Dualities of strings and branes

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Chapter 1

Introduction

Twentieth century theoretical physics has been dominated by two major achievements which both revolutionized the way of thinking in physics: quantum mechanics and the theory of relativity.

The *theory of relativity*, written down by Einstein between 1905 and 1916, states that the laws of physics should be the same for all observers in the universe and must therefore be formulated in a covariant (observer independent) way. The theory of relativity consists of two parts: the theory of special relativity, which reformulates and corrects Newtonian mechanics at relativistic velocities (near the speed of light) and the theory of general relativity, which describes gravity in an observer independent way by introducing the concept of curved spaces.

Quantum mechanics, formulated in the nineteen twenties and thirties, is the theory that describes the behaviour of particles at (sub)-atomic scales, and is therefore the theory to be used if one is dealing with elementary particles. The main point of quantum mechanics is that some quantities in Nature do not have the continuous behaviour as described in classical mechanics, but turn out to be quantized, i.e. they can only take some discrete values. Furthermore there exist fundamental uncertainty relations: physical quantities can no longer be determined with the same accuracy as in classical mechanics, but due to quantum fluctuations the theory should be formulated in terms of probabilities.

Both theories, relativity and quantum mechanics, have become the pillars of modern physics and (general) covariance and quantum behaviour should be the basic ingredients of every fundamental theory. However, each of the two theories is only valid in its own specific range: relativity does not incorporate quantum effects needed to describe elementary particles at relativistic velocities, nor has quantum mechanics the necessary covariant formulation in order to be observer independent. It is therefore logical to look for a better formulation, a new theory which incorporates these two properties and of which both relativity and quantum mechanics are special limits.

Quantum field theory (formulated between the thirties and the sixties) was a first attempt to formulate a relativistic description for elementary particles. It made use of the concept of *gauge invariance*, which was inherited from classical electromagnetism. Gauge invariance is a symmetry which states that there are more degrees of freedom (fields) in the theory than physically relevant variables: the physically relevant variables are built from the degrees of freedom, the so-called gauge fields, but different gauge fields give rise to the same expression for the physical variables. Gauge transformations relate the gauge fields that build up the same physical variable, thereby dividing them into equivalence classes of identical physics. The physical variables, and therefore the theory, are (by construction) invariant under the gauge transformations. A gauge invariant formulation is thus in a sense a kind of over-description of the theory, but it can be used as a tool to calculate the physical quantities: at every point in space one can choose the form of the gauge field that is most convenient to solve a particular problem.

In the nineteen seventies, the Standard Model took form as the generally accepted way to describe elementary particles and their interactions: quarks and leptons were identified as the basic constituents of matter and photons, gluons and vector bosons as gauge particles that transmit the strong and electro-weak interactions. These interactions are governed by the gauge group $SU(3) \times SU(2) \times U(1)$, which at low energies is spontaneously broken to $SU(3) \times U(1)$ via the Higgs mechanism. The Higgs boson, of which the potential is responsible for the spontaneous symmetry breaking, accounts for the masses of the particles in the Standard Model.

The Standard Model is a very successful model, for various reasons. It gives an elegant and powerful description of the strong and the electro-weak interactions, making use of the principle of gauge invariance. Furthermore it agrees to a very high accuracy with experimental results, and made some predictions which were later verified in experiments.

In spite of this success, there are reasons to believe that the Standard Model is not the end of the story. These are not experimental reasons, since the Standard Model agrees very well with experiments, but theoretical reasons to believe the theory is not complete. A first indication comes from the theory itself: if the Standard Model is really the final and fundamental theory of particles and interactions, how come that there are still so many free parameters left? The masses of the particles, the mixing angles and the coupling constants of the interactions, all play an important role in the Model, but are not predicted by it. Their exact values are inserted by hand in order to agree with experiment. A fundamental theory would be more convincing if it could explain why all these parameters have the values we measure.

