Introduction and summary

In the not too distant past, our theorizing about the nature of the Universe we live in, was not much limited by observational constraints. Consequently, no true science could be developed dealing with the nature of the Universe at large: its origin, its present state and its future. This was the realm of religion and philosophy. In this century, revolutionary developments in physics have provided the framework within which to describe the Universe as a whole and which finally made it possible to obtain tentative answers to questions we have only recently learned to ask.

In this thesis, I present investigations that deal with a small part of the theory of cosmology. In particular, I have investigated certain aspects of the theory of structure formation in the Universe. This subject has been extensively studied in the last few decennia. It originated from the realization that the Universe has not always been the same as observed at present. The Universe as we observe it today is filled with objects of a great variety of sizes and shapes. In the 2nd and 3rd decade of this century Hubble discovered that our Universe is expanding. This implies that in the past the Universe was smaller and therefore denser. All the structures we observe nowadays, if also existing in the past, would have been closer and at some time would have touched and overlapped. Furthermore, the theories that were developed to describe such an expanding Universe in quantitative detail, required that the Universe be homogeneous and isotropic, i.e. it should look the same at every position and in every direction. All mass and radiation must once have been distributed uniformly throughout space. With these theories, Gamov (1946, 1948ab) predicted that in the past the Universe must have been much hotter than presently, and that the afterglow of this epoch should still be observable as a faint radio signal at a temperature a few degrees above the absolute zero point. In the early sixties, Penzias and Wilson discovered the corresponding radiation field, at a temperature of roughly 3K (Penzias & Wilson, 1965). It soon appeared that this microwave background radiation was isotropic to a high degree, which confirmed the assumptions made about the homogeneity of the early Universe.

At present however, we see that the Universe is no longer featureless and smooth. Starting from the smallest scales we see matter organized in structures up to very large scales: from planets to stars to stellar systems to galaxies to groups and clusters of galaxies, up to super-clusters, where clusters and galaxies are organized in the largest structures known. Somewhere during the evolution of the Universe, these structures must have developed out of the featureless, uniform sea of matter and radiation. Various different theories have been developed to explain the emergence of structure, but in this thesis I will concentrate exclusively on the most generally accepted theory, that of gravitational instability. In this theory
it is assumed that in the early Universe, small fluctuations in the density were present, and these would grow under the influence of gravity towards the presently observed structures. There is actually a rather complete theory of the early stages of this process, that regime where these deviations from homogeneity are small. In that case, the inhomogeneous field may be seen as a small disturbance to the uniform model, and the standard apparatus of perturbation theory may be applied. In this thesis I investigate the later stages of this process of structure formation, where the fluctuations have grown to such a size that this ‘linear’ perturbation approach breaks down.

There is as yet no comprehensive model describing this ‘nonlinear’ regime as successfully as the linear theory describes the early stages of structure formation. Instead, the problem is approached from many different directions, using different, approximate models for describing the dynamics and other techniques for describing the resulting patterns in the matter distribution. Throughout this thesis I will argue that in fact this is the greatest hindrance for progress in this field; namely, the dynamics of the matter distribution and its structural characteristics are described using different techniques, and it is difficult to translate the results from one into the other. In the rest of this Introduction I will explain how this comes about, using the example of the linear regime, where this discrepancy does not yet exist. First I will give a more detailed description of the homogeneous Universe and then apply the perturbation approach to derive the equation governing the evolution of the density fluctuations in the linear regime. From these it is easy to see how the development of nonlinearities spoils the unity between description and dynamics, and in the following section I will give a short description of some of the models that have been used to treat this regime. At the end of this Introduction I will then give an overview of the work presented in this thesis.

1 The homogeneous universe

Since there are many excellent monographs on cosmology (e.g. Weinberg, 1972; Peebles, 1993), I will give but a brief and limited account of this field. In particular I will not discuss the very early Universe, or those scales where one needs a fully relativistic description of its evolution. I will here and in the rest of this thesis limit myself to the Newtonian approximation, both in the description of the evolution of the homogeneous Universe itself and in the treatment of the perturbed Universe. Furthermore, it will be assumed that the energy density of the Universe is dominated by matter. The Friedman-Lemaître equations governing the expansion of the Universe may then be derived very simply. Consider a sphere of radius $R$, in a uniform Universe of average density $\rho$. A point on the edge of the sphere feels an acceleration towards the center of the sphere of magnitude

$$\ddot{R} = -\frac{4\pi G R^3 \rho}{3 R^2} = \frac{GM}{R^2}.$$  

(0.1)

