Inductive types in constructive languages

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Abstract

Logic grammar is used to partly define a formal mathematical language "ADAM", that keeps close to informal mathematics and yet is reducible to a foundation of Constructive Type Theory (or Generalized Typed Lambda Calculus). This language is employed in making a study of inductive types and related subjects, as they appear in languages for constructive mathematics and lambda calculi. The naturality property of objects with type parameters is described and employed.

Cover diagram

Behold the mathematical universe,
developing from original unity
into categorical duality.

The central beam contains
the initial and the final type,
together with the remaining flat finite types.

It is flanked by the dual principles
of generalized sum and product,
and of initial and final fixed point construction.

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door

Peter Johan de Bruin

geboren op 20 september 1964 te Harderwijk

Promotor: Prof. dr. G. R. Renardel de Lavalette

Preface

The fascination for mathematical truth has been the driving force for my research as it had been for my master's thesis written in Nijmegen. Being amazed at the lack of a general language for the irrefutable expression of mathematical argument, I looked for it in the direction of what is called Type Theory. I started my research in Groningen in 1988 under supervision of Roland Backhouse, and enjoyed his discussion club with Paul Chisholm, joyous Grant Malcolm, Albert Thijs, and broadly interested Ed Voermans. When Backhouse moved to Eindhoven in 1990, I and Thijs remained in Groningen and our roads parted.

The advent in 1991 of Gerard Renardel who accepted to take up my supervision with fresh interest, started a new period of seeking to assemble all available pieces. I owe him much for his constant trust and support in keeping up courage, his help in formulating ideas, his effort to curtail outgrowing branches, and for leaving the choice to carry this work through entirely to me. I also thank Jan Terlouw for his modest cooperation, and my fellow Ph.D. students for their good company and friendship.

Now my heart has turned from abstract truth to living Truth, and since 1993 I'm living in community *Agapè* of the Blessed Sacrament Fathers in Amsterdam. I am glad to thank its members, Aad, Eugène, Gerard, Herman, Jan, Paul, Pieter, and Theo, for their support while finishing this thesis. I thank the members of the Ph.D. committee, Roland C. Backhouse (Eindhoven), Wim H. Hesselink (Groningen), Gerard R. Renardel de Lavalette (Groningen), and Michel Sintzoff (Louvain-la-Neuve), for accepting this duty and providing kind comments on the manuscript, especially Wim who gave detailed comments which helped me to prepare the final text. Though it is tough material, my presentation sometimes being terse and not all ideas given sufficient scientific support, I hope you will get a catch of its beauty.

Peter de Bruin

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