in Fig. 6.1 and 6.3 as cylindrically averaged PS. A fraction of this power leaks into Stokes $I$ because of the polarized primary beam. The fractional leakage ($L_I$) has been calculated through simulations for three FoV and two fields, as listed in Table 6.3 and 6.4. The observed $P_P$ and model $L_I$ (according to the model beam) can be used to calculate the RMS of the leakage into Stokes $I$ that should be expected in the two fields. Mathematically, the RMS of the expected leakage

$$L_{I}^{\text{exp}} = \sqrt{P_P^{\text{obs}} \times L_{I}^{\text{model}}}.$$  \hspace{1cm} (6.7)

We have seen that $L_I$ varies very little over the instrumental $k$-space, and hence the leakage PS can be considered to be an almost scaled-down version of the polarization PS. The same scaling relationship is also seen seen in Fig. 4.4 of Chapter 4.

In Fig. 6.10, we show the variance of the expected leakage, $[L_{I}^{\text{exp}}]^2$, in the 3C196 (blue lines) and NCP (red) fields within $4^\circ \times 4^\circ$ (dotted) and $9^\circ \times 9^\circ$ (solid). To calculate $[L_{I}^{\text{exp}}]^2$ for a certain observing field and field of view, the corresponding medians of $L_I$ were used from Table 6.3 and 6.4. The figure shows that leakage is lower in the $9^\circ \times 9^\circ$ fields. The NCP field suffers from the same level of leakage as the 3C196 field within $4^\circ \times 4^\circ$, but within $9^\circ \times 9^\circ$ NCP leakage is higher. This is of course coincidental. As noted earlier, the NCP field exhibits higher power at high $k_\parallel$ in the cylindrical PS, and hence even if the expected level of leakage is similar in both fields, the ‘EoR window’ would suffer more from leakage-contamination in the NCP field.

One limitation of our results regarding $L_I$ is that we do not know the accuracy of the LOFAR model beam very well outside the first null, which has a diameter of $6^\circ.4$ at 150 MHz. The accuracy of the beam model within this limit is presented in Chapter 4 of this thesis. It was found that the beam has 10% error in predicting polarization leakage within the first null. We do not expect the same level of accuracy outside the field of view, but more work is needed to quantify this. In this chapter, the beam model out to a maximum diameter of $15^\circ$ has been used, and we are bound to be affected by beam model errors to some extent.
6.5 Conclusion

In this chapter, we have presented cylindrically and spherically averaged power spectra (PS) of the observed Galactic diffuse polarized emission in the 3C196 and NCP fields of the LOFAR-EoR key science project. The PS have been produced from an 8-hour synthesis observation of the 3C196 field, and a 13-hour observation of the NCP field for 50 spectral subbands ranging from 150 to 160 MHz. A version of the 3C196 PS was presented in Chapter 4, but unlike that chapter, here we have created the PS directly from the observed image cubes without re-convolving them with the primary beam. The main difference between the PS of the two fields seen in Fig. 6.1 is that, in ‘3C196’ polarization power is restricted within a ‘wedge’ at low-$k_\parallel$ and high-$k_\perp$, whereas in ‘NCP’ power is more distributed and one can see a considerable amount of power at the high-$k_\parallel$, low-$k_\perp$ corner of the PS, i.e. in the ‘EoR window’. This is because the diffuse emission in ‘NCP’ has more spectral fluctuations than that of ‘3C196’. The PS were produced for two different fields of view: $9^\circ \times 9^\circ$ and $4^\circ \times 4^\circ$. Power slightly decreases within the larger field of view, because the region outside a diameter of $4^\circ$ is dominated by noise.

We have also determined the fraction of polarized power that would leak into Stokes $I$ PS of the two fields, contaminating the EoR window, for three different fields of view: $15^\circ \times 15^\circ$, $9^\circ \times 9^\circ$ and $4^\circ \times 4^\circ$. A leakage PS of the 3C196 field was calculated in Chapter 4 by simulating a sky modeled from real observations of Galactic diffuse polarized emission. To avoid the effect of noise, here we have calculated the PS for both ‘3C196’ and ‘NCP’ by simulating LOFAR observations of a model Galactic-diffuse-polarized emission created by Jelić et al. (2010). The model emission has significant fluctuations along frequency due to Faraday rotation, and the spectrally fluctuating emission contaminate the EoR window of the PS considerably. The level of leakage is higher in ‘NCP’ than in ‘3C196’ (compare Fig. 6.7 and 6.6), because the simulated polarized emission is also higher in ‘NCP’. The squared-root of the ratio of the leakage and linear polarization powers, i.e. fractional leakage ($L_f$), has been found to vary very little over the instrumental $k$-space. Histograms of $L_f$ (Fig. 6.9) show
6.5 Conclusion 113

this more clearly. Although the $L_I$ was found to be lower in the NCP field compared to the 3C196 field, higher leakage power was observed in the former field, because the polarized power was also much higher in the former.

We have taken the median of the fractional leakage as the best estimate of $L_I$. One very interesting result of this chapter is that, the median of $L_I$ depends very weakly on the field of view. This might be due to the fact that, although fractional leakage increases with distance from the phase center, polarized power decreases due to attenuation by the polarized primary beam, and the two cancel each other out. The resulting fractional leakage has a median of 0.35% in the 3C196 field, and 0.27% in the NCP field.

After calculating the PS from observation, and the fractional leakages from model beam simulation, we have predicted the level of leakage to be expected in the EoR windows of the two fields for different fields of view (see Fig. 6.10). The leakage into Stokes $I$ is lower in the $9^\circ \times 9^\circ$ fields compared to the $4^\circ \times 4^\circ$ fields. Within $4^\circ \times 4^\circ$, leakage is almost same in the two fields, but within $9^\circ \times 9^\circ$, it is higher in the NCP field. The differences are not very high, and in any case they are coincidental.

One limitation of this work is that we have used fields much wider than the width of the primary beam for our simulations, but we do not know the accuracy of the beam very well outside the first null. Moreover, we did not investigate yet the mathematical explanation of why the fractional leakage depends very weakly on the FoV. Answering this question is a part of our ongoing and future work. In Chapter 7, we will come back to the future perspectives of our work.
Conclusions and Summaries

7 Conclusions and outlook ........................ 117
  7.1 Conclusions
  7.2 Outlook

Bibliography ......................................... 129

English summary ................................. 131

Nederlandse samenvatting ..................... 137

Bengali summary ................................. 145

Acknowledgments ............................... 151
7. Conclusions and outlook

The standard cosmological model of our age, in contrast to the models of the ancients, have passed innumerable observational tests. But one can still say that our observational knowledge of the universe is rather limited. We have pictures of the universe only as a baby and as an adult; snapshots of the whole adolescence period are missing. The Cosmic Microwave Background (CMB) is the snapshot of the baby universe, and large portions of the adult universe have been mapped by astronomical surveys throughout the electromagnetic spectrum. During the so-called adolescence period, galaxies, stars and black holes formed, and gradually re-ionized almost all hydrogen gas in the universe. Neutral hydrogen is the most promising beacon of this period, because they emit a signal of 21-cm wavelength. Modern low-frequency radio telescopes, such as the GMRT, LOFAR, MWA, PAPER, are trying to detect this 21-cm signal coming from the epoch of reionization (EoR) which ended about one billion years after the big bang.

The 21-cm signal is expected to be much weaker than the extragalactic and Galactic foregrounds, and the system noise in the current telescopes. The detection of the EoR signal will largely depend on a successful removal of the foregrounds. The extragalactic foregrounds appear as compact sources at the resolutions relevant for EoR observations, and thus easily removable. The Galactic foregrounds are diffuse, but their total intensity can be removed utilizing its spectral smoothness. However, the leakage of spectrally structured polarized Galactic foregrounds into total intensity could be more difficult to remove. This leakage is caused by systematic effects of the instrument. In this thesis, we have studied the observed polarized Galactic emission, the systematic effects of LOFAR, calibration errors and polarization leakage removal methods. In this chapter, we give a summary of our main results, and then discuss the outlook of our work. The major conclusions that one should take
7.1 Conclusions

The motivation behind this thesis was to analyze the systematic effects in general and polarization systematics of LOFAR high-band antennae (HBA) in particular on EoR observations. Some key questions were presented in Chapter 1, Section 1.3.1. This thesis was an attempt to answer these questions. For this purpose, we created a simulation pipeline, by assimilating existing calibration, imaging and simulation software\(^1\), that can simulate LOFAR observations from any compact or diffuse sky model taking into account the direction independent and dependent (model primary beam) systematic effects. The wide-band simulated data can be analyzed in rotation measure, image, or power spectrum (PS) space using this pipeline. The PS can be produced either from images or from visibilities. The mathematical formalism of this pipeline is described in Chapter 2, and the parts of the pipeline that were added later in this thesis are described in the subsequent chapters.

7.1.1 Primary beam model of LOFAR

We described the spatial, temporal and spectral structure of the full-polarization nominal model primary beam of LOFAR\(^2\) using Mueller matrices in Chapter 2. We found that the polarization leakage predicted by the model primary beam of LOFAR increases with distance from the phase center of the field, and also with distance of the field from the local zenith. The components of the Mueller matrix responsible for leakage were found to be 3–4 orders of magnitude lower than the Stokes I beam in power. The FWHM and spatial structure of the beam model change smoothly with frequency. The LOFAR beam model has an FWHM of 3.8\(^\circ\) at 150 MHz and decreases with increasing frequency.

We quantified the polarimetric performance of the beam model using IXR\(_M\), the Mueller matrix version of the intrinsic cross-polarization ratio, a standard figure of merit for measuring the polarimetric performance of low-frequency arrays (see, e. g., de Lera Acedo et al., 2015). Fig. 5.3 and 5.4 show that the polarimetric performance of low-frequency aperture arrays such as LOFAR is best near the phase center of the field and when the field is close to its culmination point. However, narrowing the field of view or filtering out the observations close to horizon result in reduced 12-cm power spectrum sensitivity and a balance between data filtering and calibration and modeling of the systematic errors needs to be maintained.