Here the mass $M$ is a constant, since

$$\rho \propto R^{-3}.$$  

(0.2)
in a matter dominated universe. Integrating this equation once gives

\[ \frac{1}{2} \dot{R}^2 = \frac{4\pi G \rho R^2}{3} + E = \frac{GM}{R} + E. \]  

(0.3)

The integration constant \( E \) may be interpreted as the energy of the point at \( R \) with respect to the center of the sphere. \( E \) corresponds to the constant \( k \) in the Robertson-Walker metric, that is used in the relativistic derivation of the Friedman equation (0.3) (e.g. Weinberg, 1972, § 15). In the fully relativistic treatment this same equation is derived, but the interpretation of the scale \( R \) is less trivial than in the Newtonian derivation. Since absolute coordinates do not exist in the general relativistic description of space-time, \( R \) is a scale factor, which governs the proper distance between points at constant expanding or comoving coordinate positions (see any text on cosmology). More general forms for the energy density are also allowed now. For example, in a radiation dominated universe, the density would scale with the radius as

\[ \rho \propto R^{-4}. \]  

(0.4)

Also, one may add a term \( \Lambda / 3 \), corresponding to the so-called cosmological constant, to the right-hand side of Eq. 0.1. Although this term had been out of favor since Hubble discovered the expansion of the Universe, in recent years it has gained in popularity again, due to problems with standard models for structure formation. This will be more fully discussed in Chapter 6 of this thesis.

For the matter dominated model, the Friedman equation has three different types of solutions, depending on the sign of \( E \). For \( E < 0 \) and \( E > 0 \) these can only be given in parametric form.

\[ E < 0 : \quad R(\eta) = \frac{GM}{2|E|}(1 - \cos(\eta)) \]

\[ t(\eta) = \frac{GM}{2|E|^{3/2}}(\eta - \sin(\eta)) \]  

\[ E = 0 : \quad R(t) = (9GM/2)^{1/3}t^{2/3} \]  

\[ E > 0 : \quad R(\eta) = \frac{GM}{2E}(\cosh(\eta) - 1) \]

\[ t(\eta) = \frac{GM}{2E^{3/2}}(\sinh(\eta) - \eta). \]  

(0.5)

(0.6)

(0.7)

These solutions correspond to the closed, the critical and the open cosmologies respectively. For more general models, containing mixed ratios of radiation and matter, possibly with a non-vanishing cosmological constant, Eq. 0.3 can not be solved analytically.

The critical or flat solution is the most popular models from a theoretical perspective, apart from being the simplest solution from a mathematical viewpoint. The main reason for this is philosophical. Observations tell us that our Universe is close to critical, if not exactly so. In the language of Eq. 0.3 this implies that in the present Universe, the kinetic and the potential energy, given by the term on the left-hand side and the first term on the right-hand side respectively, almost exactly cancel each other. This must then also have been so in the early Universe, when both the kinetic and the potential energy were far larger.
then they are at present. Consequently, in relative terms, these two contributions to the total energy were much closer to each other. Specifically, at the time of recombination the kinetic energy and the absolute value of the potential energy were equal to less than one part in a hundred. At even earlier times, presumably the times when the parameters of the present Universe were fixed, a mismatch of only a fraction of the value of the kinetic energy as compared to the potential energy would at present either lead to a Universe that had collapsed already a long time ago, or otherwise would have reached its asymptotic linear expansion rate, leaving no room for structure formation as will be seen below.

It was to explain this fine-tuning, among various others reasons, which led Guth (1981) to propose the inflationary universe. In this model, the very early Universe went through a phase of exponential expansion, which forced the kinetic and potential energies to be equal to less than one part in \(10^{15}\) and so solved the flatness problem (Guth, 1981; for a thorough discussion of inflation and its modern variants see Linde, 1990). A surprising aspect of the inflationary model is that it provides a way to generate density fluctuations of the desired type and amplitude to act as the seeds of the structures observed in the present Universe. It is beyond the scope of this thesis to discuss the mechanism by which this is achieved. We will instead now turn to its consequences for the theory of structure formation.

2 The inhomogeneous universe

2.1 The linear regime

On the subject of the inhomogeneous universe, there are also several excellent texts available, in particular Peebles (1980, 1993) and Padmanabhan (1993). Most of the discussion in this section will be based on the extensive review paper by Efstathiou (1990). Again we will limit ourselves to the Newtonian approximation. A detailed relativistic treatment of linear perturbations in a Friedman universe is for instance given in Weinberg (1972) and Peebles (1980).