A second reason is more fundamental: although the Standard Model is the quantum field theory of the strong and electro-weak interactions, and therefore incorporates both a quantum and a (special) relativistic description of these interactions, it does not take the gravitational interactions into account. In other words, the Standard Model is a successful unification of special relativity and quantum mechanics, making use of gauge theories, but not of general relativity. There still does not exist a good theory which deals with the quantum aspects of gravity, or vice versa, gives a good description of the gravitational interaction between two elementary particles. This problem, the search for the theory of quantum gravity, has become an important challenge in modern high energy physics.

From the point of view of experiment, there is no direct problem: the gravitational interaction is much weaker than the strong or the electro-weak interaction so that in any realistic experiment it can be ignored completely. This is the reason why the Standard Model agrees so well with experiment: gravitational effects on elementary particles can simply not be measured with the present technology.

Yet, on theoretical grounds, one can argue that at a certain point the present theory will no longer hold: at higher energy scales, the gravitational interaction becomes increasingly important, and near the Planck-mass M_P it can no longer be ignored. The Planck-mass is the energy scale at which the Schwarzschild radius $R_S = 2mG_N/c^2$ of a particle becomes equal to its Compton wave length $\lambda_C = h/mc$:

$$M_P \;=\; \sqrt{\frac{hc}{2G_N}} \;\sim 10^{19}\ \text{GeV}/c^2\ . \tag{1.1}$$

The Schwarzschild radius of an object of mass m is the limit beyond which the object has to be compressed in order to become a black hole and the Compton wave length is a measure for the quantum uncertainty in the position of a particle. So at the Planck-scale the structure of space-time gets interwoven with quantum uncertainties and a theory of quantum gravity is needed. Note the the Planck-mass can be expressed in terms of three fundamental constants of Nature: Planck's constant h, the speed of light c and Newton's gravitational constant G_N.

The energy scales corresponding to the Planck-mass are many orders of magnitude beyond the reach of present accelerators (the LHC, being built in Geneva, will be able to perform experiments at energies around 1.5×10^4 GeV), so in constructing a theory of quantum gravity, one will have to rely strongly on theoretical arguments and intuition, instead of following experimental indications.

The reason why it is so difficult to construct a quantum theory of gravity is that gravity is *not renormalizable.* Newton's gravitational constant, which is the coupling constant of gravity, has dimensions of $(\text{mass})^{-2}$ (in units where $c = \hbar = 1$), such that the effective, dimensionless coupling constant $G_N E^2$ is proportional to the square of the energy of a given process. For higher and higher energies, the coupling will grow arbitrarily large and lead to divergences in perturbation theory that become larger in every order and make the theory difficult to handle at high energies. Effectively this means that something goes wrong in the short distance (= high energy) behaviour of the theory and that there is a cut-off beyond which the theory is no longer valid.

A parallel can be drawn between the non-renormalizability of gravity and of the four-fermi theory for the weak interaction. Both theories suffer from the same kind of problems due to a dimensionful coupling constant. In the case of the weak interactions the problem was solved by replacing the four-fermi theory by the $SU(2) \times U(1)$ gauge theory of electro-weak interactions, where the divergences were smoothed by the introduction of gauge bosons that spread out the interaction and weaken the short distance behaviour.

A logical attempt therefore would be to write gravity as a gauge theory for a new kind of symmetry, namely *supersymmetry*. This is a symmetry that relates bosons and fermions and associates to each particle a new particle of the opposite type. There is still no experimental evidence for the existence of supersymmetry: none of the extra particles predicted by supersymmetry has ever been observed. Therefore, if it exists, it has to be broken at low energies.

In spite of the lack of experimental evidence, many physicists believe that supersymmetry is an important ingredient for a description of quantum gravity: a local version of supersymmetry induces invariance under general coordinate transformations and thus leads to a theory with dynamical gravitational fields. In other words, a locally supersymmetric quantum field theory is a supersymmetric version of general relativity. Field theories with local supersymmetry are generally called *supergravity theories*.