Accuracy of the model beam

In Chapter 5, we assessed the accuracy of the model beam of LOFAR by comparing the leakages predicted by the beam model with the actual leakages observed in LOFAR observations of the 3C295 field. We found that the leakage prediction of the beam, in 68% of the cases, will have a relative error of $\leq 10\%$, i. e. if the predicted leakage is 1%, the actual leakage might be between 0.9% to 1.1%. Therefore, if the differential beam effects are taken out

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\(^1\)The software packages used in this pipeline are BBS, SAGECAL, AWIMAGER, CASA, EXCON, the RM-synthesis code by Brentjens & de Bruyn (2005), the diffuse foreground simulations by Jelić et al. (2010), DS9, and the python packages numpy, scipy, matplotlib, pyrap, pyfits.

\(^2\)The model is created from the EM simulations of the ASTRON antenna group (Hamaker, 2011).
perfectly using the model beam, the errors in the correction will be \( \leq 10\% \), i.e. the residual leakage in Stokes \( Q, U \) in that case will be \( 10^{-3} \) of Stokes \( I \) flux.

The result of this study using the \( I \rightarrow Q, U \) leakages should hold true even for the \( Q, U \rightarrow I \) leakages, as their relationship is symmetric for both the on-axis (Sault et al., 1996) and off-axis (e.g. see Fig. 2.2) beams. Therefore, we can say that the beam model used to predict the \( Q, U \rightarrow I \) leakage in Chapter 4 and Chapter 6 has a 10\% error, and if the leakage could be removed, this error would be one of the constituents of the residual.

### 7.1.2 Extragalactic foreground

In Chapter 3, we simulated an observation with DI-errors by assuming them to be random at every timestep and the rms of the random numbers drawn from a Gaussian distribution with zero mean is dubbed the ‘RMS DI-error’. We find that self-calibration can solve for these errors to an extremely high accuracy if the sky model is perfect. Because, in that case the information provided by an interferometer is highly redundant. For an RMS DI-error of \( 10^{-3} \), the selfcal error is less than 0.002\% and the corresponding error in the rms of the resulting residual image is less than 0.005\% (Fig. 3.2).

In this chapter, we also simulated LOFAR observations of extragalactic unpolarized point sources in the 3C196 field taking into account the time-frequency-baseline dependent DD effects, i.e. the primary beam model described in Chapter 2. We estimated the flux and position errors due to self-calibration with incomplete sky models, and the percentage of \( I \rightarrow (Q, U) \) leakage of the brightest sources (see Fig. 3.4). We see that the errors decrease significantly as the sky model is improved. 

_EoR observations will not be severely affected by leakage of intrinsically polarized point sources into Stokes I, because the point sources seen at low frequencies are very weakly polarized._

### 7.1.3 Galactic polarized foreground

In Chapter 4, we took the real LOFAR observations of Galactic diffuse polarized emission in the 3C196 field and created a mock sky model where \( I = V = 0 \) to more precisely quantify the leakages from \( Q, U \) to \( I, V \) caused by the beam model. An RM-synthesis of the beam-corrupted polarization image cubes showed that in this particular field polarization peaks within the Faraday depths (\( \Phi \)) of -1 and +5 rad/m\(^2\). From the effective Stokes I Faraday dispersion images we saw that the instrumental polarization leakage is localized around \( \Phi = 0 \) (Fig. 4.2), because DD-effects (beam) themselves do not have any rapid variation along frequency. In the 3C196 field, maximum leakage was found to be around 15 mK which could be comparable to the EoR signal (Fig. 4.1) in magnitude.

In Chapter 6, we presented cylindrically and spherically averaged PS of the Galactic diffuse polarized emission in the 3C196 and NCP observing fields. The PS have been produced from an 8-hour synthesis observation of the 3C196 field, and a 13-hour observation of the NCP field for 50 spectral subbands from 150 to 160 MHz. A version of the 3C196 PS was presented in Chapter 4, but unlike that chapter, in Chapter 6 we have created the PS directly from the observed image cubes without re-convolving them with the primary beam. The main difference between the PS of the two fields seen in Fig. 6.1 is that, in ‘3C196’ polarization power is restricted within a wedge-shaped region at low-\( k_{\parallel} \) and high-\( k_{\perp} \), whereas
in ‘NCP’ power is more distributed and one can see a considerable amount of power at the high-$k_\parallel$, low-$k_\perp$ corner of the PS. This is because the diffuse polarized emission in ‘NCP’ has a higher level of spectral fluctuations than that of ‘3C196’. The PS were produced for two different fields of view: $9^\circ \times 9^\circ$ and $4^\circ \times 4^\circ$. Power slightly decreases within a larger field of view, probably because of averaging the polarized emission with anti-correlated or random position angles.

7.1.4 Effects of leakage on EoR window

A fraction of the polarized power described above leaks into the total intensity PS because of the polarized primary beam. The low-$k_\perp$, high-$k_\parallel$ corner of a 2D PS is called the ‘EoR window’, because this region is expected to be least affected by total intensity foregrounds. In this thesis, we studied the effects of leakage on the EoR window of the 3C196 field.

In the 3C196 PS presented in Chapter 4 (Fig. 4.3), a clear ‘EoR window’ can be defined in terms of polarization leakage above the wedge and below $k_\parallel \sim 0.5$ Mpc$^{-1}$. Within this window, the EoR signal dominates the polarization leakage and the window takes up the whole $k$-space at $k_\parallel < 1$ after removing $\geq 70\%$ of the leakage. The fraction of polarized power that leaks into Stokes $I$ can be found by taking the ratio $L_I = \sqrt{P_I/P_P}$ where $P_I$ and $P_P$ are the Stokes $I$ and $P = Q + iU$ power spectra, respectively. We saw that $L_I$ varies by a factor of $\sim 2$ and ranges from 0.2% to 0.4%. We compared this leakage with the PS of the expected 21-cm differential brightness temperature at $z = 9$ simulated by Mesinger et al. (2011) and saw that the region above the PSF-induced wedge and below $k_\parallel \sim 0.5$ Mpc$^{-1}$ is dominated by the cosmic signal (Fig. 4.3d) and hence defines a potential ‘EoR window’.

7.1.5 Leakage within wider fields

In Chapter 4, leakage was predicted from observed polarized emission. To see the effect of RMS fractional leakage within wider fields of view (FoV), in Chapter 6 we have calculated the PS for both ‘3C196’ and ‘NCP’ by simulating LOFAR observations of a Galactic-diffuse-polarized emission model simulated by Jelić et al. (2010). The PS were created for three different FoV: $15^\circ \times 15^\circ$, $9^\circ \times 9^\circ$ and $4^\circ \times 4^\circ$. The model emission has significant fluctuations along frequency due to Faraday rotation, and the spectrally fluctuating emission contaminate the EoR window of the PS considerably. The level of simulated leakage is higher in ‘NCP’ than in ‘3C196’ (compare Fig. 6.7 and 6.6), because the simulated polarized emission is also higher in ‘NCP’. $L_I$ has been found to vary very little over the instrumental $k$-space, as expected.

In Chapter 6, the histograms of the values of $L_I$ show that the best estimator of $L_I$ is the median. We see that the median of the fractional leakage is lower in ‘NCP’, but even then higher leakage power was observed in ‘NCP’ because the model polarized power was also higher in ‘NCP’ than in ‘3C196’. One very interesting result of this chapter is that, the level of fractional leakage depends weakly on the field of view. This might be due to the fact that, although fractional leakage increases with distance from the phase center, polarized power decreases due to attenuation by the primary beam. Moreover, the polarization vectors might add incoherently lowering the leakage within a wider field of view. The resulting fractional leakage is around 0.35% in the 3C196 field, and around 0.27% in the NCP field.
After calculating the PS from observations, and the fractional leakages from beam model simulation, we have predicted the level of leakage to be expected in the EoR windows of the two fields for different fields of view (see Fig. 6.10). The leakage into Stokes $I$ is lower in the $9^\circ \times 9^\circ$ fields compared to the $4^\circ \times 4^\circ$ fields. The levels of leakage in the 3C196 and NCP fields are comparable in the $4^\circ \times 4^\circ$ FoV. This is of course coincidental.

### 7.1.6 Removing or avoiding polarization leakage

We tested various strategies for removing polarization leakage of both point sources and diffuse emission. Two methods for removing point source leakage were tested, one uses AW-projection, and the other uses DD calibration. AW-projection is implemented in AWIMAGER and, in Chapter 3 we found that it can remove up to 80% of the $I \rightarrow Q,U$ leakage. The DD-calibration was tested in Chapter 5, and it was found that for sources with sufficiently high SNR, more than 80% of the flux could be removed and the residuals were generally very close to the local noise level. AW-projection and DD-calibration have similar efficiency in removing polarization leakage of point sources.

However, our main concern is leakage in the opposite direction, i.e. from Stokes $Q,U$ to $I$, and this leakage is mostly contributed by diffuse polarized emission. There can be two strategies for dealing with such leakage: leakage removal and avoidance. If the leakage has similar spectral structure as the expected EoR signal, then avoiding the leakage would be impossible, and removing it would become difficult. In such cases, we could either model the polarized emission in RM-space and remove it following a deconvolution procedure (Geil et al., 2011), or correct the power spectrum for the bias caused by the leakage as shown in Chapter 6.

