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# A numerical study of large deformations of low-density elastomeric open-cell foams

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## Abstract

A numerical study is presented of the mechanical properties of low-density open-cell polymer foams subjected to large deformations. The foams are modelled as three-dimensional frameworks of slender struts. Regular as well as random foams are analyzed, where the latter are generated using the Voronoi technique. The macroscopic mechanical properties are determined for various types of struts properties through unit-cell analyses containing many foam cells per unit-cell. The computations make use of standard Finite Element (FE) techniques. Bending of the struts dominates the mechanical foam response at low strains. Axial deformation of the struts becomes the dominant mechanism at larger tensile strains. Strut buckling becomes the main mechanism at larger compressive strains, and causes a significant decrease in load carrying capacity of the foam. The large strain mechanical behavior of foams is found to be dependent on the weakest cross-section of the foam appearing in the random foam structure, the so-called “minimum effective cross-section”. The minimum effective cross-section determines the tangential foam modulus at large tensile strains. Regular foam structures have a uniform unit-cell cross-section and, as a result, a higher minimum effective cross-section than regular foam structures and, therefore, a higher tangent modulus in the large strain range. © 1998 Elsevier Science Ltd. All rights reserved.

*Keywords:* Open-cell foam; Modelling; FE; Mechanical properties; Nonlinear analysis; Voronoi tessellation; 3D

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## 1. Introduction

Foams can be considered as a system consisting of two phases (solid and gas). Gas bubbles nucleate and grow in the liquid material during production. After solidification of the material (often a polymer), foam has a microstructure which can be represented as a stacking of randomly distributed cells of various shapes and sizes which fills the space completely. In closed-cell foams, most cells

have walls (thin membranes) shared by the two neighboring cells, and each cell wall is bounded by struts. After removal of the cell walls by chemical treatment or reticulation, an open-cell foam is obtained, which only comprises a framework of struts being interconnected at the vertices.

Open-cell foams can be classified according to their relative density. In high-density foams, more than 10% of the foam volume is occupied by the solid phase which is primarily concentrated in the vertices. In low-density foams, the solid material is distributed more uniformly along the struts. Fig. 1 shows an image of a low-density open-cell foam, where it is seen that the struts are relatively

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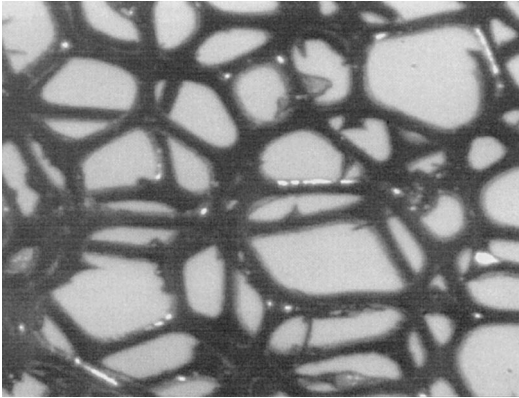


Fig. 1. Image of a low-density open-cell foam.

slender and have a more or less constant cross-section. This paper concentrates on the mechanical properties of low-density open-cell foam.

In the literature, many simplified models of open-cell foams have been suggested to estimate or understand the mechanical characteristics of open-cell foams. Most models are mainly based on the geometrical features of the foam structure on one hand, and on the mechanical properties of the solid phase on the other hand. Such models attempt to address a number of aspects of foam, like the stress–strain relation or the elastic collapse stress as a function of the foam density. The most well-known model is the regular cubic model of Gibson and Ashby (1988), which is illustrated in Fig. 2. This simple foam model elucidates that small tensile or compressive deformations are governed by bending of the struts. Application of this model gives insight into the elastic response of open-cell foams depending on the initial foam density. When strut bending is the dominant deformation mechanism, a quadratic relation between foam stiffness and density is expected (Gibson and Ashby, 1988):

$$\frac{E_{f,i}}{E_s} \approx \left( \frac{\rho_f}{\rho_s} \right)_i^2, \quad (1)$$

where  $E_{f,i}$ ,  $E_s$  and  $\rho_f$ ,  $\rho_s$  are Young's moduli and mass densities of the foam and the solid polymer, respectively. Subscript  $i$  refers to the initial state at zero strain. When the tensile strains of the foam become sufficiently large (when the tensile strain  $\varepsilon_f$  exceeds  $\sim 0.3$ ), the struts become oriented in the

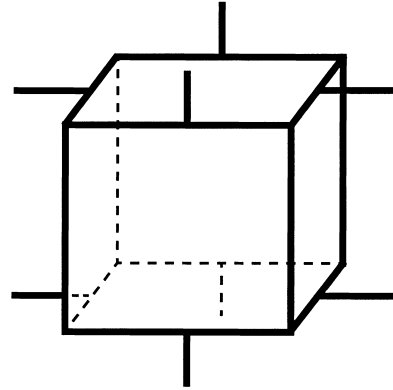


Fig. 2. The cubic cell representing the most important mechanical aspects of open-cell foams by Gibson and Ashby (1988), e.g., bending of the struts at small straining of the foam.

loading direction. Consequently, the axial deformation of the struts increasingly dominates the foam response. The foam elastic response in this regime, in terms of the foam tangent modulus, depends on the density according to

$$\frac{E_{f,t}}{E_s} = C_1 \left( \frac{\rho_f}{\rho_s} \right)_i, \quad (2)$$

where  $C_1$  is a constant. Gibson and Ashby (1988) accepted  $C_1 \approx 1$  as a fair approximation. Eq. (2) reflects a direct relation of the macroscopic foam deformation and the local tensile deformation of struts.

In case of compressive deformations of the foam, some struts will buckle and henceforth initiate the collapse of the foam. Gibson and Ashby (1988) expected the elastic collapse stress  $\Sigma_{el}$  to be expressible as

$$\frac{\Sigma_{el}}{E_s} = C_2 \left( \frac{\rho_f}{\rho_s} \right)_i^p, \quad (3)$$

where  $C_2$  and  $p$  are coefficients. It was found from experiments (Gibson and Ashby, 1988) that  $C_2 \approx 0.05$  and  $p \approx 2$ .

A geometrically more realistic model of the foam microstructure can be based upon the Voronoi technique (Voronoi, 1908). The Voronoi tessellation of space has been applied by Van der Burg et al. (1997) to obtain a cellular microstructure which better resembles the morphology of real foam microstructures. This technique is based on the distribution of nuclei, which mimics the gas

bubble nuclei in a liquid polymer during production of foam. If the nuclei in the model are distributed periodically, the subsequent cellular microstructure of foam is regular. A random distribution of nuclei, just like in reality, is modelled with a random space tessellation. In this way, unit-cells of a foam have been generated (Van der Burg et al., 1997) with many hundreds of randomly distributed foam cells, and where the struts are intersections of three cell faces, just like in real foam (see, for instance, Williams, 1968). Important geometrical foam features, like the four-way strut connectivity, are automatically achieved.

This microstructural model was applied (Van der Burg et al., 1997) for the initial linear elastic response of foam. The Finite Element (FE) method was performed. The effect of the relative density was investigated by three-dimensional FE analysis. In these analyses, the slender struts were modelled as beams. The transition from regular to completely random geometrical microstructures was made to investigate its effect on the mechanical response. It was shown that randomness increased the foam stiffness at small foam deformations by the occurrence of percolating chains of struts which deform solely by axial strain.

In the analyses (Van der Burg et al., 1997), the deformations of the foam remained small and the strut material responded linear elastically. In the present paper, the response of an elastomeric foam during large deformations is analyzed, allowing the strut material to become nonlinear elastic. The geometrical nonlinearity also appears to play a considerable role at large deformations of the foam. To get a good understanding of the polymer foam behavior at large deformations, a numerical large deformations analysis has been performed. Tensile as well as compressive strains are studied using the foam model based on the same 3D Voronoi tessellation of space as used by Van der Burg et al. (1997).