Furthermore it was observed that supersymmetry softens the divergences in a quantum field theory. Since fermionic contributions to perturbative loop calculations have opposite signs compared to bosonic contributions, it was hoped that in this way the different divergences might cancel each other and give a finite result. However this turned out much more difficult to show than was first thought and people have more or less abandoned the idea that divergence cancellation might work in quantum field theory.

Since local supersymmetry alone is not sufficient to remove the divergences in gravity, a bigger step is needed. This is done by *string theory*, a theory that has as a starting point the idea that all elementary particles are not point-like, as we intuitively used to think, but one-dimensional objects, strings with a certain spatial extension. The theory has a natural cut-off built in at short distances, since the interactions are now spread out over the length of the string. In this way the short-distance behaviour is softened. The different oscillation modes of the string should correspond to the various elementary particles that we know from the Standard Model, that are predicted by supergravity and many more.

Introducing supersymmetry in string theory, one obtains the so-called *superstring*. The reason for introducing supersymmetry is that the simplest model, the bosonic string, which has only bosonic degrees of freedom, contains “unphysical” states in its spectrum. These are called tachyons and have the strange property that their mass squared is negative. However, this undesirable feature can be eliminated by introducing supersymmetry. Indeed, the superstring does not suffer from this problem, and at the same time it contains fermionic degrees of freedom, which the bosonic string did not have.

The short distance behaviour of superstring theory is better than that of most quantum field theories: it can be shown [32] that the superstring scattering amplitudes are ultraviolet finite. Whereas in quantum field theory perturbative calculations are done by computing Feynman diagrams, the perturbation expansion in string theory is a sum over the topologies of the two-dimensional world sheet which the string sweeps out in space. This means that in every order there is only one “diagram” to be considered (in a theory of closed strings), this in contrast to quantum field theory, where the number of diagrams increases rapidly with the order.

Important progress was made when it was realized [166, 136] that the superstring theory has massless states of spin two, which could be identified as the gravitons, the gauge particles of gravity. The identification of the graviton in the string spectrum also sets the length scale of the string: the typical size of a string should be of the order of the Planck-length L_P, the Compton wave length of a particle with mass equal to the Planck-mass:

$$L_P = \sqrt{\frac{G_N h}{c^3}} \ \sim \ 10^{-35} \text{ m}. \tag{1.2}$$

Note that present accelerators can probe distances down to about 10^{-18} m, so this explains why nothing has been noted of the stringy extendedness of elementary particles.

After what became known as the first superstring revolution (1984-1985), string theory really began to be considered as a serious candidate for a unifying theory. It turned out that there exist (only) five consistent versions, called the Type I string, the Type IIA, Type IIB, Heterotic $E_8 \times E_8$ and Heterotic $SO(32)$ string, which all have well-defined perturbation expansions and differ in their field content and the amount of space-time supersymmetry. Consistency in the quantization requires each of the five string theories to live in a ten-dimensional space-time.

The fact that the space-time is required to have ten dimensions, and not four, as we are used from general relativity or quantum field theory, is not such a big problem as it might seem. The explanation is that six of the ten dimensions are compact and very small (in fact of the order of the Planck-length [99]), so they cannot be detected at low energies. A technique, called *dimensional reduction*, is known to rewrite the ten-dimensional theory as an effectively four-dimensional one in order to make contact with our experimentally observable world.

Depending on how this dimensional reduction is performed, all kind of gauge symmetry groups can appear, some of which resemble the Standard Model at low energies. But there are many different reductions possible, leading to many low energy effective theories and many different vacua, and it is not at all clear why the universe as we see it has precisely four dimensions (and not any other number smaller than ten) and why precisely one particular reduction scheme should be preferred to others. A fundamental theory like string theory should be able to give a natural answer to these questions.

Another problem of string theory (which is maybe related to the previous ones) is that little more of it is known than a perturbative description. Glances into the non-perturbative regime have only recently become possible, since what is called the second superstring revolution, which started in the mid nineties. Then a new concept was introduced in string theory, namely the *duality symmetries*. In fact dualities might be one of the fundamental principles to understand string theory.