On the other hand, if the leakage does not mimic the EoR signal, we can either avoid it, as it will be confined within a small region in the PS-space, or try to remove it to decontaminate even the low-$k_\parallel$ scales. In contrast to the NCP field, the leakage in the 3C196 field does not mimic the EoR signal in the PS-space. So we tried to remove the leakage in the latter field using GMCA which is being used to remove diffuse total-intensity foreground from the LOFAR-EoR data. From the cylindrical PS of the residual left after the removal of foreground leakage components by GMCA, we saw that (Fig. 4.3f,g) at $k_\parallel < 0.1$, i.e. in the high SNR regime, GMCA could reduce the leakage by up to two orders of magnitude while the region above that scale was left completely untouched. For a more realistic analysis, we added 60 mK noise to the Stokes $I$ leakage maps, reran GMCA on it and saw that (Fig. 4.3h) in this case almost no leakage was removed, not even in the relatively high SNR region. A DDE-blind foreground removal method like GMCA is not ideal for removing leakage of diffuse polarized emission, as the level of leakage is lower than the current noise level in the LOFAR observations.

### 7.2 Outlook

This is the best of times for observational 21cm cosmology. Some upper limits on the EoR signal have already been published using the GMRT, MWA, PAPER, and LOFAR telescopes, and this is only the beginning. Most 21-cm EoR experiments are still limited by total intensity foregrounds and systematics. Once these limitations are overcome, polarization leakage
will become an important issue. To understand the leakage, the polarized emission at low frequencies will have to be studied carefully, and thus EoR experiments will also boost low frequency polarization studies. In this section, we describe future prospects of the results of this thesis, and also our future work.

7.2.1 Future prospects

Antennas for the future arrays such as SKA, that have the 21-cm EoR signal detection as one of the main scientific objectives, are being designed in such a way that their polarimetric performance is good enough to be able to minimize the effects of polarization leakage. A recently proposed figure of merit for quantifying the polarimetric performance is the intrinsic cross-polarization ratio (IXR) which, in Mueller formalism, can be directly related to the instrumental polarization. In the case of LOFAR, we found (Chapter 2–4) an instrumental polarization of around 0.3% (Fig. 4.3e) within the FWHM of the nominal station beams, i.e. within a FoV of ~4°. This corresponds to an IXR_M of 25 dB, or equivalently an IXR_J of 56 dB, and if the leakage can be reduced by more than 70%, IXR_M will improve to 35 dB.

Our calculations of the polarimetric performance of LOFAR can give some insights into the polarimetric performance of future telescopes, such as SKA. We can say that if SKA has a minimum IXR_M of 25 dB within the central ~4° of its nominal station beams, then even a modest polarimetric calibration (~70% leakage removal) will ensure that the polarization leakage remains well below the expected EoR signal at the scales of 0.02–1 Mpc\(^{-1}\). However, if the IXR_M is lower within a FoV of 4°, more leakage needs to be removed to reach the same level as before in relation to the EoR signal in the power spectra, e.g. if the IXR_M is 20 dB, 91% leakage has to be removed, and if it is 15 dB, 97% has to be removed.

7.2.2 Future work

We could calculate the accuracy of the beam model only up to the first null; accuracy of the sidelobes of the model could not be calculated for two interconnected reasons. First, the beam model under-predicts leakage on the sidelobes to some extent which can be seen by comparing the observed (Fig. 5.6a) and the simulated (Fig. 5.6b) images. In the former figure, some sources can be seen on the sidelobes, whereas in the latter all sources are within the FoV (note that the FoV would also change with frequency). Of course, the accuracy of the model beam could still be calculated, if we could quantify the under-prediction, and that’s where the second reason comes in. The Stokes I fluxes of the sources in the sidelobes were already very low as they were attenuated by the primary beam, and when we predicted leakage from these "faint" sources, the resulting leakage was even lower. So, we could not find compact sources bright enough to give rise to a detectable polarization leakage, even after the under-prediction of the beam, that would make the calculation of the accuracy possible at these distances from the phase center. Due to this limitation, we claim our measurement of the accuracy of the beam model to be reliable only within the FoV. However, in the future we will take into account the accuracy of the beam model farther away from the phase center, as it is crucial for EoR experiments.

Also, more study is needed to see how DDE-blind correction of leakage compare with the correction using a model beam. We have compared the modeling and DD-calibration approaches by testing the effectiveness of AW-projection (using AWIMAGER) and SAGECAL in
removing linear polarization leakage. However, both AWIMAGER and SAGECAL can remove only the leakages of compact sources from Stokes $I$ to $Q, U$, whereas for the EoR project we are interested in the leakages of diffuse emission from Stokes $Q, U$ to $I$. In the future, we will test the effectiveness of removing this leakage using an RM-model of the diffuse emission and the beam model of an instrument.

In Chapter 6, we have found some paradoxical results. For example, we saw that the RMS of the fractional polarization leakage depends very weakly on the FoV. This is especially important as the other telescopes being used to detect the EoR signal have larger fields of view than LOFAR. We have thought about two reasons for the weak-dependence of leakage on FoV. First, although the leakage term of the polarization beam increases, the power of the polarized emission itself rapidly decreases with distance from the phase center, and the two effects might lead to a converging level of polarization leakage. Second, the polarized emission with anti-correlated or random position angles might average down more in the larger FoV. One of the main focus of our future work is to test these hypotheses further using simulations and observations.
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Let us put our poetic-naturalist hat for a second and imagine that our universe is a living being on an evolutionary journey from birth to death. It had an extremely restless, chaotic, playful (even happy!) childhood. When the universe was a child, it did not know that it was a child. But now, after a long 13.8 billion years of journey, the grown-up universe knows that it once was a child, that it even had a dark teen age, that its wayward clouds were first dispersed by the bright shafts of light from stars and black holes. How does the universe know so much? Did it know that it would one day know? It knew nothing, not even the potentiality of its knowledge. But on an insignificant bend of its evolutionary path, that wayward child has come to terms with its life by accident, it has met its greatest creation, because what it has produced through random musings now dare to know itself, dare to unveil all its dark veils. That random creation, that proud knower of truth is the human animal; and other animals of that sort, if they really exist.

The life story of the universe is a most unusual one. It started in a grand explosion—called ‘big bang’—of something resembling existence itself, and is expanding and cooling following the laws of general relativity and thermodynamics ever since. The origin of big bang is still a mystery, but its aftermath has been so well understood by humans that their cosmologists boast of their modern theory as one of the greatest ever devised. Another one of the greatest human-made theories say that all matter are made of atoms, that are in turn made of subatomic wave-particles, mysterious entities that are waves in reality and particles in appearance. Ancient Roman poet Lucretius believed in the eternity of atoms, but contemporary humans know better: the building blocks of atoms—mainly quarks and electrons—were created during the first second after big bang. The creators were photons, carriers of energy and light. Matter and antimatter particles created from photons annihilated by colliding with each
Figure 7.1: The 13.8 billion years of history of the universe—from the big bang to the present day following the arrow of time. Different telescopes are suited for observing different epochs—BICEP2 is aiming to indirectly detect the signature of big-bang-producing inflation, whereas the ground based optical telescopes can probe only the recent history. The low-frequency radio telescope LOFAR is on the verge of observing the epoch of reionization. Adapted from an original graphics of NAOJ.

other, recreating the creator photons in an impressive feat of irony. As the universe expanded and cooled, the temperature became insufficient to keep the creator-creation-creator cycle going, and almost all matter and antimatter particles annihilated for the last time without being created again. Almost, because there was a billion and one matter particles for every billion antimatter particles, and this excess meant the universe, at the end of the ultimate annihilation, consisted of photons and matter in the form of leptons (e.g. electrons) and quarks. The quarks glued together to form protons during the first three minutes.

The universe was dominated by photons when it was less than 380,000 years old. Electrons and protons were independent, free beings, and the freedom of electrons meant the captivity of photons, however dominant they may be. Photons were unable to follow their natural ‘straight path’, because they were being continually distracted and scattered by untaken electrons. But when the universe was roughly 380,000 years old, the electrons were taken by protons and together they formed a soup of neutral hydrogen in an event called ‘recombination’. Bound electrons were no longer attracted to photons, and the photons propagated everywhere freely on a straight line without any swerves. We can detect these emancipated photons as the cosmic microwave background (CMB) radiation of 2.7 K. Planck satellite has created by far the most sensitive maps of CMB, and these maps are the greatest treasure trove of observational cosmology to date. They are a snapshot of the infant universe, when it was only 380,000 years old, 0.003% of its current age.

After the recombination of hydrogen, the universe entered into its teen age which was very dark, because there was no source of light. The first stars and black holes soon formed but even they could not light things up. Because they emitted mostly ultraviolet light, and the hydrogen-bound electrons absorbed the energy of this light to free themselves from the relationship with protons. Although the old photons of the recombination era were relatively free from the influence of electrons, the new photons produced from stars and black holes
were not. When photons cannot propagate freely, we get a foggy, smoggy atmosphere. Therefore, one can say that the universe entered into a dark, cold, winter smog of reionization, from the bright, hot, summer smog of pre-recombination. The epoch of reionization is only theoretically somewhat known to humans, but so far they have not been able to observe it directly, nor have they been able to observe the dark ages and the cosmic dawn of the first stars. If a person has a photograph of her in the womb, and then when she is an adult, we would not be able to guess how she looked and behaved during her childhood and teenage; people, not unlike the universe, change a lot. This is exactly the condition of the universe as revealed to humans. They have a snapshot of the infant universe through CMB, and then they have many pictures of the adult universe, but the dark ages, the cosmic dawn, and the epoch of reionization are still a mystery.

