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Miura, Kohtaroh; Lombardo, Maria Paola; Pallante, Elisabetta

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Thermodynamic Study for Conformal Phase in Large $N_f$ Gauge Theory

Kohtaroh Miura*
INFN Laboratori Nazionali di Frascati, I-00044, Frascati (RM), Italy
E-mail: Kohtaroh.Miura@lnf.infn.it

Maria Paola Lombardo
INFN Laboratori Nazionali di Frascati, I-00044, Frascati (RM), Italy
Humboldt-Universität zu Berlin, Institut für Physik, D-12489 Berlin, Germany
E-mail: Mariapaola.Lombardo@lnf.infn.it

Elisabetta Pallante
Centre for Theoretical Physics, University of Groningen, 9747 AG, Netherlands
E-mail: e.pallante@rug.nl

We investigate the chiral phase transition at finite temperature ($T$) in colour SU($N_c = 3$) Quantum Chromodynamics (QCD) with six species of fermions ($N_f = 6$) in the fundamental representation. The simulations have been performed by using lattice QCD with improved staggered fermions. The critical couplings $\beta^L_c$ for the chiral phase transition are observed for several temporal extensions $N_t$, and the two-loop asymptotic scaling of the dimensionless ratio $T_c/\Lambda_L$ ($\Lambda_L =$ Lattice Lambda-parameter) is found to be achieved for $N_t \geq 6$. Further, we collect $\beta^L_c$ at $N_f = 0$ (quenched), and $N_f = 4$ at a fixed $N_t = 6$ as well as $N_f = 8$ at $N_t = 6, 12$, the latter relying on our earlier study. The results are consistent with enhanced fermionic screening at larger $N_f$. The ratio $T_c/\Lambda_L$ depends very mildly on $N_f$ in the $N_f = 0 - 4$ region, begins increasing at $N_f = 6$, and significantly grows up at $N_f = 8$, as $N_f$ reaches to the edge of the conformal window. We discuss the interrelation of the results with preconformal dynamics in the light of a functional renormalization group analysis.

*Speaker.
1. Introduction

Emergence of a conformal symmetry and a preconformal (walking) behavior in strongly flavored non-Abelian gauge theories has received much attention. Walking dynamics near the infrared fixed point has been advocated as a basis for strongly interacting mechanisms of electroweak symmetry breaking. Lattice Monte-Carlo simulations are expected to provide a solid theoretical base to understand the (pre-)conformal nature in the gauge theory.

A second zero of the two-loop beta-function of massless QCD with \( N_f \) flavours implies, at least perturbatively, the appearance of an infrared fixed point (IRFP) at \( N_f \gtrsim 8.05 \) with the restoration of conformal symmetry before the loss of asymptotic freedom (LAF) at \( N_f^{LAF} = 16.5 \). Conformality should emerge when the renormalized coupling at the would be IRFP is not strong enough to break chiral symmetry. This condition provides the lower bound \( N_f^c \) of a so called conformal window in the flavor space, and we find elaborated analytic predictions \([3, 4]\): for instance, the functional renormalization group method \([5]\) suggests \( N_f^c \sim 12 \). Before the emergence of conformal symmetry, a qualitative change of dynamics is claimed at \( N_f = 6 \) based on instanton study \([6]\).

Recent lattice studies \([7]\) focused on the computation of the edge of the conformal window \( N_f^c \) and the analysis of the conformal window itself, either with fundamental fermions \([8–15]\), or other representations \([16]\). Among the many interesting results with fundamental fermions, we single out the observation that QCD with three colours and eight flavours is still in the hadronic phase \([8, 10]\), while \( N_f = 12 \) seems to be close to \( N_f^c \), with some groups favouring conformality \([8, 9, 11, 12]\), and others chiral symmetry breaking \([14]\). The onset of new strong dynamics at \( N_f = 6 \) has been implied via an enhancement of the ratio of chiral condensate to cubed pseudoscalar decay constant \([17]\).

Using the thermal transition as a tool for investigating preconformal dynamics has been largely inspired by a renormalization group analysis \([5]\). The critical temperature for the chiral phase transition has been obtained as a function of \( N_f \). Then the onset of the conformal window has been estimated by locating the vanishing critical temperature. The phase transition line is almost linear with \( N_f \) for small \( N_f \), and clearly elucidates the universal critical behaviour at zero and non-zero temperature in the vicinity of \( N_f^c \). Thus, it would be a promising direction to extend the knowledge of finite \( T \) lattice QCD to the larger \( N_f \) region, by using the FRG results as analytic guidance.

In this proceedings, we investigate the thermal chiral phase transition for \( N_f = 6 \) colour SU(\( N_c = 3 \)) QCD by using lattice QCD Monte Carlo simulations with improved staggered fermions based on our recent study \([1]\). \( N_f = 6 \) is expected to be in the important regime as suggested by the results in Refs. \([3, 8]\). We also compute the critical couplings for \( N_f = 0 \) (quenched) and \( N_f = 4 \) at \( N_t = 6 \), and use the results from Ref. \([10]\) for \( N_f = 8 \). Then we investigate \( N_f \) dependences of the chiral phase transition.