The contents of the Universe will be described by a pressureless fluid of density \(\rho(t, \mathbf{r})\), and velocity field \(\mathbf{v}(t, \mathbf{r})\). The evolution of this fluid is governed by the equations of fluid dynamics:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad : \text{continuity equation,} \\
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla \Phi \quad : \text{Euler equation,} \\
\nabla^2 \Phi = 4\pi G \rho \quad : \text{Poisson equation.}
\]

In the Euler equation, for simplicity we have left out a pressure term, that should be included in case the matter is 'warm', i.e. has appreciable random motions. The potential \(\Phi\) is ill-defined in an infinite homogeneous universe, which is why Jeans, in his treatment of instabilities of a static universe, had to introduce his 'swindle', setting \(\Phi\) to zero in the unperturbed situation. In an expanding universe one does not need this trick. We will rewrite the fluid equations in comoving coordinates \(\mathbf{x}\), defined by

\[
\mathbf{r} = R \mathbf{x},
\]
where $x$ is the comoving coordinate and $R$ is the scale factor. It is assumed that $R$ solves the Friedman equations of the previous section. The velocity can be written as

$$v = \dot{r} = \dot{R} x + R \dot{x},$$

where $R \dot{x} \equiv R u$ is called the peculiar velocity, which vanishes in the unperturbed universe.

In the inhomogeneous universe we will separate out the homogeneous part from the density and the potential as follows

$$\rho(t, x) = \bar{\rho}(t)(1 + \delta(t, x))$$  \hspace{1cm} (0.13)

$$\Phi(t, x) = \Phi_0(t) + \phi(t, x).$$  \hspace{1cm} (0.14)

Using these definitions we may rewrite the fluid equations to (Efstathiou, 1990)

$$\frac{\partial \delta}{\partial t} + \nabla_x \cdot u + \nabla_x \cdot (u \delta) = 0,$$  \hspace{1cm} (0.15)

$$\frac{\partial u}{\partial t} + 2 \frac{\dot{R}}{R} u + (u \cdot \nabla_x) u = -\nabla_x \phi / R^2,$$  \hspace{1cm} (0.16)

$$\nabla_x^2 \phi / R^2 = 4\pi G \bar{\rho} \delta.$$

Making a Fourier expansion of the various perturbed quantities, e.g.

$$\delta(t, x) = \sum_k \hat{\delta}(t, k)e^{i k x},$$

these equations reduce to

$$\frac{\partial \hat{\delta}(t, k)}{\partial t} + bk \cdot \hat{u}(t, k) + \sum_{k'} i \hat{\delta}(t, k')(k \cdot \hat{u}(t, k - k') = 0,$$  \hspace{1cm} (0.19)

$$\frac{\partial \hat{u}(t, k)}{\partial t} + 2 \frac{\dot{R}}{R} \hat{u}(t, k) + \sum_{k'} i \left[ \hat{u}(t, k') \cdot (k - k') \right] \hat{u}(t, k - k') = 0,$$  \hspace{1cm} (0.20)

$$\hat{\phi}(t, k) / R^2 = -4\pi G \bar{\rho} \frac{\hat{\delta}(t, k)}{|k|^2}.$$

Assuming that all fluctuation amplitudes are small, $\delta(t, k) \ll 1$ etc., and assuming that the Fourier modes for different values of $k$ are independent, the complicated mode-coupling terms in the summations vanish to first, linear order. The individual Fourier modes therefore evolve independently. This is the cause of the great simplicity of the linear regime, when compared to the nonlinear regime, where these summations cannot be excluded. In the linear regime we may now derive a single equation for the density perturbation:

$$\frac{\partial^2 \hat{\delta}(t, k)}{\partial t^2} + 2 \frac{\dot{R}}{R} \frac{\partial \hat{\delta}(t, k)}{\partial t} - 4\pi G \bar{\rho} \hat{\delta}(t, k) = 0.$$  \hspace{1cm} (0.22)

This equation is linear in $\hat{\delta}$ and due to the decoupling of the different modes it will therefore also hold for the field $\delta(t, x)$ itself.
The behaviour of the scale factor $R$ in time determines the evolution of $\delta$. In the simplest case, where $\Omega = 1$ and the universe is matter dominated, $R \propto t^{2/3}$ and one finds

$$\tilde{\delta}(t, k) = a_k t^{2/3} + b_k t^{-1}.$$  
(0.23)

