Fundamentals of grinding
Hegeman, J

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Chapter 3

MODELLING OF THE GRINDING PROCESS

In this chapter a new grinding model is presented based on the three dimensional profile of the diamond abrasive wheel. The model is used to simulate grinding forces, residual stresses induced in the workpiece and the full topography of ground surfaces. The roughness properties are calculated from the simulated surfaces. In addition, the relation between the grinding parameters and the roughness is studied in this chapter. Some of the parameters needed in the model are experimentally measured.

3.1 INTRODUCTION

Already since the late sixties, attempts were made to simulate the surface profile of the workpiece after grinding [1, 2, 3, 4, 5, 6]. The first approaches were based on line profiles of the grinding wheel. Mostly, a simple grain geometry was applied and uniform distributions of abrasives over the wheel surface were assumed (see also chapter 2). The results of those models were somewhat empirical because of the assumptions made. Some models were designed to simulate the elastic modulus of the wheel and the contact length between the grinding wheel and the specimen [7, 8, 9, 10]. Also completely different approaches, like using fuzzy logic have been tried to predict the surface roughness after grinding [11]. Furthermore, attempts were made in order to calculate and to determine subsurface effects such as residual stress, phase transformation and thermal damage from the grinding forces [12, 13, 14, 15, 16]. More recently due to the development of 3D surface characterisation techniques, such as optical profilometry, and the improvement of computers, three-dimensional models were developed [17, 18, 19, 20]. These models were
then used to calculate roughness values of the workpiece after grinding versus the processing parameters.

This chapter concentrates on the formation of the surface morphology during grinding. An in-plane grinding operation with a super-abrasive wheel was chosen because of a rather simple overall geometry. However, the complexity of the local geometry, i.e. a random distribution of cutting points, makes this operation far from trivial. As a consequence, many of abrasive and cutting processes can be described in using a similar approach. This emphasises on the formation of the workpiece topography during the process and on the prediction of quality of the surface finishing.

Every abrasive processing includes the engagements of (randomly) distributed cutting points of a tool. The whole process then can be described as a cumulative action (superposition principle) of unit events. The so-called unit event is an engagement of one, individual grain with the surface of material processed. The understanding of this unit event and the effect of it on the surface integrity is therefore of essential importance in the grinding model. Single scratch experiments can be performed to simulate the unit event and so the material response during the abrasive processing can be studied.

This process description can easily be adapted to different types of the overall tool/workpiece geometry. Moreover, because of a random distribution of cutting points, the present modelling can readily be reformulated for other situations describing the interaction between the surface of material and abrasive units. A well-established analysis of an abrasive action of regularly distributed cutting points will be explored [21]. Recently, an attempt was made to extend that approach to the case of random engagements of the abrasive grits with the material surface [18]. The plane grinding operation was considered, for which an effective number of cutting points was determined and the outgoing topography was constructed in 3D. Further elaboration on this model extended this approach to the calculation of forces that arise during impact of the abrasive units [19]. However, some aspects were not addressed properly so far. These include that during abrasion different cutting points may come in an engagement with the same local area of the surface altering the surface topography accordingly. This will have consequences for the calculation of the forces as well.
3.2 THE KINEMATIC GRINDING MODEL

In the kinematic grinding model, described in this chapter, single pass plane surface grinding will be considered. The process is schematically displayed in figure 3.1. In the figure, the main parameters that are used in the experiments and in the simulation are shown. These include the processing parameters: rotational speed of the grinding wheel $v_s$, speed of the workpiece/specimen $v_w$ and the depth of cut $d_{cut}$. The so-called grinding direction is the direction of the speed of the workpiece, as is indicated in figure 3.1.

To model the grinding process several steps should be included such as the generation of the grinding wheel, transformation of the wheel topography to the surface of the workpiece and modelling of the materials properties of both the specimen and the grinding wheel. Finally, for the understanding of the changes in materials properties, such as magnetic permeability (chapter 5) or piezoelectric constants, the grinding forces and the residual stresses are calculated. The change in magnetic permeability, piezoelectric properties or mechanical strength can then be related to the grinding induced residual stresses. However, first some assumptions are made in order to simplify the complex machining process.

**figure 3.1:** Schematic picture of the grinding process.

A diamond abrasive wheel is rotating at a spindle velocity $v_s$. The workpiece is moving along the y-direction, the so-called grinding direction with a speed $v_w$. The relative distance between the wheel surface and the surface of the workpiece is $d_{cut}$. Notice that the real dimension of the grinding wheel is much bigger than in the figure relative to the depth of cut $d_{cut}/R_s<<1$. The radius $R_s\approx20$ cm and $d_{cut}\approx10 \, \mu m$. 
The wear of the wheel surface is neglected when a super-abrasive wheel is used. Because it is assumed that the diamond abrasive wheel is much tougher and more wear resistant than the specimen. In addition, the tool is assumed to be perfectly rigid and hence possible deflections of abrasive grains during their engagements with the material processed are not taken into account. However, possible overall deflections (or vibrations of the grinding machine) and roundness errors of the grinding wheel are included by on-line recording of those using a force platform. The grinding process is then described by the machining parameters and the average characteristics of the wheel topography. The wheel is characterized by the average concentration of the diamond grains in the wheel surface, the average size of these cutting grains and their statistical distribution and it is supposed that the abrasive grains do not overlap. All these parameters can be obtained by SEM, optical and mechanical profilometry techniques.

The removal of material from the workpiece is based on a rigid-perfectly plastic material behaviour, i.e. the cutting edges will remove all material that they encounter on their path. Although the elastic deflection of the workpiece will reduce the actual cutting depth, the possible pile-up of material and the deflections are neglected. It is also assumed that the material removal behaviour is not different for ‘up-grinding’ ($v_s$ and $v_w$ at the surface of the specimen have opposite directions) and ‘down-grinding’ ($v_s$ and $v_w$ are parallel) in spite of the fact that both methods will cause a different stress field in the specimen. For up-grinding, more fracture of material is expected and for down-grinding, more ploughing is expected. Another assumption that is made is that there is no grinding fluid or lubrication present during the process, which might change the material removal behaviour, i.e. if a lubricant is present a more ploughing behaviour is expected. Despite of the absence of cooling fluid no (local) heating of material is supposed. However, during the grinding experiments, grinding fluid is used to avoid burnout or thermal damage and the assumption is made that the grinding behaviour is the same for dry grinding with an alternative way of cooling.

