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Symmetries in string theory

Boonstra, Harm Jan Hugo

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Chapter 1

Introduction

The physics of elementary particles is currently described in terms of a very successful theory called the standard model. It describes all known elementary particles and their interactions except gravitational interactions. The standard model accommodates the quarks and the leptons which are the constituents of matter, the vector particles that mediate the strong and electroweak forces, and the Higgs boson which is expected to account for the masses of the particles. So far, the standard model agrees, often to great accuracy, with all experimental findings.

The only phenomena that are known not to fit within the standard model are those involving gravity, for which there is another successful theory: Einstein's general relativity. Because of its weakness, gravity is completely negligible in all particle scattering experiments that are done to test the standard model. It is because gravity is a long-range attractive force that we can see it act between macroscopic systems.

In both general relativity and the standard model, the interactions are governed by the principle of gauge invariance. Gauge invariances are symmetries of the equations of the theory under a group of local transformations, i.e. transformations that may vary from point to point in space-time. One can think of this as the freedom to choose a reference frame (in space-time or in some internal space) independently at each space-time point. For general relativity the underlying gauge invariance is the symmetry under general space-time coordinate transformations. In the standard model, the gauge symmetry is internal, i.e. it acts on fields rather than on space-time coordinates. The gauge group of the standard model that governs the strong and electroweak forces is $SU(3)_s \times SU(2)_{ew} \times U(1)_{ew}$. It is spontaneously broken to $SU(3)_s \times U(1)_e$ at low energies by the Higgs mechanism whereby three vector bosons of the electroweak gauge group acquire a mass and the photon of the electromagnetic interaction that corresponds to the unbroken $U(1)_e$ remains massless. A consequence of gauge invariance is that certain degrees of freedom in the description are redundant, i.e. not physical. However, they are needed for a symmetric description of the theory.

An essential difference between the standard model and general relativity is that the standard model is formulated as a quantum field theory whereas Einstein's general relativity is a classical field theory. This causes conceptual difficulties in trying to combine particle physics with gravity. One might try to couple quantum matter to classical gravity, but this seems a strange thing to do and it might not even be consistent. On the other hand, attempts to formulate gravity as a quantum field theory have failed because of the nonrenormalizability of gravity. The nonrenormalizability of gravity stems from the fact that Newton's constant G_N has negative mass dimension. (In units where $\hbar = c = 1$, G_N has dimension $(\text{mass})^{-2}$, see equation (1.1) below.) In the usual perturbative approach, dimensional analysis then shows that in higher loop Feynman diagrams involving gravitons, divergences become more and more serious.

Still, one can argue that quantum gravitational effects become important at the energy scale set by the Planck mass, which is the mass constructed out of Planck's constant, the speed of light and Newton's gravitational constant,

$$M_P = \sqrt{\frac{\hbar c}{G_N}} \approx 10^{19}\,\mathrm{GeV}/c^2\,. \tag{1.1}$$

Although such energies are far beyond those that one can hope to reach in particle accelerators, theorists are still very much interested in finding a consistent theory of quantum gravity. One of the motivations is the hope that a quantum theory of gravity might be united with particle physics into a single theory of all particles and forces. For other motivations and a comprehensive review of several issues in quantum gravity, see reference [120].

Notwithstanding the great experimental successes of the standard model, there are several reasons to expect that it is not the ultimate theory for particle physics. First of all, as we discussed above, quantum effects of general relativity can no longer be ignored as we approach the Planck scale, and gravity should somehow be coupled to the rest of the theory. Secondly, there are reasons that stem from the standard model itself. The standard model depends on many parameters, such as the different particle masses, mixing angles and coupling constants, whose values have to be put in by hand to fit experimental data. This is not something expected (or at least wanted) from a fundamental theory. Instead, one might believe the standard model to be an effective low-energy theory of a more fundamental underlying theory. More fundamental means that the underlying theory has a larger domain of validity, extending to smaller distances, or equivalently, to higher energies that have not (yet) been explored by particle accelerators. An effective low-energy theory can (in principle) be obtained from its underlying theory by integrating out all fluctuations above a certain energy.