Dualities are symmetry transformations that relate different compactifications of a theory, different regimes and even different string theories to each other. There are many different types of duality transformations, but the ones we treat in this thesis are the most important ones: T-duality and S-duality. The other types of dualities can mostly be related to combinations of these two.

T-duality stands for target space duality, the duality on the space-time through which

the string moves. It relates small volumes to large ones and therefore physics of small scales to physics of large scales. Suppose one of the dimensions of the target space is rolled up into itself (for example as in a process of dimensional reduction) and forms a circle of radius R. A string running around in this compact dimension will have discrete momentum and energy states. In particular, the smaller the radius of the circle, the higher the energy of the string states. On the other hand, the string can also wind a number of times around this compact dimension. Since the energy of the string is also proportional to its length, the string winding states become more and more energetic as the radius of the circle becomes bigger and the string itself longer.

Now it turns out that energy levels of a string moving around on a circle with small radius correspond exactly to the energy levels of a string wound around a large circle and vice versa. In general, a string which is moving with momentum m and is wound n times around a circle of radius R, is equivalent to another string, moving with momentum n and wound m times around another circle of radius $\tilde{R} = \alpha'/R$, where α' is a constant related to the length of the string.

The duality transformation that relates these two descriptions is called T-duality and the two backgrounds (one with a compact dimension of radius R and the other of radius α'/R) are called T-dual. The string (and hence the observer) doesn't see whether it is in the first or in the second case, so it seems that there exists a kind of symmetry $R \rightarrow \alpha'/R$ between large and small scales. If the size of a compact dimension shrinks beyond a certain size ($R = \alpha'$), the theory behaves essentially as if in a dual description the dimension would be increasing again. This is an indication that the space-time at the Planck-scale may be very different from what we are intuitively used to.

It is clear that in this way many different compactifications can be related. If we perform a dimensional reduction over d dimensions of radii R_a ($a = 1, ..., d$), the obtained vacuum is physically equivalent to a dimensional reduction over coordinates of radii $\tilde{R}_a$, if the radii R_a and $\tilde{R}_a$ are related via T-duality and permutations in the index a. In this way T-duality divides the different vacua into equivalence classes and, although it does not say which vacuum is preferred to others, at least it reduces the problem significantly.

Not only can different compactifications of a specific theory become equivalent via T-duality, also the different theories themselves can be related via this procedure. As we will show later on in this thesis, the dimensionally reduced version of one theory compactified over a circle of radius R can be mapped onto the dimensionally reduced version of a different theory, which has been compactified over a circle of radius α'/R. In this way the Type IIA and the Type IIB theory and the Heterotic $E_8 \times E_8$ and $SO(32)$ can be related to each other: one theory compactified on a small volume is equivalent to the other theory compactified on a large volume.

Another duality that has been conjectured to exist, is the strong/weak coupling duality or *S-duality*. In perturbation theory only the weak-coupling regime of string theory can be explored but as the coupling grows too strong perturbative calculations break down and trustworthy results are hard to obtain. S-duality might give insight in the strong-coupling regime since it relates the strong and weak coupling regions of theories to each other. If the S-duality conjecture holds, a string theory A with fields ϕ_A and coupling constant g_A can be rewritten in terms of another string theory B with dual fields $\tilde{\phi}_B$ and coupling constant $g_B = 1/g_A$. In this way, non-perturbative calculations in one theory can be translated into perturbative calculations in the other theory. There are strong reasons to believe that in this way the strong coupling limit of the Type I theory corresponds to the weak coupling limit of Heterotic $SO(32)$ theory (and vice versa) and that the Type IIB theory is S-self dual, i.e. S-duality relates the strong and the weak coupling limits within the same theory.