## 2. Method of analysis

### 2.1. Generation of foam microstructure

The low-density open-cell foam is constructed from a unit-cell, assuming that each unit-cell

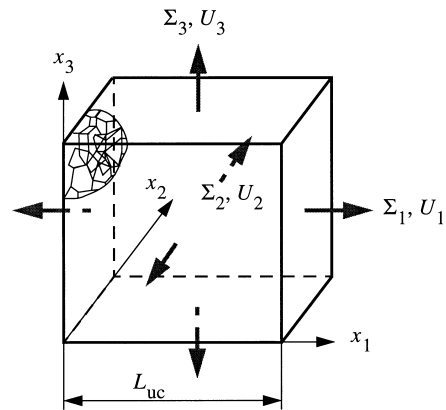


Fig. 3. Unit-cell of the foam. The complete foam is constructed from this cell by reflection with respect to each of the cell boundaries. Deformation is prescribed in terms of uniform displacement increments normal to the cell boundaries. The response is characterized by the three principal macroscopic stresses  $\Sigma_1$ ,  $\Sigma_2$  and  $\Sigma_3$ .

boundary is a plane of reflective symmetry. Loading is applied through uniform normal displacements  $U_1$ ,  $U_2$ ,  $U_3$  in the three directions (see Fig. 3); the average corresponding tractions on the cell faces define the macroscopic principal stresses  $\Sigma_1$ ,  $\Sigma_2$ ,  $\Sigma_3$  acting on the foam.

The strut framework used to model the foam is generated on the basis of the same Voronoi tessellation of space technique as used by Van der Burg et al. (1997). This technique allows one to generate a 3D network of foam cells, derived from the randomly distributed points (nuclei), which become the centers of the foam cells. The flat cell boundary faces appear, where two neighboring cells come into contact. The edge where three cell faces meet is the location of the future struts after removal of the faces. As an analogous example, a Voronoi tessellation of a plane is shown in Fig. 4. The initial distribution of the nuclei completely determines the final geometry of the Voronoi tessellation and, hence, the strut framework.

To create a regular space-filling strut framework, a body-centered cubic (bcc) and a face-centered (fcc) close packed distribution of nuclei are taken here, as displayed in Figs. 5(a) and (b). Their subsequent regular cells are the tetrakaidcahedron and the rhombic dodecahedron. After

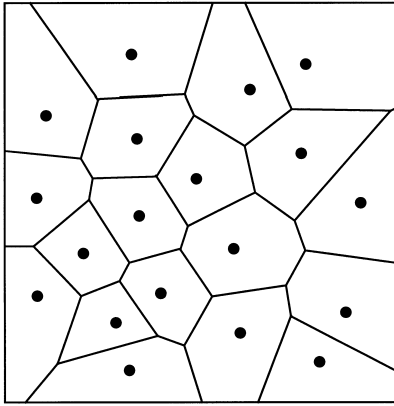


Fig. 4. A 2D Voronoi tessellation of a plane. The black dots are the initial nuclei of the cells. Since all cells grow with the same rate, the 2D cell boundaries are straight.

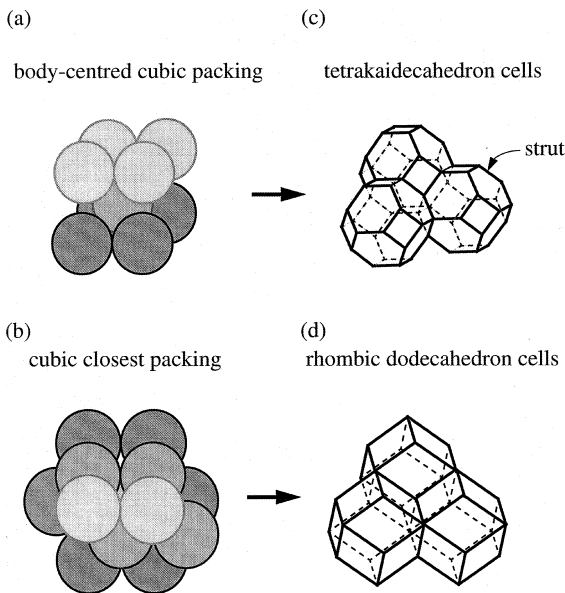


Fig. 5. (a) Tetraikadehedron based on body-centered cubic (bcc) distribution of nuclei and (c) its subsequent strut framework. (b) Rhombic dodecahedron based on a face-centered cubic (fcc) closest nuclei distribution and (d) its subsequent strut framework.

removal of the cell walls, the regular strut framework remains, as shown in Figs. 5(c) and (d).

To obtain a more realistic geometry of the strut framework, the nuclei are randomly distributed inside a box with edge length  $L$ , as shown in

Fig. 6(a). The number of nuclei has been limited by demanding a minimum spacing  $d$  between neighboring nuclei. The nuclei generator places a new nucleus randomly inside the box and then checks whether the distance to the nearest nucleus does not exceed the minimum allowed distance  $d$ . If the distance is too small, the nucleus is removed. The nuclei generator repeats this procedure 2 orders of magnitude more times than the expected final number of nuclei in the box. The nuclei distribution obtained inside the box is then the input for the Voronoi procedure which subdivides the box into cells, where the cell wall boundaries define the random strut framework. This strut framework exhibits some peculiarities at the box boundaries. Therefore, all struts associated with these peculiarities have been removed from the box. The resulting structure is geometrically close to the real foam morphology, e.g., four struts intersect in every vertex and every strut is the result of a junction of three walls. The unit-cell needed for the foam analysis is cut out of the above mentioned box and has an edge length of  $L_{uc}$ , where  $L_{uc} < L$ . In order to ensure that the unit-cell boundaries are planes of symmetry, the struts crossing the unit-cell boundary are rearranged in such a way that these struts become normal to the boundary. The result of this procedure is displayed in Fig. 6(b).

## 2.2. FE analysis

The FE analyses are performed using the standard program MARC. Geometrical nonlinearity occurs at large foam deformations. To obtain a good description of, for example, the buckling of the struts, all struts are meshed in relation to their length. This allows one to achieve a rather uniform distribution of the element length throughout the unit-cell. In this way, shorter struts, which need quite high compressive load before they buckle, contain less elements than the longer struts which will buckle first and, hence, are subdivided into more beam elements. A view of a cubic unit-cell in 3D with the finite elements distribution is given in Fig. 6(c).

To save computing time, the strut framework is swept. The sweep mechanism is extensively

