Multi-level ILU preconditioners and continuation methods in fluid dynamics

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Chapter 7

Conclusions

In this thesis we have considered preconditioners based on an incomplete LU factorization, and applied them in solving problems originating from the field of computational fluid dynamics. The flows that we have computed are described by the Navier-Stokes equations. Instead of using the popular pressure-correction approach, we have solved the equations fully coupled. With this approach non-symmetric large sparse systems have to be solved. With the increase of computer power and the development of fast linear solvers this approach has become feasible.

A good preconditioner is essential when solving large sparse systems, originating for instance from the discretization of partial differential equations, iteratively. An important class of preconditioners is formed by the (modified) incomplete LU factorizations ((M)ILU). The quality of these preconditioners strongly depends on the ordering of the unknowns and the dropping strategy employed during the factorization.

In Chapter 2 and 3 we have considered block (M)ILU factorizations with respect to a repeated red-black ordering. The ordering in such factorizations is fixed, and the dropping is based only on the position of the fill and not on the size of the elements. This type of preconditioners is attractive because the construction of the factorization is cheap and can easily be vectorized.

In Chapter 2 we have applied several variants of the block RRB preconditioner to some test cases. We used either an ILU or MILU factorization, of which the last level was factorized exactly or approximated by a block diagonal matrix. The fastest convergence was obtained with the MILU factorization with an exact factorization of the last part. For the Poisson system this method was almost grid independent. However, for the (Navier-)Stokes equations the convergence stagnated when the number of levels was taken too high in this factorization.

In Chapter 3 we have given theoretical estimates for the condition number of the preconditioned system. Furthermore, we have estimated for some test cases the eigenvalues of the preconditioned system, and with these results explained the convergence behaviour observed in Chapter 2.

The factorizations can be improved by using in the MILU factorizations a relaxed modification, and by replacing the diagonal approximation at the last level with a more advanced one. Furthermore, the method can be made more competetive when a time derivative is added to the system. A disadvantage of this method is that the ordering is fixed. Therefore, the grid has to be structured. Another disadvantage is that the dropping is based on the position of the fill and not on its size. The factorization will become worse when the elements of the matrix will vary largely in size.

In Chapter 4 we consider the MRILU factorization. With this method the ordering of the unknowns and the dropping are determined during the factorization, and are both based on the size of the elements in the factorization. This factorization has succesfully been applied in a variety of problems. Because both the ordering and dropping are based on the matrix, the method can handle matrices with a general sparsity, for instance matrices stemming from the discretization on an unstructured grid. For the Poisson problem the method shows grid independent convergence. We have applied the MRILU factorization in solving the Navier-Stokes equations. We observed that the performance of the factorization is better when the diagonal of the system is stronger, which can be obtained by using a suitable discretization for the convection terms or adding a time derivative to the system. In the future the MRILU factorization will be improved further.

Continuation methods are used to perform a bifurcation analysis of a parameter dependent system. Until recently, these methods have only be applied to low-dimensional systems (about 10 degrees of freedom). The bottle-neck in computations on high-dimensional systems (about 10^5 degrees of freedom) is formed by the necessity to solve large linear systems and eigenvalue problems. With the development of fast linear solvers and eigenvalue solvers such computations have become feasible.

In Chapter 5 we have used a pseudo-arclength continuation method to perform a bifurcation analysis of the Rayleigh-Bénard problem. For this problem the solutions are stationary. The MRILU factorization has been used to compute the solutions, and turned out to be very robust. The Jacobi-Davidson QZ method has been used to solve the eigenvalue problem which determines the stability of the solutions and the position of the bifurcation points. This method allows to compute a few eigenvalues near a user specified target, and hence is well suited for application in a continuation method. The MRILU factorization can be used as preconditioner in the JDQZ method.

We have used the Newton-Picard method in Chapter 6 to perform a bifurcation analysis of the lid-driven cavity problem. With this method both stationary and periodic solutions of high-dimensional systems can be computed and their stability determined.

For the time discretization of the Navier-Stokes equations we have used an implicit method, which is possible when a good preconditioner as MRILU is used. We have used a second-order spatial discretization with no artificial diffusion.

On a 128×128 grid we found that the first Hopf bifurcation is at about Re=8375. The next Hopf bifurcations are at about Re=8600, 9000, 9100, 10,000. The frequencies corresponding to the corresponding unstable modes are also encountered on the branch of periodic solutions occurring at Re=8375. This periodic solution is stable up to Re=9150, and no new unstable modes occur before Re=10,000. The periodic branch emerging from the second Hopf bifurcation point at Re=8600 is unstable up to Re=8800, and thereafter stays stable to at least Re=10,000. Our results are in good comparison with results obtained by others.

More accurate results can be obtained when using a higher-order discretization and a finer grid. To make the latter possible the Newton-Picard method has to become faster, especially the convergence of the invariant subspace has to be improved.