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Condition-based maintenance for complex systems

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CBM for a parallel system with economic dependence

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Abstract

This chapter focuses on a two-component system subject to structural dependence (through a parallel setting) and (positive) economic dependence. We continue work on the CBM optimization approach proposed by B. Castanier, A. Grall, C. Bérenguer, A condition-based maintenance policy with non-periodic inspections for a two-unit series system, Reliability Engineering & System Safety 87 (1) (2005) 109-120. Their approach is advanced, compared to those proposed by others, in that it optimizes both the inspection moments and the condition thresholds (on which the planning of maintenance actions is based) simultaneously. It considers a discrete-time system with two machines that operate in series, where the objective is to minimize the long-run average maintenance cost per time unit. We analyze an adapted version of their system where two machines operate in parallel rather than in series, and provide new insights on CBM for such systems. Furthermore, an extensive comparison to other (classical) maintenance policies, such as failure-based maintenance, block replacement, and CBM with periodic inspections, is included.

4.1. Introduction

This chapter considers the joint optimization of the (aperiodic) inspection moments and maintenance decisions for a system with two components that are functioning in parallel (i.e., structural dependence) and subject to (positive) economic dependence. For systems with multiple components subject to economic dependence, it might be rewarding to combine maintenance actions. This means that the optimal maintenance policy for one component might not be optimal for the complete system [1]. Economic dependence can for example result from set-up costs which need to be paid exactly once if maintenance is performed, independent of the number of components that are maintained. These costs can consist of ordering spare parts, shutting down the system, or traveling to the right location. Examples of articles that consider maintenance grouping for multi-component systems with economic dependence are given by [2–4], while a CBM strategy is considered for such systems by, e.g., [1, 5].

A shortcoming that applies to most existing literature on CBM is that either optimization of the critical level at which a preventive replacement should be initiated given the (periodic) inspection intervals, or optimization of the inspection intervals given the critical level is considered. The combination of the two, so optimizing the maintenance policy with respect to both the inspection intervals and the critical preventive maintenance level, has only been studied for a dynamic maintenance grouping policy by [6] and for a multi-threshold maintenance policy by [1, 7–12] (see also Chapter 3). In fact, a single-component system is considered by [7–10], while the policy is extended to a two-component series system by [1, 12]. Moreover, a series-parallel system (i.e., a system with a number of subsystems in series, each containing multiple parallel components) is studied by [11]. Of these contributions, aperiodic inspection moments are considered by [1, 7, 8, 11, 12]. An advantage of using aperiodic inspection moments is that it takes into account that when a system is relatively new, inspections are required less often than when the system is approaching the end of its life. This can reduce costs substantially. The authors explain that the performances of their models are promising, even better than the classical preventive maintenance methods, although this has not been tested with real data.

In this chapter, we focus on the discrete-time CBM model described by [1]. They consider a two-component series system, whereas we consider a system with two components that are functioning in parallel instead of in series. In this way, the system will not completely fail as soon as at least one of the components fails as is the case in the article, but a component failure will result in some lost revenue. Consider, for example, a factory which has two machines to produce

output. A failure of one of the machines will not halt production completely, but will reduce the amount of output produced. We will gain insight in such systems by assessing the performances of our model, and comparing these with those of classical preventive maintenance strategies. Note that [11] developed a maintenance optimization method for a series-parallel system based on the work of [1]. However, they restricted the components to discrete deterioration levels, while we are interested in continuously deteriorating components. Furthermore, [11] use a numerical simulation to evaluate their performances, while we are using an exact method.

The remainder of this chapter is organized as follows. Section 4.2 describes the system, starting by the deterioration model and then continuing to describe the possible maintenance actions and corresponding costs. Next, the multi-threshold policy under consideration is explained in Section 4.3, followed by the mathematical model in Section 4.4. The performances are assessed for an example case in Section 4.5, and compared to those of classical (corrective and preventive) maintenance policies. Section 4.6 concludes the chapter.