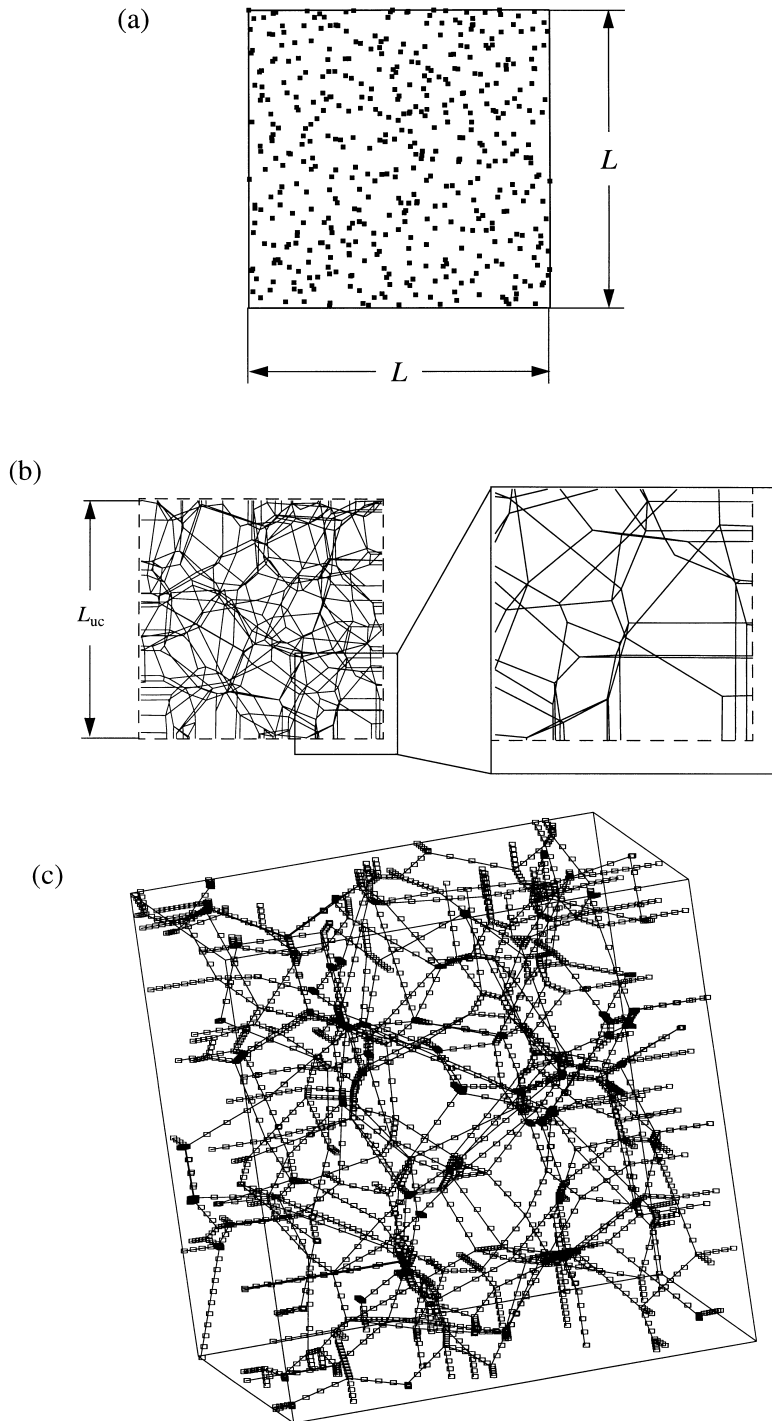


Fig. 6. Unit-cell generation. (a) Randomly distributed nuclei in box with the edge length  $L$  (projection). (b) Unit-cell ( $L_{uc} < L$ ) with arranged boundaries (projection). (c) Spatial picture including an example of the finite element mesh of the unit-cell.

described by Van der Burg et al. (1997). It involves elimination of struts which are shorter than a minimum allowed strut length  $l_{\min}$  and merging of both ends of the removed struts. These struts are eliminated since small struts do not influence the mechanical properties significantly. However, the sweep procedure does affect the accuracy of the results to some degree. It was shown by Van der Burg et al. (1997) that sweeping with  $l_{\min}$  being 15% of the average initial strut length leads to a considerable reduction of the computation time, while the accuracy remains satisfactorily high (within 3% for the Young's modulus). Also in this paper, this  $l_{\min}$  is used.

One of the important simplifications made in the model here concerns the strut cross-section. In real foams, the cross-sectional area along the strut has a triangular shape, the so-called Plateau–Gibbs border (Kann, 1989), as reminiscent of the three cells and cell walls during foam production. In the foam model, all struts are simplified to have the same and constant circular cross-section with diameter  $D$  and area  $A$ . The total strut length in combination with the strut cross-sectional area  $A$  determines the (initial) relative foam density  $(\rho_f/\rho_s)_i$ . The relative foam density of the model can be changed by changing the cross-sectional area of struts in the same unit-cell.

In addition to the foam geometry, the behavior of the solid material in the struts is of the great importance for the macroscopic behavior of the foam. In the analyses, the solid material can have various types of constitutive behavior. Three types of idealized behavior are used in the present analyses. Linear elastic (A) and two nonlinear elastic

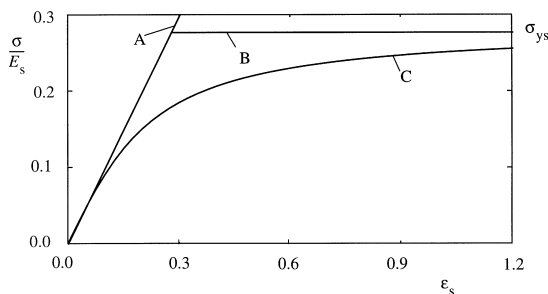


Fig. 7. Three different solid material behaviors of the struts: A – linear elastic; B – bilinear; C – nonlinear.

types of behavior (B, C) are displayed in Fig. 7. Material A from Fig. 7 is used in the geometrically nonlinear analysis. In the nonlinear analysis, the behavior of the solid material of the struts is described either by the bilinear curve (material B) or the nonlinear constitutive relations (material C). Both descriptions involve a limit stress  $\sigma_{ys}$ , which is here taken to have the value  $0.28E_s$ . The material C is characterized by the uniaxial stress–strain law

$$\frac{\sigma}{\sigma_{ys}} = \frac{2}{\pi} \operatorname{atan}\left(\frac{\varepsilon_s}{\varepsilon_{s0}}\right),$$

with  $\varepsilon_{s0} = 0.18$ . These two strut material behaviors are idealized representations of a real rubber-like strut material of open-cell foams. Additionally, the material behavior is assumed to be the same in tension and compression.

During large deformations, especially in compression, contact between struts may arise. This contact problem has not been incorporated in the numerical analysis and, therefore, the densification region of the foam observed during compression cannot be investigated.

### 3. Results

#### 3.1. Sensitivity to unit-cell size

First, a number of unit-cell analyses are performed in order to determine the minimum size of the unit-cell that supplies an accurate solution. The unit-cells are created by the Voronoi tessellation technique for randomly distributed nuclei sets. The dimensions of the unit-cell for the detailed analyses are chosen on the basis of preliminary computations of unit-cells with various sizes. The number of foam cells per unit-cell is controlled by the size parameter  $L_{uc}/d$ , which is here chosen to be 2, 4 or 6. To demonstrate the reproducibility of the random model, the Voronoi tessellation is applied for at least five different random nuclei distributions per  $L_{uc}/d$  and the corresponding unit-cells are generated for each size parameter. The corresponding unit-cells contain on the average 63, 383 and 616 struts for  $L_{uc}/d = 2, 4$  and 6, respectively. The solid material is assumed to behave linear elastically (material A) for the moment, and