The strong coupling limits of Type IIA and Heterotic $E_8 \times E_8$ are even more surprising: although these two theories are both ten-dimensional, as are all other string theories, in their strong coupling limit an extra, eleventh dimension appears. This is possible because this extra eleventh dimension is compact and its size is related to the ten-dimensional coupling constant. So at weak coupling the eleventh dimension is very small and in fact invisible, but as we let the coupling grow this extra dimension unfolds.

This discovery drew the attention back to eleven-dimensional supergravity, a theory which was known already from the times before string theory, when people still thought supergravity might lead to the theory of quantum gravity. However, eleven-dimensional supergravity was always neglected because of its possible non-renormalizability and the fact that one cannot obtain a chiral spectrum as in the Standard Model, where left and right handed components of fields behave differently under symmetry transformations. With the rise of string theory, it was considered an irrelevant curiosity, since all string theories live in ten dimensions and no connection to eleven-dimensional supergravity was found. Now that it turns out that some string theories have an eleven-dimensional limit, $D = 11$ supergravity gains importance as a possible low-energy effective theory for this strong-coupling limit.

We see that the duality symmetries weave a web of duality transformations between the different string theories and even eleven-dimensional supergravity: they are all interconnected via T or S-duality. This feeds the idea that the various string theories are in fact not the really fundamental theories, but rather different perturbation expansions around different vacua of one and the same underlying theory. This is an attractive and elegant idea, that explains both the wide variety of duality relations between the different theories, as well as the fact why we find no less then five versions of what we thought was the unifying, fundamental theory.

However, the other side of the picture is that it is not clear at all what this underlying theory looks like. It is usually referred to as *M-theory* (where the M can stand for many things, such as Membrane, Mother, Matrix, ...), but little more of it is known then that it has eleven-dimensional supergravity as its low-energy effective theory and that it is supposed to be the strong coupling limit of Type IIA and Heterotic $E_8 \times E_8$ theory. A lot of work is done nowadays to get a better picture of what M-theory actually is.

An important role in checking the M-theory conjecture and the duality relations between the different theories is played by the solutions of the equations of motion of the theory. In general, they appear as extended objects, objects with one or more spatial extension and are referred to as *p-branes*, where p stands for the dimensionality of the object: $p = 0$ is a particle, $p = 1$ is a string, $p = 2$ a membrane, ... Many of these

p-branes occur as solitons in the theory, i.e., not as solutions of perturbative calculations, but as topological defects which are very heavy and strongly interacting at weak coupling.

An example of such a brane is the solitonic five-brane, an object that has five spatial directions and carries a magnetic charge. Isolated magnetic charges have never been observed but occur typically in solitonic objects. This is in contrast to electrically charged objects which are considered to be the fundamental objects of the theory, since they appear in perturbation theory: they are light and weakly interacting at small coupling.

In an early version of S-duality a conjecture was made stating that, for a theory of electrically and magnetically charged particles, a dual formulation exists where the role of fundamental and solitonic particles is reversed: in the dual formulation the fundamental particles are the ones with magnetic charge, while the solitons are electrically charged. Furthermore, since the Dirac quantisation condition states that electric charge e and magnetic charge q are related via their inverses $q \sim 1/e$, the strong coupling limit of one theory corresponds to the weak coupling limit of the dual theory and vice versa: strongly interacting solitons in the fundamental theory can be viewed as weakly interacting fundamental particles in the dual theory.

In string theory, the fundamental, electrically charged object that interacts weakly at small coupling is the fundamental string, while the heavy, strongly interacting magnetic object is the solitonic five-brane. The string theory version of the electric/magnetic duality conjecture is the string/five-brane duality, which states that the strongly interacting string is dual to the weakly interacting five-brane. Instead of starting off with a theory for strings, we could have written down a theory for elementary five-branes that has string-like solitons (however, the problem with this dual formulations is that it is not clear how to quantize such an elementary five-brane). More generally, every p-brane in D dimensions has a dual $(D-p-4)$-brane of the opposite charge (electric vs. magnetic) and coupling (strong vs. weak).

