Condition-based maintenance for complex systems

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CBM for a redundant, parallel system with economic dependence and load sharing

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Abstract

In this chapter, we consider a multi-component system subject to structural dependence (through an active redundant, parallel setting), stochastic dependence (through load sharing), and (positive) economic dependence. Redundancy is often essential for achieving high system availability. An additional benefit of installing redundant components is that the total system load can be shared among components, thus preventing fast deterioration. On the one hand, this provides an incentive to replace failed components as soon as possible, as a component failure increases the load on the remaining components. On the other hand, however, redundancy gives rise to maintenance clustering and postponement opportunities, to reduce the maintenance frequency and thereby lower downtime and maintenance set-up costs. We are the first to investigate this trade-off under a CBM regime. We formulate our system as a Markov Decision Process, and obtain the optimal replacement decisions that minimize the long-run average cost per time unit. Through a numerical investigation and a sensitivity analysis, in which we vary the degree of load sharing and the maintenance set-up cost, we obtain key insights into the optimal policy structure. Standard threshold policies, that replace a component as soon as its deterioration exceeds a certain threshold, can be far from optimal, while ignoring or misinterpreting the load sharing effects between components can also lead to a significantly more expensive maintenance policy.

6.1. Introduction

This chapter investigates the joint effects of structural, stochastic, and economic dependence on the structure of the optimal CBM policy. This research is inspired by the following real-life example that we encountered at a gas company that pumps up and distributes gas. Storage options for gas are very limited, so the company must continuously produce gas to meet demand from both companies and households. To ensure a high availability of the pumps, redundant components are included for the most critical and failure-prone components, which are placed in a parallel setting. An additional benefit of installing redundant components is that the load (the amount of gas to be distributed) can be shared, thus reducing the load on each individual component, thereby implying a lower failure rate. On the one hand, this provides an incentive to replace failed components as soon as possible. On the other hand, high maintenance set-up costs are often involved when replacing a component. This economic dependence on a parallel system indicates that postponing and clustering corrective maintenance on a failed component can be profitable, rather than performing immediate corrective maintenance. To date, this trade-off has not been investigated when it is possible to monitor component conditions (i.e., CBM).

An additional trade-off concerns the decision to add an extra redundant component to the parallel setting. On the one hand, this extra component both lowers the probability of a system failure and contributes to the load sharing. On the other hand, the maintenance costs will be increased from maintaining this extra component. This trade-off has also not been researched yet in a CBM setting.

CBM has been considered by others for systems with either redundancy, load sharing, or economic dependence. For example, redundancy is studied in the form of a k-out-of-N system by [1–4] (see also Chapter 5). Furthermore, load sharing is considered for a parallel system without redundancy by [5–7], for a series system by [8], and for a series-parallel system by [9]. Economic dependence has for example been studied for a two-component series system by [10–12] (see also Chapter 3), for a two-component parallel system by [13, 14] (see also Chapter 4), and for a series-parallel system by [15]. Also [4, 5, 8] consider economic dependence. To the best of our knowledge, however, no research has yet been performed on the interface of redundancy (structural dependence), load sharing (stochastic dependence), and economic dependence.

In this chapter, we develop a CBM policy for a system with multiple components in an active redundant, parallel setting, which are subject to both economic dependence and stochastic dependence through load sharing. By formulating our system as a Markov Decision Process, we are able to obtain structural insights into the optimal maintenance policy and investigate the influence of the degree of economic dependence and load sharing on this structure. In addition, we compare our results to a simple threshold CBM policy (where maintenance is initiation upon reaching a certain deterioration threshold). Results indicate that both the degrees of load sharing and economic dependence significantly influence the optimal CBM policy. A threshold CBM policy cannot capture the optimal policy properties, and can thus result in a significantly higher cost rate.

The remainder of this chapter is organized as follows. The system is described in Section 6.2. Section 6.3 provides the Markov Decision Process formulation of our model, after which numerical experiments (including a sensitivity analysis on the degree of economic dependence) are performed in Section 6.4. Section 6.5 concludes the chapter.

6.2. System description

6.2.1. Deterioration model

We consider a discrete-time system consisting of N identical components, which are subject to economic dependence and stochastic dependence through load sharing. The components are placed in a parallel setting, which means that the system functions as long as at least one component functions (properly). We consider active redundancy, i.e., all non-failed components are fully operational and subject to deterioration. Let x_j denote the (discrete) state of component j, and let L denote the fixed failure level of a component. If component j is in state 0, i.e., $x_j = 0$, component j is as-good-as-new, while component j has failed if $x_j \geq L$. A replacement, which is assumed to be instantaneous, can be performed at the start of any time unit. We use the Poisson distribution to model the deterioration processes. This is also done in, e.g., [4, 10].