2. Simulation setups

Simulations have been performed in the same as in the study used for \( N_f = 8 \) in Ref. \([1]\): We have utilized the publicly available MILC code \([18]\) with the use of an improved version of the staggered action, the Asqtad action, with a one-loop Symanzik \([19, 20]\) and tadpole \([22]\) improved
gauge action. The tadpole factor $u_0$ is determined by performing zero temperature simulations on the $12^4$ lattice, and used as an input for finite temperature simulations.

To generate configurations with mass degenerate dynamical flavours, we have used the rational hybrid Monte Carlo algorithm (RHMC) [21]. Simulations for $N_f = 6$ have been performed by using two pseudo-fermions, and subsets of trajectories for the chiral condensates and Polyakov loop have been compared with those obtained by using three pseudo-fermions with the same Monte Carlo time step $d\tau$ and total time length $\tau$ of a single trajectory. We have observed very good agreement between the two cases for both evolution and thermalization. We have monitored the Metropolis acceptance and reject ratio, and adjusted $\tau = 0.2 – 0.24$ and $d\tau = 0.008 – 0.016$ to realize the best performance.

Measured observables are the expectation values of the chiral condensate and Polyakov loop,

$$a^3 \langle \bar{\psi} \psi \rangle = \frac{N_f}{4N_s^4N_c^3} \langle \text{Tr}[M^{-1}] \rangle, \quad L = \frac{1}{N_cN_s^3} \sum_x \text{Re} \langle \text{tr}_c \prod_{j=1}^{N_c} U_{4,x} \rangle,$$

where $N_c$ ($N_f$) represents the number of lattice sites in the spatial (temporal) direction, $U_{4,x}$ is the temporal link variable, and $\text{tr}_c$ denotes the trace in colour space. The output of this measurement is the critical coupling $\beta_L^c$ for the chiral phase transition.

3. Results

All results have been obtained for a fermion bare lattice mass $am = 0.02$. In the left panel of Figs. [1], the expectation values of the chiral condensate $a^3 \langle \bar{\psi} \psi \rangle$ are displayed as a function of $\beta_L$ for several $N_f$. It is found that different $N_f$ give a different behaviour of $a^3 \langle \bar{\psi} \psi \rangle$. The asymptotic scaling analysis below will confirm that it corresponds to a thermal chiral phase transition (or crossover) in the continuum limit.

All values of the critical lattice coupling $\beta_L^c$ are summarized in Table [1]. For larger $N_f$, the signal for the chiral phase transition becomes less clear, hence we investigate the histogram of the chiral condensate: The histogram for $N_f = 8$ exhibits the double-peak structure at $\beta_L = 5.2$, i.e., the competition between chirally symmetric and broken vacua. The critical coupling can be estimated as $\beta_L^c = 5.225(25)$ for $N_f = 8$. For $N_f = 12$, we also observe the double-peak structure in the histogram of the chiral condensate around $\beta_L = 5.45$.

These results can be analyzed and interpreted in terms of the two-loop asymptotic scaling. Let us consider the two-loop lattice beta function,

$$\beta(g) = -(b_0 g^3 + b_1 g^5), \quad (b_0, b_1) = \left(\frac{(11-2N_f/3)}{(4\pi)^2}, \frac{(102-38N_f/3)}{(4\pi)^4}\right),$$

for fundamental fermions in colour $SU(3)$. From Eq. (3.1), we obtain the well known two-loop asymptotic scaling,

$$\Lambda_L a(\beta_L) = (2N_c b_0/\beta_L)^{-b_1/(2b_0^2)} \exp\left[-\beta_L/(4N_c b_0)\right].$$

Here, $\Lambda_L$ is the so-called lattice Lambda-parameter, and $\beta_L = 2N_c/g^2$, with $g = \sqrt{2N_c/10 \cdot g_L}$. This definition effectively takes account of the improvement of the staggered lattice action when comparing to the asymptotic scaling law, see Ref. [10]. We insert $\Lambda_L$ to the definition of temperature
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\[ T \equiv [a(\beta_L)N_f]^{-1}, \]

\[ N_f^{-1} = (T_c/\Lambda_L) \times \left( \Lambda_L a(\beta_L^c) \right), \]  \hspace{1cm} (3.4)

and extract the physical quantity $T_c/\Lambda_L$ by substituting the simulation outputs $\beta_L^c$ for Eq. (3.4). This ratio must be unique as long as the asymptotic scaling Eq. (3.3) is verified for a given $\beta_L^c$.