4.2. System description

4.2.1. Deterioration model

We consider a discrete-time system consisting of two components, which deteriorate independently. The random variable X_k^i is used to describe the deterioration level of component i at time k , for $i = 1, 2$ and $k \in \mathbb{N}$. The random deterioration increments $\Delta_{(k,k+1)}X^i$ in each time interval must be nonnegative to ensure that it is useful to perform preventive maintenance. Each component is either functioning properly or has failed. Both at the start of the process (at time 0) and after a replacement (say at time t_r), component i is as-good-as-new, i.e., $X_0^i = 0$ and $X_{t_r}^i = 0$, while component i fails as soon as its deterioration level reaches the fixed failure level L_i , for $i = 1, 2$. Failures can only be observed during an inspection, until which the component remains unavailable. At the start of each time unit, it is possible, but not mandatory, to perform a system inspection, thus allowing for an aperiodic inspection schedule. Let f_i denote the probability density function of the deterioration increments $\Delta_{(k,k+1)}X^i$ of component i , for $i = 1, 2$ and for $k \in \mathbb{N}$. We assume that the deterioration increments are stationary and exchangeable, so the degradation increments satisfy the memoryless property and the distribution functions f_i are infinitely divisible [13]. This is for example the case for the gamma distribution, which is the most appropriate choice for our deterioration model [14].

Nomenclature

$\pi(x_1, x_2)$	Long-run probability that components 1 and 2 are in states x_1 and x_2 , respectively, at the start of an inspection
$\xi_j^{(i)}$	Inspection thresholds for component i ($j = 0, 1, \dots, n - 1$)
$\xi_n^{(i)}$	Preventive replacement threshold for component i
ζ_i	Opportunistic replacement threshold for component i
$C(t)$	Cumulative operating costs up to time t
C_∞	Long-run average operating costs per time unit
c_c^i	Cost of a corrective replacement for component i
$C_C^{(i)}(t)$	Cumulative corrective replacement costs of component i up to time t
c_n	Cost of a system inspection
$C_N(t)$	Cumulative inspection cost up to time t
c_p^i	Cost of a preventive replacement for component i
$C_P^{(i)}(t)$	Cumulative preventive replacement costs of component i up to time t
c_s	Fixed set-up costs for a replacement
c_u^i	Unavailability cost rate, per time unit that component i is unavailable
$C_U(t)$	Cumulative system unavailability costs up to time t
$E_\pi[\cdot]$	Expected value with respect to the stationary law π
$f_i(\cdot)$	Pdf of the deterioration increments of component i
$f_i^{(l)}(\cdot)$	Pdf of the cumulative deterioration increments over l time units for component i
L_i	Failure-level of component i
n	Number of inspection threshold values for each component
$N_{SR}(t)$	Total number of complete system replacements up to time t
S	Length of a semi-regeneration cycle in steady state
X_k^i	Condition of component i at time k

4.2.2. Maintenance actions and corresponding costs

An inspection, which costs c_n , reveals the deterioration level of each component. If component i has failed, it needs to be replaced correctively and the corrective replacement cost c_c^i is incurred. If, on the other hand, component i is still func-

tioning, it can be replaced preventively, for which the preventive replacement cost c_p^i is incurred. In case at least one component is replaced, the set-up cost c_s is incurred. Since failures can only be observed during an inspection, component i could endure some unavailability time before being replaced. The unavailability cost c_u^i is incurred per time unit that component i is unavailable. The two components are in parallel, meaning that the system keeps functioning as long as at least one component functions, but these unavailability costs can arise for example from losses in revenue if one of the components is unavailable.

4.3. Multi-threshold maintenance policy

The maintenance decisions for each component are based on comparing the observed deterioration level with a set of threshold values, which are defined as $\xi_0^i, \xi_1^i, \dots, \xi_n^i$ (with $\xi_0^i \leq \xi_1^i \leq \dots \leq \xi_n^i \leq L_i$) and ζ_i (with $0 \leq \zeta_i \leq \xi_n^i$) for component i . The fixed number n is chosen in advance, and represents the number of inspection threshold values, i.e., the next inspection is scheduled at most n periods later. Since a component starts with a deterioration level of zero, we set $\xi_0^i := 0$ for all components. Figure 4.1 schematically represents the maintenance decisions that should be taken for component i , provided that an inspection is performed at time k . This inspection reveals the deterioration level of the component, which is then compared with the threshold values on the vertical axis: if the deterioration level exceeds the failure level L_i , component i has failed, and requires a corrective replacement. If component i has not failed, it *must* still be preventively replaced if its deterioration level exceeds the preventive replacement threshold ξ_n^i . Moreover, even if the deterioration level is below ξ_n^i but above the opportunistic replacement threshold ζ_i , the component *can* be replaced preventively, but only if the other component requires a replacement.