6.2.2. Maintenance actions and corresponding costs

The state of each component is known at the start of each time unit. In case all components have failed, the system is shut down and a penalty cost p is incurred. Next, replacements can be performed. A preventive replacement, on a component that still functions, costs c_p, while a corrective replacement, on a failed component, incurs a cost c_c. Generally, replacing a failed component is more expensive than performing a preventive replacement, i.e., $c_c \geq c_p$. The economic dependence is included in the form of a fixed set-up cost for maintenance c_s. This set-up cost needs to be paid once if at least one component is replaced (either preventively or correctively). This means that the higher this set-up cost for maintenance, the stronger the economic dependence.

Nomenclature

δ_j	Binary variable indicating whether or not component j is replaced
μ	Deterioration rate of a single, functioning component
c	Load sharing factor
c_c	Cost of a corrective replacement on a component
c_p	Cost of a preventive replacement on a component, $c_p \leq c_c$
c_s	Fixed set-up cost for maintenance
$g(k)$	Dependence function for k functioning components
L	Fixed failure level of a component
$\mathbf{L}$	Failure state of a component (equivalent to $x_j \geq L$)
N	Number of identical components in the system
p	Penalty for a system failure
$R(x_1, x_2, \ldots, x_N)$	System availability, with deterioration levels of $x_1, x_2, \ldots, x_N$ for components $1, 2, \ldots, N$, respectively
$R_j(x_j)$	Availability of component j, given a deterioration level of x_j
T_R	Preventive replacement threshold
x_j	State of component j

6.2.3. Load sharing

If the system consists of a single functioning component, then we assume that this component deteriorates with rate μ. However, we incorporate redundancy in our model by including extra components. Besides reducing the probability of a system failure, these components allow the total system load to be shared to a certain extent, thus leading to slower deterioration. To incorporate this, we apply the so-called redundant dependence as defined in [16], by setting the failure rate with k functioning components to $\mu \cdot g(k)$, where $g(k)$ is defined as follows:

$$g(k) = \left(\frac{1}{k}\right)^c, \quad \text{for } k \geq 1,$$

where c can be interpreted as the load sharing factor. In [16], the term failure dependence or redundant dependence is used rather than load sharing. Note that if $k = 1$, only one component is functioning, which is subject to its nominal failure rate μ as $g(1) = 1$. A higher value of c means that redundant components benefit more from sharing the load, and thus are subject to a lower deterioration rate (if $k \geq 2$). According to [16], some special cases exist:

$c = 0$ (no load sharing):
No load sharing exists between the components, so they are always subject to deterioration rate μ (i.e., $g(k) = 1$ for any k).

$0 < c < 1$ (weak load sharing):
The system load is shared, but less than proportional to the number of components (i.e., $1/k < g(k) < 1$).

$c = 1$ (proportional load sharing):
The load is proportional to the number of components (i.e., $g(k) = 1/k$).

$c > 1$ (strong load sharing):
Including an extra component has a strong influence on the failure rates (i.e., $g(k) < 1/k$).

In Figure 6.1, we show $g(k)$ for different realizations of the number of functioning components k and degrees of load sharing c. Of course, when only one component is functioning, it is subject to a deterioration rate of μ, independent of the degree of load sharing. When at least two components are functioning, however, we observe that a positive degree of load sharing c significantly influences the failure rates (given by $\mu \cdot g(k)$). Although the case of strong load sharing ($c > 1$) represents an extreme case, it can apply to single-component, overloaded systems with high individual deterioration rates μ. Adding a second component will relieve the first component and significantly reduce the deterioration rates, as can be seen in Figure 6.1. In line with this, the added benefit of installing an extra component decreases as the system size increases.

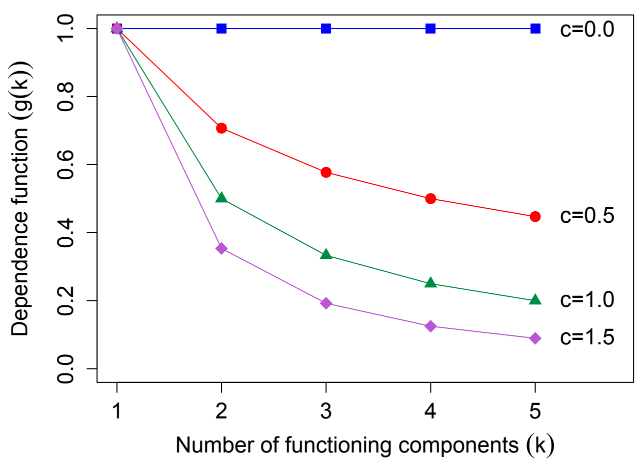

Figure 6.1. The dependence function $g(k)$ for different values of k and c.

Deterioration process
Let X_k be distributed according to a Poisson process with parameter $\mu \cdot g(k)$, i.e., $X_k \sim \text{Poisson}(\mu \cdot g(k))$, for $k = 1, 2, \ldots, N$. Then, provided that k components are functioning, component j will move from state x to state y with probability $\text{P}(X_k = y - x)$.