For the evolution of $\delta$ for different choices of the background see the review by Efstathiou (1990). Assuming that the growing mode will soon dominate the evolution, $\tilde{\delta}$ will thus grow proportional to the scale factor. This is the central problem of structure formation in the linear regime. In an expanding universe density perturbations only grow as a power law in time, compared to exponential growth in a static universe. One therefore needs substantial fluctuations in the early universe to obtain the structures we see today. It is clear that these need to be much greater than the typical $\sqrt{N}$ fluctuations one would naively expect in a uniform universe. Until the advent of the inflationary model no realistic mechanism for generating such fluctuations was known, but soon after Guth’s original paper it was realized that quantum fluctuations would be scaled up during the inflationary phase together with the other scales in the universe (Guth & Pi, 1982).

These inflationary models also allow a prediction of the statistical characteristics of the resulting fluctuation field to be made. The predictions are that the density perturbation field follows the characteristics of a so-called Gaussian random field. Such a field is defined by its Fourier expansion as in Eq. 0.18 and the constraint that the different Fourier modes are uncorrelated. The field is then completely specified by one function, the power-spectrum$^1$,

$$P(k) \equiv |\tilde{\delta}(k)|^2.$$  
(0.24)

The predicted statistical characteristics of the density perturbation field are the same that allow the linearized dynamical equations to be solved. This coupling disappears as soon as the density perturbations reach nonlinearity. Both the facts that quadratic terms can no longer be discarded and that modes become coupled complicate matters so much that a description in terms of Fourier modes is no longer useful. To date no analogue of this coupling has been developed for the nonlinear regime. Attempts at treating that regime analytically will be described in the next section.

### 2.2 The nonlinear regime

The breakdown of the linear approximation shows itself most clearly through the development of well defined, more or less isolated individual objects. For these objects the relative density contrast against the background $\delta >> 1$. This also implies that the Fourier modes of the perturbation field must be strongly coupled. Both these effects lead to the breakdown of the simple state of affairs that existed in the linear regime. There all the statistical characteristics of the complete matter distribution were described by one function, the power spectrum $P(k)$, the evolution of which could be calculated by calculating the evolution of single modes. In the non-linear regime, all modes are connected and both the dynamics and

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$^1$The study of Gaussian random fields as applied to structure formation in the Universe has been intensive and fruitful (e.g. Doroshkevich, 1970; Bardeen et al., 1986; Bond et al., 1991), but is not directly relevant for the rest of this thesis and will not be discussed here.
The statistical characteristics of the matter distribution can no longer be described in full detail. To evade these problems, models have been developed that concentrate on certain aspects of the matter distribution, both dynamically and statistically. In this section I will describe the most important of these.

The simplest and therefore most useful model is the so-called top-hat model (e.g. Gott & Rees, 1975). It describes the collapse of a uniform, spherically symmetric density perturbation in an otherwise unperturbed universe. Its evolution can be calculated along lines similar to the derivation of the Friedmann equations in § 1, where for the mass we must now take \( M = 4\pi R^3 \bar{\rho}(1 + \delta)/3 \). Usually it is assumed that initially the overdense region expands with the background. It then follows that every positive density perturbation in a critical universe will have negative total energy and will stop expanding after a finite time, \( t_{\alpha} \propto 1/\sqrt{G \bar{\rho}_0 \delta^2} \). The structure will then turn around and recollapse. This recollapse phase is not described in any detail in this model. It is usually assumed that the object will contract by a factor of order two, which will be the case when it ends up in virial equilibrium and remains uniform and spherically symmetric.

When these last two conditions are relaxed, one may still obtain some analytical results. First, Lynden-Bell (1964) and Lin et al. (1965) showed that spherically symmetric collapse is unstable under non-radial perturbation. A uniform ellipsoid will remain uniform during collapse, but its ellipticity will increase such that its shortest axis will collapse first, leading to a flat structure (see also Chapter 6). Second, a large number of authors, starting with Gunn & Gott (1972) and Gunn (1977), have investigated spherically symmetric collapse of structures with a non-trivial radial density profile. Particularly, in the case of a power-law density perturbation profile,

\[
\left( \frac{\delta \rho}{\rho} \right)(r) \propto r^{-\gamma},
\]

the equations of motion can be simplified sufficiently to allow analytical results to be obtained. It turns out that the qualitative aspects of the top-hat model are still true, but since the density perturbation is now dependent on the radius, the turn-around and collapse time will depend on radius as well. Inner radii stop expanding earlier than outer ones, but as Fillmore & Goldreich (1984) and Bertschinger (1985) have shown, at least for a range of values for \( \gamma \), all radii recollapse by the same factor afterwards, resulting in power-law density profiles.

