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## Symmetries in string theory

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# Chapter 6

## Discussion

The main part of this thesis, and indeed the main part of my research in the last four years, has been concerned with the investigation of  $W$ -strings. Whereas the world-sheet description of ordinary strings is based on conformal invariance,  $W$ -strings are based on higher-spin extended conformal invariance. We only considered a special class of  $W$ -strings: those based on free scalar realizations of  $W_N$  algebras. One of our main results is that we found redefinitions of the classical constraint algebras associated with  $W_N$  gauge symmetry that simplify the BRST analysis. These redefinitions can also be interpreted as canonical transformations in the extended phase space including the ghosts. In the case of the critical  $W_3$  string [135], the non-critical  $W_3$  string [21, 18] and the critical  $W_4$  string [20, 40] the simplifications have also been found to apply at the quantum level. In particular, this elucidated a similarity of critical  $W_N$  strings to (non-critical) Virasoro strings. From this and especially from a lot of work that had been done before on the  $W_3$  string, the overall picture of  $W$ -strings, at least concerning a number of issues such as the spectrum, seems to be clear. Summarizing, we can say the following about the status of these  $W$ -strings:

- The only known realizations with critical value of the central charge in terms of scalar fields only, are the Miura realizations or multi-scalar generalizations thereof. In particular, it is impossible to build a genuine *critical*  $W$ -string based on string coordinates without background charges.
- It is not clear whether  $W$ -strings can be interesting from a phenomenological point of view. For example, space-time interpretations are not straightforward like those for the bosonic and superstrings. However,  $W$ -strings and especially non-critical  $W$ -strings may allow one to study two-dimensional ( $W$ -)gravity coupled to matter with central charge greater than one, or non-critical strings in  $D > 2$ . An example is the  $D = 4$   $W_3$  string discussed extensively in [47].
- The spectra of critical  $W$ -strings show a remarkable resemblance to Virasoro string spectra. This is mostly due to the special structure of the Miura realizations of

$W$ -algebras. Nevertheless, the spectra have quite a rich structure. The spectrum of a critical  $W_N$  string is closely connected to the spectra of non-critical  $W_n$  strings for all  $n = 2, 3, \dots, N$  where the matter sector is the  $(N, N + 1)$  unitary  $W_n$  minimal model.

- It would be interesting to find different realizations of  $W$ -algebras such that the corresponding string spectra do not reduce to several effective Virasoro string sectors. However, this seems impossible to realize with just scalar fields [83]. Other types of realization are then needed. Perhaps such new realizations can be found using the linearizing conformal algebras of [130].

The relations that have been found between  $W_N$  strings for different  $N$  are in some sense similar to the Berkovits-Vafa construction and its generalizations to hierarchies of superstrings and strings based on linear  $W$ -algebras. In both cases, certain strings in special realizations are shown to be related to other strings based on different world-sheet gauge symmetries. In the Berkovits-Vafa construction the relation has been shown to be an equivalence, at least concerning the physical spectrum [121]. In this light, some recent observations in supersymmetric field theory are also worth mentioning. It has been argued for certain  $N = 1$  supersymmetric field theories that different dual descriptions of the same theory can have a different gauge symmetry [177]. All this reminds us of the fact that gauge symmetry is not a physical symmetry but rather a symmetry of the description. For example, the Hilbert space of physical states of a theory comes in representations of the global symmetry group but not in representations of any gauge group. Indeed, the gauge symmetry is divided out to obtain the physical spectrum.

Canonical or similarity transformations have proved to be a useful tool to show relations among string theories. Also in the context of duality symmetries canonical transformations may elucidate certain equivalences in string theory (see e.g. [4, 125, 95]). For example, in section 5.2 we described all  $O(d, d; \mathbb{Z})$   $T$ -duality transformations for a toroidally compactified string in the language of canonical transformations.

However, canonical transformations in the two-dimensional sigma model description of strings can probably not say much about possible nonperturbative duality transformations. Nowadays there are many examples of nonperturbative strong/weak coupling dualities, and they have passed many non-trivial tests. In section 5.3 we discussed two examples of strong/weak coupling duality. In the first example [42] we studied, at the level of low-energy effective actions, the relation between string  $S$ -duality and fivebrane  $T$ -duality and vice versa. It turns out that vector fields in the ten-dimensional effective action destroy this relation. See also [176, 180]. In the second example [24] we studied a six-dimensional reduction of the type IIB effective theory. We used an  $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$  symmetry of this model to generate dyonic string solutions with nonzero Ramond Ramond charges. These solutions interpolate between fundamental string and solitonic string (compactified fivebrane) solutions.

New results on dualities are appearing at a fast rate and now the view is favoured that the known superstring theories correspond to different perturbative descriptions of the same nonperturbative ‘superstring’ theory. Interestingly, other extended objects

and eleven-dimensional supergravity seem to play prominent roles in this picture. For a relatively recent review on string dualities, we refer to [157]. It seems that these developments are already leading to a better understanding of (nonperturbative) string theory.