How fortunate for humans that they can now dare to map the whole universe in both space and time! Because the happy families that electrons and protons produced during recombination do give away some detectable vibes, however weak that may be compared to the strong signal of the craving for freedom via reionization. It is produced due to an interplay between electrons and protons, who have a special directedness called ‘spin’. When the partners in a hydrogen family are spinning in the same direction, they have higher energy as opposed to when they are spinning in the opposite direction. Let us call the hydrogen families with ‘aligned’ spin ‘happy’ and the ones with ’anti-aligned’ spin ‘sad’, for no other reason than being a bit poetic again. The sad families play around with other families or lonely photons and become happy after gaining some extra-familial energy, but when the phase passes and they become sad again they release the afore-gained energy as a radiation of 21-cm wavelength. It might be relevant to mention here that, in these families it is always the electron who is unstable. This 21-cm signal is so fundamental a beacon for learning about the universe that a picture of the spinning partners in a hydrogen family was put on the ‘pioneer plaque’ in the Pioneer 10 spacecraft by humans, so that other intelligent animals, if there are any, can understand what they are talking about.

Another big stroke of luck for humans is that the speed of light is constant, otherwise they would not come into existence, let alone observe their creator. Because of this constancy, the farther they look in space, the earlier they see in time. Reionization of hydrogen by stars and black holes was probably complete when the universe was 1 billion years old. If humans look far enough in space, they can see that adolescent universe through the 21-cm signal. If they look even farther, they can even observe what was happening during the cosmic dawn and the dark ages. Huge low-frequency radio telescopes have been and are being built on Earth to do just this. Because of the expansion of the universe, the 21-cm signal is stretched out to almost 4 to 1 meters depending on how far into the history one is looking. For this reason, the telescopes are tuned to low frequencies, equivalent to large wavelengths. One such telescope is LOFAR (LOw Frequency ARay, centered around a core near Exloo, Netherlands), that detects all extraterrestrial and terrestrial signals with wavelengths between 1 and 30 meters. Human-created terrestrial signals are filtered out, and the remaining haystack of extraterrestrial radiation is combed thoroughly to find the 21-cm signal lying deep in the noise.

The 21-cm signal is elusive. First of all, one cannot really see the absolute signal, what is observable is the relative intensity of the signal with respect to CMB, called the differential brightness. And even the actual value of the differential brightness is not observable; telescope arrays like LOFAR can only see the spatial fluctuations of this brightness. But one can learn a
Figure 7.2: How the beacon from reionization might get obstructed by the inherent imperfections of human-made telescopes.

lot just from these fluctuations of the differential brightness.

From theoretical calculations, the period from the dark ages to the epoch of reionization has been characterized by three temperatures: kinetic, spin, and radiation. Radiation temperature has to do with the energy of CMB, and kinetic temperature is a measure of the motion of atoms. Spin temperature is the measure of the relative number of atoms with aligned and anti-aligned spins. These three temperatures engaged in a ménage à trois. In the beginning of the dark ages they were living and falling together with the cosmic expansion, but around 10 million years after the big bang, kinetic temperature broke away from the other two and started falling rapidly. The spin temperature was vacillating between the other two, but remained close to the radiation temperature. It was less than the radiation temperature in the beginning resulting in a 21-cm signal in absorption, but around 100 million years after the big bang it surpassed the radiation temperature producing a 21-cm signal in emission. Finally, the 21-cm signal disappeared when all hydrogen in the intergalactic medium became ionized releasing the electrons from their protonic shackles once and for all. These theoretical predictions, among many other, remain to be proved, disproved or better constrained by the forthcoming observations of the 21-cm signal.

But the observation is one of the most difficult ever attempted by humankind. Just as humans can only think of the form of a perfect circle, but can never reproduce that ideal form in reality, so the telescopes they build are bound to be flawed reproductions of the eye of a flawless observer. Understanding these flaws flawlessly has never been as dire a necessity as in the case of detecting the 21-cm signal. This thesis is concerned with understanding some of the errors of LOFAR in detecting the 21-cm signal coming from the epoch of reionization (hereafter, the ‘EoR signal’). However imperfect their material creations may be, humans are excellent at imagining the perfect, ideal form behind those creations. In fact, they cannot mold anything imperfect without a perfect model. And there is a perfect model describing the ‘form’ of the ideal observer which is, fortunately, also very straightforward to describe pictorially within the scope of this summary. In the remainder of this summary, I will do just that referring to Figure 7.4.
Figure 7.4 shows three mathematical forms: the observed Stokes vector on the left hand side ($S_O$), the true Stokes vector on the right hand side ($S_T$), and a $4 \times 4$ matrix (known as Mueller matrix, $M$) representing the flawed observer in between. Between the truth and the appearance, there is always an observer. Each term in the vectors and the matrix spans a field of view of $20^\circ \times 20^\circ$. An observation $S_O$ is equivalent to a matrix multiplication of the true object $S_T$ with the flawed observer $M$. But there are more complications, because the truth observed here by LOFAR is not the desired truth. As mentioned above, $S_T$ here is a great haystack of extraterrestrial radiation from which all unwanted parts need to be combed out. For example, all extragalactic and Galactic foreground radiation lying in the intervening medium need to be removed to get at the EoR signal. Extragalactic foreground appears as tiny compact sources and are comparatively easy to remove. But the Galactic foregrounds arise mostly from relativistic electrons spinning around magnetic field lines in the interstellar medium and are diffused throughout the fields of view of our telescope. Fortunately, there is a way to identify the Galactic diffuse foreground too: it is smooth along frequency, unlike the EoR signal. After removing the frequency-sensitive components, the residuals are expected to contain the EoR signal, the random and systematic noises, and another subtle effect called ‘polarization leakage’, the focal point of this thesis. To explain this leakage, we have to delve into Figure 7.4 again and explain the phenomenon of polarization.

Astronomy is concerned with detecting and analyzing electromagnetic waves coming from space. These waves are created by oscillations of electromagnetic field. Looking at an open ocean on a sunlit day, we recognize only one wave, that of the ocean. But if we think deeply enough, we come to the realization that the light engulfing us and enabling us to see the water waves is itself a kind of wave, not wave on the water but on the electromagnetic field. The ocean waves can look very well structured when they are about to break on the shore, but in general they are haphazard, sloshing this way and that way. Likewise, the electric (and/or magnetic) field can oscillate in a specific direction, or haphazardly in all directions. The former is called polarized, and the latter unpolarized. The polarization characteristics of these waves can be completely described by the four Stokes parameters shown by the Stokes vectors in Figure 7.4. Stokes $I$ represent the total intensity, $Q$ and $U$ linear polarization and $V$ circular polarization. Low-frequency radio emission from our galaxy is polarized, but it gets depolarized to a large extent on the way to us. The linear polarization ($Q$, $U$) fraction that we detect in the end is, however, might not be small enough to discard. Especially because the polarized emission might mimic the EoR signal in its spectral structure.

The EoR signal will be detected in total intensity, i.e. in Stokes $I$, but a small fraction of the polarized emission can leak into Stokes $I$ via the $M$ matrix shown in Figure 7.4—Stokes $I$ in $S_O$ will have contributions from all elements of $S_T$ and the elements of the first row of $M$. If the polarization indeed mimics the signal in frequency, then its leakage would be very difficult to separate from the signal. If we know both the polarized emission and the $M$ matrix, we can hope to disentangle the two, although it will still be difficult as $M$ varies with time, frequency, observing antenna, pointing direction, and field of view.

In this thesis, we have characterized the linearly polarized emission in the 3C196 and NCP fields, two observing windows of the LOFAR-EoR project. After creating a model of this polarized emission, we have predicted the level of leakage into Stokes $I$, for a standard one-night observation, using the time-frequency-antenna-direction dependent $M$ matrices created from the standard primary beam model of LOFAR. We have shown that the leakage is lower than one of the simulated EoR signals at larger angular scales, but surpasses the signal
at smaller scales. The leakage was found to mimic the EoR signal to some extent in the NCP field, but not in the 3C196 field. We have determined the accuracy of the primary beam model of LOFAR using an one-night full polarization observation. The accuracy was found to be around 10% and it helped us better constrain the predicted level of leakage. The fields of view of LOFAR antennae ranges from around 4 to 5 degrees at higher frequencies. However, this inner region is affected by leakage and sidelobes from farther away. We have shown that the effect of leakage from outside the field of view is negligible, because although leakage increase as one goes away from the phase center, the intensity of the emission decreases, and the two effects cancel each other out to a certain extent.

Another focus of this thesis was to test different strategies of removing polarization leakage. Existing algorithms can only deal with removing leakage of compact sources from total intensity into polarization, whereas we are interested in the opposite case. These strategies, the direction dependent calibration and AW-projection, have been tested in any case, and we have found them almost equally capable of doing the job—both can remove more than 80%–90% of the leakage if the signal to noise ratio is sufficiently high. Ultimately, we want to be able to remove leakage of diffuse polarized emission into total intensity, and the works presented in this thesis will be helpful in this regard. Every scientific work has some serendipitous elements. In our case, we can say that a work that started as an attempt to characterize the level of polarization leakage also enabled us to detect polarized compact sources previously un-observed at these frequencies, and quantify the accuracy of the primary beam model of LOFAR.

The level of polarization leakage is comparable to the EoR signal at some scales and toward some directions, and it is much lower than the noise level. Therefore, for the current generation telescopes like LOFAR leakage might not be one of the main concerns. This effect will be much more relevant and interesting for the more sensitive future telescopes like SKA. We have only just started to characterize the polarized emission at such low radio frequencies, and what we will be able to discover in the future is the unknown. This thesis is one of many stepping stones toward that age of low-frequency astropolarimetry.
Laten we ons universum op een poetisch naturalistische wijze bekijken en het zien als een levend wezen op haar evolutieaire reis van de wieg tot het graf. Het had een ongelofelijk rusteloze, chaotische, levendige en misschien zelfs wel gelukkige jeugd. Toen het universum een kind was, had het geen idee dat het een kind was. Maar vandaag de dag, na een reis van 13,8 miljard jaar, weet het universum dat het ooit een kind was, dat het zelfs donkere tienerjaren beleefde en dat haar nukkige mistflarden pas verdwenen door het velle licht van sterren en zwarte gaten. Hoe weet het universum zo veel? Ooit wist het van niks, niet eens van de potentie van haar kennis. Maar door een onbeduidende bocht in haar evolutieaire reis is het nukkige kind per ongeluk in het reine gekomen met haar verleden. Het heeft haar grootste creatie ontmoet. Want wat het geproduceerd heeft, heeft door willekeurige overpeinzingen zichzelf en haar gedurft te leren kennen, gedurft al haar donkere sluiers te ontsluiten. Die ene willekeurige creatie, die trotse kenner van waarheid is de menselijke diersoort; en andere dieren van dat soort, mochten ze echt bestaan.