Although the dynamics of these models is well understood, clearly they are very limited. They do not take into account the effects of structures outside the object under consideration, nor do they allow the amount of substructure that is expected in realistic initial conditions. Nevertheless, the predictions of these models have been used in various approaches, aimed at providing a more complete description of the whole matter distribution. One of the most successful of these is the approach pioneered by Press & Schechter (1974). They used the statistical properties of the initial linear perturbation field together with the dynamical properties of the top-hat model to determine the mass spectrum of collapsed objects as function of time. The form of this mass spectrum which they obtained bears much resemblance to the Schechter luminosity function (Schechter, 1976). This approach is limited to initial conditions in which the amplitude of the fluctuations is a decreasing function of the scale over which they are determined. Power-law power spectra, \( P(k) \propto k^N \)
with \( n > -3 \), are in this class of so-called hierarchical spectra, as is for instance the popular cold dark matter spectrum (Bond & Efstathiou, 1984; see also Blumenthal et al., 1984, and Bardeen et al., 1986). The main characteristic of this class of initial conditions is that structure builds up from small to larger scales, which is why they are also known as ‘bottom-up’ models.

The so-called hot dark matter spectrum (Bond et al. 1980; Bond & Szalay, 1983) is not hierarchical. This spectrum has a low mass cutoff which implies that structures on scales smaller than this cut-off must be formed by fragmentation of structures on larger scales. For such ‘top-down’ initial conditions another approximation into the nonlinear regime is more appropriate. This approximation was first introduced by Zel’dovich (1970), who rewrote the equations for the linear perturbations in a Lagrangian form. These can be traced into the nonlinear regime without running into conceptual problems such as the fact that the density becomes less than zero for \( \delta < -1 \). With this approximation Zel’dovich could show that gravitational collapse generically produces flat, two-dimensional structures, just as in the case of the uniform ellipsoid. Although this approach is really only valid in the mildly nonlinear regime, \( \delta \sim 5 \), it does explain the flattened and filamentary features in the large-scale galaxy distribution as observed in extensive galaxy redshift surveys (de Lapparent et al., 1986; Haynes & Giovanelli, 1986).

So far, none of the models above are complete in the sense of giving a fully consistent description of the matter distribution in terms of statistics that are easily accessible by observations. In cosmology, the standard approach to describe the clustering properties in the nonlinear regime is to use N-point correlation functions (Peebles, 1980 and references therein). Essentially, these functions describe the joint probabilities for finding \( N \) values for the density, \( \rho_1 \) to \( \rho_N \), at the points \( x_1 \) to \( x_N \). In particular the two-point correlation function, \( \xi_2 \), is defined by

\[
<(\rho(x_1) - \bar{\rho})(\rho(x_2) - \bar{\rho})> = \bar{\rho}^2(1 + \xi_2(|x_1 - x_2|)) .
\]  

(0.26)

Because of the assumed homogeneity and isotropy of the matter distribution the two-point correlation function only depends on the distance between the two points \( x_1 \) and \( x_2 \). A complete description of the dynamics of the density field in terms of the N-point correlation functions is provided by the so-called BBGKY hierarchy (Peebles, 1980). The equations in this hierarchy express the evolution of \( \xi_N \) in terms of the correlation functions up to \( \xi_{N+1} \). Thus, for instance, for calculating the evolution of the two-point function, one needs full information on the three-point function etc. To solve that equation one must close the hierarchy, either by finding a cut-off or by some assumption on the form of \( \xi_{N+1} \) in terms of the lower order functions. While the former of these possibilities can be achieved in the linear regime, the latter is the only option available for the nonlinear part of the evolution. The best studied assumption for the form of the higher order uses the hierarchical ansatz in which all the correlation functions of higher order than two are expressed in terms of the two-point function. For example, the three-point function may be written as

\[
\xi_3(x_1, x_2, x_3) = Q_1(\xi_2(x_1, x_2)\xi_2(x_1, x_3) + \xi_2(x_1, x_2)\xi_2(x_2, x_3) + \xi_2(x_1, x_3)\xi_2(x_2, x_3)) + Q_2(\xi_2(x_1, x_2)^2 + \xi_2(x_1, x_3)^2 + \xi_2(x_2, x_3)^2) ,
\]  

(0.27)
while the general N-point function would contain products containing N-1 two-point functions

\[ \xi_N \sim \xi_2^{N-1} . \]  

(0.28)

Using this and other assumptions, Davis & Peebles (1977) were able to solve the BBGKY equation for \( \xi_2 \), but as will be shown in Chapter 4, their conclusions are incorrect. Since also much higher orders of the correlation hierarchy are needed for describing the observed structures in the Universe than the two or three-point functions, there is not much hope of extending the analysis of the BBGKY hierarchy.