If the surface topography of the grinding wheel could be described and rendered on the basis of these assumptions adequately, then the influence of the machining parameters on the grinding performance, surface finishing and surface integrity may be studied. The model can predict changes in materials properties by the calculation of machining induced residual stresses. Using the model, the performance of the abrasive machining can be optimised.
3.2.1 Generation of the grinding wheel

The surface of a grinding wheel usually consists of hard abrasive particles randomly distributed and randomly oriented in a metal binder. The wheel can be described by its radius, the average size of the abrasive grains, the shape of the grains, the distribution of grains in radial, tangential and lateral direction and by the materials properties. Various methods have been proposed in order to model and characterise the topography of a grinding wheel [3,22, 23, 24, 25, 26, 27]. During the simulation of the grinding wheel, most researchers used random distributions for the position and size of the abrasive grains in the wheel. Some of them included the actual dressing process of the grinding wheel in order to describe the exact shape of the grains [3]. Others used various shapes like octahedrons, cuboids, tetrahedrons or spheroids to represent the cutting points in the wheel surface [19]. In the grinding model presented in this chapter, a Monte Carlo-like concept was used to distribute the abrasive grains randomly over the surface of the grinding wheel.

For small depth of cut, the basic shape of an abrasive grain can be assumed a spheroid. This assumption becomes even more valid when the average size of the cutting grains is much larger than the depth of cut $d_{cut}$. The axis $R_x$, $R_y$ and $R_z$ of the spheroid, which in general are different, are varied using Gaussian distributions for each axis. $R_z$ is chosen to be perpendicular to the wheel surface. The basic spheroidal shape of the grain can mathematically be formulated as follows:

$$\Omega(x_s, y_s) = R^g_z \sqrt{1 - \left(\frac{x_s}{R^g_x}\right)^2 - \left(\frac{y_s}{R^g_y}\right)^2}$$  \hspace{1cm} (3.1)

where $\Omega$ is the height of the grain as function of $x_s$ and $y_s$ and $R^g_i$ with $i=x, y, z$ are the principal axes of the spheroid.

Since the abrasive grains are randomly oriented relative to the normal of the wheel surface, additional shape factors are added to allow the grain to rotate around its centre of gravity. This shape factor can be simulated using a random 3D periodic wave function $\left(\cos(\omega_i x_s + \varphi_i) + \cos(\omega_i y_s + \varphi_i)\right)$ imposed on the axis in $z$-direction, where $\omega_i$ and $\varphi_i$ are random numbers. By this method, the skewness/orientation of the grains relative to the wheel surface can be controlled and randomised.
Due to the dressing process of the grinding wheel and due to wear of (diamond) abrasive grains during ‘run-in’ of the wheel, the global shape of the cutting points might be changed. Therefore, some additional details should be added to the smooth spheriodal shape of the simulated abrasive particles. These finer details are modelled by superimposing a stochastic periodic function $f(x_s,y_s)$ on the spheroid that describes the cutting grain. This procedure is similar to the method described in [27] but here it is generalised in 3D. As a result, the total grain shape can be described by equations (3.2) and (3.3)

$$z_s(x_s,y_s)=\begin{cases} f(x_s,y_s) & \text{outside grain} \\ f(x_s,y_s)+\Omega(x_s,y_s) & \text{inside grain} \end{cases}$$

(3.2)

where $f(x_s,y_s)$ is the stochastic fluctuation representing the fine structure of the binder material and cutting grains. $\Omega(x_s,y_s)$ is the overall grain shape and can be expressed with the following equation:

A typical spheroidal grain with different radii in the x-, y- and z-direction as simulated by the model. A stochastic function was added to model the finer details of the abrasive grain. The radii of the grain are $R_{x_s} = 28 \, \mu m$, $R_{y_s} = 45 \, \mu m$, $R_{z_s} = 56 \, \mu m$ in the x-, y- and z-direction respectively. Notice that the scale in z-direction is different than in the x- and y-direction which gives the impression that the grain is elongated in the z-direction.

**Figure 3.2: Simulated abrasive grain.**

A typical spheroidal grain with different radii in the x-, y- and z-direction as simulated by the model. A stochastic function was added to model the finer details of the abrasive grain. The radii of the grain are $R_{x_s} = 28 \, \mu m$, $R_{y_s} = 45 \, \mu m$, $R_{z_s} = 56 \, \mu m$ in the x-, y- and z-direction respectively. Notice that the scale in z-direction is different than in the x- and y-direction which gives the impression that the grain is elongated in the z-direction.
MODELING OF THE GRINDING PROCESS

\[
\Omega(x_s, y_s) = \left[ R^g_s + \xi_s \left( 1 - \frac{\alpha_s \cos(\omega_s x_s + \varphi_s) + \alpha_y \cos(\omega_y y_s + \varphi_y)}{2} \right) \right]
\]

(3.3)

\[
1 - \left( \frac{x_s - l_x + \xi_x}{2 R_x^g + \xi_x} \right)^2 - \left( \frac{y_s - l_y + \xi_y}{2 R_y^g + \xi_y} \right)^2 \right]^{1/2}
\]

where \( z_s \) is the height as a function of \( x_s \) and \( y_s \) (associated with the wheel surface), \( \xi_i \)'s are random numbers using a Gaussian distribution describing deviations from averaged characteristics (\( R^g_i \)) of a grain, \( \zeta_i \)'s are random numbers (using a uniform distribution) giving an arbitrary position of a grain within a chosen unit-cell, and \( \alpha_i, \omega_i \) and \( \varphi_i \) are random numbers simulating stochastic deviation of grains’ overall shape. In figure 3.2, a typical simulated grain is shown.

At this stage, only one single abrasive grain randomly positioned in a cell of size \( l_x \) by \( l_y \) is simulated. In order to generate the whole wheel surface a regular 2D mesh with cell sizes of the same characteristic dimension \( l_x \) by \( l_y \) was applied.

![Random lattice for the positioning of grains.](image)

The abrasive grains are randomly positioned within a cell with characteristic dimensions \( l_x \) and \( l_y \) and they may not exceed the cell boundaries. The regular lattice is randomised perpendicular to the grinding direction (x direction) by a distance \( 0 < \delta < l_x \) to avoid artificial effects in the resulting profile transverse to the grinding direction.
on a flat surface. It is assumed that only one cutting grain may occupy an arbitrary position within such a unit cell (see previous pages) and that the grain does not cross the cell boundaries. This quasi-random method is used to reduce the calculation time to simulate a grinding wheel.

Although the grains are randomly located within the unit cells, they are still distributed on a regular grid. Regarding the plane grinding operation, such a distribution along the grinding direction does not really influence the results, because the movement of the abrasive grains is in this direction and a possibly artificial pattern would be removed by successive grains. However, in the perpendicular direction with respect to the grinding direction, i.e. the $x$-direction, such a regular grid will certainly impose a non-natural distribution of grooves over the ground workpiece, because the grains do not cross the borders of the cells, i.e. there is a minimum at the cell boundaries.