Het levensverhaal van het universum is een zeer bijzondere geschiedenis. Het begon met een grote explosie, genaamd de oerknal, van iets dat lijkt op het bestaan zelf en sindsdien expandeert en koelt volgens de wetten van de algemene relativiteitstheorie en de thermodynamica. De oorsprong van de oerknal is nog steeds een mysterie, maar haar nasleep is zo goed begrepen door mensen, dat cosmolologen hun moderne theorie bestempelen als een van de voornaamste theorien ooit bedacht. Een andere van deze voornaamste door mensen ontwikkelde theorien zegt dat alle materie bestaat uit atomen, die op hun beurt weer bestaan uit subatomaire golf-deeltjes, mysterieuze entiteiten die in werkelijkheid golven zijn, maar in hun verschijnen zich gedragen als deeltjes. De klassieke Romeinse dichter Lucretius geloofde in de eeuwigheid van atomen, maar hedendaagse mensen weten beter: de bouwstenen van

Het heelal werd overheerst door fotonen tijdens de eerste 380.000 jaar. Elektronen en protonen waren onafhankelijke, vrije wezens. De vrijheid van elektronen betekende de gevangenschap van fotonen, hoe dominant deze ook waren. Fotonen waren niet in staat hun natuurlijke rechthoekige beweging uit te oefenen, omdat ze continu afgeleid en verstrooid werden door ongevangen elektronen. Toen het universum ongeveer 380.000 jaar oud was, werden de elektronen gegrepen door de protonen om samen een brij van neutraal waterstof te vormen in een gebeurtenis die recombinatie heet. De gebonden elektronen werden niet langer aangetrokken door fotonen en de fotonen konden zich vrij bewegen in rechthoekige bewegingen zonder omzwervingen te maken. We kunnen deze vrij gemaakte fotonen waarnemen als de kosmische achtergrondstraling (in het Engels Cosmic Microwave Background (CMB) radiation) van 2,7 K. De Planck satelliet heeft verreweg de meest fijngevoelige kaart van de CMB gemaakt. Deze kaarten zijn tot nu toe de grootste schatkamer voor observationele kosmologie. Het zijn momentopnames van het universum als baby, toen het nog maar 380.000 jaar oud was, 0,003% van haar huidige leeftijd.
Na de recombinatie van waterstof ging het heelal door haar tienerjaren, die erg donker waren door het totale gebrek aan lichtbronnen. De eerste sterren en zwarte gaten vormden snel, maar zelfs zij konden geen verlichting brengen. Dit kwam doordat ze voornamelijk ultraviolet licht uitstraalden. De energie afkomstig hiervan werd door aan waterstof gebonden elektronen geabsorbeerd om zichzelf te bevrijden uit de greep van de protonen. Hoewel de oude fotonen uit het recombinatietijdperk relatief gevrijwaard waren van de invloed van elektronen, waren de nieuwe elektronen dat niet. Als fotonen zich niet vrij kunnen bewegen, ontstaat er een mistige, rokerige atmosfeer. Daarom kan gezegd worden dat het heelal een donkere, koude wintersmog van reionizatie betrad, vanuit de felle, hete zomersmog van pre-recombinatie. Het tijdperk van reionizatie is enkel theoretisch enigzins bekend voor mensen, maar tot dusverre zijn ze nog niet in staat geweest om het rechtstreeks waar te nemen, noch de zogenaamde dark ages en de kosmische dageraad van de eerste sterren. Als een mens een foto heeft van zichzelf in de baarmoeder en als volwassene, zouden we niet kunnen raden hoe ze eruit zag en zich gedroeg tijdens haar kinder- en tienerjaren. Mensen, net als het heelal, veranderen veel. Dit is exact hetzelfde als de toestanden van het heelal die geopenbaard zijn voor mensen. Ze hebben een momentopname van het universum als baby door het CMB en ze hebben vele opnames van het volwassen heelal, maar de dark ages, kosmisch dageraad en het tijdperk van reionizatie zijn nog steeds een mysterie.

Wat een geluk voor mensen dat ze nu kunnen dromen over het in kaart brengen van het heelal in zowel ruimte en tijd! Dit is mogelijk doordat de blije gezinnen gevormd door elektronen en protonen tijdens recombinatie waarnembare golven uitzenden. Hoe zwak ook ten op zichte van het sterke signaal van de hunkering naar vrijheid door reionisatie. Het wordt geproduceerd door een samenspel van elektronen en protonen, die een speciale gerichtheid hebben genaamd ‘spin’. Als de partners in een waterstofgezin in dezelfde richting spinnen hebben ze een hogere energie dan als ze in tegengestelde richting zouden spinnen. Laten we de waterstofgezinnen met uitgelijnde spin blij noemen en met anti-uitgelijnde spin somber, enkel om weer een beetje poetisch te zijn. De sombere gezinnen kunnen met andere gezinnen rondhangen of met eenzame fotonen en blij worden door het winnen van wat buitengezins-energy. Als deze fase voorbijgaat dan ze weer somber worden, zullen ze eerder gewonnen energie wegstralen in de vorm van 21-cm straling. Het is misschien belangrijk te melden dat in deze gezinnen het altijd het elektron degene is die onstabiel is. Dit 21cm signaal is zo fundamenteel voor kennis van het universum, dat mensen een afbeelding van de spinnende partners in het waterstofgezin op de ‘pioneer plakette’ hebben gezet in de Pioneer 10 ruimtesonde, zodat andere intelligentie dieren, als ze er zijn, kunnen begrijpen waar mensen het over hebben.

Met de constante lichtsnelheid hebben mensen ook veel mazzel gehad, anders zouden ze niet bestaan en zeker niet in staat zijn om hun schepper te observeren. Door deze constante kijk je terug in de tijd als je verder de ruimte in kijkt. Reionisatie van waterstof door sterren en zwarte gaten was waarschijnlijk klaar toen het heelal een miljard jaar oud was. Als mensen ver genoeg kijken, zien ze het heelal tijdens haar puberteit in 21cm straling. Als ze nog verder kijken, kunnen ze zelfs waarnemen wat er gebeurde tijdens de kosmische dageraad en de dark ages. Enorme lage frequentie radiotelescopen zijn en worden op aarde gebouwd om dit te doen. Door het uittreden van het heelal is het 21cm signaal uitgerukt tot bijna 4 tot 1 meter, afhankelijk van hoe ver je terugkijkt in de geschiedenis van het universum. Dit is de reden dat de telescopen afgesteld zijn op lage frequenties, die overeenkomen met lange golflengtes. Een van deze telescopen is LOFAR (LOw Frequency ARray, gebouwd met vele
stations rond het centrum nabij Exloo in Drenthe), die alle aardse en buitenaardse signalen met golvengtes tussen 1 en 30 meter waarneemt. Door mensen geproduceerde aardse signalen worden weggefilterd, waarna de overgebleven hooiberg aan buitenaardse signalen doorgezeefd wordt op zoek naar het 21cm signaal, dat diep verborgen ligt in de ruis.

Het 21cm signaal is moeilijk te vangen. Allereerst kun je het absolute signaal niet echt waarnemen. Wat wel waar te nemen is is de relatieve intensiteit van het signaal ten opzichte van de CMB, genaamd de differentiële helderheid. En zelfs de waarde van de differentiële helderheid is niet waarneembaar: radio interferometrische telescopen zoals LOFAR kunnen alleen de ruimtelijke fluctuaties van deze helderheid waarnemen. Het is echter mogelijk om veel te leren van deze fluctuaties van de differentiële helderheid.

Uit theoretische berekeningen blijkt dat de periode van de dark ages tot het tijdperk van reionisatie wordt gekarakteriseerd door drie verschillende temperaturen: kinetische, spin- en stralingstemperatuur. Stralingstemperatuur heeft te maken met de energie van de CMB en de kinetische temperatuur is een maat voor de bewegingen van atomen. De spintemperatuur is een maat voor het relatieve aantal atomen met uitgelijnde en anti-uitgelijnde spintoestand. Deze drie temperaturen zijn deelnemers aan een ménage à trois. Aan het begin van de dark ages leefden en daalden ze samen met de kosmische uitdijing. Ongeveer 10 miljoen jaar na de oerknal ging de kinetische temperatuur een andere weg en begon snel te dalen. De spintemperatuur kon niet kiezen tussen de twee, maar bleef dichtbij de stralingstemperatuur. Het was lager dan de stralingstemperatuur in het begin, waardoor het 21cm signaal in absorptie verscheen. Na ongeveer 100 miljoen jaar na de oerknal werd de spintemperatuur hoger dan de stralingstemperatuur, wat zorgde voor een 21cm signaal in emissie. Uiteindelijk verdween het 21cm signaal toen alle waterstof in het intergalactisch medium geioniseerd werd, wat de elektronen voor eens en voor altijd bevrijdde van hun protonenboeien. Deze theoretische voorspelling, net als vele andere, dienen nog bewezen, weerlegd, of nog beter, haar parameters ingeperkt te worden door de aanstaande waarnemingen van het 21cm signaal.