As possible underlying theories, so-called grand unified theories (GUTs) have been proposed. A grand unified theory unifies the strong, weak and electromagnetic forces in the sense that they all become part of a larger gauge group with a single coupling constant. One indication for grand unification comes from renormalization group calculations, which show that the three effective (running) coupling constants corresponding to the three factors of the standard model gauge group meet approximately at a single energy $M_{GUT} \approx 10^{15}\,\mathrm{GeV}$. Below this scale, the GUT gauge group should be broken to the standard model gauge group in much the same way that the electroweak gauge group $SU(2)_{ew} \times U(1)_{ew}$ is broken to the electromagnetic gauge group $U(1)_e$ below the weak scale $M_W \approx 100\,\mathrm{GeV}$. However, in [6] it was calculated, starting from more precise data for the coupling constants at the weak scale, that the running couplings in the standard model miss each other by several standard deviations. It was also shown that in a modified version of the standard model, the minimal supersymmetric standard model (MSSM), the three running coupling constants meet quite accurately at an energy scale of about $10^{16}\,\mathrm{GeV}$, thus supporting supersymmetric grand unification[1]. This somewhat higher energy scale is also favoured in view of the present experimental limit on the proton lifetime. Note also that it is 'only' three orders of magnitude below the Planck energy.

The minimal supersymmetric standard model has a particle content which is richer than that of the standard model. This is due to supersymmetry, which associates to any boson a fermion and to any fermion a boson. Thus, in the MSSM all particles of the standard model are accompanied by a superpartner of opposite statistics, such that the theory is invariant under their interchange. However, none of these superpartners has ever been observed. Therefore, supersymmetry, if it exists, has to be broken at low energies. The favoured energy at which this is supposed to take place is on the order of $1000\,\mathrm{GeV}$.

One of the original motivations for supersymmetry is that it softens the divergences of quantum field theory. This may make the renormalization procedure seem less artificial. For example, in the standard model the Higgs boson is expected to have a mass not too far from the weak scale, but it gets radiative corrections of leading order Λ, which is the cut-off representing the scale at which new physics appears, say M_{GUT} or the Planck scale. This is not inconsistent but it is unnatural, since renormalization requires fine tuning of parameters to obtain the correct physical value. Supersymmetry solves this unnaturalness problem, since the leading divergent contribution to the boson's mass vanishes as a consequence of cancellations between bosonic and fermionic contributions. From a technical point of view, let us mention that supersymmetry often makes a theory more tractable. Nonrenormalization theorems and other special properties induced by supersymmetry may sometimes be used to compute exact results for the quantum theory. A recent example that received a lot of attention is [178].

In a (supersymmetric) grand unified theory, the strong and electroweak gauge interactions are unified. However, they still contain many a priori undetermined parameters and moreover, gravity is still not taken into account. In summary, we are looking for a unified theory for all interactions including (quantum) gravity with as few parameters as possible that reproduces the standard model at low energies and is preferably supersymmetric. Superstring theory seems to be the best candidate for such a theory. So let us review some basic properties of string theory.

String theory is an approach to describe all phenomena in terms of strings, i.e. one-dimensional objects. This is to be contrasted with the usual picture of quantum field theory as a second quantized theory of point-particles, described as elementary excitations of the fundamental fields. String theory introduces one new parameter: the string scale M_{str}. Its inverse[2] is the typical length of an oscillating string. However,

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[1] See [77] for a recent account.

[2] In units with $\hbar = c = 1$ length (or time) has dimension $(\mathrm{mass})^{-1}$.

string theory contains no other parameters like those of the standard model. Unlike a point particle, a string carries many degrees of freedom, corresponding to its possible modes of oscillation. An interesting idea of string theory is that different elementary particles are manifestations of a single string oscillating in different modes, much as different vibrations of a violin string give rise to different tones. This is a very elegant way of unifying all elementary particles. Clearly, this means that strings should be very tiny since there are no indications for any stringy extendedness from experiment. A lot of excitement was caused by the recognition [197, 170] that one of the excitations of a closed string corresponds to a massless spin-two particle with the properties of a graviton. For this particle to have also the correct coupling strength of gravity, the length scale of the string should be of order the Planck length (the inverse of the Planck mass (1.1) in $\hbar = c = 1$ units), $l_P \approx 10^{-33}\,\mathrm{cm}$. As string theory is a quantum theory, it naturally includes quantum gravity.

Strings can be either open or closed. Open strings have two ends whereas closed strings are closed loops without ends. A further differentiation among string theories is provided by the internal degrees of freedom they may carry. The simplest string theory is the bosonic string. As its name suggests, its spectrum (the Hilbert space of allowed excitations) contains only bosonic degrees of freedom. Therefore, it is impossible for this simplest string theory to accommodate a theory like the standard model. However, the bosonic string is studied extensively because it is a good starting point to learn about string theory and to investigate some of its general properties. The consistency of the bosonic string can be questioned though, since its spectrum contains a particle of negative mass squared, a tachyon.