6.3. Markov Decision Process formulation

In this section, we provide the Markov Decision Process (MDP) formulation of our model. At the start of each time unit, the system can be in a set of states $\mathcal{I}$. Depending on the current state $i \in \mathcal{I}$, a set of actions $\mathcal{A}_{\{i\}}$ can be performed, including the option to not perform any maintenance. The system then moves from state $i \in \mathcal{I}$ to some state $\bar{i} \in \mathcal{I}$ under action $a \in \mathcal{A}_{\{i\}}$ with a certain transition probability $p^a(i, \bar{i})$, while a cost of $c^a(i)$ is incurred. Below, we define this state space and action space, and provide an expression for the transition probabilities and the expected cost function.

State space The CBM replacement decisions are based on the complete system state. For that reason, we keep track of the states of all N components in our state space, i.e.,

$$\mathcal{I} = \{(x_1, x_2, \ldots, x_N)\},$$

where $x_j \in \{0, 1, \ldots, L-1, \mathbf{L}\}$ denotes the state of component j, for $j = 1, 2, \ldots, N$. Since L denotes the failure level of each component, we assume that state $\mathbf{L}$ denotes the failed state of a component, i.e., where $x_j \geq L$. In this way, we truncate the state space.

Action space At the start of each time unit, we need to decide which components will be replaced. Thus, the action space is defined as

$$\mathcal{A} = \{(\delta_1, \delta_2, \ldots, \delta_N)\},$$

where, for $j = 1, 2, \ldots, N$,

$$\delta_j = \begin{cases} 1, & \text{if component } j \text{ is replaced,} \\ 0, & \text{otherwise.} \end{cases}$$

At each state, any component can be replaced. However, as we are dealing with identical components, we can limit the set of possible actions without affecting the results. If two (or more) components are in the same state, replacing any one of these components will have the same effect on the system state. We do not need to decide which of the components will be replaced, but can instead assume that, if component i and j are in the same state, and only one of them is replaced, then that will be component i if $i < j$, i.e.,

$$\mathcal{A}_{\{(x_1,x_2,\ldots,x_N)\}} = \{(\delta_1,\delta_2,\ldots,\delta_N) : \delta_i \geq \delta_j \text{ if } x_i = x_j,\ \forall i,j \in \{1,2,\ldots,N\} \text{ s.t. } i < j\}.$$

This assumption will also prove useful for deciding whether it is beneficial to include an additional redundant component to the system, as this component may or may not be kept in the failed state. Furthermore, this assumption is of use for solving the MDP, as explained later in this section. Note that, immediately following possible replacements, component j will be in state $(1-\delta_j)\cdot x_j$, provided that the component was in state x_j at the start of the time unit.

Before we define the transition probabilities and the expected costs, we first introduce the function R_j, which equals one if component j is functioning, and zero otherwise, i.e.,

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$$R_j(x_j) = \begin{cases} 1, & \text{if } x_j < L, \\ 0, & \text{if } x_j = \mathbf{L}, \end{cases}$$

for $j = 1,2,\ldots,N$. In addition, we define R as the system availability, which indicates whether the complete system is functioning or not, i.e.,

$$R(x_1,x_2,\ldots,x_N) = \begin{cases} 1, & \text{if } \sum_{j=1}^{N} R_j(x_j) \geq 1, \\ 0, & \text{if } \sum_{j=1}^{N} R_j(x_j) = 0. \end{cases}$$

Note that if the system is in state $(x_1,x_2,\ldots,x_N)$, then the number of functioning components is equal to $\sum_{j=1}^{N} R_j(x_j)$.

Transition probabilities Let $p_j(x;y|k)$ denote the probability that component j moves from state x to state y, provided that k components are functioning. Because we truncated the state space (by assuming that $x_j = \mathbf{L}$ is equivalent to $x_j \geq L$), it follows that $p_j(x;y|k)$ is defined as

$$p_j(x;y|k) = \begin{cases} \mathrm{P}(X_k = y-x), & \text{if } x \leq y < L, \\ \mathrm{P}(X_k \geq y-x), & \text{if } x < y = \mathbf{L}, \\ 1, & \text{if } x = y = \mathbf{L}, \\ 0, & \text{if } x > y, \end{cases}$$

for $j = 1, 2, \ldots, N$. Observe that, as we consider identical components, this expression is independent of j. The transition probabilities (of moving from state $(x_1, x_2, \ldots, x_N)$ to state $(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_N)$ under action $(\delta_1, \delta_2, \ldots, \delta_N)$) are then given by

$$p^{(\delta_1,\delta_2,\ldots,\delta_N)}((x_1, x_2, \ldots, x_N); (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_N)) = \\ \prod_{j=1}^{N} p_j \left((1-\delta_j)\cdot x_j; \bar{x}_j \middle| \sum_{m=1}^{N} R_m((1-\delta_m)\cdot x_m) \right).$$

Expected costs The expected cost per time unit of performing a certain action $(\delta_1, \delta_2, \ldots, \delta_N)$ in state $(x_1, x_2, \ldots, x_N)$ consists of the penalty cost for a system failure, the set-up cost for maintenance, and the preventive and corrective replacement costs as follows.