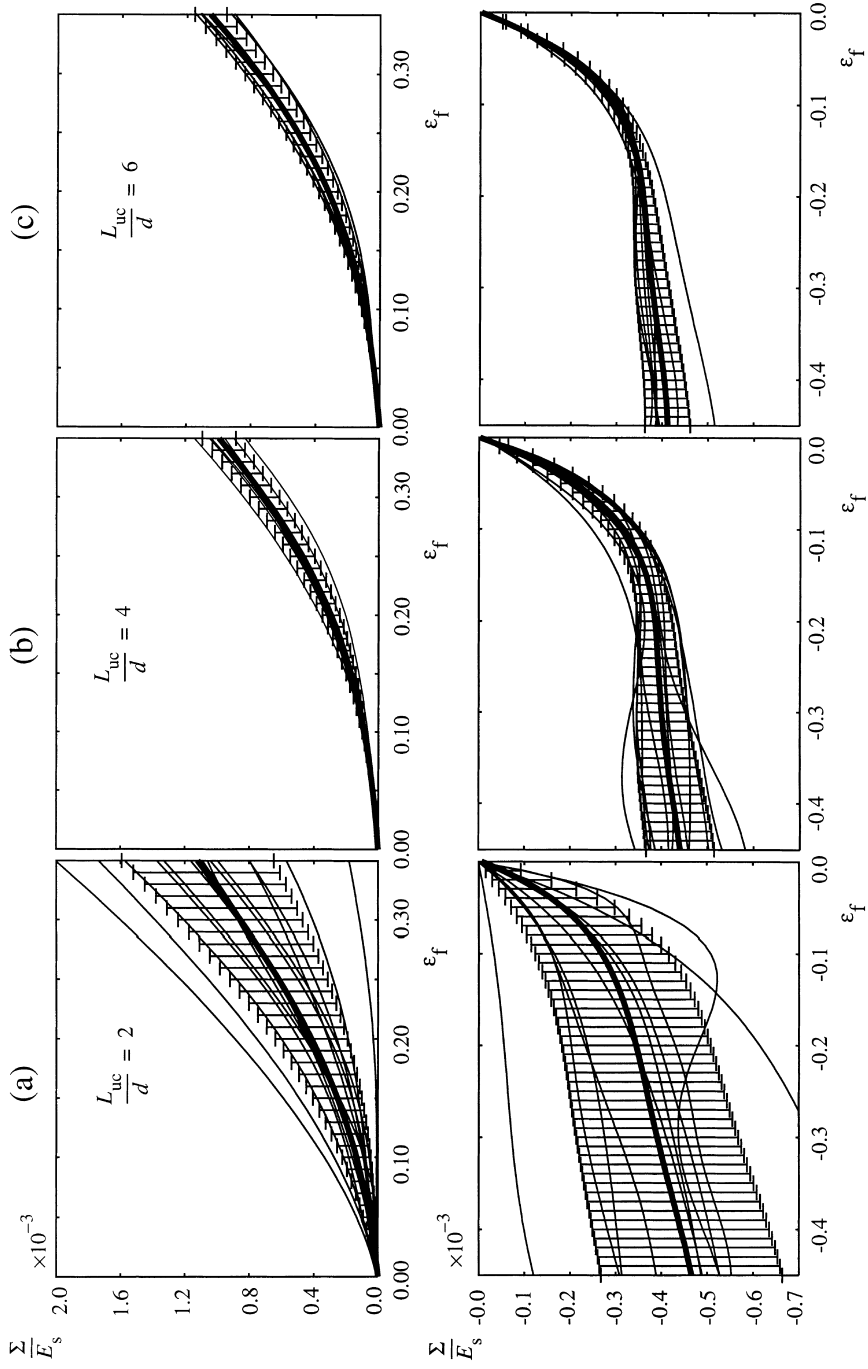


Fig. 8. Stress-strain diagrams and mean value (thick line) with standard deviation for different random realizations per size parameter: (a)  $L_{uc}/d = 2$ ; (b)  $L_{uc}/d = 4$ ; (c)  $L_{uc}/d = 6$ .



the relative foam density  $(\rho_f/\rho_s)_i$  is taken equal to 0.025. This corresponds to the range of low-density foams. To achieve a good convergence during buckling of the struts, a minimum number of 4 and a maximum number of 8 elements per strut are chosen. Uniaxial tensile or compressive loading is applied in an incremental manner through displacement control.

Figs. 8(a)–(c) show stress–strain curves under tension or compression of unit-cells with the size parameters  $L_{uc}/d$  being 2, 4 or 6. The smallest unit-cell with nearly 10 nucleation centres ( $L_{uc}/d = 2$ ) shows a rather wide variation in the predicted stress–strain curves [see Fig. 8(a)]. This scatter can be explained by the small sizes of the unit-cell. Some unit-cells exhibit stiff behavior due to the influence of the rearranged boundaries which may reinforce the model. An increase of the unit-cell size to  $L_{uc}/d = 4$  with nearly 50 nucleation points per unit-cell leads to a considerable reduction of the scatter and a clear plateau in the stress–strain compression diagram. A further increase of the parameter  $L_{uc}/d$  to 6 (128 cells) shows a raise of the computation times with a factor 10 in comparison with the unit-cell with  $L_{uc}/d = 4$ , while the results for tension and compression remain almost the same [compare Figs. 8(b) and (c)]. Obviously, the unit-cell should be sufficiently large compared to  $d$ . Based on the shown results, the unit-cell with  $L_{uc}/d = 4$  can be recommended to be used in the random foam modelling as the smallest unit-cell that combines small scatter with low computing times.

### 3.2. Geometrically nonlinear model

#### 3.2.1. Tension

The above study on the effect of the unit-cell size presents a reference for the foam response based on linear elastic strut material. In the FE analysis, the tensile stress–strain response of the foam is determined for the linear elastic strut material (line A in Fig. 7). Three random models with  $L_{uc}/d$  equal to 2, 4 and 6 having behavior closest to the averaged behavior from Fig. 8 are chosen as representative models.

Additionally, two regular foam models, fcc- and bcc-based, with the same relative foam density are

analyzed. As shown in Fig. 9(a), both regular structures (bcc and fcc) show an initial stiffness that is lower than the stiffness of the random structures. Moreover, the stiffness of the fcc-based microstructure at large strains is much larger than the bcc-based. This can also be seen in Fig. 9(b) where the tangent modulus  $E_{f,t}$  of the foam is plotted as a function of the logarithmic strain of foam,  $\varepsilon_f = \ln(1 + U_i/L_{uc})$ . For both regular microstructures, three distinct deformation regions can be seen in Fig. 9(b). In the first region, the stiffness changes hardly and bending of struts is mainly responsible for the deformation. This is the so-called ‘strut bending region’. Subsequently, the axial deformation of struts starts to play an increasing role in the global deformation of the foam. The increasing stiffness can be explained by the gradual reorientation of struts towards the direction of the global stress  $\Sigma_i$ . This stage corre-

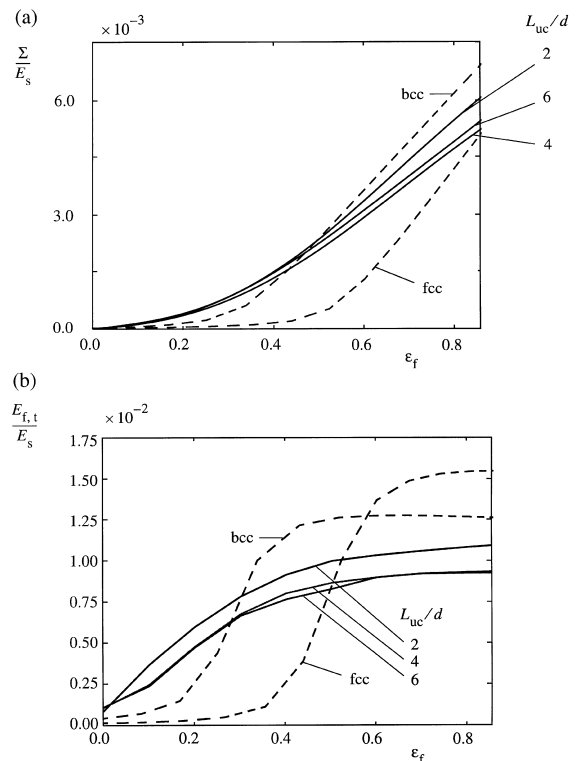


Fig. 9. (a) Stress–strain diagrams for the geometrically nonlinear analysis. (b) Corresponding tangent moduli.

sponds to the second, transitional region with a highly variable tangent modulus due to the mixture of bending and axial deformation in the struts. At a strain of  $\varepsilon_f \approx 0.4$  for the bcc-based model and  $\varepsilon_f \approx 0.7$  for the fcc-based structure, the third region starts. In this region, the foams deformation is almost completely determined by axial deformation of the struts that are aligned with the macroscopic stress.