The tachyon is eliminated by the introduction of supersymmetry in string theory. Hence, supersymmetry is probably required in a consistent string theory. The superstring also necessarily contains fermionic degrees of freedom. Moreover, certain versions of the superstring turn out to contain 'standard model like' theories as their four-dimensional low-energy field theory limits. The superstring has many attractive features. Its ultraviolet properties are better than those of most field theories. It is known that one-loop amplitudes in superstring theory are finite. For higher loops there are no complete results, but supersymmetry properties provide good hope that the superstring is finite to all orders. This is good, especially since one might fear renormalization problems in a theory that contains quantum gravity. An intuitive way of understanding the mild ultraviolet behaviour of string theory is to think of the interactions among strings to be spread out along the length of the strings. In this sense string theory has a natural short-distance cut-off built in. Instead of a perturbation theory in terms of Feynman diagrams, the perturbation series of string theory consists of a sum over topologies of two-dimensional surfaces (note that as a string moves through space-time it sweeps out a two-dimensional surface, the world-sheet). Whereas in quantum field theory the number of Feynman diagrams grows rapidly with the order in the perturbation parameter, in string theory there is only one process to consider at any order (at least for a theory containing closed strings only).

To give an honest picture of string theory, we should also mention some problems encountered in its study. The most common criticism is the lack of possible experiments to test string theory. This is a problem, since the genuinely string-like aspects of the theory are believed to appear only near the Planck scale. Nevertheless, string theory makes predictions also for the low-energy theory, for example that supersymmetry exists. It might be that supersymmetric partners will be detected by the next generation of particle accelerators (e.g. CERN's LHC starting around 2004 will look for supersymmetric partners besides the Higgs boson(s)). Other exotic particles may also be predicted by string theory. This takes us immediately to another, this time calculational problem of string theory. There are many ways of obtaining realistic low-energy effective theories from string theory. However, there is no principle within the present formulation of the theory that selects one possibility out of the many. If we knew this principle we would probably also know the precise low-energy predictions made by string theory and experiments could be planned to verify or falsify the theory. It is also not clear how supersymmetry should be broken and how the cosmological constant can be small or zero after supersymmetry breaking. As string theory contains quantum gravity and a priori makes no reference to four dimensions, it should even 'explain' why we experience the world as four-dimensional and not otherwise. All these questions cannot yet be answered in string theory.

Most of these problems are probably due to our present imperfect formulation of string theory. In the usual world-sheet approach to string theory, one describes a first-quantized string moving in a target space-time of fixed geometry. Eventually however, the geometry should be determined dynamically by the strings themselves. Perhaps these matters have to await the formulation of a string field theory. Another shortcoming of the present understanding of string theory is the fact that it is only known perturbatively in the number of quantum loops (number of handles of the world-sheet). On the other hand, the fact that the string perturbation series diverges (at least for the bosonic string) should be understood as an indication that nonperturbative effects are very important [105].

In this thesis we discuss some of the different symmetries that occur in string theory. We have seen that in the search for a possible unified theory more and more symmetry is introduced to improve certain properties of a theory or to decrease the number of parameters. String theory has many symmetries that play an important role. In the world-sheet approach one considers a two-dimensional field theory which is reparametrization invariant. This can be considered a gauge symmetry of two-dimensional coordinate transformations, i.e. the gauge symmetry of two-dimensional gravity. It is natural to ask if we can base a string theory on an even larger world-sheet gauge invariance. In this thesis we address this question for a certain class of extended world-sheet symmetries. For simplicity, we only consider bosonic string theories. These involve bosonic degrees of freedom only. String theory also has many space-time gauge symmetries. Undeniably, there is some relation between world-sheet and space-time gauge symmetries. For example, world-sheet reparametrization invariance imposes certain restrictions on the physical spectrum of excitations of space-time fields. These give the massless spin-two particle the gauge properties of a graviton. Besides gravity, there are many more space-time gauge symmetries in string theory. For example, the massless sector also includes the gauge symmetry of an antisymmetric tensor field and possible Yang-Mills fields. Recently, there has been much interest in what are believed to be discrete gauge symmetries of string theory. These so-called duality symmetries map a string theory in one background to the same string theory in another background or even one string theory to another. In the last chapter of this thesis some of these duality symmetries are discussed.