The previous example serves to emphasize the problems for finding a consistent approach, that would treat both the statistical description and the dynamics of the matter distribution in the nonlinear regime. It may also serve to emphasize the fact that even on the phenomenological side the nonlinear regime is incompletely known. It is hard to assess the relevance of models that should describe limited aspects of the dynamics, for the actual evolution of clustering in the Universe. And while an essentially complete statistical approach is provided by the correlation function hierarchy, it is difficult to estimate N-point correlation functions already for \( N \gtrsim 4 \), while much higher orders are needed for the description of the clustering process alone. To cure this last problem, many different statistics have been proposed and applied to observations and results of N-body simulations. Most of these are aimed at describing specific aspects of the observed galaxy distribution. Some examples are percolation and genus statistics (e.g. Gott et al., 1987; Shandarin & Zel’dovich, 1989, and references therein), for analyzing the connectivity of the filamentary clustering pattern, fractal and multi-fractal statistics (Coleman & Pietronero, 1992; Martínez et al., 1990), for analyzing its scaling and self-similar properties and void spectra (Kauffman & Fairall, 1991), for analyzing the properties of the extended regions of low galaxy density (not the void probability distribution). In contrast to these other statistics, the void probability distribution essentially gives a complete description of the clustering process (White, 1979; Chapter 2). For this reason and for its ease in estimating it in practice, the void probability has received increasing attention in recent years. And while the other statistics have no dynamical basis, a theory has been developed for the void probability, which predicts its form from thermodynamical arguments applied to gravitational clustering (Saslaw & Hamilton, 1984). This result will be discussed further in the next section, in which I will give an overview of the work presented in this thesis.

2.3 This thesis: summary of main results

As explained in the previous section, we do not have a working theory that explains the characteristics of the galaxy distribution from dynamical arguments, using the same statistical measures that quantify these characteristics. Such a framework is not developed in this thesis either. Instead I have tried to add to the understanding of the phenomena, which ultimately have to find a place in such a theory of the nonlinear clustering regime.

This thesis can be subdivided in two parts, corresponding to the two sides of the problem. The first three chapters deal with the statistics of the clustered matter distribution, the last three chapters deal with its dynamics. On the statistical side one may again distinguish two distinct subjects. One deals with developing statistical measures for describing the galaxy
distribution, either completely or partially. The other deals with the problems associated with estimating these statistics in practice. The first chapter deals with the last problem, the second and third with the first.

In Chapter 1 the conditional density is investigated as an alternative to the two-point correlation function. The conditional density gives the average run of the density around a typical galaxy. It has been argued that the conditional density is superior to the two-point correlation function in providing a more sample-independent description of galaxy correlations and the approach to homogeneity (Pietronero, 1987). This is demonstrated by applying both statistics to two different synthetic distributions of points: a Lévy flight fractal with no intrinsic scale and a Voronoi tessellation with points distributed uniformly on the walls of cells having a well defined mean size. In the first case, the conditional density is a well-defined power law out to the limits where it can be reliably estimated, while the two-point correlation function approaches zero on a length scale which is sample dependent giving a false indication of homogeneity. In the second case the conditional density is a power law with the correct exponent (-1) out to the mean cell-size where it flattens correctly indicating the approach to homogeneity, while the two-points correlation function yields an incorrect estimate of the exponent (-1.8) and no obvious feature connected with the mean cell size. These synthetic distributions are also used to test two suggested methods of correcting for the boundaries of a finite sample, and it is shown that the standard method of normalizing pair counts by a Monte Carlo routine is the most accurate and unbiased. This method is then applied to the CfA-14.5 catalogue where the edge correction allows us to extend the estimate of the conditional density beyond that of previous work (Coleman et al., 1988) to scales comparable to the depths of various volume limited subsamples. The result is that the galaxy distribution apparently crosses over to homogeneity on a scale of roughly 30 Mpc and that on smaller scales the conditional density is a power law with an exponent closer to -1 (as for the Voronoi tessellation) rather than the traditional -1.8.