Another type of extended objects that appear in string theory is the so-called Dirichlet-brane, or short D-brane. D-branes arise in the T-dual formulation of open strings: it turns out that open strings can also be described as strings whose end-points are attached to these D-branes. All D-branes are related to each other via T-duality and the strings attached to them make it possible to study their dynamics using familiar string perturbation theory. Furthermore, upon dimensional reduction to lower dimensions, D-branes might give insight into the microscopic description of the quantum states of black holes.

It was also realised that if Type IIA theory is really a compactified version of an eleven-dimensional theory, all solutions of Type IIA should be interpretable as reductions of eleven-dimensional objects. Indeed, it turns out that the fundamental object of $D = 11$ supergravity is a membrane, rather than a string, but upon dimensional reduction over one of the directions in which this membrane is oriented, a string-like object is found, which can be identified with the fundamental string solution of the Type IIA theory. Also the other Type IIA solutions (the solitonic five-brane, the D-branes, etc.) can be obtained from eleven-dimensional objects after dimensional reduction.

The fact that all these types of p-branes turn up in string theory has given rise to questions concerning the very nature of the theory: why call it string theory if there exist dual, equivalent formulations in terms of (for example) five-branes and if one of the theories entering in the duality web, $D = 11$ supergravity, does not even have a string-like solution, but a membrane? Why should in such a variety of objects, strings be more fundamental then other ones? Terms such as "p-brane democracy" and "Is string theory a theory of strings?" have become common amongst string theoreticians. It is hoped that M-theory will deal with these questions, but one of the reasons why it is so hard to formulate it, is that we do not know in terms of which objects the description is best given.

In this thesis, some of the aspects of the duality symmetries within string theory are discussed. This is done by looking at three main parts: the target space theory, the solutions and the world volume theory.

The *target space action* is the low-energy effective action of string theory, as seen from the space-time in which the theory lives, if one integrates out the massive modes. The action one obtains is one of supergravity theories, so that supergravity can be seen as a low-energy approximation of string theory. We will work often with these target space actions. They have many symmetries, which help us to understand the full string theory, even if these symmetries may not be completely conserved up to the level of the full theory. Also indications of the existence of the duality transformations between the various string theories are already present in the low-energy effective actions.

The equations of motion of the low-energy effective action give rise to the *solutions*. The duality transformations between these solutions, discussed above, are manifestations of the duality relations between the different string theories. Using the dualities on the solitonic solutions, one can get insight in the non-perturbative regime of the theory, while on the other hand looking at the duality relations between the solutions one can perform tests to check the conjectured dualities between the theories.

The dynamics of these solutions is described by the *world volume actions*. So in order to get a good understanding of the solutions it is necessary to look at their world volume actions. Also here there exist all kinds of duality relations between the world volume actions, much as they exist between the solutions themselves.

This thesis is organized as follows: in Chapter 2 we give a general introduction to string theory, the world volume theory and the dynamics, the different types of string theories, the target space action and the different solutions. In Chapter 3 we present T- and S-duality and show how they act at the level of the world volume, the target space actions and the solutions. We determine the strong coupling behaviour of the different theories and sketch the duality web between the actions and the solutions. We also explain how dimensional reduction is performed.

After these two introductory chapters, we will look in more detail at the different aspects of string theory. In Chapter 4 we study the target space actions, their symmetry groups and the duality relations, both in ten, nine, six and five dimensions. Chapter

5 is about the solutions of the target space actions and more specifically intersections of two or more p-branes. First a kind of stability condition is determined for an intersection of two such branes, and then this condition is used to construct and classify intersections consisting of more then two intersecting branes. Dimensional reductions of these intersections lead to new solutions in lower dimensions. The world volume theory is studied in Chapter 6. An overview is given of the world volume actions of the different solutions and the duality relations between them will be demonstrated at the level of these world volume actions. At the end of this chapter, the world volume action of one particular solution, namely the Kaluza-Klein monopole, is constructed, making use of the duality relations between the monopole solutions of the solitonic five-brane solution.