After possible maintenance actions have been performed, the (new) deterioration level of the component is again compared with the threshold values to decide on the next inspection moment. For a deterioration level between ξ_l^i and ξ_{l+1}^i , the next inspection is scheduled after $n - l$ time units. In this way, we obtain an aperiodic inspection schedule in which components are inspected sooner when they are approaching a failure than when they are as-good-as-new. Since both components should be inspected at the same time to be able to combine maintenance actions, and since the costs of an inspection are typically independent of the number of components inspected, it is reasonable to assume that both components are inspected as soon as one of them requires an inspection.

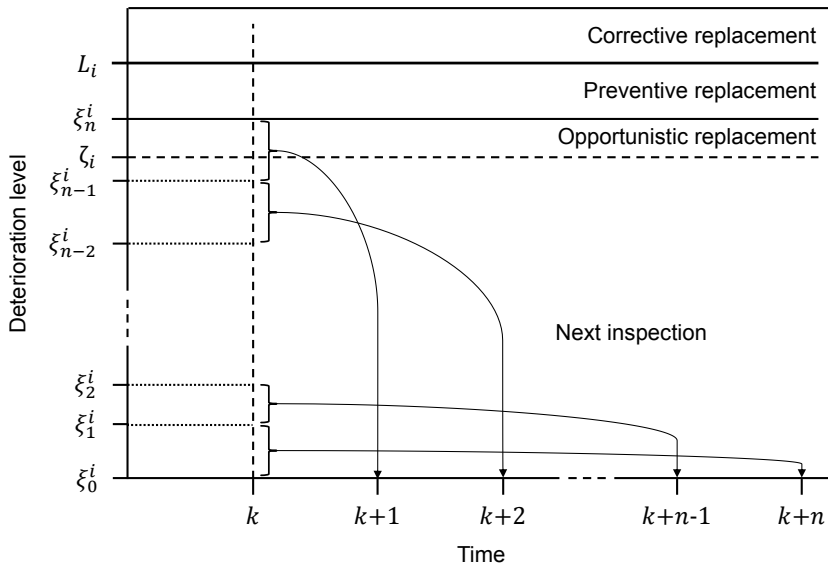


Figure 4.1. Maintenance decisions for component i , given an inspection at time k .

4.4. Mathematical model

Since a complete system replacement, either preventively or correctively, transforms the system state to the as-good-as-new state, and since deterioration does not depend on the past, we can view our deterioration process as a regeneration process [7]. Hence, we can consider the long-run behavior of the process in a single renewal cycle, i.e., the period between two successive system replacements. Moreover, we can view the period between two successive system inspections as a semi-regeneration cycle [1], since the decisions to be made during an inspection solely depend on the system state at that point in time. The existence of semi-regeneration cycles enables us to construct a probabilistic law $\pi(x_1, x_2)$ to describe the probability of being in a certain state at the moment of an inspection. Similarly to [1], this probabilistic law can be calculated as the probability of being in state (y_1, y_2) multiplied by the probability of moving from state (y_1, y_2) to state (x_1, x_2) during an inspection cycle. For ease of notation, we introduce the function F as follows.

$$F(x_1, x_2, l) = f_1^{(l)}(x_1) \cdot f_2^{(l)}(x_2)$$

Then, the probabilistic law π can be expressed by distinguishing four cases; no replacement, only component 1 is replaced, only component 2 is replaced, and a

complete system replacement.