Both the conditional density and the two-point correlation function provide only a limited description of the statistical characteristics of the galaxy distribution. As explained above, using the technique of counting probabilities should help to obtain information also about the higher order clustering properties. White (1979) showed how to relate the single-volume counting probabilities to the correlation functions. In Chapter 2 this description is completed. There I derive the relation between the correlation functions and the multi-volume counting probabilities. It is shown that these probabilities are generated by the multi-volume void probability functions, just as this was the case for single volumes. This gives an illustration and extension of the well known result, that the void probability function describes the statistical properties of a point process completely (Daley & Vere-Jones, 1988).

These results are then used to derive various quantities of statistical interest, namely the moment structure and the characteristic and probability generating functions. These last are used to investigate the continuum limit, the limit where the expected number of points approaches infinity. It is shown, for instance that one may thus reproduce the whole probability structure of Gaussian random fields. It is then shown how the formalism of multi-volume counting probabilities may be easily applied to derive some interesting relations.
First, the Limber equation, relating angular to spatial correlation functions, is rederived. A modification to the standard derivation is made, by explicitly taking into account the fact that galaxies on the sky are observed from within a galaxy. It is shown that this introduces extra terms in the usual Limber equation. These correction terms are estimated to be less important for deeper samples, but an analysis of two synthetic point sets shows that some effect remains, even at depths large with respect to the correlation length. Formally these terms must be taken into account when inverting the observed angular correlation function to obtain information about the spatial function. Second, the continuum limit for Gaussian point processes is used to calculate the probability distribution for the density of volumes which are imbedded within larger volumes. It is shown that this offers an elegant way to reproduce the results by Bower (1991) in his analysis of the cloud-in-cloud problem, which furthermore should make it possible to extend his results to include multiple mergers. Such an extension might be included in the Monte Carlo approach to merging histories as developed for instance by Kauffmann & White (1993).

Another application from the relation between the correlation functions and the counting probabilities is introduced in Chapter 3. There it is shown how to use these results to constrain stochastic models for the counting probabilities. Although each point process has associated to it a full set of counting probabilities, it is not true that to each model for the counting probabilities one can associate a point process, let alone that it may be of cosmological interest. It is in general not easy however to find constraints which allow one to assess such stochastic models for their cosmological applicability. It appears that the relation in which the probability distribution is generated by the void probability function can be interpreted as just such a constraint. In Chapter 3 this relation is further investigated and interpreted. Then it is applied to two of the most popular models for the single-volume counting probabilities for galaxies in the Universe, namely the ‘gravothermal’ distribution (GTD) and the negative-binomial distribution (NBD). The GTD was derived by Saslaw & Hamilton (1984), from a thermodynamical theory of gravitational clustering in the non-linear regime. The NBD does not have a physical justification and is mainly used as a fitting model.

Here it is shown that both models can not be linked to cosmologically interesting point processes. In Chapter 3 it is shown that the GTD as originally proposed by Saslaw & Hamilton (1984) can only be realized by a Poisson distribution of point clusters. Such a process is explicitly excluded from their derivation, and their thermodynamic theory of galaxy clustering can thus not be correct. But even generalized versions of the GTD-model are inconsistent with the constraints imposed by the relations in Chapter 2. Nevertheless, the GTD gives surprisingly good fits to the counting probabilities of one of the cosmological simulations, namely the one with white-noise initial conditions. It is shown in an Appendix to Chapter 3, that this may be understood with a discrete version of the Press-Schechter theory. The NBD, which fits the simulations less well than the GTD, is also shown to be linked to non-interesting point processes. Here two possible realizations may be found, one a Poisson point cluster process, the other a so-called mixed Poisson process, where each individual realization is a Poisson process, but the average density is chosen randomly.

In Chapter 4 I start the investigation of the dynamics of nonlinear gravitational clustering. In that chapter, the previrialization hypothesis is investigated, in which it is claimed
that the top-hat model does not give an adequate description of gravitational collapse of clusters which contain abundant sub-structure and lie imbedded in larger structures. Analyzing two N-body simulations using three different approaches, I find this claim to be unjustified. First of all, from the evolution of global statistics and secondly from the collapse of individual clusters I find evidence that both qualitative and quantitative predictions of the top-hat model are borne out very well. This includes various scaling relations that may be calculated from the statistics of the initial density perturbation field. In a third approach the clusters are compared to isolated collapse simulations from the same initial conditions in order to remove the effects of larger scale structures. To remove the effects of internal structure I also evolve these proto-clusters after they were averaged on spherical shells. Both the evolution of characteristic radii and the final radial structure of the clusters are very similar, further strengthening the conclusion that previrialization is not a significant effect.