To avoid this artefact, the regular lattice is shuffled in the $x$-direction perpendicular to the grinding direction, see figure 3.3. The origin of the

![Figure 3.4: Distribution of the cutting grains generated in the simulation.](image)

Part of the simulated surface of the grinding wheel showing the distribution of abrasive grains over the surface. The cell size of this specific wheel is chosen to be five times the average grain size ($c=5$). This number $c$ determines the concentration of grains on the wheel surface. $z_s$ is calculated using equation (3.2). Notice that the scales in $x$-, $y$- and $z$-direction are different which might give the impression that the abrasive grains do not have a spheroidal shape.
simulated wheel is positioned at the first layer of cells. Then every next layer \( i \) is moved with a (uniform) random step \( \delta_i \) (with \( 0 < \delta_i < l_c \)) with respect to the origin. After creating such a random lattice and positioning the grains in each cell, a rectangular area representing the bearing width of the grinding wheel is cut-out in order to get a homogeneous grain distribution at both ends of the wheel as well. The surface constructed in this way is plotted in figure 3.4 to illustrate the grain distribution of our model description.

**Experimental determination of the wheel parameters**

Many different experimental techniques and methods have been used to characterise the grinding wheel \([23, 24, 26, 28, 29]\). The static techniques include 2D and 3D profilometry, contact and optical profilometry, scanning electron microscopy and confocal scanning optical microscopy. Usually the profiles are recorded directly on a wheel or on an imprint of a wheel. The advantage of confocal scanning optical microscopy compared to other profilometer techniques is that this relative new method enables us to record surfaces with much steeper surface slopes. The latter are often observed on grinding wheels because of the fracture of abrasive grains. Further, there are dynamic methods such as acoustic emission, thermocouple methods and scratching techniques to characterise the number of cutting points. Usually the number of static cutting points is larger than the number of dynamic cutting points (\( C_{stat} \geq C_{dyn} \)), because

![figure 3.5: SEM micrographs of the grinding wheel.](image)

*Scanning electron microscopy was used to study the position of abrasive grains in the grinding wheel. Also the shape of the diamond grains was studied. In these pictures the typical surface of a D140 grinding wheel with a radius \( R_s = 1.4 \) cm is shown at low magnification and a typical diamond abrasive grain in the metal binder at higher magnification.*
during the grinding process not all abrasive grains will come into contact with the workpiece [30]. Shaw [31] argued that the dynamic number of abrasive grains is depending on the grinding conditions where the static number is independent since some material may already be removed by preceding cutting points.

Here, it is sufficient to restrict the analysis to the main wheel/grain parameters only, i.e. the overall grain concentration, their average size distribution and standard deviations when using the model described above to generate a grinding wheel. By minimising the surface in the kinematic grinding model (see next paragraphs), it is not necessary to account for the difference in number of cutting edges. The static number of grains \( C_{\text{stat}} \) and hence the full topography of the grinding wheel can be used. Only the cutting points that actually hit the surface will perform the grinding process and this can be calculated by taking the kinematic trajectories of the cutting points. In fact, using the kinematic paths of the abrasive grains is analogous to calculating the dynamic number of grains \( C_{\text{dyn}} \) from the static number of grains \( C_{\text{stat}} \). In some particular situations, this restricted number of parameters (or statistical distribution) characterizing the wheel topography should be enhanced to refine this model.

### Table 3.1: Parameters of the Abrasive Grain Distribution Measured by Scanning Electron Microscopy

<table>
<thead>
<tr>
<th>Wheel Parameter</th>
<th>D140, ( R_s=1.4 ) cm</th>
<th>D54, ( R_s=1.4 ) cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{\text{grain}} )</td>
<td>6.9 ± 0.8 mm(^2)</td>
<td>41 ± 6 mm(^2)</td>
</tr>
<tr>
<td>( l_x=l_y )</td>
<td>381 ± 22 µm</td>
<td>156 ± 11 µm</td>
</tr>
</tbody>
</table>

The measurements were carried out on grinding wheels with different grain sizes and on their replicas. The wheels were dressed and ran-in first before studying the wheel parameters. This is because dressed and ran-in wheels are used during the grinding experiments. Using both, the imprint and a small wheel allows us to employ different techniques such as scanning electron microscopy (SEM), mechanical stylus profilometry and confocal scanning optical microscopy. (see explanation of these techniques in chapter 2). Therefore, it is possible to extract all parameters of the wheel surface that are needed. Imprints were made using Technovite 4000, consisting of polyester.
resin and anti-shrinkage powder with particles of approximately 1 µm diameter. The parameters of the distribution of the position of grains over the surface were mainly studied using SEM. In figure 3.5, the typical morphology of the grinding wheel and an individual grain directly measured on the wheel (D140, with diameter $R_s = 1.4$ cm) is represented. The grain distribution and concentration was measured by counting the number of grains per unit area. The results are listed in table 3.1. The cell size, used in the model to generate the grinding wheel, can be calculated from the overall grain density using equation (3.4):

$$l_x = l_y = \frac{1}{\sqrt{C_{\text{grain}}}}$$

Confocal scanning optical microscopy with the objective chosen such that the grain fits within the image field, was used to record 3D images of individual abrasive grains. The shape parameters, like average grain size, were determined from these images. Figure 3.6 shows an individual diamond cutting grain directly measured on the wheel surface.

<table>
<thead>
<tr>
<th>grain sizes</th>
<th>directly on wheel</th>
<th>using imprint</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{x,y}$</td>
<td>$37.6 \pm 8.8$ µm</td>
<td>$28.7 \pm 7.1$ µm</td>
</tr>
<tr>
<td>$R_z$</td>
<td>$34.2 \pm 5.1$ µm</td>
<td>$44.6 \pm 8.6$ µm</td>
</tr>
</tbody>
</table>

In Table 3.2, typical results of measurements are summarized. From the analyses based on those results, the conclusion may be drawn that there is no perfect correspondence between average parameters of the wheel and its imprint, which means that the shape of the individual grains cannot be characterised by the imprint method. However, statistical characteristics (such as the concentration of grains and standard deviations from averages) gathered from the wheel replica give representative results (see table 3.1). The replica method is often used to study the morphology of the grinding wheel since the radius of these wheels are usually too large to do a proper analysis using the
techniques mentioned above. Another advantage of the replica technique is that the overall curvature of the wheel, which makes direct measurements on the wheel quite complicated, can be avoided by flattening the imprint.