$$\begin{aligned}
\pi(x_1, x_2) = & \\
& \sum_{l_1=0}^{n-1} \sum_{l_2=0}^{n-1} \int_{\xi_{l_1}^1}^{\xi_{l_1+1}^1} \int_{\xi_{l_2}^2}^{\xi_{l_2+1}^2} \pi(y_1, y_2) \cdot F(x_1 - y_1, x_2 - y_2, n - \max\{l_1, l_2\}) \, dy_2 dy_1 \\
& + \sum_{l_2=0}^{n-1} \int_{\xi_n^1}^{\infty} \int_{\min\{\xi_{l_2}^2, \zeta_2\}}^{\min\{\xi_{l_2+1}^2, \zeta_2\}} \pi(y_1, y_2) \cdot F(x_1, x_2 - y_2, n - l_2) \, dy_2 dy_1 \\
& + \sum_{l_1=0}^{n-1} \int_{\min\{\xi_{l_1}^1, \zeta_1\}}^{\min\{\xi_{l_1+1}^1, \zeta_1\}} \int_{\xi_n^2}^{\infty} \pi(y_1, y_2) \cdot F(x_1 - y_1, x_2, n - l_1) \, dy_2 dy_1 \\
& + \left(\int_{\xi_n^1}^{\infty} \int_{\zeta_2}^{\infty} \pi(y_1, y_2) \, dy_2 dy_1 + \int_{\zeta_1}^{\xi_n^1} \int_{\xi_n^2}^{\infty} \pi(y_1, y_2) \, dy_2 dy_1 \right) \cdot F(x_1, x_2, n)
\end{aligned}$$

Using this probabilistic law, it is possible to find an expression for the long-run average operating costs per unit of time, which is denoted by C_∞ . Our goal is to minimize these costs by optimizing the threshold values on which the maintenance actions are based. In that way, both the decisions on when to perform maintenance and when to inspect are optimized simultaneously.

Let $C(t)$ denote the cumulative operating costs up to time t . These operating costs consist of the cumulative costs up to time t involved in performing inspections (denoted by $C_N(t)$), preventive replacements (denoted by $C_P^{(i)}(t)$ for component i), corrective replacements (denoted by $C_C^{(i)}(t)$ for component i), and the unavailability costs due to a failure summed over all components (denoted by $C_U(t)$). Note that both $C_P^{(i)}(t)$ and $C_C^{(i)}(t)$ include the set-up costs for a replacement, without considering possible cost savings from combining maintenance on both components. Therefore, the total number of complete system replacements up to time t is denoted by $N_{SR}(t)$, and is used to subtract the set-up costs that are saved in case of a system replacement. Similarly to [1], the cumulative operating costs up to time t can now be expressed as follows.

$$C(t) = C_N(t) + \sum_{i=1}^2 C_P^{(i)}(t) + \sum_{i=1}^2 C_C^{(i)}(t) - c_s \cdot N_{SR}(t) + C_U(t)$$

If we denote the length of an inspection cycle by S , we can express the long-run average operating costs per unit of time as follows [1].

$$C_\infty = \lim_{t \rightarrow \infty} \frac{E[C(t)]}{t} = \frac{E_\pi[C(S)]}{E_\pi[S]}$$

Although our analysis shows similarities to that of [1], our model differs in two aspects. First, we do not include the inspection costs in the set-up costs for a replacement, but instead assume that these need to be paid exactly once during each inspection cycle. This assumption simplifies the expression for the long-run average operating costs. Hence,

$$E_{\pi}[C_N(S)] = c_n.$$

Second, we consider a different structure for the two components. Whereas in case of a two-component series system an unavailability cost rate needs to be paid for the time during which at least one of the components has failed, for a parallel system it holds that the system keeps functioning as long as at least one component is functioning. We therefore include an unavailability cost rate for each component separately, which could result from for example losses in revenue. The expression for this part of the cost function (denoted by $E_{\pi}[C_U(S)]$) is included in Appendix 4.A. Compared with the two-component series system developed by [1], these two changes in the analysis of the model facilitate easier generalization to systems containing three or more components. The expression for the expected unavailability cost can easily be extended by summing over all components rather than two, while the expression for the inspection cost does not depend on the number of components at all.

The other parts of the cost function remain unchanged. Hence, the preventive replacement cost (including the set-up cost) for component i during an inspection cycle can be obtained as follows, for $i = 1, 2$ and $j \neq i$.

$$E_{\pi}[C_P^{(i)}(S)] = (c_s + c_p^i) \cdot \left(\int_{\xi_n^i}^{L_i} \int_0^{\infty} \pi(x_1, x_2) dx_j dx_i + \int_{\zeta_i}^{\xi_n^i} \int_{\xi_n^j}^{\infty} \pi(x_1, x_2) dx_j dx_i \right)$$