$$c^{(\delta_1,\delta_2,\ldots,\delta_N)}(x_1, x_2, \ldots, x_N) = \\ p\cdot(1-R(x_1, x_2, \ldots, x_N)) + c_s \cdot \left(1 - \prod_{j=1}^{N}(1-\delta_j)\right) + c_p \cdot \sum_{j=1}^{N} \delta_j \cdot R_j(x_j) \\ + c_c \cdot \sum_{j=1}^{N} \delta_j \cdot (1-R_j(x_j))$$

Performance criterion As a performance criterion, we are interested in minimizing the long-run average cost per time unit. In this way, we can quantify the impact of adding an additional component, and find the most cost-efficient maintenance policy. We are dealing with a finite-sized state space and action space. Moreover, our cost function is bounded by definition. Due to our restriction of the set of possible actions, our MDP model does satisfy the Weak Unichain Assumption[1] as defined in [17]. In [18], this assumption is also applied to a CBM setting, where both the spare parts and maintenance decisions are condition-based and optimized simultaneously (see also Chapter 7). We therefore choose to solve our MDP by applying the Value Iteration algorithm (see [18, 19]).

6.4. Numerical investigation

From Figure 6.1, observe that adding one component to a single-component system has a relatively large impact on the effect of load sharing, while the effect

[1] For each average cost optimal stationary policy, the associated Markov chain has no two disjoint closed sets [17].

Table 6.1. Minimal long-run average cost per time unit for a system with $N = 2$, $p = 300$, $c_s = 4$, $c_p = 5$, $c_c = 11$, $\mu = 0.7$, and $L = 5$, for different values of c.

Degree of load sharing	Minimal average costs
$c = 0.0$	3.42
$c = 0.5$	2.33
$c = 1.0$	1.60
$c = 1.5$	1.10

of adding a component decreases as the number of functioning components increases. For that reason, we start our analysis with a system consisting of two components, i.e., we set $N = 2$. Similar to [4], we select a penalty cost of $p = 300$, a set-up cost of $c_s = 4$, a preventive replacement cost of $c_p = 5$, and a corrective replacement cost of $c_c = 11$, while we select a nominal failure rate of $\mu = 0.7$ and a failure level of $L = 5$ (see also Chapter 5). We consider four different values for the degree of load sharing c: 0.0 (no load sharing), 0.5 (weak load sharing), 1.0 (proportional load sharing), and 1.5 (strong load sharing). The resulting minimal costs are shown in Table 6.1. Recall that a high degree of load sharing c corresponds to a lower deterioration rate in case both components are functioning. Figures 6.2a-6.2d show the optimal replacement policies for the different degrees of load sharing. In these figures, we represent the optimal replacement decisions as $\delta_1\delta_2$ for every possible system state (x_1, x_2), so 01, for example, means that only component 2 is replaced. First, consider the case with no load sharing ($c = 0.0$). As in Chapter 5, we observe that corrective maintenance on a failed component is postponed in some cases to allow for clustering (consider e.g. the case where $x_1 = 0$ or 1 and $x_2 = 5$). As we introduce load sharing, we introduce an incentive to perform corrective replacements immediately. Indeed, in case $c = 0.5$, we observe that corrective maintenance is only postponed if the other component is as-good-as-new (consider e.g. $x_1 = 0$ and $x_2 = 5$), while corrective replacements are no longer postponed for higher degrees of load sharing c.

6.4.1. Adding additional components

If we include an additional component in our two-component system, we benefit on the one hand from the decreased probability of a system failure and the increased load sharing between components, but on the other hand incur an additional cost from maintaining this extra component. To investigate this trade-off, we now consider a similar system as before, but with $N = 3$ rather than $N = 2$. The results are shown in Table 6.2. From this table, we observe that including a third component is not beneficial for low degrees of load sharing ($c = 0.0$ and $c = 0.5$),

x_1 \ x_2	0	1	2	3	4	5
0	00	00	00	00	01	00
1	00	00	00	00	01	00
2	00	00	00	00	01	01
3	00	00	00	11	11	11
4	10	10	10	11	11	11
5	00	00	10	11	11	11

(a) $c = 0.0$

x_1 \ x_2	0	1	2	3	4	5
0	00	00	00	00	01	00
1	00	00	00	00	01	01
2	00	00	00	00	01	01
3	00	00	00	11	11	11
4	10	10	10	11	11	11
5	00	10	10	11	11	11

(b) $c = 0.5$

x_1 \ x_2	0	1	2	3	4	5
0	00	00	00	00	01	01
1	00	00	00	00	01	01
2	00	00	00	00	01	01
3	00	00	00	00	11	11
4	10	10	10	11	11	11
5	10	10	10	11	11	11

(c) $c = 1.0$

x_1 \ x_2	0	1	2	3	4	5
0	00	00	00	00	01	01
1	00	00	00	00	01	01
2	00	00	00	00	01	01
3	00	00	00	00	11	11
4	10	10	10	11	11	11
5	10	10	10	11	11	11

(d) $c = 1.5$

Figure 6.2. Optimal replacement policy for a system with $N = 2$, $p = 300$, $c_s = 4$, $c_p = 5$, $c_c = 11$, $\mu = 0.7$, and $L = 5$, for different values of c.

as the optimal policy is to keep the third component in the failed state, which reduces the system to a two-component system. The benefit from load sharing does outweigh the increase in maintenance costs for $c = 1.0$ and $c = 1.5$, for which the average cost decreases with 9 and 24 percent, respectively, to 1.45 and 0.84 per time unit. The optimal replacement policies for a three-component system with $c = 1.0$ and $c = 1.5$ are shown in Figure 6.3. The optimal replacement decisions

Table 6.2. Minimal long-run average cost per time unit (decrease compared to $N = 2$) for a system with $N = 3$, $p = 300$, $c_s = 4$, $c_p = 5$, $c_c = 11$, $\mu = 0.7$, and $L = 5$, for different values of c.