Deze waarnemingen zijn echter sommige van de moeilijkste die ooit gepoogd zijn te
ondernemen. Net zoals mensen enkel kunnen denken over de vorm van een perfecte cirkel en het ze nooit zal lukken deze perfecte vorm te tekenen, zo zullen telescopen die door mensen gebouwd worden altijd een gebrekkige weergave zijn van het oog van een perfecte waarnemer. Het perfect begrijpen van deze imperfecties is nog nooit zo van belang geweest als bij het waarnemen van het 21cm signaal. Dit proefschrift heeft betrekking op het begrijpen van een paar van de gebreken van LOFAR bij het waarnemen van 21cm straling afkomstig van het tijdperk van reionisatie (in het Engels Epoch of Reionization, EoR). Hoe imperfect hun fysieke creaties ook zijn, mensen zijn heel goed in het inbeelden van het perfecte beeld achter deze creaties. In werkelijkheid zijn ze niet in staat om iets imperfects te modelleren, als ze geen perfect model hebben. Ook voor deze samenvatting is er zo’n perfect model dat een ideale waarnemer in beelden beschrijft beschikbaar. Voor de rest van deze samenvatting zal ik verwijzen naar deze afbeelding: 7.4.


Sterrenkunde houdt zich bezig met het waarnemen en analyseren van elektromagnetische straling afkomstig uit het heelal. Deze golven worden veroorzaakt door oscillaties in het elektromagnetische veld. Als we naar de oceaan kijken op een zonnige dag, kunnen we enkel één golf zien, de golf van de oceaan. Maar als we diep genoeg nadenken, realiseren we ons dat het licht waarin we baden en hetgeen het ons mogelijk maakt de watergolven te zien, zelf ook een soort golf is, maar dan van het elektromagnetische veld. Golven op de oceaan kunnen er zeer gestructureerd uitzien als ze op het punt staan op het strand te breken. Over het algemeen zijn ze echter willekeurig, ze klotsen eerst zo en dan weer zo. Op dezelfde wijze kan het elektrische (en/of magnetische) veld in een bepaalde richting oscilleren, maar ook willekeurig in alle richtingen. Het eerste heet gepolariseerd, het tweede ongepolariseerd. De polarisatie-eigenschappen van deze golven kunnen volledig beschreven worden door de vier Stokes parameters. Deze zijn in figuur 7.4 te zien als de Stokes vectoren. Stokes I staat voor de totale intensiteit, Q en U voor lineaire polarisatie en V voor circulaire polarisatie. Lage frequentie radiostraling van onze Melkweg is gepolariseerd, maar wordt grotendeels
gedepolariseerd op de weg naar ons toe. De hoeveelheid lineaire polarisatie, $Q$ en $U$, die nog over is zou voor ons echter wel eens te groot kunnen zijn om te negeren. Vooral omdat de gepolariseerde straling de spectrale structuur van het EoR signaal na zou kunnen bootsten.


In dit proefschrift hebben we de linear gepolariseerde straling in de 3C196 en NCP velden, twee van observatievelden van het LOFAR-EoR project, in kaart gebracht. Nadat we een model van deze gepolariseerde straling hebben gemaakt, hebben we voorspeld hoeveel lekkage naar Stokes $I$ er aanwezig is voor een normale observatie van één nacht, met de tijd, frequentie, antenne en richtingsgerelateerde $M$ matrices. Deze matrices hebben we afgeleid uit het algemene primaire bundel-model van LOFAR. We hebben laten zien dat de hoeveelheid lekkage lager is dan een van de gesimuleerde EoR-signalen op grote hoekafstanden, maar hoger op kleine hoekafstanden. De lekkage bleek het EoR signaal tot op zekere hoogte na te bootsten in het NCP veld, maar niet in het 3C196 veld. We hebben de nauwkeurigheid van het primaire bundel-model bepaald aan de hand van een observatie van één nacht met volledige polarisatie. De nauwkeurigheid van het model bleek binnen de tien procent te vallen. Deze observaties hebben ons geholpen om het mogelijke bereik voor de voorspelde hoeveelheid lekkage beter in te perken. De gezichtsvelden van de LOFAR-antennes variëren tussen vier en vijf graden bij hogere frequenties. Echter, dit binnenste gebied wordt beïnvloed door lekkage en bijbundels verder weg. We hebben laten zien dat het effect van lekkage buiten het gezichtsveld verwijderbaar is. Hoewel lekkage toeneemt als je verder bij het fasecentrum weggaat, neemt de intensiteit van de straling af. Deze twee effecten heffen elkaar tot op zekere hoogte op.

Een ander speerpunt van dit proefschrift is het testen van verschillende strategieën om polarisatielekkage te verwijderen. Bestaande algoritmes kunnen enkel lekkage van compacte bronnen van totale intensiteit naar polarisatie verwijderen. Wij zijn geïnteresseerd in het tegenovergestelde. We hebben deze strategieën, richtingsafhankelijke calibratie en AW-projectie, getest en zijn tot de conclusie gekomen dat ze ongeveer net zo capabel zijn. Beide verwijderen meer dan 80% tot 90% van de lekkage, als de signaal-ruisverhouding voldoende hoog is. Ons ultieme doel is om lekkage van diffuse gepolariseerde straling naar totale intensiteit te verwijderen. Het werk gepresenteerd in dit proefschrift zal hiervoor nuttig zijn. Elk wetenschappelijk werk heeft enkele toevalst treffers. In ons geval, heeft een exercitie om de hoeveelheid polarisatie lekkage te bepalen ons in staat gesteld om gepolariseerde, compacte bronnen waar we nemen die voorheen niet zichtbaar waren voor LOFAR en om de nauwkeurigheid van de primaire bundel van LOFAR te bepalen.

De hoeveelheid polarisatielekkage is vergelijkbaar met het EoR-signaal op sommige schalen en in sommige richtingen en is bovendien veel lager dan het ruinsniveau. Daarom kunnen we concluderen dat voor de huidige generatie radiotelescopen, zoals LOFAR, lekkage niet tot de belangrijkste redenen tot zorgen behoort. Polarisatielekkage zal veel relevanter en
interessanter zijn voor de gevoeligere, toekomstige generatie van radiotelescopen zoals SKA. We zijn nog maar net begonnen met het bepalen van de eigenschappen van gepolariseerde straling op zulke lage frequenties. Wat we nog kunnen ontdekken in de toekomst is een grote onbekende. Dit proefschrift is een van vele stapjes in de richting van lage frequentie astropolametrie.
নিজেদের কবি-প্রকৃতিবিদ মন্টা দিয়ে একবার ভাবা চেষ্টা করা যাক আমাদের মহাবিশ্ব একটি জীবন্ত সত্য যে জন্য থেকে মৃত্তিকায় একটি বিবর্তনীয় যাত্রা করে যাচ্ছে। মহাবিশ্বের একটি খুব অস্তিত্ব, বিপুল সংখ্যায় আমাদের (এমনকি সৃষ্টিতে) শৈলের ছিল। সে বখন শিখে ছিল তখন দেখাত না যে যে একটি শিখ। কিন্তু এখন ১৩৮৫ কোটি বছরের দীর্ঘ যাত্রা শেষে সে জানে একসময়ে যে শিখ ছিল, তার একটি আন্তর্জাতির ভৌগলিক ছিল, তার বিবাহধর্ম মনে প্রথম অপসরা হওয়ার কন্যা ও বুদ্ধিবোধের মৃত্তিকায় উত্থিত করা।

মহাবিশ্বের কিংবদন্তি বলে এখন এককাম তারা জানত? সে কি জানে যে একসময়ে যে জানতেন? জানত না; এমনকি তার জানার সৃষ্টি-ক্ষমতাও তার অপরাধি ছিল। কিন্তু নিজের বিবর্তনীয় পথের এক নগণ বিকে এরা যে স্থায়িত্ব শিখত তার জীবনের সাথে বেঁধে পড়েছে, তার প্রতি পুনরায় মুখ্যমুখি হয়েছে, কারণ ঘটুকি কন্যা অপরাধের মাধ্যমে এমন একটি সত্য তৈরি করেছে যে নিজেকে জনার সেবা করে, মহাবিশ্বের সর অজ্ঞপর্যন্ত সংগঠন করার সাহস করে। সেই যাদুজ্জিল সৃষ্টি, সেই প্রকৃতি সত্যনুসরণী হচ্ছে মানুষ নামক প্রাণী, যা একসময় অন্য সব প্রাণী, যদি তারা আসলে থেকে থাকে।