Similarly, the corrective replacement cost (including the set-up cost) for component i during S are given by

$$E_{\pi}[C_C^{(i)}(S)] = (c_s + c_c^i) \cdot \int_{L_i}^{\infty} \int_0^{\infty} \pi(x_1, x_2) dx_j dx_i.$$

The expected number of complete system replacements during an inspection cycle equals the probability of a system replacement during S . Hence,

$$E_{\pi}[N_{SR}(S)] = \int_{\xi_n^1}^{\infty} \int_{\zeta_2}^{\infty} \pi(x_1, x_2) dx_2 dx_1 + \int_{\zeta_1}^{\xi_n^1} \int_{\xi_n^2}^{\infty} \pi(x_1, x_2) dx_2 dx_1.$$

For obtaining an expression for the long-run average length of an inspection cycle we again distinguish four cases; no replacement, only component 1 is replaced, only component 2 is replaced, and a complete system replacement. Note that a complete system replacement immediately implies an inspection cycle of length n .

$$\begin{aligned}
 E_{\pi}[S] &= \sum_{k=1}^n k \cdot P(S = k) = \sum_{k=1}^n k \cdot \left(\int_{\xi_{n-k}^1}^{\xi_{n-k+1}^1} \int_0^{\xi_{n-k+1}^2} \pi(x_1, x_2) dx_2 dx_1 + \int_0^{\xi_{n-k}^1} \int_{\xi_{n-k}^2}^{\xi_{n-k+1}^2} \pi(x_1, x_2) dx_2 dx_1 \right. \\
 &+ \int_{\xi_n^1}^{\infty} \int_{\min\{\xi_{n-k}^2, \zeta_2\}}^{\min\{\xi_{n-k+1}^2, \zeta_2\}} \pi(x_1, x_2) dx_2 dx_1 \\
 &+ \int_{\min\{\xi_{n-k}^1, \zeta_1\}}^{\min\{\xi_{n-k+1}^1, \zeta_1\}} \int_{\xi_n^2}^{\infty} \pi(x_1, x_2) dx_2 dx_1 \\
 &\left. + I_{\{k=n\}} \cdot \left(\int_{\xi_n^1}^{\infty} \int_{\zeta_2}^{\infty} \pi(x_1, x_2) dx_2 dx_1 + \int_{\zeta_1}^{\xi_n^1} \int_{\xi_n^2}^{\infty} \pi(x_1, x_2) dx_2 dx_1 \right) \right)
 \end{aligned}$$

where $I_{\{\cdot\}}$ denotes the indicator function, which equals one if the expression between brackets is true, and zero otherwise.

4.5. Illustrative example

In this example, we consider a system consisting of two identical components, which allows us to drop the superscripts denoting to which component a certain parameter or threshold value corresponds. Moreover, we only have to optimize one set of threshold values, which reduces calculation time dramatically. Furthermore, results are easier to interpret while using identical components. For determining reasonable values for the cost and deterioration parameters, we consider [1, 8]. For the performance assessments of their policies these authors choose to use (among others) the following costs: an inspection costs $c_n = 1$, a preventive replacement costs $c_p = 40$, a corrective replacement costs $c_c = 100$, and the set-up cost for performing at least one replacement is equal to $c_s = 20$. We do consider a larger unavailability cost rate $c_u = 1000$, to avoid unrealistic situations in which it is cheaper to let the system be in the failed state than to maintain the components. The exponential distribution is a common choice for a distribution function describing the deterioration of a component during one time unit [1, 8]. It is a special case of the gamma distribution that allows the

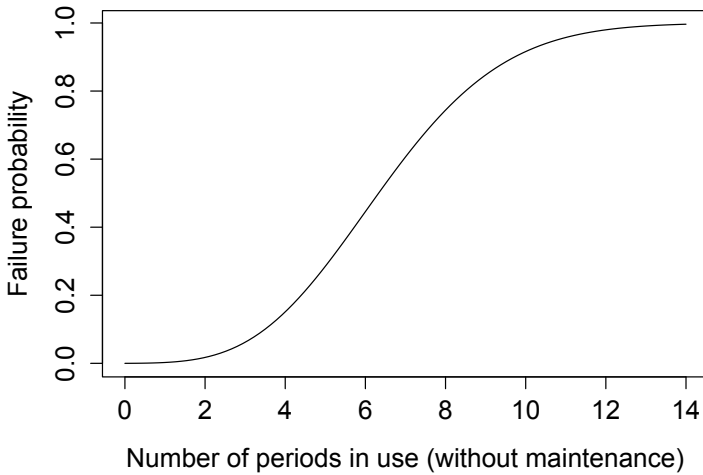


Figure 4.2. Failure probability over time of a new component that is not subject to maintenance.