Degree of load sharing	Minimal average costs
$c = 0.0$	3.42 (- 0%)
$c = 0.5$	2.33 (- 0%)
$c = 1.0$	1.45 (- 9%)
$c = 1.5$	0.84 (-24%)

$x_1 = 0$

x_2 \ x_3	0	1	2	3	4	5
0	000	000	000	000	001	000
1	000	000	000	000	001	000
2	000	000	000	000	000	000
3	000	000	000	000	011	000
4	010	010	000	011	011	011
5	000	000	000	000	011	011

$x_1 = 1$

x_2 \ x_3	0	1	2	3	4	5
0	000	000	000	000	001	000
1	000	000	000	000	001	000
2	000	000	000	000	000	000
3	000	000	000	000	011	000
4	010	010	000	011	011	011
5	000	000	000	000	011	011

$x_1 = 2$

x_2 \ x_3	0	1	2	3	4	5
0	000	000	000	000	000	000
1	000	000	000	000	000	000
2	000	000	000	000	000	000
3	000	000	000	000	000	000
4	000	000	000	000	011	011
5	000	000	000	000	011	011

$x_1 = 3$

x_2 \ x_3	0	1	2	3	4	5
0	000	000	000	000	101	000
1	000	000	000	000	101	000
2	000	000	000	000	000	000
3	000	000	000	000	111	000
4	110	110	000	111	111	111
5	000	000	000	000	111	111

$x_1 = 4$

x_2 \ x_3	0	1	2	3	4	5
0	100	100	000	101	101	101
1	100	100	000	101	101	101
2	000	000	000	000	101	101
3	110	110	000	111	111	111
4	110	110	110	111	111	111
5	110	110	110	111	111	111

$x_1 = 5$

x_2 \ x_3	0	1	2	3	4	5
0	000	000	000	000	101	101
1	000	000	000	000	101	101
2	000	000	000	000	101	101
3	000	000	000	000	111	111
4	110	110	110	111	111	111
5	110	110	110	111	111	111

(a) $c = 1.0$

$x_1 = 0$

x_2 \ x_3	0	1	2	3	4	5
0	000	000	000	000	001	000
1	000	000	000	000	001	000
2	000	000	000	000	001	000
3	000	000	000	000	011	000
4	010	010	010	011	011	011
5	000	000	000	000	011	011

$x_1 = 1$

x_2 \ x_3	0	1	2	3	4	5
0	000	000	000	000	001	000
1	000	000	000	000	001	000
2	000	000	000	000	001	000
3	000	000	000	000	011	000
4	010	010	010	011	011	011
5	000	000	000	000	011	011

$x_1 = 2$

x_2 \ x_3	0	1	2	3	4	5
0	000	000	000	000	001	000
1	000	000	000	000	001	000
2	000	000	000	000	000	000
3	000	000	000	000	000	000
4	010	010	000	000	011	011
5	000	000	000	000	011	011

$x_1 = 3$

x_2 \ x_3	0	1	2	3	4	5
0	000	000	000	000	101	000
1	000	000	000	000	101	000
2	000	000	000	000	000	000
3	000	000	000	000	000	000
4	110	110	000	000	111	111
5	000	000	000	000	111	111

$x_1 = 4$

x_2 \ x_3	0	1	2	3	4	5
0	100	100	100	101	101	101
1	100	100	100	101	101	101
2	100	100	000	000	101	101
3	110	110	000	000	111	111
4	110	110	110	111	111	111
5	110	110	110	111	111	111

$x_1 = 5$

x_2 \ x_3	0	1	2	3	4	5
0	000	000	000	000	101	101
1	000	000	000	000	101	101
2	000	000	000	000	101	101
3	000	000	000	000	111	111
4	110	110	110	111	111	111
5	110	110	110	111	111	111

(b) $c = 1.5$

Figure 6.3. Optimal replacement policy for a system with $N = 3$, $p = 300$, $c_s = 4$, $c_p = 5$, $c_c = 11$, $\mu = 0.7$, and $L = 5$, for different values of c.