মহাবিশ্বের জীবনকাহিনীর চেয়ে অসুন্দ আর কিছু হতে পারে না। তার জন্য হয়েছিল যথেষ্ট অধিকের মতো একটা জিনিসের এক জাকালো বিজ্ঞানীদের (মহাবিশ্বের) মাধ্যমে যার পর থেকে যে তাপসিতিবিদ্যা ও সাধারণ আপেক্ষিকতার নিম্ন মনে সংরক্ষিত ও শীতল হতে থাকে। মহাবিশ্বের উৎপত্তি আজো রহস্যময়, কিন্তু তার পরিণতি মানুষ এত ভালবাসে যুদ্ধে যা তাদের বিশ্বাসকরা তাদের আধারণত অদৃশ্য কেন্দ্রদিকে যে অথবা উম্মাদিত সর্বমোচ তত্ত্বের একটি মোট করে। মানুষের আরেকটি অন্যতম সৃষ্টি তাত বলে যে পথ পথ Baltimore দিয়ে তৈরি, যারা আবার অতিরিক্তপরমাণুকে তৃতীয়-কণা দিয়ে গঠিত যাত্রা রংরঙে তথ্য কিন্তু বাদ্যরেখার কণ। প্রাচীন ভৌগর্ভ কবিতায় দৃব্ধিতের বিষয়ে বিভিন্ন করত, কিন্তু অধুনিক মানুষেরা বেশি জানে: পরমাণুর গাঠনিক উপাদানগুলো (প্রাচীন কোয়ান্টাম এবং ইলেক্ট্রন) সৃষ্টি হয়েছিল মহাবিশ্বের পর প্রথম সেকেন্দ্রে। স্পষ্ট ছিল সোনাট, যারা শিক্ষা ও আলোর বাহক। কোটন থেকে তৈরি পথের প্রতিযোগিতার কারণ একে অপরের সাথে সংঘর্ষ করে ধর্ষণ হয়ে যাচ্ছিল, যার ফলে পুনরুদ্ধার হচ্ছিল স্থান ফোটাও।
এরপর মহাবিশ্ব আরো প্রসারিত ও পীতল হওয়ার পর প্রাকৃত-পৃষ্ঠের চর্চাটি আর চলতে পারেনি, এবং এক মূলে প্রায় সব পদাথ্য ও প্রতিপদাথ্য পোষার মতো বিলুপ্ত হয়েছে, এবং তাদের পুনর্নির্দেশ আর সম্ভব হয়নি। প্রায় কলাম কারণ 'প্রতি একশ' কোটি প্রতিপদাথ্যের বিপরীতে একশ' কোটি একটি পদাথ্য ছিল। এবং এর ফলে চূড়ান্ত ধ্বংসযুক্তির পর মহাবিশ্বে অবশিষ্ট ছিল কোনো আরেকটি মেটন (মেটন, ইলেকট্রন) ও কেনার রূপান্তরের পদাথ্য ছিল। তারপর কেনার রূপান্তর তিন মিনিটে মিলিত হয়।

ছবি 4: মহাবিশ্বের ১৩৮০ কোটি বছরের ইতিহাস–মহাবিশ্বক্ষেত্র থেকে সময়ের অনুসরণ করে বর্তমান পর্যন্ত। বিভিন্ন দক্ষিণ বিভিন্ন রূপ পর্যবেক্ষণ করার জন্য উপযুক্ত–এক প্রকার বাইসেপ্পোল মহাবিশ্বক্ষেত্র-উপাদানকৃত স্তরিত মিশন হাউস চেকা করা যাচ্ছে, আর আনা প্রাপ্ত বুদ্ধিমান আলোক দৃষ্টিনিবদ্ধ কোনো পদাথ্যকে ইতিহাস দেখতে পারে। লোকার মতো ধর্ম কলামের নিবন্ধ হিসেবে দৃষ্টিকূট দর্শনগুলো পুনরায়নন্ত (reionization) রূপ পর্যবেক্ষণ করতে পারে।

মহাবিশ্বের বয়স ৩ লক্ষ ৭০ হাজার বছরের কম তখন তাতে ফোটনের দাপ্ত ছিল সবচেয়ে বেশি।

হাইড্রোজেনের পুনর্নির্দেশের পর মহাবিশ্ব তার কীর্তিতে প্রবেশ করেছে—অন্যকার হলে কীবোর, কারণ তখন তাতে কোনো আলোর উৎস ছিল না। প্রথম নক্ষত্র ও কৃষ্ণকালের চাইতেই ঘটে নেয়, বিভিন্ন তাতের আলো সে অন্যকার দমনের জন্য ব্যাপ্ত ছিল না। কারণ তাতে মূলত অর্থক্ষণ নিঃশব্দ নিঃশব্দ করত যা শোষণ করে নিয়ে হাইড্রোজেন-রক্ষা ইলেকট্রন পুনরুজ্জব হয়ে যায়।

মহাবিশ্বের পুনর্নির্দেশের পর মহাবিশ্ব তার কীর্তিতে প্রবেশ করেছে এবং নবাগত ধর্ম কলামের রূপ পর্যবেক্ষণ করে নিয়ে নিজস্ব হাইড্রোজেন-রক্ষা ইলেকট্রন পুনরুজ্জব হয়ে যায়। পুনর্নির্দেশের পর মহাবিশ্বের প্রাক্কালের উদ্ভাস, উদ্বোধ হয়।
নারকার, পীড়ণ বৌদ্ধিক বাস্তব হয়। পুনরায়ন রূপ সম্পর্কে মানবের কেবল তাত্ত্বিক ধারণা আছে, কিন্তু এখানে তারা এটি দেখে পারলেন, ঠিক যেমন অধঃপত্নী ও মহিলাদের উপায়ও তারা এখানে সাধারণ দেখতে পারেন। যদি একজন মানবের মাত্রায় পেতে যথার্থতা ছবি থাকে, এবং তারপর কেবল পূর্বাপেক্ষা অবস্থা ছবি থাকে তাহলে সে পৃষ্ঠপোষক দেখতে-নেত্র করেছিল তা ঠিকভাবে বলা সম্ভব নয়; মানবিক ও মানুষ ছাড়াই অনেক পাল্টায়।

মানবের দেখা মহিলাদের দাঁড়িও ঠিক এমন। মানুষের কাছে পাকিস্তান দিয়ে তোলা দুর্গ মহাবিশ্বের একটি স্বর্ণতাত্ত্বিক আছে, এবং তারপর প্রাক্তন মহিলাদের অনেক ছবি আছে, কিন্তু আঁধাররুপ, উত্তালন, এবং পুনরায়ন রূপ যে আঁধারে সে অঁধারেই রয়ে গেছে।

মানুষের কোন কোন বর্তমানে তারা স্বাভাবিক এবং কোন দিকে দিকেই মূলত মানবিক মানিতের ভিত্তি সাহং করতে পারে কোন পাপিনদের সময় সে পৃথিবী হাইড্রোজেন পরিবারগুলোর জন্য হয়েছিল তারা একটি সংখৈত দেখ, যা পুনরায়নের মাধ্যমে পাম যথার্থতায় লাভের সংক্রান্ত চর্চা যত কিছু থাকে না কেন। ইলেকটর ও প্রাচীন স্পিনি' নামে একটি বিশ্বের দিকপ্রতিশোধি আছে যার কারণ এই সংক্রান্ত তাত্ত্বিক হয়। হাইড্রোজেন পরিবারের সময় যখন একই দিকে ধুল তখন তাদের প্রতি বিচিত্রতা দিয়ে ধুল সময়ের চোখে বেশি থাকে। কেবল একটি কাছাকাছি জন্য যাদের স্পিন ‘একমুক্তী’ তাদেরকে ‘পুঞ্জী’ এবং যাদের স্পিন ‘বিপরীততমুক্তি’ তাদেরকে ‘হৃষ্টী’ বলা যায়। হৃষ্টী পরিবারগুলোর অন্য পাপিন বা নিশ্চিত কোন সাধারণ সাথে মূল্যায়ন করা একটি পরিবার-বিতর্ক শক্তি অবস্থায় মাধ্যমে তৃষিত হয়ে যায়, কিন্তু যখন সুপরীণ বেস্ট কে যায় এবং তাদের সূত্রে আর বস্ত নেমে আসে তখন তারা সেই অজ্ঞাত শিকারুক্তি ২১ সেকেন্টমিটার তমাদ্বারের নিহিত সিঙ্গ বিসন্ন করে।

মানবের আচরণ দেখা যে অনেকে এখানে পড়ে যায়। যাদের তারা জানাতেই গেছে তাদের পাপিন স্বপ্নের সাথে পাপিন একটি হবে মানুষ তাদের ‘পাপাননারা’ ২১ নেয়ায় যে একটি চর্চায়, যাতে যুদ্ধান্ত বৃত্তি প্রাপ্ত (যদি তারা আলেমে থাকে)। কেনেত পাগলান তারা কোন কাঠামো প্রতি পাকিস্তান করা থাকে।

মানুষের আচরণ বড় সৌভাগ্য যে আলেমে এখানে পড়ে যায়। যাদের তারা জানাতেই গেছে তাদের পাপিন স্বপ্নের সাথে পাপিন একটি হবে মানুষ তাদের ‘পাপাননারা’ ২১ নেয়ায় যে একটি চর্চায়, যাতে যুদ্ধান্ত বৃত্তি প্রাপ্ত (যদি তারা আলেমে থাকে)।
তাপমাতা অন্য হটর মাঝে আনাগনা করলেও সবসময় বিকিরণ তাপমাতার বেশি কাছাকাছি ছিল। পুরুষতে সে বিকিরণ তাপমাতার মাঝে ছোট ছিল যা-কারণ তখনকার এককে সংকেত একটি বিখ্যাত সংকেত। মাত্রার ক্ষেত্রের অনুসারী ১০ কোটি বছর পর সে বিকিরণ তাপমাতার আকারী যাওয়া একুশে সংকেতটি নিসরণশীল হয়ে যায়। অনেকে আকারশীলতার মাধ্যমে প্রাপ্ত সব হাইড্রোজেন পুনরায়নিত হয় যাওয়ার পর একুশে সংকেতটি মিলিয়া যায়। এগুলো এবং এরকম আরো অনেক তাপমাতা তত্ত্বাবধারণ প্রমাণ অপ্রমাণ, যা আরে সুপ্রতিষ্ঠিত করার জন্য একুশে বিকিরণ পর্যবেক্ষণ করার কোনো বিকল্প নেই। পর্যবেক্ষণগুলো ও তত্ত্বতাত্ত্বিকতার একুশে সংকেত দেখার আশায় তাই হিন পুণ্যে।

ছবি ২: কিতারা মন্ত্রণনিমিত গার্বনিয়ের অন্তরিত হটর কারণে পুনরায়ন বৃষ্টির একুশে সংকেত দেখায় বিজ্ঞ ঘটতে পারে।

কিন্তু একুশে সংকেত সন্দর্ক করার চেয়ে কর্তিন করা সম্ভব। মানুষ মেজা একটি বিষয়ক বৃষ্টির ধারণা তোলে পারে, কিন্তু বায়ুর বেল তার একটি কৃত্রিম মূর্তির তেরিন করতে পারে, থিক মেমনি তাদের তেরিন বুঝিকারছো একটি কৃত্রিম পর্যবেক্ষণ্যর চেয়ে স্বল্পরে একটি কৃত্রিম মূর্তির।