mathematical analysis of the model to be simplified. Hence, f is assumed to be exponential with deterioration parameter α . This implies that the sum of l exponentially distributed variables, or the deterioration gained in l consecutive time units, follows an Erlang distribution with parameters α and l . For this example, we set the deterioration parameter to $\alpha = 3$, and assume a fixed failure level of $L = 2$. This means that a component fails as soon as its deterioration level exceeds the value of two. The failure probability of a component that is as-good-as-new at time 0 and that does not undergo any maintenance actions will evolve as depicted in Figure 4.2. In this setting, the average time to failure is equal to six time units, while Figure 4.2 confirms that n should not be chosen too high, as this increases the probability of a failure. Hence, we will consider $n = 2$ and $n = 3$.

We start by considering $n = 2$, which means that the next inspection is always scheduled either one or two time units later, and that the threshold values that need to be optimized to minimize the long-run average operating costs per time unit are given by ξ_1 , ξ_2 , and ζ . We calculate the long-run average operating costs per period for $\xi_1, \xi_2, \zeta \in \{0, 0.2, 0.4, \dots, L (= 2)\}$, with $0 \leq \xi_1 \leq \xi_2$ and $0 \leq \zeta \leq \xi_2$, in order to construct Figure 4.3. From this figure, we conclude first of all that each graph is rather smooth. In fact, the minimal long-run average costs given a fixed value of the inspection threshold ξ_1 are strictly increasing, implying that it is profitable to perform inspections quite often. This is due to the high unavailability cost rate and the relatively low costs corresponding to an inspection. Furthermore, the preventive replacement threshold ξ_2 should not be chosen too high, as this will increase the unavailability and hence the costs. On the other hand, costs

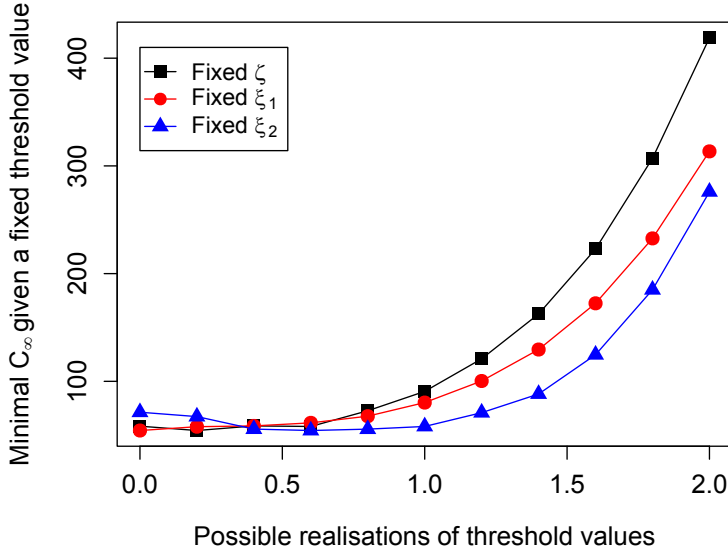


Figure 4.3. Minimal long-run average operating costs for different values of ξ_1 , ξ_2 , and ζ for $n = 2$.

also increase if ξ_2 is chosen close to zero, by scheduling maintenance more often than required. Also the opportunistic replacement threshold ζ should not be chosen too large, as a high ζ enforces a high ξ_2 (since $\xi_2 \geq \zeta$), thus increasing the unavailability and increasing the operating costs.

Results indicate that, in this setting, the minimal long-run average cost is equal to 54.3 per time unit, for $\xi_1 = 0$, $\xi_2 = 0.6$ and $\zeta = 0.2$. However, if we allow inspections to be postponed by one more time unit, i.e., by selecting $n = 3$ instead of $n = 2$, the long-run average costs can be reduced to 35.4 per time unit. The corresponding set of threshold values is $\xi_1 = 0$, $\xi_2 = 0.2$, $\xi_3 = 0.6$ and $\zeta = 0.2$. Hence, the next inspection is scheduled after three, two, and one time units if the deterioration level equals 0, is between 0 and 0.2, and between 0.2 and 0.6, respectively. Furthermore, a component is preventively replaced if its deterioration level exceeds 0.6, while an opportunistic replacement is performed for a deterioration level exceeding 0.2 (in case the other component is replaced). We can conclude that both allowing for aperiodic inspections and including opportunistic replacements are profitable.