Further, the conclusion can be drawn that the actual average size of a cutting grain is of about 25-30% of the size of grains used for the wheel manufacture, so only a part of the grain is protruding out of the wheel surface. The volume concentration of abrasives in the wheel can be calculated from the measured surface concentration (table 3.1) and grain size at the surface (table 3.2). The actual radius of the abrasive grains can be calculated from the mean intercept length ($2R_{\text{measured}}$ in table 3.1) by $\% R_{\text{grain}} = 2R_{\text{measured}}$. The volume concentration can be estimated from the concentration of grains at the surface using the total volume of grains $N\% \pi R_{\text{grain}}^3$ in a slice with thickness: the protruding height (30%) of grains at the surface $0.30*2R_{\text{grain}}$. The volume concentration is approximately 15%, which is consistent with the specifications of the wheel and other observations [28,29]. SEM images and confocal microscopy images of single abrasive grains shown that many grains are

![3D measured diamond abrasive grain](image)

**Figure 3.6 3D measured diamond abrasive grain.**

Confocal scanning optical microscope image of a typical diamond abrasive grain directly measured on the grinding wheel D140 with radius $R_w=1.4$ cm. A 50x/0.80 objective was used with a field size of 280 $\times$ 264 $\mu$m and a lateral resolution of $\sim$0.53 $\mu$m. The optimal vertical resolution is approximately 30 nm (see also chapter 2). NB: The magnification in z- and x-, y-direction is different!
blunted at the top. That suggests large attack (indentation) angles, i.e. an individual engagement may be modelled invoking a blunt indenter analogy, at least at a small depth of cut. This justifies the assumption that the grain may be simulated by a spheroid.

3.2.2 Kinematics of the plane grinding process

The kinematics of the abrasive process was described by Inasaki [18] who has adopted the approach of Reshetov and Portman [21]. In this thesis, a similar method was followed. The movement of the cutting grains on the grinding wheel relative to the workpiece and the kinematic interaction of these grains with the material can be described by a simultaneous translational and a rotational motion as shown in figure 3.1. All the movements can be transformed from the wheel’s coordinate system (indicated by the subscript s) to system of the workpiece (subscript w corresponds to the workpiece). The set of equations describing the trajectories of the cutting points may be written as follows. Note that this equation is very similar to equation 2.3:

\[
\begin{align*}
    x_w &= x_s \\
    y_w &= (R_s + z_s) \sin(\omega_s t + \theta_{s,\text{grain}}) + v_w t \\
    z_w &= \mp (R_s + z_s) \cos(\omega_s t + \theta_{s,\text{grain}}) - (R_s + d_{\text{cut}}) \\
\end{align*}
\]

where \( \omega_s = \frac{v_w}{R_s} \) and \( \theta_{s,\text{grain}} = \frac{y_w}{R_s} \),

\( v_s \) is the speed of the wheel surface, \( v_w \) is the in-feed velocity of the specimen ground, \( t \) is the time and \( R_s \) is the wheel radius (see figure 3.1). The “-” and “+” signs in \( z_w \) are corresponding to up- and down-grinding, respectively, i.e. it determines the rotational direction. The function \( z_s(x,y) \) is given by equation (3.2). \( d_{\text{cut}} \) is the depth of cut, either relative to the average height of the grains \( z_s \) or to the maximum height of the abrasive grains \( z_{\max} \). If the material behaves as a rigid-perfectly plastic material, abrasive grains will produce ideal grooves in the existing workpiece topography, according to their trajectories. This assumption is satisfied for most materials if the degree of penetration \( D_p \approx d_{\text{cut}}/R_{\text{grain}} \) (chapter 2, figure 2.7) is higher than \( D_p > 0.30 \) because for these values microcutting is predicted. Hence, the depth of cut \( d_{\text{cut}} \) relative to the abrasive grain size \( R_{\text{grain}} \) must be large enough.
In general $v_s \gg v_w$, this means that there is a probability that a few different abrasive grains may engage the surface at the same, local position. As a consequence, the profile created at some moment can be modified by subsequent incoming grains at a later moment. In this way, trajectories of all grains scratching the surface are compared with each other, and the surface level is determined from the collection of those trajectories in order to minimise the surface topography. In the simulation a fixed time step $dt$ is used. The surface of the workpiece is then discretised with a grid size $s = v_w dt$. The movements during the grinding process are separated in two parts. First, the wheel moves along the $y$-direction with a step $s_{dt}$. Then the wheel rotates through an angle $\theta = dt v_s / R_s = s_{dt} v_s / v_w R_s$. During this rotational movement the actual removal of material takes place by modifying the profile of the workpiece at position $y_w(t_0 + dt)$ as is demonstrated in figure 3.7. The minimisation (which is similar as that is used by Bhateja\cite{5} in chapter 2) is performed over the angle $\theta_{dt}$. The whole grinding process is considered as a repetition of such steps and rotations. In this way, the discretisation is actually only involved into the translational part, i.e. the movement along $y$ direction. To minimize the error introduced by this approach, the step $s_{dt}$ and hence the time step $dt$ is chosen as small as possi-

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{simulation_method.png}
\caption{Schematic picture of the simulation method.}
\end{figure}

The translational movements and the rotational movements are separated in the simulation. First the wheel moves a step $s_{dt}$ corresponding to a time step $dt$, along the $y$-direction. Then the wheel rotates through the angle $\theta_{dt}$. The union of all profiles within this angle, i.e. the effective, cutting profile will be transferred to the workpiece at position $y_w(t_0 + dt)$. The whole ground surface is rendered by repeating such steps and rotations.
The angle $\theta_{di}$ should be at most corresponding to the average distance $L$ between the abrasive grain and therefore the upper limit of the step size is $s_{di} \leq v_w L / v_s$ (equation (2.2)). A too fine step may increase the computational time tremendously.

A brief summary: the present model is based on the grinding wheel topography, that is simulated using equation (3.3). The ‘transition’ to the workpiece topography is realised via equation (3.5) where an ideal ‘removal’ process is assumed. So far, this method is similar to the approach used in one-dimensional models (chapter 2). However, in these models, equally spaced abrasive grains were used, all of the same height. The set of equations (3.3) and (3.5) allows the instantaneous rendering of the surface topography. Therefore, the real area of contact between the grinding wheel and the specimen can be derived, in contrast to the one-dimensional model where it is not possible to calculate real contact areas. From the real area of contact the forces acting during the process can be computed which, in turn, constitutes a basis for analysis of the surface integrity of a workpiece (and related properties like strength and permeability).

Figure 3.8 Simulated engagement of a single abrasive grain.