একুশে সংকেত সন্দর্ক করার হলে এই রেটিকুলো যথাযথত কৃত্রিমভাবে বোঝাটা পুরুষতাপ। এই সিস্টেম পুনরায়ন বৃষ্টি থেকে আসা কৃত্রিম বিকিরণ সন্দর্ক করার ক্ষেত্রে লোকার ক্ষেত্রির উপর এই বীমার নিয়ে আলোচনা করা হয়। মানুষের তৃতীয় যদিও কৃত্রিম দেখা না চান তারা সেলে সূক্ষ্ম পিণ্ডের কৃত্রিম রূপান্তর নকশা তেরিন করতে হবে পড়া। আলে কৃত্রিম একটি মূলে ছাড়া তারা কৃত্রিম কিছু বানানো পারে না। নিউক্লিয় পর্যবেক্ষণের সরকার একটি মূলে আছে, এবং তারা বলে মূলটা এই সার্বভৌম ব্যাপিত মধ্যে চরমে বর্ধিত করাটাও বেশ সৌজন্য। বা কি অংশ ২ নং হবিন মাধ্যমে থিক সেই কাজটাই করব।

ছবি ২-৩ তিনটি গার্বনিয়ের রূপ দেখালো হবে: বাম পাশে পর্যবেক্ষণের পর পাওয়া কৃত্রিম কৃত্রিম ডেস্ট্র (S), ডান পাশে প্রকৃত কৃত্রিম ডেস্ট্র (S), এবং মাঝারে কৃত্রিম পর্যবেক্ষনের প্রতিরোধিতিক রূপ ৪×৪ মিটির যাকে মূলার মেটাক্স (M) বলা হয়। সত্য ও বায়ুর মাধ্যম সরাসরি থেকে একটি পর্যবেক্ষণ ডেস্ট্র এবং মেটাক্স প্রতি উপাদান আকারের ২০২০ দিন এলাকা দেখাচ্ছে। প্রকৃত আকারের (S) সাথে পর্যবেক্ষনের কৃত্রিম মেটাক্স গুণের মাধ্যমে কৃত্রিম পর্যবেক্ষণ (S) পাওয়া যায়। কিন্তু তার উপরেও অনেক সমস্যা

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আচ্ছা। কারণ লোকার যে প্রকৃতি আকাশ দেখে তা পুঞ্জিপুরি কাজীকাত আকাশ নয়। আলে যমন বলা হচ্ছে, এখানে $S_1$ অনগ্রিজ আলেঞ্জের এক বিবলি ক্রান্তায় যা থেকে সব আকাশজ্ঞ অংশ বা দিতে হচ্ছে। এখানে, একুশে সংক্ষেপে পুঞ্জাদান যুগ এবং আমাদের মাঝারির সব গালালাক্ষ ও বিগ্যালালাক্ষ বিকিরণ (যারেখক 'পুঞ্জিপুরি'র বল) বাদ দিতে হচ্ছে। বিগ্যালালাক্ষ বিকিরণ প্রথমে বিদ্যুত চোট নিয়েছে যেখান যেখান বাদ দেয়া তুলনামূলক সহজ। কিছু গালালাক্ষ বিকিরণ আলে আমাদের গালালাক্ষ (আকাশগুলি) সত্যি বিকির্ত চোট ক্ষেত্রের চারিদিকে অপসরী গতিতে পুঞ্জামান ইলেকট্রন থেকে, যার নির্দেশনাটি বিকিরন। তথ্যের উদ্ভিদের তড়িত বিকিরণ এই বিকিরণ বাদ দেখায় একটি উপায় আছে: এরা ক্ষেত্রের সাধারণ মূল, অংশ এক কপালক থেকে আকাশ ত্রিকোণক এর উপস্থাপনা খুব ধীরের পাল্টায়। কিন্তু একুশে সংক্ষেপে এখন মূল নয়। সুতারে ক্ষেত্রের সাধারণ মূল অংশটি বাদ দেখার পর অনিস্বার্থে থাকানি সিকেটিক্রিটিক ও বৈবিত্ব একুশে সংকেট এবং অন্তরীক দৃষ্টি যিনি নাম 'সমবর্ত্তন লিকি'। এই লিকিনগত এই বিজ্ঞিৎসের মূল বিস্ময়টি। বায়োপার্ট্রা সংস্থা করার জন্য আমার ২ নং হ্রাসিত লিপিতে সমবর্তন সিকেটিক্রিটিক্রিটিউ ব্যাখ্যা করে।

ত্রিসিকেনের প্রথম কাজ মূলায় থেকে আস সব তন্ত্রীকৃতকাত তরল সন্দেহ এবং বিলোপকরণ। এই তরলগুলো ত্রিতিরি হল তন্ত্রীকৃতকাত্তিক্রিটিক্রিটিউ ক্ষেত্রের আদেশের মাধ্যমে। ত্রিত্রীকৃতক করা হল একটি সমান্তরাল মুখ ত্রিতিরির মুক্তির ক্ষেত্রের আদেশের মাধ্যমে। ত্রিত্রীকৃতক নির্দেশনাতে একটি সমান্তরাল হয়ে যায়, যে আলে আলে দিতে যেখান এবং আদেশের মুক্তির মুখ ত্রিতিরি তরল-তরলণ নয়, তন্ত্রীকৃতের তরল। ত্রিতিরির মুখ ত্রিতিরির আদেশের মাধ্যমে। ত্রিতিরির মুখ ত্রিতিরির আদেশের মাধ্যমে অতিপাল্টান হল একটি মুখ ত্রিতিরির আদেশের মাধ্যমে। ত্রিতিরির মুখ ত্রিতিরির আদেশের মাধ্যমে অতিপাল্টান হল একটি মুখ ত্রিতিরির আদেশের মাধ্যমে। ত্রিতিরির মুখ ত্রিতিরির আদেশের মাধ্যমে।
বিষয়বস্তু এবং ফলাফল

লিকের পরিমাণ আরো ঠিকভাবে নির্ধারণ করতে সাহায্য করেছে। তুলনামূলক উচ্চ কমিশন লোকারের 
এক্রনাশ্লের দৃষ্টিকোণে আনুমানিক ৪ থেকে ৫ ভিত্তি। কিন্তু দৃষ্টিকোনের বাইরের বিকিরণের সাইডলাইব এবং লিক 
দৃষ্টিকোনের ভিত্তির পর্যাবেক্ষণে কল্পিত করতে পারে। আমরা দেখিয়েছি, দৃষ্টিকোনের বাইরে থেকে আসা লিকের 
পরিমাণ নগণ, কারণ কোনো থেকে বাইরের দিকে লিক বাড়িও স্বাঙ্গ বিকিরণের তীর্থা কম যায়, এবং টো কিয়া 
একে অপরকে অনেকটা বাংলা করে দেয়।

এই সিসিসের আরেকটা বিষয়বস্তু ছিল সমবর্তন লিক বাদ দেয়া বিভিন কৌশল। বর্তমানে বাদ দেয়ার সেসব 
কৌশল আছে সেগুলো কেবল মোট তীর্থা থেকে সমবর্তনের দিকে লিক নিয়ে কাজ করতে পারে, যেখানে আমাদের 
চিক্টা বিষয় উপস্ত দিকের লিক। তারপরে আমরা তোমরা হাটা কৌশল পরস্পর দেখেছি: দিকনির্দেশ কালিকারণ 
এবং একটি-প্রজেকশন। হঠাৎ যাদের পরস্পর প্রায় একই রকম—নয়নের তুলনায় সংকলন যথাক্রমে বেশি হল হাটা 
কৌশলের প্রায় ৫০% লিক সম্মান করতে পারে। প্রথম যে যাওয়া তা হলো সুবিধাজনক সমবর্তন বিকিরণ থেকে 
মোট তীর্থার দিকে লিক বাদ দেয়া, এবং এই সিসিসের কাজ বস্তা উপকারে নামে। প্রতিটি কার্যক্রমেরই 
কিছু ঘটনাচক্রক উপরি পাওয়া থাকে। আমাদের ক্ষেত্রে এমন হাটা উপরি পাওয়া হল নির্দেশ কমিশন ওয়াং 
পরস্পরের করা যায়নি এমন কিছু সমবর্তন বিদ্য-উৎস সম্পত্তি করা এবং লোকারের মূখ্য বিন্যাসের মডেলে কমট নির্দেশ 
বিশার করা।

লিকের পরিমাণ কিছু কৌশল কৌশল এবং আকাশের কৌশল কৌশল অঞ্চল একুশ সঞ্চারের প্রায় সমান, 
কিন্তু নয়নের মধ্যে অনেক কম। সুতরাং লোকারের মতো সমাকালীন ছাড়ে কৌশলের প্রায় এবং মাথা বায়ের 
সংকলন ব্যাগ করণকালোর একটি হবে না। কিন্তু SKA-এর মতো আনন্দমূলক ছাড়ে কৌশলের সংকলন লিক 
আরো আকর্ষণীয় এবং প্রাপ্তি হয়ে উঠবে। এটি সিসিসকে সমবর্তনের চিত্র ধারণ করার কাজ আমরা সবে শুরু 
করেছি, এবং ভবিষ্যতে এ বিষয়ে কিছু সমান যায় তার আকাশের অনেকটা অনিয়ম। এই সিসিস সেই নির-কমিশনের 
জ্যোতিষভবন মিটির যুগের দিকে যাতার অনেকগুলো ধাপের একটি।
This PhD has changed me so much in so many different ways that I can hardly remember let alone recognize my four-years-ago self. I would like to thank all the people who supervised, helped, inspired and/or supported me during this period.

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