Comparing with the regular models, the first deformation region is not present for all the random unit-cells. This indicates that even in the initial deformation of the random model, axial deformation influences the overall foam behavior. The large initial stiffness of the random model is explained by percolation of oriented chains of struts loaded mainly in tension. This effect has been discussed by Van der Burg et al. (1997). These percolations are absent in regular structures. However, due to the strain-induced strut reorientation, the same effect occurs in regular models at large deformations. The bcc-based structure shows this effect earlier than the fcc-based model. Regular foams exhibit the alignment of many struts at the same time, thus explaining the reversed trend at large deformations. Regular foams are stiffer than random foams at large strains because of simultaneous strut alignment. Further considerations of the large strain stiffness will be presented in a forthcoming section.

To investigate the influence of the initial relative foam density  $(\rho_f/\rho_s)_i$  on the mechanical properties of the random model, the random unit-cell with  $L_{uc}/d = 4$  is loaded uniaxially in tension. The various relative foam densities are taken to be 0.0125, 0.0250, 0.0375 and 0.0500, and obtained by changing the diameter  $D$  of the struts. Fig. 10(a) shows the stress–strain responses for the various densities. In Fig. 10(b), the tangent moduli  $E_{f,t}/E_s$  are displayed as function of the foam strain. Again, the second and the third regions can be distinguished clearly. A first conclusion from this figure is that the stress–strain curves become linear at a certain strain. Moreover, this strain becomes larger with increasing density. This is due to the fact that with increasing strut diameter, the bending stiffness of struts increases as  $D^4$  while its axial stiffness increases as  $D^2$ .

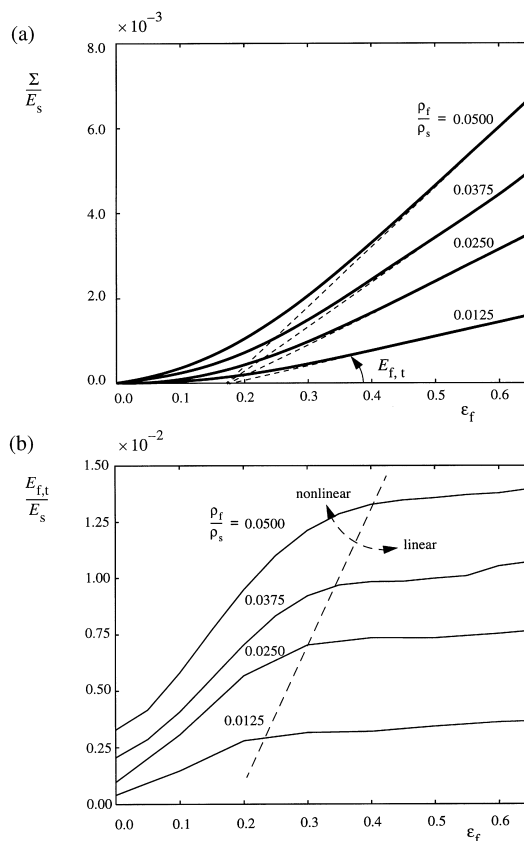


Fig. 10. (a) Stress–strain diagrams for the geometrically nonlinear analysis of the random foam model with various densities. (b) Corresponding tangent moduli.

The results for  $E_{f,t}/E_s$  of the random model in the small deformation region (to be precise, at  $\varepsilon_f = 0.05$ ) from Fig. 10(b) are given as a function of  $(\rho_f/\rho_s)_i^2$  in Fig. 11(a). The results of the cubic model of Gibson and Ashby (1988) [see Eq. (1)] for the initial strain are plotted also, as well as  $E_{f,t}/E_s$  of the random model in the undeformed state ( $\varepsilon_f = 0$ ), which are taken from Van der Burg et al. (1997). If bending is the main deformation mechanism, the FE results would be on the Gibson–Ashby model line. Indeed, at 0% strain, the FE analyses show that bending is the main deformation mechanism. However, at higher densities, axial deformation becomes more important. At a relatively small strain of 5%, the importance of the axial deformation in struts in the random model has increased substantially in comparison

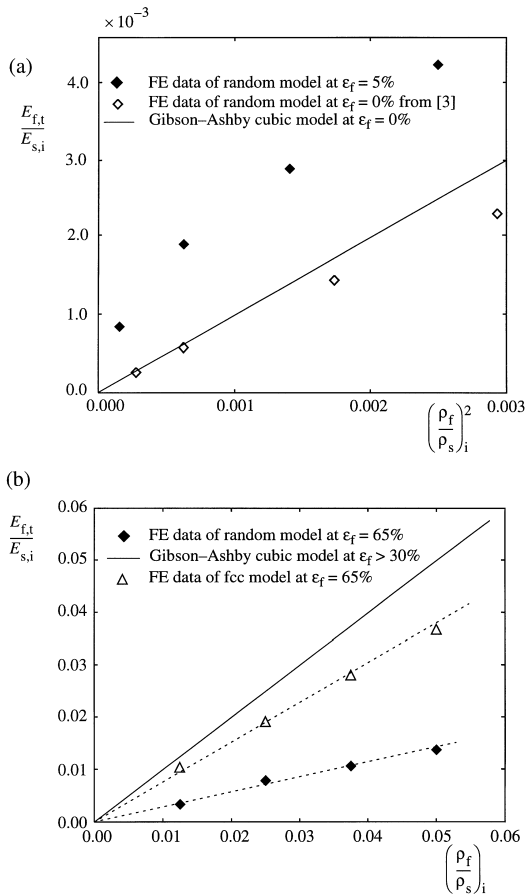


Fig. 11. Tangent modulus of the fcc-based and random models with linear elastic material: (a) at small strain and (b) at large strain as a function of the relative foam density.

with that at  $\epsilon_f = 0$ . Due to the rapidly increasing importance of the axial deformation, the cubic model of Gibson and Ashby loses accuracy with increasing strain.

At large strains of the foam ( $\epsilon_f > 0.4$ ), the random model exhibits an asymptotic stiffness due to the alignment of several strut chains in the direction of maximum principal stress. The values of  $E_{f,t}/E_s$  for the random unit-cell and for the fcc-based regular model at large strains are plotted against the initial relative foam density  $(\rho_f/\rho_s)_i$  in Fig. 11(b). The random model is less stiff in this region than the fcc-based unit-cell and both models do not reach the stiffness given by the cubic model of Gibson and Ashby by Eq. (2). If defor-

mation is purely by uniaxial tension of the struts,  $E_{f,t}/E_s$  should be linear with  $(\rho_f/\rho_s)_i$ . Any deviation is due to bending. The lower lines indicate that bending is still of importance or that not all struts deform axially. In the fcc-based micro-structure, severe bending of the struts takes place at the ends of the struts, close to the vertices. Finally, the relative tangent modulus of a foam model under a large tensile strain is linear with respect to  $(\rho_f/\rho_s)_i$ . It is clear from Fig. 11(b) that neither the fcc-based nor the random model reaches such a high modulus; therefore,  $C_1 < 1$  in Eq. (2).

### 3.2.2. Compression

Similarly, the fcc-based model and the random unit-cell built up of linear elastic struts are loaded in compression. The initial density of the foam is again  $(\rho_f/\rho_s)_i = 0.025$ . The two foam models predict a very different behavior, as can be seen in Fig. 12. The random model is initially much stiffer than the regular one and exhibits a “maximum”, which is termed the elastic collapse stress  $\Sigma_{el}$ . Fig. 13(a) depicts the random unit-cell in the post-collapse regime when a considerable number of struts is buckled. In contrast, the local buckling of struts does not occur in the fcc-based model what is also seen in Fig. 13(b), showing bending of the struts only. This explains the absence of a collapse stress.