are presented as $\delta_1\delta_2\delta_3$ for each system state (x_1, x_2, x_3). Both for $c = 1.0$ and $c = 1.5$, we observed in Figure 6.2 that corrective maintenance is never postponed when $N = 2$. In Figure 6.3, however, we observe that corrective maintenance is postponed regularly for a system with three components. Consider for example Figure 6.3a, with proportional load sharing. If component 1 is failed, i.e., $x_1 = 5$, it will not be replaced until at least one other component is in state 4 or 5. The same holds for Figure 6.3b, with strong load sharing. Nevertheless, we do observe some differences between the optimal policies for the two degrees of load sharing. Consider for example $(x_1, x_2, x_3) = (2,0,4)$ or $(2,1,4)$. For proportional load sharing ($c = 1.0$), no maintenance is performed, while component 3 is preventively replaced for $c = 1.5$. In the latter case, the failure of component 3 should be avoided due to the strong load sharing benefits. On the other hand, when $(x_1, x_2, x_3) = (3,3,4)$, $(3,4,3)$, or $(4,3,3)$, all components are preventively replaced for $c = 1.0$, while no maintenance is performed for $c = 1.5$. Due to the stronger degree of load sharing, the components deteriorate at a lower rate for $c = 1.5$ than for $c = 1.0$. In these cases, maintenance can thus be postponed at a lower risk of failure. We can conclude that the optimal policy depends heavily on the degree of load sharing.

It is important to keep in mind that we do not consider the purchase or investment cost of including additional components, as our focus is on the structure of the optimal maintenance policy rather than system design. In cases where including an additional component reduces operational costs, that reduction should be traded off against the increased investment (in a net present value analysis), which is beyond the scope of this research.

6.4.2. Comparison to a threshold policy

In the literature, CBM is often implemented in the form of a deterioration threshold that is used to schedule maintenance (see e.g. [9, 20–22]). Such a threshold policy is, however, not necessarily optimal for a system with economic dependence and redundancy. To investigate this, we compare our performances to those of a threshold policy. We define T_R as the threshold for the state of a component at or above which a maintenance action is triggered. As we consider identical components, it is common to use a single threshold that applies to all components, rather than one per component. Table 6.3 shows the costs corresponding to the threshold policy, obtained through simulation, along with the optimal threshold and the percentage increase in cost compared with our optimal CBM policy, for $N = 2$ and $N = 3$. Results indicate that, similar to the optimal replacement policy, two components are sufficient for the cases where $c = 0.0$ and $c = 0.5$. The

Table 6.3. Minimal long-run average cost per time unit of the threshold policy (increase compared to the optimal policy) for different values of c.

(a) $N = 2$.

Degree of load sharing	Average cost	Optimal threshold
$c = 0.0$	3.77 (+10%)	$T_R^* = 3$
$c = 0.5$	2.52 (+ 8%)	$T_R^* = 4$
$c = 1.0$	1.69 (+ 6%)	$T_R^* = 4$
$c = 1.5$	1.16 (+ 5%)	$T_R^* = 4$

(b) $N = 3$.

Degree of load sharing	Average cost	Optimal threshold
$c = 0.0$	4.89 (+43%)	$T_R^* = 4$
$c = 0.5$	2.80 (+20%)	$T_R^* = 4$
$c = 1.0$	1.60 (+10%)	$T_R^* = 4$
$c = 1.5$	0.92 (+10%)	$T_R^* = 4$

minimal average costs corresponding to the threshold policy increase as a third component is added to the system, because the maintenance costs of this extra component do not outweigh the gain from load sharing and redundancy. For $c = 1.0$ and $c = 1.5$, however, the average cost per time unit is decreased when a third component is added. Nevertheless, we observe that significant cost savings can be obtained by applying our optimal CBM strategy; the costs corresponding to the threshold policy are about 10 and 8 percent higher for $c = 0.0$ and $c = 0.5$, respectively, for $N = 2$, while for $N = 3$, the threshold policy is about 10 percent more expensive than our optimal policy for both $c = 1.0$ and $c = 1.5$.

6.4.3. Sensitivity analysis with respect to maintenance set-up cost

So far, we considered a maintenance set-up cost of $c_s = 4$. For a larger set-up cost, we expect replacements to be clustered more often. To investigate this, we now consider the same case as before, but we increase the maintenance set-up cost to $c_s = 8$. In Table 6.4, we show the minimal long-run average cost corresponding to the optimal policy for $N = 2$ and $N = 3$, for different degrees of load sharing c. We observe that adding a third component to the two-component system results in a cost decrease of 0, 3, 18, and 31 percent for $c = 0.0, 0.5, 1.0$, and 1.5, respectively. These cost decreases are larger than those we observed in Section 6.4.1 for a set-up cost of $c_s = 4$. We thus observe that adding a third component is more rewarding for a higher maintenance set-up cost. Indeed, maintenance actions are clustered more often for larger systems, and the high set-up cost needs to be paid less often.

Table 6.4. Minimal long-run average cost per time unit (decrease compared to $N = 2$) for different values of c and N, for $c_s = 8$.