Example of a single abrasive grain encountering the surface of the workpiece and removing the material by ideal cutting at normal processing conditions: $v_s=30 \text{ m/s}$, $v_w=12.2 \text{ m/min}$, $d_{cut}=2 \mu\text{m}$. 

Figure 3.8 Simulated engagement of a single abrasive grain.
3.2.3 Materials response and force calculations

The material removal behaviour during the engagement of a single abrasive grain can be simulated by using a scratch test. The material removal behaviour, pile-up and cracking of the material can be studied by such scratch experiments under various conditions like different attack angle or various loads. Since perfectly plastic material behaviour is assumed in the grinding model, it is important to quantify the deviation with respect to this removal behaviour.

The forces acting during a scratch test are theoretically estimated by a simple ploughing model which is based on the hardness of the material \cite{33}. The normal force $F_n$ between the indenter and the specimen is given by:

$$F_n = S \cdot H_v$$

(3.6)

where $H_v$ is the Vickers hardness and $S$ is the projected contact area between both bodies. For a spherical indenter this contact area is $S = \pi r^2 / 2$ (division by two because only half of the indenter is in contact during scratching, see figure 2.7 & 3.9) with $r$ the radius of the cross-section of the contacting area.

The tangential force $F_t$ exerted during the movement of a scratching indenter, can be split in two terms of different physical origin. The first term is depending

\[ \text{figure 3.9: Schematic figure of a scratch and the important variables} \]

When a scratch is made on a material the following parameters can be identified. R is the radius of the indenter, r is the radius of the imprint and d is the depth of the groove. The projected contact zone during scratching is gray-shaded
on the normal force $F_n$ applied to the indenter. The second part is a shearing term (sticking) and is proportional to the shear stress $\gamma$ as a fraction of the yield strength in shear $\gamma_m$ of the material. The shear strength is often approximated by $\gamma = \sigma_y/2 \approx H_v/6$. The tangential force for a spherical indenter can be written as [64]:

$$F_t = H_v R^2 \left[ \arcsin \left( \frac{r}{R} \right) - \frac{r}{R} \sqrt{1 - \left( \frac{r}{R} \right)^2} \right] + 2\gamma R^2 \left[ 1 - \sqrt{1 - \left( \frac{r}{R} \right)^2} \right]$$

(3.7)

where $R$ is the radius of the indenter, $r$ is half of the projected contact width (refer to figure 3.9) and $H_v$ is the hardness of the material being scratched. The friction coefficient $\mu$ is usually defined as the ratio between the tangential force $F_t$ and the normal force $F_n$ exerted by the indenter during the scratch tests. For a spherical indenter $\mu$ can be expressed as [64]:

$$\mu = 2\pi \left( \frac{R}{r} \right)^2 \left[ \arcsin \left( \frac{r}{R} \right) - \frac{r}{R} \sqrt{1 - \left( \frac{r}{R} \right)^2} \right] + \frac{\gamma}{3\gamma_m} \left[ 1 - \sqrt{1 - \left( \frac{r}{R} \right)^2} \right]$$

(3.8)

The $\gamma/\gamma_m$ ratio varies for lubricated and unlubricated sliding contact and is usually less than 0.3 for the lubricated system and close to unity for the unlubricated system. Equations (3.6) - (3.8) were derived for a spherical indenter but a similar approach can be used to calculate the friction coefficient for other indenter geometries.

The real contact area between the grinding wheel and the workpiece was calculated as a function of time in order to determine the forces between the wheel and the workpiece during the grinding process. As mentioned before, the grains are assumed to be blunt because of the shallow depth of cut compared to the grain size. Therefore, the force applied by a grain can be described by a ploughing contact between a sphere (using an effective radius) and the workpiece using equation (3.6). The total normal force as a function of time $F_n(t)$ is then given by the superposition of the forces exerted by each individual abrasive grain and which will result in the following equation:

$$F_n(t) = \frac{\Gamma}{2} H \sum_i S_i = \frac{1}{2} HS(t)$$

(3.9)
where $S_i$ is the real contact area of an individual grain per unit of length, see figure 3.7. $\Gamma$ normalises the force per unit width and the factor $\frac{1}{2}$ emphasises that the contact area in scratching mode is half of the contact area for indentation, see figure 3.9.

During the simulations some aspects that were not mentioned up to now may affect the force and topography calculations. The elastic deflection of the grinding machine, the elastic deflection of individual abrasive grains and roundness errors of the grinding wheel may influence the force measurements during the grinding experiments significantly. However, it was shown that the overall stiffness of the grinding machine, grinding wheel and deflections of abrasive grains might be approximated by $\delta h \sim F_n^{\frac{1}{2}}$ and the error made was estimated and in the order of 10% [35]. Therefore, the grinding machine and wheel are considered to be rigid and the deflections of individual grains are neglected during the simulations. Partly, the roundness errors, i.e. an elliptical wheel shape, can be removed by measuring the wheel deflections and instantly changing the wheel radius during the simulation or by filtering out the fluctuations from the resulting topography assuming a rigid–perfectly plastic material behaviour. Using a slightly different method, these topographical errors can be determined from the measured grinding forces afterwards. The normal force between a spherical indenter encountering the surface of the workpiece is linearly proportional to the depth of cut for a small $d_{\text{cut}}$, i.e. $F_n = k d_{\text{cut}}$. Therefore, the topographical fluctuations are proportional to the force fluctuations, which are measured during the grinding experiments.

3.2.4 Prediction of the change in functional materials properties

Residual stresses

The residual stress induced by the grinding process can be evaluated using an analysis proposed by Hill [36] and Johnson [37, 38, 39, 40], which is described in appendix 3.1. The method is based on the analysis of the contact between a rigid body with a cylindrical shape and a flat, elastic-plastic deformable surface. The material behaviour can be divided in three different regimes, depending on the distance $r$ from the indenter. Near the indenter, a hydrostatic core is assumed of constant pressure. Far away from the indenter, the material behaviour is pure elastic and in between, an intermediate behaviour is expected, i.e. an elastic-plastic zone. The residual stress can be determined from
the size of the hydrostatic core and the elastic-plastic zone. The stress components are recalculated for the normal and tangential directions. The total stresses are calculated using the superposition principle under the assumption that the stress does not relax during the cutting of a subsequent incoming abrasive grain. Here, the stresses in tangential direction, i.e. perpendicular to the grinding direction, will almost be cancelled out and therefore the residual stress is found to be compressive. Subsequently, a reasonable estimate (upper-limit) of the residual stress induced by the grinding process can be made.