To determine the coefficients  $C_2$  and  $p$  appearing in Eq. (3), numerical experiments with unit-cells of various relative densities are performed here. The coefficient  $p$  appears to depend on the ratio between the two deformation mechanisms in the struts, namely axial deformation ( $p=1$ ) and bending ( $p=2$ ). In a foam under global compressive stress, the buckling of struts is responsible for the maximum stress. For this reason,  $p$  is expected to be close to but lower than 2. Fig. 14 shows how the elastic collapse stress of the foam model under compression depends on the relative foam density. Based on these FE results, the coefficient  $C_2$  in Eq. (3) is found to be equal to 0.057 if  $p=2$ . Similar experimental results have been obtained by Gibson and Ashby (1988) in Eq. (3). The minor discrepancy with the numerical results can be explained from the fact that the material in the struts

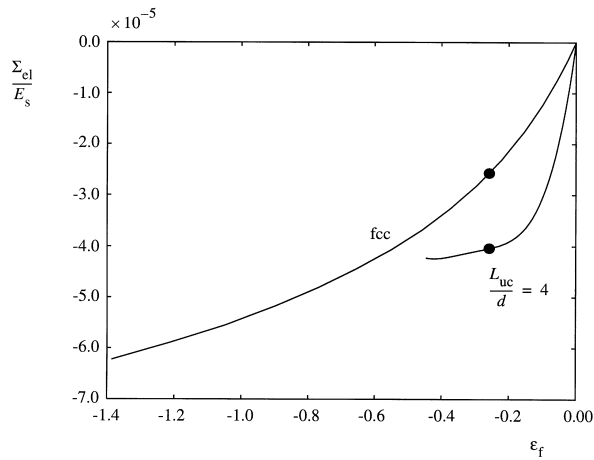


Fig. 12. Stress–strain diagram of the foam models under uniaxial compression. The deformed networks at the strains indicated by the dots are shown in Fig. 13.

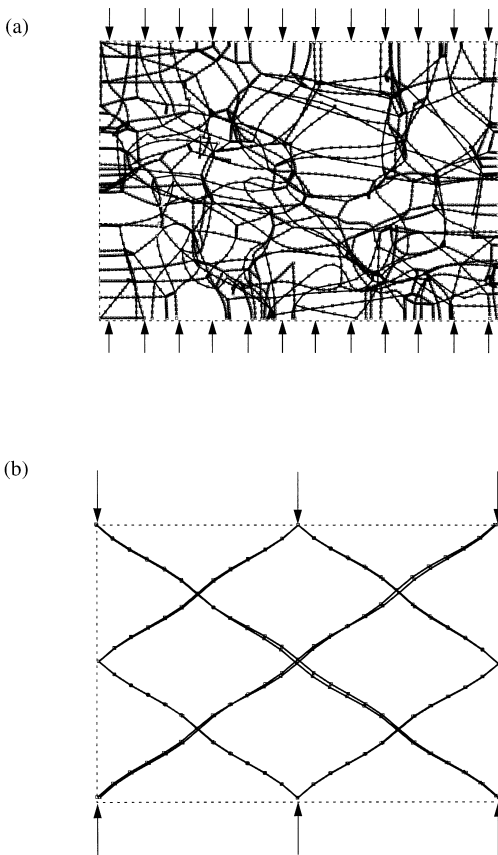


Fig. 13. (a) The random and (b) the fcc-based foam models at a compressive strain of  $\epsilon_f = -0.25$  (see Fig. 12).

of the model behave linear elastically. The model of Gibson and Ashby is implicitly linear elastic and nonlinear behavior of solid has been obscured. Real material behavior in the model would cause a lower elastic collapse stress of the model in Fig. 14. If the strut diameter increases with the relative foam density, it becomes more likely that the material in struts will behave in a nonlinear elastic fashion during buckling (it corresponds to the region of the solid material behavior when  $\epsilon_s > 0.1$  in Fig. 7). In the region of relatively high

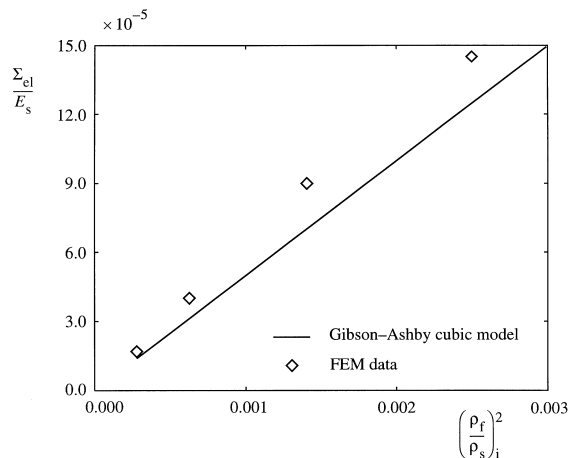


Fig. 14. Elastic collapse stress as a function of the relative foam density.

foam densities, for example, the numerical simulation point in Fig. 14 with  $(\rho_f/\rho_s)_i = 0.05$ , the relative error becomes too high and the nonlinear behavior of the solid material in the struts may not be neglected anymore.

### 3.2.3. Nonlinear elastic material in struts

In order to study foams with nonlinear strut material (lines B and C in Fig. 7), three-dimensional finite strain beam analyses must be possible with such constitutive behavior. Unfortunately, most FE codes do not have this capability. To make numerical analysis with nonlinear material behavior possible, the cross-section of the strut with radius  $r$  is discretized in radial and tangential directions, as described in Appendix A. Application of the nonlinear constitutive behavior to the solid material in the standard FE program used caused a number of numerical problems, especially when strut buckling would occur. Therefore, to investigate the influence of the constitutive behavior of the solid material in the struts, only tensile deformation of the representative random foam model with  $L_{uc}/d = 2$  has been analyzed.

The results of the simulations for the bilinear elastic material (curve B in Fig. 7) and nonlinear function (curve C in Fig. 7) are displayed in Fig. 15. Both these curves will ultimately approach the same maximum value of the stress. The effect

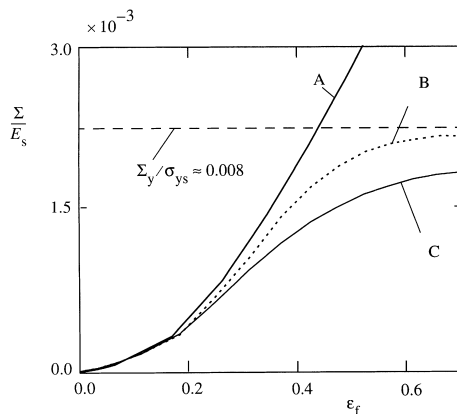


Fig. 15. Stress–strain diagram of the random model with various constitutive behaviors.

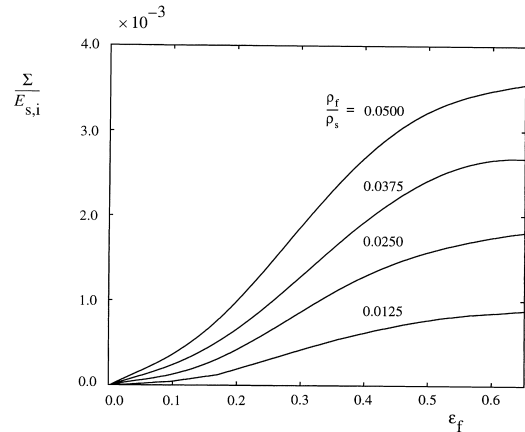


Fig. 16. Stress–strain diagrams for the nonlinear analysis.

of nonlinear solid behavior is reflected in the foam properties in a similar way as may be expected. Contrarily to the nonlinear elastic solid behavior (curve A), where the foam stiffness increases monotonically with strain (Fig. 9), the presence of a limit stress  $\sigma_{ys}$  in the solid response induces a maximum of the stiffness after some strain level. The global tensile stress asymptotically approaches some maximum value  $\Sigma_y \approx 2.3 \times 10^{-3} E_s \approx 0.82 \times 10^{-2} \sigma_{ys}$ .