Degree of load sharing	$N = 2$	$N = 3$
$c = 0.0$	4.29	4.29 (- 0%)
$c = 0.5$	3.02	2.94 (- 3%)
$c = 1.0$	2.09	1.72 (-18%)
$c = 1.5$	1.46	1.01 (-31%)

In addition, Figure 6.4 shows the optimal policy for $N = 2$ for different values of c. When increasing the set-up cost for maintenance, the economic dependence between the components becomes stronger. Clustering maintenance thus becomes much more rewarding. In Figure 6.4, we observe that all maintenance actions are indeed clustered for $c = 0.0$, i.e., without load sharing. Preventive replacements are no longer performed solely to prevent corrective replacements,

x_1 \ x_2	0	1	2	3	4	5
0	00	00	00	00	00	00
1	00	00	00	00	00	00
2	00	00	00	00	11	11
3	00	00	00	11	11	11
4	00	00	11	11	11	11
5	00	00	11	11	11	11

(a) $c = 0.0$

x_1 \ x_2	0	1	2	3	4	5
0	00	00	00	00	01	00
1	00	00	00	00	01	00
2	00	00	00	00	11	11
3	00	00	00	11	11	11
4	10	10	11	11	11	11
5	00	00	11	11	11	11

(b) $c = 0.5$

x_1 \ x_2	0	1	2	3	4	5
0	00	00	00	00	01	00
1	00	00	00	00	01	01
2	00	00	00	00	11	11
3	00	00	00	00	11	11
4	10	10	11	11	11	11
5	00	10	11	11	11	11

(c) $c = 1.0$

x_1 \ x_2	0	1	2	3	4	5
0	00	00	00	00	01	01
1	00	00	00	00	01	01
2	00	00	00	00	11	11
3	00	00	00	00	11	11
4	10	10	11	11	11	11
5	10	10	11	11	11	11

(d) $c = 1.5$

Figure 6.4. Optimal replacement policy for a system with $N = 2$, $p = 300$, $c_s = 8$, $c_p = 5$, $c_c = 11$, $\mu = 0.7$, and $L = 5$, for different values of c.

Table 6.5. Minimal long-run average cost of the threshold policy (increase compared to the optimal policy) for different values of c, for $c_s = 8$.

(a) $N = 2$.

Degree of load sharing	Average cost	Optimal threshold
$c = 0.0$	5.15 (+20%)	$T_R^* = 4$
$c = 0.5$	3.40 (+13%)	$T_R^* = 4$
$c = 1.0$	2.34 (+12%)	$T_R^* = 4$
$c = 1.5$	1.62 (+11%)	$T_R^* = 4$

(b) $N = 3$.

Degree of load sharing	Average cost	Optimal threshold
$c = 0.0$	6.52 (+52%)	$T_R^* = 4$
$c = 0.5$	3.84 (+31%)	$T_R^* = 4$
$c = 1.0$	2.24 (+30%)	$T_R^* = 4$
$c = 1.5$	1.31 (+30%)	$T_R^* = 4$

as clustering maintenance has become more rewarding ($c_s + c_p > c_c$). When we do include load sharing (i.e., when we select $c > 0$), a preventive replacement becomes more rewarding, as deterioration is slowed down when both components are functioning. We thus observe that not all replacement actions are clustered for $c > 0$, though it does happen more often than in Figure 6.2, for a set-up cost of $c_s = 4$.

Table 6.5 shows the long-run average cost per time unit obtained with the threshold maintenance policy for $N = 2$ and $N = 3$, along with the optimal threshold values, and the percentage increase in costs compared with our optimal policy. We observe that the threshold policy results in significantly higher costs than our optimal CBM policy for all considered values of c. Whereas two components are sufficient for $c = 0.0$ and $c = 0.5$, a cost decrease is observed for $c = 1.0$ and $c = 1.5$ when adding a third component. For these cases, the threshold policy is 30 percent more expensive than the optimal policy. Compared with Section 6.4.2, where we considered a maintenance set-up cost of $c_s = 4$, we observe that the threshold policy performs worse for a higher set-up cost. Indeed, in the case of strong economic dependence, clustering maintenance actions becomes more rewarding. The threshold policy is not able to capture this behavior.

6.4.4. Ignoring the stochastic dependence through load sharing

In practice, observing load sharing effects between multiple components can be challenging. Even if such stochastic dependence within a system is recognized, further difficulty arises in estimating the actual degree of load sharing. Failure data are often lacking or incomplete, preventing the maintenance managers from justifying their assumptions. To investigate the consequences of ignoring or misinterpreting the load sharing effects between components, we apply the optimal policy for independent components ($c = 0.0$) to the cases where load sharing exists ($c > 0$). We consider the same cases as before, and use simulation to find the corresponding costs. The results are summarized in Table 6.6. For a weak degree of load sharing ($c = 0.5$) and a low degree of economic dependence ($c_s = 4$), we observe that ignoring the load sharing will result in a 1 percent more expensive policy, while for $c_s = 8$ this difference increases to 3 percent for $N = 3$. For stronger degrees of load sharing, however, we find that ignoring the stochastic dependence will increase costs by 3 to 8 percent for $N = 2$. For $c = 1.0$ and $c = 1.5$, we found in Section 6.4.1 that adding a third redundant component can reduce costs significantly. When ignoring the load sharing effects in these cases, this extra component will be kept in the failed state, which increases the costs by up to 54 percent. We can thus conclude that weak load sharing can be ignored, but a stronger degree of load sharing has to be taken into account. This holds in particular for systems with a strong degree of economic dependence.