To get more grip on a complicated theory, an approach that is often useful is to reduce the number of degrees of freedom in such a way that as many as possible features of the original theory are preserved. In string theory, a way to do this is to reduce the number of dimensions of the target space-time in which the string moves. Usually, consistency demands that a string can only move in a target space-time of definite 'critical' dimension; $D = 26$ for the bosonic string or $D = 10$ for the superstring. However, it is possible to reduce the number of space-time dimensions D at the cost of introducing a new field which should be regarded as a component of the two-dimensional world-sheet metric [159]. The resulting string theories are called non-critical. So far, only results for non-critical strings in $D \leq 2$ have been obtained. In this case, even nonperturbative calculations can be performed using the discrete approach of matrix models. It would be very interesting to see if such results could be extended to higher values of D. The problems for $D > 2$ can be traced back to the representation theory of the conformal algebra or Virasoro algebra, a remnant of world-sheet reparametrization invariance. The representation theory of certain extended conformal algebras known as W-algebras suggests that the equivalent of the $D = 2$ barrier for these extended algebras takes place at higher values of D. This is one of the motivations to study string theory based on such extended conformal symmetries. In the main part of this thesis, we will look at string theories with W-symmetry ('W-strings') and compare their properties with those of the ordinary (non-critical) string that is based on conformal symmetry only. It turns out that many relations exist between strings based on different world-sheet gauge symmetries.

The W-algebras that we use in our construction are nonlinear algebras. As a consequence of these nonlinearities, explicit computations can be quite complicated. We improve on this situation by introducing transformations of the variables that simplify many calculations. These simplifications are particularly welcome in the BRST formalism, a framework for quantization of gauge theories. In string theory, the BRST formalism has proved to be very useful. We use it to compute the physical spectrum of a string based on the W_4 algebra, a typical example of a W-algebra.

The other symmetries that we consider in this thesis are the duality symmetries in string theory. In the past few years, duality transformations have been a subject of intense study, both in string theory and field theory. Some duality transformations relate strings moving in different space-time backgrounds. Such dualities are referred to as target-space dualities or T-dualities. The best known example is the duality transformation which shows the equivalence of a string moving in a background with one coordinate compactified on a circle of radius R and the same string moving in the same background but with a radius l_{str}^2/R, where l_{str} is the typical length of a string. This suggests that in string theory there is a notion of minimal distance near the Planck length and that our usual view of space-time should not be relied upon too much for distances near or below the Planck scale. We describe T-dualities in the language of canonical transformations in the two-dimensional world-sheet theory. Also, we discuss the low-energy limit of string theory (which is represented as a field theory) and review its symmetries associated to T-duality.

Many other duality symmetries have been conjectured, and among them are dualities that interchange weak and strong coupling. These are particularly interesting since they provide information on the strong coupling behaviour of certain string theories. Moreover, the strong coupling limit of one string theory turns out in many cases to be another string theory (in weak coupling) or even a field theory or something else. This has raised questions such as 'Is string theory unique?' or 'Is string theory a theory of strings?'. Anyhow, it is hoped that a better understanding of string dualities gives us more control over the vast number of four-dimensional effective theories that seem to be realizable within string theory. We investigate a possible strong/weak coupling duality between a string and another extended object called a fivebrane. Strong/weak coupling dualities acting within the string theory itself are also known. These are referred to as S-dualities. In particular, we look at a possible interchange of T and S-dualities under string/fivebrane duality. Also, we use certain duality symmetries in one of the superstrings, the type IIB string, to obtain new solutions of the low-energy theory from known ones.

The organization of this thesis is as follows. First, in chapter 2, we introduce the classical bosonic string in the usual formulation of a reparametrization invariant two-dimensional action. Choosing a specific gauge leads to a conformal field theory. We describe some general properties of two-dimensional conformal field theory, which plays an important role in string theory. We also briefly review some well-known variants of string theory and start to discuss the possibility of extended world-sheet gauge symmetries, in particular those described by W-algebras. Then in chapter 3 we discuss the quantization of the string, starting with the ordinary bosonic string. We use the BRST formalism and we start this chapter by introducing this formalism. Then we apply it to W-strings. We report on the construction of the BRST operators and on realizations for W-algebras needed for quantization of the W-string. In chapter 4, the physical spectra of string theories will be investigated. We start again with the ordinary bosonic string and then proceed to a discussion of W-string spectra. At the end of this chapter, we mention some relations that have been found between string theories based on different world-sheet gauge symmetry. Chapter 5 is about duality symmetries. First we review how low-energy effective actions are obtained, and a simple compactification procedure called dimensional reduction is explained. Then we discuss T-duality from the point of view of the low-energy effective theory as well as from the world-sheet point of view. We also consider strong/weak coupling dualities, in particular one relating strings and fivebranes. At the end of this chapter, we use duality transformations in the type IIB string to obtain some new solutions. Among them are solutions that have both 'electric' and 'magnetic' charges. We conclude with a short discussion (chapter 6).