4.5.1. Comparison to classical maintenance policies

Several classical preventive maintenance strategies, such as failure-based maintenance (FBM), block replacement (BR), and CBM with periodic inspections, can be seen as special cases of our policy. First, FBM is a strategy in which no preventive maintenance is performed, but each component is correctively replaced upon failure. Since failures can only be observed upon inspection, we can obtain this policy by setting n equal to 1 (i.e., to check for failures during every time unit), and setting both ξ_1 and ζ equal to the failure level L (i.e., to only perform corrective maintenance). No parameters need to be optimized for this policy. Second, BR is a strategy in which the complete system is replaced with a certain periodicity P , independent of the number of intervening failures. This policy can be obtained by setting n equal to the desired periodicity P , and setting all threshold values $(\xi_1, \xi_2, \dots, \xi_n, \zeta)$ equal to zero (to ensure that the system is replaced at each inspection). For this policy, only the periodicity P needs to be optimized. Third, CBM with periodic inspections only differs from our multi-threshold policy in the sense that inspections must be performed with a certain periodicity P . This can be achieved by setting n equal to this periodicity P , and choosing the inspection thresholds $(\xi_1, \xi_2, \dots, \xi_{n-1})$ equal to the preventive replacement threshold ξ_n (to ensure that the next inspection is always scheduled after P time units). For this policy, the periodicity P , the preventive replacement threshold ξ_n , and the opportunistic replacement threshold ζ need to be optimized. Table 4.1 summarizes the minimal costs obtained with the different maintenance policies, along with the corresponding optimal parameters. From this table, we can conclude that FBM is the most expensive strategy, which can be explained by the fact that it is the least advanced one. Furthermore, both with BR and CBM with periodic inspections, the minimal long-run average costs per time unit are much higher than the costs that we found with our multi-threshold strategy (which equal 35.4 per time unit). This leads to the conclusion that CBM indeed can significantly outperform

Table 4.1. Minimal costs for different maintenance policies (increase from CBM with aperiodic inspections), along with optimal parameter(s).

Maintenance policy	Minimal average costs	Optimal parameter(s)
Failure-based maintenance	206.7 (+484%)	-
Block replacement	71.3 (+101%)	$P = 2$
CBM with periodic inspections	58.2 (+ 64%)	$P = 1, \xi_1 = 0.6, \zeta = 0.4$
CBM with aperiodic inspections	35.4	$n = 3, \xi_1 = 0, \xi_2 = 0.2, \xi_3 = 0.6, \zeta = 0.2$

more traditional policies, but also that allowing for aperiodic inspections can be profitable.

4.6. Conclusion

This chapter considers the joint optimization of the inspection moments and (condition-based) maintenance decisions for a two-component system subject to both structural dependence (through a parallel system structure) and economic dependence (through a fixed maintenance set-up cost). To this extent, we considered the CBM optimization approach developed by [1] for a discrete-time system consisting of two components. We adapted this model such that it considers a system with two components that are functioning in parallel instead of in series, such that the system does not completely fail as soon as at least one of the components fails. We observed that a number of classical maintenance policies can be viewed as special cases of our policy, implying that our model is widely applicable and of value to the maintenance literature. Furthermore, we observed that it can be profitable to allow for aperiodic inspections, but also that clustering maintenance activities can reduce costs.

For future research, it could be interesting to consider an uncertain failure level rather than a fixed failure level such as in this chapter, while also an optimization procedure can be developed in order to determine the optimal threshold values in an efficient way. This is especially useful when extending the model to three or more components. Furthermore, it could be interesting to test the CBM optimization approach on real data.