**Magnetic permeability**

The surface roughness and the induced residual stresses may change the magnetic permeability at the machined surface and will therefore change its magnetic performance. The change in magnetic permeability is simulated by dividing the cross-section of the ground material in three different layers. The first layer represents the rough surface; the following layer is affected by the residual stresses because of the magnetostriction and the lowest layer is assumed not to be influenced by the grinding process, i.e. it has the original bulk permeability of the material. The effective magnetic permeability of such layered media may be calculated by:

\[
\mu_{\text{eff}}^{\text{tot}} = \frac{1}{A_{\text{tot}}} \sum_{i=1}^{3} \mu_i A_i
\]

(3.10)

where \( A_{\text{tot}} \) is the area of the cross-section, \( A_i \) corresponds to the area of the cross-section of layer \( i \) and \( \mu_i \) is the magnetic permeability of that layer. The analysis of the permeability of the rough layer is complicated because the magnetic field lines may vary with the surface slope. Therefore, the magnetic field lines are supposed to be parallel to the surface and pass through a medium with alternating magnetic permeability, i.e. the asperities are assumed to be of air with a different magnetic permeability. In other words, the material is assumed to consist of a serial stacking of two materials with different permeability. In addition, this layer is influenced by the residual stress and therefore an effective permeability is used for this layer. However, it should be noted here that the residual stress might be relaxed at the rough layer because of the free surface. The effective permeability in this rough layer is given by [41]:

\[
\mu_{\text{eff}}^{\text{tot}} = \frac{1}{A_{\text{tot}}} \sum_{i=1}^{3} \mu_i A_i
\]
\[
\tilde{\mu}^\text{eff} = \frac{\int \frac{dl}{A}}{\int \frac{dl}{\mu A}} = \frac{\sum l_k}{A} = \frac{\sum \frac{1}{\mu_k l_k}}{A_k}
\] (3.11)

where \( l_k \) are line segments along the surface. In general, this layer is very small compared to the overall cross-section, \( A_k \ll A_\text{tot} \), and consequently makes a negligible contribution the total permeability. In appendix 3.2 a more detailed description is presented and the effect of the residual stress on the magnetic permeability is explained, see also references [42, 43, 44, 45].

### 3.3 RESULTS OF THE GRINDING MODEL

In this section, some general results of the grinding model are presented. Moreover, a comparison of these results is made with the existing one-dimensional models described in chapter 2. Since no material response, i.e. mechanical properties of the workpiece, is included in the model, the simulated topography may be valid for an arbitrary material under the condition that the

**figure 3.10: Simulated ground surface.**

This topography was simulated using a wheel speed \( v_s = 30 \text{ m/s} \). The speed of the workpiece was \( v_w = 3 \text{ m/min} \) and the depth of cut was \( d_{\text{cut}} = 5 \mu\text{m} \). A D91 grinding wheel with a radius of \( R_s = 12.5 \text{ cm} \) was simulated according the procedure described in §3.2.1. The image shows the typical grooves in the grinding direction.
MODELLING OF THE GRINDING PROCESS

response of this material approaches a rigid perfectly plastic behaviour, i.e. all material is removed from the enveloping profile.

The simulations were set up to monitor the roughness as a function of the main processing parameters, i.e. the speed of the grinding wheel $v_s$, the speed of the workpiece $v_w$ and the depth of cut $d_{cut}$. In figure 3.10 the simulated ground surface is shown using a set of standard grinding parameters. The figure shows the typical grooves of the abrasive grains parallel to the grinding direction. The roughness perpendicular to the grinding direction is higher than parallel to the grinding direction.

In figure 3.11 the roughness values, calculated from the model using the average over all lines parallel or perpendicular the grinding direction, are plotted versus the depth of cut. The error bars are the standard deviations of the average values. The calculation of the roughness is explained in chapter 2. As shown in the graph, there is a slight increase in roughness with an increase of depth of cut. This can be explained by the fact that as the depth of cut increases, more abrasive grains will become active during the grinding process. Finally, the roughness saturates because all grains on the wheel takes part in the abrasive process. The depth of cut is not included in the one-dimensional grinding models, described in chapter 2. In these models, all the abrasive grains were assumed to have the same size and are equally spaced.

![figure 3.11: $R_a$ roughness versus depth of cut.](image)

The $R_a$ roughness values as a function of depth of cut $d_{cut}$ was calculated from the simulated topography for fixed wheelspeed $v_s=30$ m/s and fixed speed of the workpiece $v_w=3$ m/min. The error bars are the standard deviations of the average roughness values.
The relation between the roughness and the speed of the grinding wheel is presented in figure 3.12a. It is shown that the roughness decreases for increasing wheel speed. In chapter 2, it has been argued that for increasing wheel speed, more abrasive grains of the wheel will hit the surface at the same local spot, and more material will be removed leaving a smoother surface. The models proposed in chapter 2 are modified and fitted to the roughness values obtained from the present model (§3.2.1 and §3.2.2). The following equation was used:

\[ R_s = A \left( \frac{v_w - L}{v_s \sqrt{R_s}} \right)^m + B \]  

(3.12)

where \( A, B \) and \( m \) are fitting parameters and \( L \) is the average distance between the grains, measured on the grinding wheel (§3.2.1). For figure 3.12 the exponent \( m \) was set equal to the original model, i.e. \( m=2 \) and \( m=0.8 \) for the roughness parallel and perpendicular to the grinding direction, respectively. The fitting parameters are \( A=3.6 \cdot 10^4 \) and \( B=0.47 \) µm for the parallel direction and \( A=29 \) and \( B=2.4 \) µm for the perpendicular direction. As demonstrated in the graphs, the agreement between the roughness values traverse to the grinding

![Figure 3.12: \( R_a \) and \( R_z \) roughness values versus the grinding parameters.](image)

The roughness values \( R_a \) and \( R_z \) calculated from the simulated ground surfaces obtained from the 3D-model versus wheel speed and speed of the workpiece. In figure a the other grinding parameters were depth of cut \( d_{cut}=8 \) µm and workpiece speed \( v_w=3 \) m/min, for figure b \( d_{cut}=5 \) µm, \( v_s=30 \) m/s. ■ corresponds to the \( R_a \) value perpendicular to and ○ is \( R_a \) parallel to the grinding direction, ▲ is the \( R_z \) value perpendicular to and ▼ is \( R_z \) parallel to the grinding direction. The data points are calculated with the 3D-model, where the lines are \( R_z \) values predicted by the model from chapter 2.
direction is reasonably good, but parallel to the grinding direction there is a mismatch between the one-dimensional model and the present three-dimensional model. This may be ascribed to the random position of the grains on the grinding wheel and the random size distribution of the grains used in the present model in contrast to the models mentioned in chapter 2. For smaller wheel speed, the influence of the random position of grains becomes more predominant because only a few grains will encounter the surface for slow wheelspeeds. For the larger speed, this effect will be cancelled out.