The stress–strain curves for several densities of the random model with nonlinear strut material behavior (curve C in Fig. 7) are shown in Fig. 16. Not unexpectedly, the one with the higher density has also the highest ultimate strength. Moreover, there is a linear correspondence between relative foam density  $(\rho_f/\rho_s)_i$  and maximum global tensile stress.

## 4. Effective unit-cell cross-section

### 4.1. Nonlinear elasticity of foam

It was shown in the previous section that alignment of struts with the tensile direction occurs with increasing strain. Fig. 17 actually demonstrates this in terms of the deformed strut network. It suggests that the percentage of struts, aligned in the direction of the maximum principal stress due to the large strain, determines the final tensile

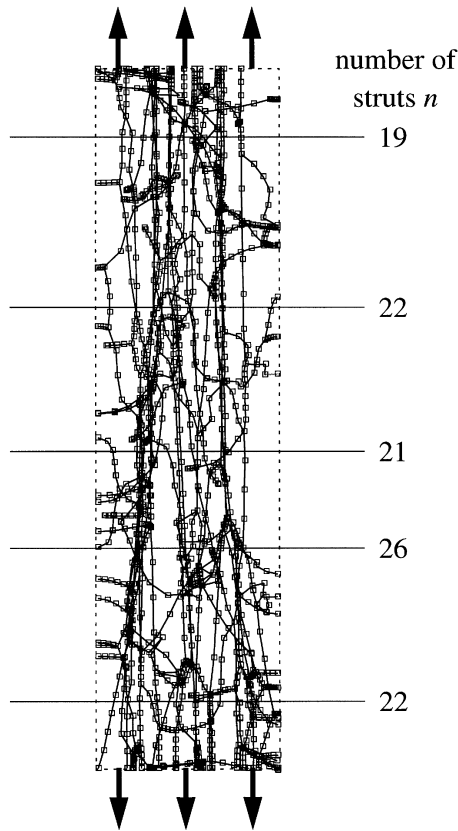


Fig. 17. Random model under a tensile strain of  $\varepsilon_f = 0.6$ , including the number of struts at various cross-sections of the model. The small squares are the nodal points in the FE mesh.

stiffness. These struts deform at large strains primarily by axial stretching.

First, the difference in stiffness of the two regular foam models – the rhombic dodecahedron (fcc) and the tetrakaidecahedron (bcc) – at large tensile strains will be explained. The first shows a much higher stiffness at large strains than the bcc-based microstructure. Then, an arbitrary cross-section through the unit-cell is made, normal to the global maximum principal stress direction. For this cross-section, the area of the solid phase in the unit-cell cross-section is the total projected area of the  $n$  struts which pass through this unit-cell cross-section and which are oriented at angles  $\alpha_i \approx 0^\circ$  to the global stress direction. This area of the cross-section is termed the effective cross-sectional area  $A_{\text{eff}} = nA$ .

In the undeformed rhombic dodecahedra unit-cell, all struts are oriented under the same angle to the global direction, which is  $54.57^\circ$ . As far as the regular structure deforms uniformly, the fcc-based strut structure at large strains has a constant effective unit-cell cross-section with area

$$A_{\text{eff}}^{\text{fcc}} = A_{\text{uc}} \left( \frac{\rho_f}{\rho_s} \right)_i$$

Alternatively, the undeformed tetrakaidecahedron contains two groups of struts oriented under  $45^\circ$  and  $90^\circ$ , correspondingly (see Fig. 5), so that the bcc-based unit-cell includes two groups of cross-sections of the unit-cell. The smallest of them is expected to determine the large deformation of foam in tension, caused by the axial deformation of the struts completely oriented in the direction of the global stress  $\Sigma_i$ . This means, that the regular tetrakaidecahedron model in tension is less stiff under the large global strains than the regular rhombic dodecahedron. The corresponding tail ends of the curves in Fig. 9(b) for the bcc- and fcc-based models support this argument.

From the above discussion it is clear that a foam model with a constant effective unit-cell cross-section will be the stiffest under large strains. As opposed to a regular model, the random model never possesses a constant  $A_{\text{eff}}$  along the unit-cell length. It is strongly dependent on the position of the cross-section in the unit-cell. It means that in any isotropic open-cell foam model, the stiffness at large strains will not exceed that of the fcc-based regular model. The effect will be more pronounced for small unit-cells. An example in Fig. 17 demonstrates various number of struts in various unit-cell cross-sections taken perpendicular to the global tensile stress direction. In this case, the model stiffness is determined by the cross-section with the lowest effective area, i.e., with the minimum number of struts in the cross-section. This is why the random model is less stiff in the large strain region than the fcc-based regular model.

The stiffness of a random model at large strains will always be lower than of the regular fcc-based model, because the random model will always contain struts that are oriented in the direction perpendicular to the principal stress. These struts

do not contribute to the effective cross-section and decrease the stiffness at large strains. For the ideal elastic model with all struts completely oriented in the principal stress under the large tensile strain, one would find

$$\frac{E_{f,t}}{E_s} = \left( \frac{\rho_f}{\rho_s} \right)_i \quad (4)$$

This is similar to Eq. (2) with  $C_1 = 1$  for a foam having a uniform cell-size distribution and no struts perpendicular to the principal stress direction. A great variation in the cell diameter can lead to a decrease of the minimum effective cross-sectional area  $A_{\text{eff}}$  and, therefore, to smaller values of the coefficient  $C_1$ . The influence of the relative foam density on the tangent modulus at large strains of the fcc-based and random unit-cells is shown in Fig. 11(b). Struts of the fcc-based structure are connected with each other in vertices as shown in Fig. 13(b) and even under large tensile strains they are bent and are not completely rotated towards the global stress direction. This leads to a lower tangent modulus of the fcc-based structure in comparison with the ideal Gibson–Ashby model with the coefficient  $C_1 = 1$  shown as the solid line in Fig. 11(b). The dashed lines in Fig. 11(b) characterize an imperfection of the model in comparison with the ideal model. In other words, the coefficient  $C_1$  in Eq. (2) can be estimated through the ratio of the minimum effective unit-cell cross-section to that of the ideal model,

$$C_1 = \frac{A_{\text{eff}}}{A_{\text{eff,ideal}}}, \quad \text{or} \quad C_1 = \frac{A_{\text{eff}}}{A_{\text{uc}}} \frac{\rho_s}{\rho_f}.$$

#### 4.2. Yielding collapse of foam

For bilinear or nonlinear elastic solid material in the struts, the insight that the struts align during deformation can be exploited to estimate the global yield stress (see Fig. 7). If the yield stress of the solid  $\sigma_{ys}$  is reached in all struts of the certain unit-cell cross-section, the macroscopic yield stress of the foam unit-cell,  $\Sigma_y$ , follows directly from equilibrium,

$$L_{\text{uc}}^2 \Sigma_y = nA\sigma_{ys}, \quad \text{or} \quad L_{\text{uc}}^2 \Sigma_y = A_{\text{eff}}\sigma_{ys}, \quad (5)$$

where  $n$  is the number of struts in the unit-cell cross-section. It means that global yield of foam in tension occurs in the unit-cell cross-section with the minimum effective area, i.e., in the cross-section with  $n = n_{\text{min}}$ . Eq. (5) can then be rewritten as

$$\Sigma_y = \frac{n_{\text{min}}A}{L_{\text{uc}}^2} \sigma_{ys}. \quad (6)$$

The normalized yield collapse stress  $\Sigma_y/\sigma_{ys}$  for the unit-cell with  $L_{\text{uc}}/d = 2$  is shown in Fig. 15 and equals nearly 0.008. Determined by Eq. (6), the foam global yield stress  $\Sigma_y$  in Fig. 15 is an asymptote for the bilinear and nonlinear elastic behavior, according to the curves B and C in Fig. 7, respectively.