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Appendix 4.A. Expression for the expected unavailability cost

It is impossible to trace the exact failure time of a component due to the discrete-time horizon [1]. Therefore, the time spent in the failed state $D_U^i(t)$ during time t for component i , $i=1,2$, needs to be estimated. It is possible to estimate this unavailability time by using an upper bound $\bar{D}_U^i(t)$ [1]. Suppose we consider the time span $[0, S]$ and moment of failure t_f (with $k \leq t_f \leq k+1$). The upper bound is then given by $S - k$, as shown in Figure 4.A.1.

The total expected unavailability cost of the complete system (denoted by $E_\pi[C_U(S)]$) is equal to the sum of the unavailability costs for the different components (denoted by $E_\pi[C_U^i(S)]$ for component i). Hence,

$$E_\pi[C_U(S)] = \sum_{i=1}^2 E_\pi[C_U^i(S)],$$

where the unavailability cost of component i can be approximated by using the upper bound as follows.

$$\begin{aligned} E_\pi[C_U^i(S)] &= c_u^i \cdot E_\pi[D_U^i(S)] \\ &\approx c_u^i \cdot E_\pi[\bar{D}_U^i(S)] \\ &= c_u^i \cdot \sum_{k=1}^n k \cdot P(\bar{D}_U^i(S) = k) \end{aligned}$$

In order to obtain an expression for $P(\bar{D}_U^i(S) = k)$, we first define $H_i(k|y; l)$ as the probability that the unavailability time of component i is equal to k time units, given that after the previous inspection and possible maintenance actions the component had a deterioration level of y and the next inspection was scheduled l

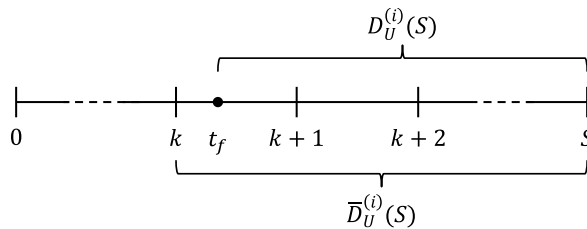


Figure 4.A.1. Exact unavailability time $D_U^i(S)$ of component i and its upper bound $\bar{D}_U^i(S)$.

time units later, i.e., $y < \xi_{n-l+1}^i$. This function is given by

$$H_i(k|y; l) = \begin{cases} 0, & \text{if } l < k, \\ \int_{L_i-y}^{\infty} f_i(t) dt, & \text{if } l = k, \\ \int_0^{L_i-y} f_i^{(l-k)}(u) \left(\int_{L_i-y-u}^{\infty} f_i(s) ds \right) du, & \text{if } l > k. \end{cases}$$

We can use this function in order to calculate $P(\bar{D}_U^i(S) = k)$, for which we distinguish four cases; no replacement, only component 1 is replaced, only component 2 is replaced, and a complete system replacement, as follows.

$$\begin{aligned} P(\bar{D}_U^i(S) = k) = & \\ & \sum_{l_1=k}^n \sum_{l_2=k}^n \int_{\xi_{n-l_1}^1}^{\xi_{n-l_1+1}^1} \int_{\xi_{n-l_2}^2}^{\xi_{n-l_2+1}^2} \pi(y_1, y_2) \cdot H_i(k|y_i; \min\{l_1, l_2\}) dy_2 dy_1 \\ & + \sum_{l_2=k}^n \int_{\xi_n^1}^{\infty} \int_{\min\{\xi_{n-l_2}^2, \zeta_2\}}^{\min\{\xi_{n-l_2+1}^2, \zeta_2\}} \pi(y_1, y_2) \cdot H_i(k|I_{\{i=2\}} \cdot y_i; l_2) dy_2 dy_1 \\ & + \sum_{l_1=k}^n \int_{\min\{\xi_{n-l_1}^1, \zeta_1\}}^{\min\{\xi_{n-l_1+1}^1, \zeta_1\}} \int_{\xi_n^2}^{\infty} \pi(y_1, y_2) \cdot H_i(k|I_{\{i=1\}} \cdot y_i; l_1) dy_1 dy_2 \\ & + \left(\int_{\xi_n^1}^{\infty} \int_{\zeta_2}^{\infty} \pi(y_1, y_2) dy_2 dy_1 + \int_{\zeta_1}^{\xi_n^1} \int_{\xi_n^2}^{\infty} \pi(y_1, y_2) dy_2 dy_1 \right) \cdot H_i(k|0; n) \end{aligned}$$