I propose and test a heuristic picture to explain this fact. In this scenario the non-radial velocities that are induced cause the growth of sub-clumps that quickly virialize themselves and afterwards move in the large scale gravitational field as individual particles, in a pattern that does not deviate much from a spherical one on larger scales. This picture is equivalent to the main assumption made in the well known Press-Schechter theory of structure formation.

In Chapter 5 the analysis of the N-body simulations is extended by investigating the structural properties of the collapsed and virialized clusters. In particular, the radial density profiles are compared to a set of models which gives a good approximation to the density profiles of elliptical galaxies (Tremaine et al, 1993; Dehnen, 1993). Recently it was shown that one of these models, earlier introduced by Hernquist (1990), also gives very good fits to the dark matter component of rotation curves of spiral galaxies (Sanders & Begeman, 1994). Such density profiles are thought to be the result of violent relaxation (van Albada, 1982). This in contrast to the results from semi-analytical asymptotic infall calculations for power-law density fluctuation spectra. These are able to follow the relaxation phase, and predict power-law density profiles for the resulting clusters, where the precise form depends on the initial conditions (Gunn, 1977; Fillmore & Goldreich, 1984; Bertshinger, 1985; Zaroubi & Hoffman, 1993).

In the previous chapter it was found that the collapse phase of the clusters is not as well described by these models as the early stages. Here we show that also the predictions for the resulting density profiles are not borne out well. Instead, the alternative view, that the final structural parameters of the clusters are determined by violent relaxation processes, resulting in de Vaucouleurs-type profiles finds support from the present work. The cluster density profiles are all described very well by the Hernquist model, independent of the precise initial conditions. We find furthermore a strong correlation between the mass and the radius of the cluster. This correlation closely resembles similar correlations in the literature, such as Fish' law for elliptical galaxies (Fish, 1964), the correlation between mass and characteristic radius for Abell clusters found by West et al (1989) and more recently the correlation between dark halo mass and radius determined by Sanders & Begeman (1994). This correlation may depend on the initial conditions, but with only two simulations this could not be investigated in more detail.
We therefore propose the following picture, where the form of the clusters (dark halos, ellipticals) is determined by violent relaxation, leading to a universal density profile, and where the precise form of the correlation between the structural parameters may be set by the initial conditions. This may also explain the earlier results on the rotation curves of halos from power-law simulations (e.g. Efstathiou et al., 1988; Warren et al., 1992). There it was seen that for higher values of the exponent $n$ in the initial power-spectrum $P(k) \propto k^n$, the rotation curves were declining more rapidly. This was interpreted as indicating power-law density profiles $\rho(r) \propto r^{-\gamma}$, with higher values of $\gamma$. This same phenomenon will also be found however with a universal density profile, but with a characteristic radius which is smaller relative to the mass, for higher values of $n$. There is some indication for this from the work presented here, but for finding this dependence on the initial conditions more simulations must be investigated, starting from a greater range of values for the power-law exponent $n$.

Finally, in Chapter 6 I investigate how we can use the characteristics of structure formation to tell us something about the background cosmological parameters. In particular I investigate the influence of a non-zero cosmological constant on the evolution of anisotropy in overdensities that grow by gravitational collapse. The claim that a positive value of $\Lambda$ might produce stronger asphericities is considered by following the collapse of homogeneous spheroids imbedded in Friedman-Lemaître backgrounds. It is shown that, whereas the calculations indeed show this effect, it is small for values of $\Lambda$ allowed by the classical cosmological tests. When we do not limit ourselves by constraints arising from a choice of an initial fluctuation spectrum, structures in an open universe ($\Omega_0 = 0.1, \lambda_0 = 0$) can be reproduced in a flat Lemaître universe ($\Omega_0 = 0.1, \lambda_0 = 0.9$) for a large part of their evolution. This includes their peculiar velocity structure. From initial conditions which are not too extreme, these two world models are both able to produce strong anisotropies that easily persist for a Hubble time. This stability of flattening is the only aspect in which these two models differ significantly from the Einstein-de Sitter model. Taking into account the crudeness of the model together with the fact that observed large scale structures are not isolated, it is concluded that using this aspect of the dynamics of structure formation is not a promising method of placing tighter constraints on the value of $\Lambda$.

Herein lies the main importance of the use of such simple models: effects such as those of the background can be easily estimated and are not drowned in the complexities of realistic structures, irrespective of how well the model approximates the structures in the real Universe.

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