When using thresholds to describe the maintenance policy, ignoring the load sharing can lead to a sub-optimal threshold. From Tables 6.3 and 6.5, we observe

Table 6.6. Costs corresponding to the optimal policy for $c = 0.0$ applied to different degrees of load sharing c, for $p = 300$, $c_p = 5$, $c_c = 11$, $\mu = 0.7$, and $L = 5$.

(a) $c_s = 4$.

Degree of load sharing	Optimal policy		Ignoring load sharing	
	$N = 2$	$N = 3$	$N = 2$	$N = 3$
$c = 0.0$	3.42	3.42	3.42 (+ 0%)	3.42 (+ 0%)
$c = 0.5$	2.33	2.33	2.35 (+ 1%)	2.35 (+ 1%)
$c = 1.0$	1.60	1.45	1.64 (+ 3%)	1.64 (+13%)
$c = 1.5$	1.10	0.84	1.16 (+ 5%)	1.15 (+37%)

(b) $c_s = 8$.

Degree of load sharing	Optimal policy		Ignoring load sharing	
	$N = 2$	$N = 3$	$N = 2$	$N = 3$
$c = 0.0$	4.29	4.29	4.29 (+ 0%)	4.29 (+ 0%)
$c = 0.5$	3.02	2.94	3.03 (+ 0%)	3.03 (+ 3%)
$c = 1.0$	2.09	1.72	2.17 (+ 4%)	2.18 (+27%)
$c = 1.5$	1.46	1.01	1.58 (+ 8%)	1.56 (+54%)

that a threshold of $T_R = 4$ is optimal for all cases except when $c_s = 4$, $N = 2$, and $c = 0.0$, for which $T_R^* = 3$. In this case, applying this threshold will increase costs by 8, 16, and 22 percent to 2.72, 1.96, and 1.41 for $c = 0.5$, $c = 1.0$, and $c = 1.5$, respectively. Compared with the optimal policy, this is even more expensive.

6.5. Conclusion

In this chapter, we consider a multi-component system which is subject to structural dependence (through an active redundant, parallel setting), stochastic dependence (through load sharing), and economic dependence (through a fixed maintenance set-up cost). On the one hand, the redundancy resulting from the parallel setting allows corrective replacements to be postponed without affecting the system performance. In this way, the corrective replacement can be combined with a preventive replacement of other components to reduce the maintenance frequency and save on the maintenance set-up cost. On the other hand, a failure of a component implies an increased load on the remaining components, thus leading to faster deterioration. This provides an incentive to perform corrective replacements as soon as possible. We are the first to explore this trade-off under a CBM strategy.

We formulated our system as a Markov Decision Process and determined cost-minimizing CBM strategies. Through a numerical study, we discovered interesting properties of the optimal policy structure, for different degrees of load sharing and economic dependence. We observed that maintenance clustering is especially beneficial for systems with a strong economic dependence and a relatively low degree of load sharing. In line with this, corrective replacements can be postponed to allow for maintenance clustering and to save on set-up costs. Furthermore, we observed that preventive replacements can best be performed at a relatively early stage for a high degree of load sharing and weak economic dependence, as more cost savings can be obtained from the load sharing than through maintenance clustering. Adding an extra, redundant component is most beneficial for systems with both a strong degree of load sharing and a strong degree of economic dependence.

By comparing the performances of our optimal policy with those of a ‘standard’ threshold CBM policy, popular in both theory and practice, we observed that significant cost savings can be obtained by basing the replacement decisions on the complete system state. The inability of the threshold policy to cluster maintenance actions by postponing corrective replacements of redundant components results in up to 30 per cent more expensive maintenance strategies. Especially for systems with a strong economic dependence and a low degree of load sharing, the threshold policy can be far from optimal.

In practice, the presence of load sharing interactions can be difficult to recognize and quantify. We therefore investigated the effects of ignoring the stochastic dependence through load sharing (even though load sharing is actually present), and found that this can lead to sub-optimal policies that are significantly more expensive. This holds in particular for systems with a strong degree of load sharing and a strong degree of economic dependence, thus stressing the importance of applying a suitable policy. Weak degrees of load sharing, on the other hand, can be ignored at a relatively small cost.

For systems consisting of a large number of components, the exact optimal maintenance policy may be too difficult to interpret and apply in practice. Nevertheless, our research does reveal the need for a custom-fit maintenance policy for systems with inter-component dependencies. Future research could thus focus on developing a heuristic that incorporates the policy properties that we describe in this chapter. Furthermore, we assume in our current model that the state of each component is known at the start of each time unit. Inspections are thus performed with a given periodicity. For future research, it could be interesting to consider a system in which the inspection interval (either periodic or aperiodic) needs to be optimized along with the replacement decisions at such an inspection. Continuous monitoring is also an interesting field of research for systems subject to both redundancy and load sharing.

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