A random unit-cell with a wide variation of  $A_{\text{eff}}$  may have poor mechanical properties at large strains, when it has a wide cell-size distribution. This effect has been observed by Gent and Thomas (1959), who characterized the non-uniformity of foam by the ratio of the largest observed cell diameter to the average diameter,  $d'_m/d_m$ . Samples with the highest  $d'_m/d_m$  ratio exhibited the lowest tensile strength and strain at failure. This effect can be explained by a local drop of the minimum effective cross-sectional area  $A_{\text{eff}}$ .

For foams with identical unit-cell geometries but with different relative densities  $(\rho_f/\rho_s)_i$ , only the strut cross-sectional area  $A$  changes. Since  $A$  is a linear function of the relative foam density, the dependence of the yield collapse stress in foam caused by strut yielding and the foam density  $(\rho_f/\rho_s)_i$  is given as

$$\frac{\Sigma_y}{\sigma_{ys}} \propto \left( \frac{\rho_f}{\rho_s} \right)_i.$$

It must be noted that these considerations apply only to an isotropic foam model. In the case of an anisotropic foam, the effective foam cross-section  $A_{\text{eff}}$  is dependent on the direction.

## 5. Conclusions

The linear elastic random model presented by Van der Burg et al. (1997) has been extended to perform nonlinear analyses. The foam microstructure has been generated by the application of

the Voronoi tessellation based on randomly distributed nuclei. The obtained microstructural geometry of the foam closely approximates the real geometry. The foams are subjected to uniaxial stress, either tensile or compressive, and the nonlinear analyses are accomplished by modelling the struts as beams and using standard FE techniques.

The nonlinear elastic analyses are applied to random and regular microstructures, where the Voronoi tessellation is based on randomly distributed nuclei and nuclei stacked according to the bcc and fcc distributions, respectively (see Fig. 5).

The stiffness of the regular foams is virtually constant when the strains remain sufficiently small. In this region, the struts deform primarily by bending. With increasing strain, the region is entered where the stiffness increases linearly. The stiffness of the foams is determined here by the continuously changing combination of bending and axial deformation of struts. It is illustrated that the reason for this lies in the gradual reorientation of the struts in the global stress direction. The influence of the axial strut deformation increases with increasing strain, and at a certain strain level, the global stiffness becomes constant. In this region, the majority of struts is aligned in the global stress direction and they deform axially only.

The axial deformations of struts in the random foam model are important even at very small strains. This is explained by the existence of chains of struts that are percolating the unit-cell and which are loaded axially already in the initial deformation stage [see also Van der Burg et al. (1997)].

At large tensile strains, the limiting stiffness is approached, which is found to be lower than that of the regular models. The effective unit-cell cross-section  $A_{\text{eff}}$  is an important factor for the foam stiffness under large tensile strains. The model with the highest minimum  $A_{\text{eff}}$ , or with the strongest “weak” place, will possess the highest stiffness in the large global deformations region. In this way, it can be understood why the regular rhombic dodecahedron model having a constant  $A_{\text{eff}}$  is the stiffest at large strains. The same high  $A_{\text{eff}}$  cannot be reached by a random foam model with the same foam density.

The above results are obtained for a linear elastic response of the struts. In the case of a nonlinear material behavior, the deformation of the struts will remain roughly the same, so that the ultimate axial stiffness of struts determines the final foam stiffness.

During the compressive deformation of foam, the elastic collapse stress of the random foam model is found to obey the estimate Eq. (2) of Gibson and Ashby (1988) which was based on experimental observations. The accuracy of the model can be improved by incorporating the nonlinear material behavior in compressed struts instead of the more simple linear elastic material behavior in compressed struts instead of the more simple linear elastic material used here. The importance of this effect grows with the increasing relative foam density.

Application of a simple bilinear constitutive model for the solid material in struts gives a more realistic elastomeric foam behavior than linear elastic. This increases the accuracy and applicability of the model.

## Appendix A

This appendix discusses the procedure adopted to carry out the FE analyses using the nonlinear material behavior of the struts (B or C in Fig. 7 in the main text). An incremental, updated Lagrange approach is used in which the incremental (or tangent) stiffness against tension and bending are determined from the integrated deformation history of the cross-section. This integration is carried out numerically by subdividing the cross-section into  $N_r$  by  $N_l$  segments as shown in Fig. 18. The axial strain  $\varepsilon^{(ij)}$  at segment  $(i, j)$ , with coordinates  $(x^{(ij)}, y^{(ij)})$ , follows from standard kinematics as

$$\varepsilon^{(ij)} = \varepsilon_a + \kappa_x y^{(ij)} + \kappa_y x^{(ij)}, \quad (\text{A.1})$$

where  $\varepsilon_a$  is the average axial strain of the strut,  $\kappa_x$  and  $\kappa_y$  are curvatures about corresponding local  $x$ - and  $y$ -axis. For the nonlinear constitutive relations in Fig. 7, the local tangent  $D^{(ij)}$  at this segment is a function of the local strain  $\varepsilon^{(ij)}$ . The local incremental response  $\Delta\sigma^{(ij)} = D^{(ij)}(\varepsilon^{(ij)})\Delta\varepsilon^{(ij)}$  at all segments is then used together with the incremental



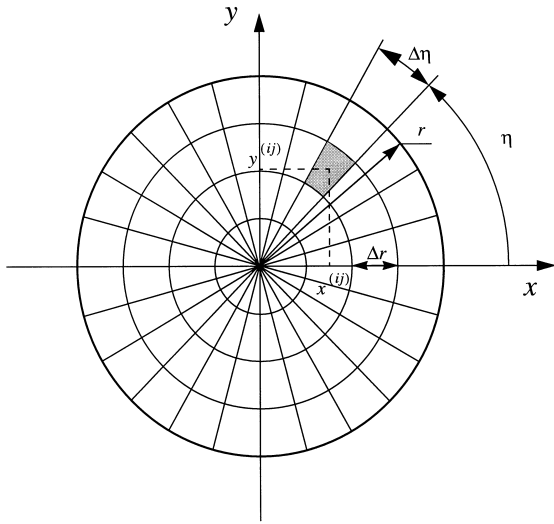


Fig. 18. Strut cross-section discretisation, where the area is divided in radial and circumferential directions. The middle of segment  $(i, j)$  (shaded gray) has coordinates  $x^{(ij)}, y^{(ij)}$ , where  $0 < i \leq N_r = r/\Delta r$  and  $0 < j \leq N_t = 2\pi/\Delta \eta$ .

form of Eq. (A.1) to determine the global incremental relations for normal force  $N$  and bending moments  $M_x, M_y$  in terms of the global incremental deformations  $\Delta \varepsilon_a, \Delta \kappa_x$  and  $\Delta \kappa_y$  through

$$\begin{aligned} \Delta N &= \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} \Delta \sigma^{(ij)} A^{(ij)}, \\ \Delta M_x &= \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} \Delta \sigma^{(ij)} A^{(ij)} y^{(ij)}, \\ \Delta M_y &= \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} \Delta \sigma^{(ij)} A^{(ij)} x^{(ij)}. \end{aligned} \quad (\text{A.2})$$

Here,  $A^{(ij)}$  is the area of segment  $(i, j)$  in the undeformed configuration (geometric nonlinearity associated with contraction of the cross-section is neglected throughout).

Because of the relatively small torsional strains found in the foam (see Van der Burg et al., 1997), material nonlinearity is neglected in the torsional resistance. Thus, the torsional stiffness is taken to be determined by the initial shear modulus  $E_s/2(1 + \nu_s)$  of the solid material.

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