The roughness as a function of the speed of the workpiece is displayed in figure 3.12b. The increase of the roughness with an increase in workpiece-speed can be interpreted using the same argument as used to explain the decrease in roughness for increasing wheel speed. More grains will remove material from the same local position for the lower speed of the workpiece. Therefore, the effective enveloping profile becomes smoother (see figure 2.5). The fit of the one-dimensional model for the perpendicular roughness values is reasonably good. However, the values predicted by the one-dimensional model parallel to the grinding direction do not fit with the present model. This can be explained using equation (3.12) which predicts an infinite surface roughness with a very

![Graph of roughness Rz versus the speed ratio vw/vs.](image)

**Figure 3.13:** Roughness Rz versus the speed ratio vw/vs.

The ratio of the speed of the grinding wheel and the speed of the workpiece is a dominant factor in grinding. From this graph the exponent m is determined. ■ and ○ correspond to the Rz values perpendicular and parallel to the grinding direction respectively, for fixed wheelspeed vw=30 m/s and depth of cut dcut=8 µm. ▲ and ▼ refer to the Rz values perpendicular and parallel to the grinding direction respectively, for fixed workpiece speed vw=3 m/min and depth of cut dcut=5 µm. The slight shift for ▼ and ○ is due to the difference in depth of cut.
high speed of the workpiece. Obviously, this cannot be the case; there is a maximum surface roughness with increasing speed of workpiece with the limit of only a few grains scratching the surface. The values calculated by the present model shown in figure 3.12 suggest that there is a trend to a maximum roughness value.

Some researchers argued that the measured roughness is less depending on the ratio \( v_w/v_s \) for a larger depth of cut. Therefore the exponent \( m \) may be smaller than derived in chapter 2 \[29\]. The dependence of the roughness determined from the present model on the ratio \( v_w/v_s \) was studied in figure 3.13 where the exponent \( m \) was determined from the log-log graph of the roughness versus \( v_w/v_s \). It was observed that the exponent is significantly lower than the values derived in chapter 2, \( m = 0.61 \pm 0.02 \) parallel and \( m = 0.055 \pm 0.002 \) perpendicular to the grinding direction. This indicates that the roughness is depending on \( v_w/v_s \) ratio, but to a smaller extent than was predicted before. Especially, the roughness in the perpendicular direction after grinding seems to be more dependent on the details of the distribution of abrasive grains on the modelled grinding wheel rather than on the processing parameters.

3.4 DISCUSSION AND CONCLUSIONS

This chapter described a three-dimensional model for the simulation of the plane grinding process. The simulation is based on a unit event model, i.e. the engagement of a single abrasive grain with the workpiece, by simulating the full topography of the grinding wheel. For each time step, an effective profile was created. This time step should be chosen small enough, in such a way that the distance through which the wheel rotates during this time is smaller than the average distance of the abrasive grains.

A rigid, perfectly plastic material behaviour, i.e. ideal cutting, is assumed in this model. As a consequence, material hardening, plastic pile-up and elastic deformation is not accounted for during the simulations. Therefore, the material response during the engagement of an individual grain should be studied and validated against the simple material response. A convenient tool may be scratch experiments under various loads and various indenter shapes. A more complicated material response can be included by calculating the grinding forces and the stresses. This will allow the determination of cracks and residual stresses. These stresses are calculated from the stress fields of the individual grains using the superposition principle. A quasi-static indentation model of a
spherical rigid indenter was used to estimate the stresses applied by a single abrasive grain. However, the stress fields may change considerably by the movement of the indenter. It should also be mentioned that stress relieve due to cracking was not included.

The depth of the subsurface damage is estimated from the induced residual stresses, using an elastic-plastic indentation model. It is also shown that the change of functional properties, e.g. magnetic permeability, can be determined from the residual stresses induced by the grinding process. The cross section was divided in three layers in which the magnetic permeability could be calculated using the magnetostriction and effective area.

The comparison between the one-dimensional models and the present model shows that the random distribution of abrasive grains on the wheel and the random size distribution have a significant influence on the final roughness of the ground surface. However, the general relations between the roughness and the grinding parameters are confirmed, but the processing parameters, like $v_s$ and $v_w$ may be of less influence on the surface condition as was predicted by the one-dimensional model, described in chapter 2. Also the work of Koshy et al. on a three dimensional grinding model has to be mentioned [20]. Although the model is similar to our work, this model does not include grinding forces, residual stresses and changes in magnetic permeability. Besides, the material removal behaviour was not studied to confirm the assumption of a rigid-perfectly plastic response. Furthermore, it is shown that the one-dimensional models fail under certain conditions. For example, they predict infinite surface roughness with infinite speed of the workpiece in contrast to the present model, which indicates that there is a trend towards a maximum roughness value. Moreover, the one-dimensional model cannot be used to predict residual stresses and changes in functional properties since the real contact area cannot be determined while in the present model this is possible.

3.5 REFERENCES

29. S. Malkin, Grinding technology: theory and applications of machining with abrasives, chapter 4.6, Society of Manufacturing Engineers (SME), Michigan, 1989
32. E.E. Underwood, Quantitative stereology, Addison-Wesley, 1970
41. E. Olsen, Grootheden uit het toegepaste magnetisme in de wisselstroomtechniek, Philips Technische Bibliotheek, 1994
43. C. Kittel, *Reviews of Modern Physics*, 21 (4) 541-582 (1949)
APPENDIX 3.A

In this appendix, the analysis of the residual stress after grinding will be explained in detail. The deformation of the material during engagement of a single abrasive grain can be described by a hemi-cylindrical contact assuming blunt grains. The deformation of the whole workpiece during the grinding process can then be simulated by the superposition of the individual hemi-cylindrical contacts. When a blunt indenter is pressed against an elastic-plastic medium, three different regions can be identified beneath the surface, see figure 3A.1 [36,37]. The three zones can be described as follows:

1. hydostatic core, for $h ≤ r ≤ a$
2. plastic zone, for $a ≤ r ≤ c$
3. elastic region, for $c ≤ r$

The stresses in the different regions are described by Hill [36] and Johnson [37,38]. The stress in the hydrostatic zone is constant and is estimated by the stress in the plastic region for $r = a$. Hill’s solution for the stresses in the three

![Figure 3A.1: Elastic plastic indentation of a hemi-cylinder.](image)

The materials response upon the indentation of a rigid hemi-cylindrical indenter can be divided in three regions. The first zone is a hydrostatic core, then there is an elastic-plastic region and below there is only a pure elastic response.
regions, for an expanding hemi-cylinder is given by:

\[ \frac{p^*}{Y} = \frac{\sigma_r}{Y} \bigg|_{r=a} = \frac{1}{2} + \ln \left( \frac{c}{a} \right), \quad \text{for} \quad h \leq r \leq a \]  

\[ \frac{\sigma_r}{Y} = -\frac{1}{2} \ln \left( \frac{c}{r} \right), \quad \frac{\sigma_\theta}{Y} = \frac{1}{2} - \ln \left( \frac{c}{r} \right) \quad \text{for} \quad a \leq r \leq c \]  

\[ \frac{\sigma_r}{Y} = -\frac{c^2}{2r^2}, \quad \frac{\sigma_\theta}{Y} = \frac{c^2}{2r^2} \quad \text{for} \quad r \geq c \]  

Equations \([3A.1]\) t/m \([3A.3]\) can be modified for a material that yields according the von Mises criterion, i.e. an effective yield strength \(Y' = Y \sqrt{\frac{2}{3}}\), but here the Tresca’s criterion \(\sigma_\theta - \sigma_r = Y\) is used. The size of the plastic zone \(c\) can be derived from expansion of a cylindrical cavity in an infinite medium, by the radial displacement of material particles which results in \([36]\)

\[ \frac{du(r)}{dc} = \frac{Y}{E} \left[ \frac{5-4\nu}{2} \frac{c}{r} \left( 1 - \frac{3(1-2\nu) r}{2c} \right) \right] \]  

Note that in general the movement of the particles is less than the movement of the core boundary. However, here the movement of the boundary of the hydrostatic core \(da\) is set be equal to the movement of the material particles \(du(a)\). Assuming conservation of volume of the core \(\pi a \ du(a) = 2a \tan \beta \ da\), and using the symmetry of the strain field \(dc/da = c / a = \text{const.}\), equation \([3A.4]\) can be solved at the core boundary, i.e. \(r=a\) which will result in:

\[ \frac{4E}{\pi Y} \tan \beta = (5-4\nu) \left( \frac{c}{a} \right)^2 - 3(1-2\nu) \]  

Solving equation \([3A.5]\) for \((c/a)\) and substituting this ratio in equation \([3A.1]\) yields

\[ \frac{p^*}{Y} = \frac{1}{2} + \frac{1}{2} \ln \left( \frac{4 \tan \beta E}{\pi (5-4\nu) Y} \right) \]  

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where \( \tan \beta \) can be approximated by \( h/a \) like in figure 2.7.

The residual stress can be obtained from the difference between the solution proposed by Hill [36,39,40], described above and the pure elastic (Hertz) solution. Because of the small depth of cut compared to the size of the abrasive grains, the slope of the surface is slowly changing in lateral direction. Therefore, the direction of the stress is almost merely in the \( z \)-direction, i.e. the angle of the stress with the \( z \)-direction is small and hence the stress in lateral direction is negligible. Consequently, the total stress can be calculated directly from the three-dimensional topography obtained from the kinematic model described in § 3.2.3. Here a simplification was made by assuming that all grains have the same average size. The total stress along the \( z \)-direction using equation (3A.5) to substitute \( c \) is then approximated by:

\[
\sigma_z \approx - \frac{Yc^2}{2z^2}, \quad \text{for } z \geq c
\]

\[
\frac{\sigma_z}{Y} = - \frac{1}{2} \ln \left( \frac{c}{z} \right), \quad \text{for } a \leq z \leq c
\]

The Hertzian solution along the \( z \)-axis for the contact between a hemicylindrical body and an elastic half space is given by:

\[
\sigma_z = - \frac{p^*}{\left( 1 + \left( \frac{z}{a} \right)^2 \right)^{3/2}}
\]

where \( p^* \) is derived from equation (3A.6). The residual stress in the regions is given by the difference between the corresponding solution, equation (3A.7) and the Hertzian equation (3A.8).
APPENDIX 3.B

Here, the evaluation of the magnetic permeability as a function of (residual) stress is explained. First, the cross-section of the ground magnetic material is divided in three regions, see figure 3A.2. The magnetic permeability of the upper region is affected by the surface roughness as well as by the residual stress. The layer in the middle is influenced only by the machining induced residual stresses and the magnetic permeability in the lowest layer is assumed to be the same as before the grinding process. In §3.2.5 the effect of surface roughness is already explained. Therefore, this appendix is focussed on the influence of residual stress on the magnetic properties.

The strain caused by the ordering of magnetic moments is called the magnetostriction, defined as \( \lambda = \Delta l / l \). In an isotropic medium, i.e. for randomly oriented magnetic domains, the saturation magnetostriction \( \lambda_s \) at any angle \( \theta \) with respect to the field direction \( H \) is given by [41,42]

\[
\lambda_s(\theta) = \frac{3}{2} \lambda_s \left( \cos^2 \theta - \frac{1}{3} \right)
\]

(3B.1)

figure 3A.2: Calculation of the magnetic permeability at the machined surface.

The cross-section is divided in three zones. The first is influenced by the roughness and the residual stress, the second is only affected by the residual stress and the grinding process does not change the last region.
where $\lambda_s$ is the saturation magnetostriction along the direction of magnetisation. The total energy of the system can be written in terms of magnetic energy $E_M = -HI_s \cos(\theta - \theta)$ and magnetic strain energy $E_\sigma = -\sigma \int_0^\theta d\lambda = 3\lambda_s \sigma \sin^2(\theta)/2$ at saturation. By minimising the total energy with respect to all possible rotations $\theta$, i.e. all possible angles between the stress and the magnetisation vector, the following expression is obtained:

$$H = \frac{3\lambda_s \sigma \sin 2\theta}{2I_s \sin(\theta_0 - \theta)} \quad (3B.2)$$

where $I_s$ the intensity of the magnetisation at saturation. The definition of the initial magnetic permeability $\mu, \mu_0 = 1 = \chi_i = dI/dH |_{\theta=0, H=0}$ is used to express the permeability for $H$ in perpendicular direction with respect to the stress, i.e. $\theta_0 = \pi/2$. The relation between the initial magnetic permeability $\mu_i$ and the (residual) stress $\sigma_z$ finally results in:

$$\bar{\mu}_i \approx \frac{I_s^2}{3\lambda_s \mu_0 \sigma_z} \quad (3B.3)$$

where $\mu_0$ is the magnetic permeability of vacuum and $\lambda_s$ the saturation magnetostriction along the direction of magnetisation. Equation [3B.3] can be used in order to estimate the change in permeability due to the residual stress induced by the grinding process.