In the right panel of Fig. 1, the slope of the line connecting the origin and the data points corresponds to $T_c/\Lambda_L$. The $N_t=6, 8, 12$ points have a common slope to a very good approximation, while the $N_t=4$ result falls on a smaller slope. The latter is interpreted as a scaling violation effect due to the use of a too small $N_t$. The existence of a common $T_c/\Lambda_L$ for $N_t \geq 6$ indicates that the data are consistent with the two-loop asymptotic scaling Eq. (3.3), confirms the thermal nature of the transition and that $N_f=6$ is outside the conformal window, as expected from a previous study [10]. A linear fit provides $T_c/\Lambda_L = 1.02(12) \times 10^3$, which can be interpreted as the value in the continuum limit for $N_f=6$ QCD.

In order to have a more complete overview, we have performed simulations for the theory with $N_f=0$ (quenched) and $N_f=4$, only at $N_t=6$. These theories are of course very well investigated, however we have not found in the literature results for the same action as ours. We note that in a previous lattice study with improved staggered fermions [23], asymptotic scaling was observed for $N_t \geq 6$ for $0 \leq N_f \leq 4$. Table 1 shows a summary of our results for the critical coupling $\beta_L^c$ of the chiral phase transition at finite temperature for $N_f=0, 4, 6, 8$ - the latter from Ref. [10].

**Figure 1:** Left: The chiral condensate $a^3 \langle \bar{\psi} \psi \rangle$ for $N_f=6$ and $am=0.02$ in lattice units, as a function of $\beta_L$, for $N_f=4, 6, 8, 12$. Error-bars are smaller than symbols. Right: The thermal scaling behaviour of the critical lattice coupling $\beta_L^c$. Data points for $\Lambda_L a(\beta_L^c)$ at a given $1/N_t$ are obtained by using $\beta_L^c$ from Table 1 as input for extracting $\Lambda_L a(\beta_L^c)$ in the two-loop expression Eq. (3.3). The dashed line is a linear fit with zero intercept to the data with $N_t > 4$.

In the left panel of Fig. 2, we display the critical values of the lattice coupling $g_c = \sqrt{2N_t/\beta_L^c}$ from Table 1 in the Miransky-Yamawaki phase diagram. Consider the $N_t=6$ results: it is expected that an increasing number of flavours favors chiral symmetry restoration. Indeed, we find that, on a fixed lattice, the critical coupling increases with $N_f$ in agreement with early studies and naive reasoning. The precise dependence of the critical coupling on $N_f$ at fixed $N_t$ is not known. It is, however, amusing to note that the results seem to be smoothly connected by an almost straight line: the brown line in the plot is a linear fit to the data. Comparing the trend for $N_f=6$ to the one for
$N_f = 8$ for varying $N_f$, one can infer a decreasing in magnitude (and small) step scaling function, hence a walking behaviour. Further study is needed at larger $N_f$, and by using the same action used for $N_f = 0 - 8$, to confirm or disprove it.

**Table 1:** Summary of the critical lattice couplings $\beta_L^c$ for the theories with $N_f = 0, 4, 6, 8, am = 0.02$ and varying $N_f = 4, 6, 8, 12$. All results are obtained using the same lattice action.

<table>
<thead>
<tr>
<th>$N_f \backslash N_f$</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
<td>7.88 ± 0.05</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>5.89 ± 0.03</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>4.675 ± 0.025</td>
<td>5.025 ± 0.025</td>
<td>5.225 ± 0.025</td>
<td>5.45 ± 0.05</td>
</tr>
<tr>
<td>8</td>
<td>-</td>
<td>4.1125 ± 0.0125</td>
<td>-</td>
<td>4.34 ± 0.04</td>
</tr>
</tbody>
</table>

**Figure 2:** Left: Critical values of the lattice coupling $g_c = \sqrt{2N_c/\beta_L^c}$ for theories with $N_f = 0, 4, 6, 8$ and for several values of $N_f$ in the Miransky-Yamawaki phase diagram. The dashed (brown) line is a linear fit to the $N_f = 6$ results. Right: The $N_f$ dependence of $R(N_f)/R(0)$ for several finite fixed $\beta_L^ref$. Here, $R(N_f) = (T_c/\Lambda_L^{ref})(N_f)$.

Next, we study the $N_f$ dependence of the ratio $T_c/\Lambda_L$ and related quantities. In addition to the scale $\Lambda_L$, we introduce more UV reference energy scale $\Lambda_{ref}^c$, which is associated with a reference coupling $\beta_L^ref$. Then Eq. (3.3) is generalized as

$$\Lambda_{ref}^c(\beta_L^c, a(\beta_L)) = \left(\frac{b_1}{\beta_0^2/\beta_L^ref + 2N_c/b_1/b_0}\right)^{b_1/(2b_0^2)} \exp\left[-\frac{\beta_L - \beta_L^ref}{4N_c/b_0}\right]. \quad (3.5)$$

At leading order of perturbation theory $b_1 \to 0$, we find $\Lambda_{ref}/\Lambda_L = \exp[\beta_L^ref/(4N_c/b_0)]$. This equation would be analogous of the ratio $\Lambda_L/\Lambda_{MS}$ derived in [24] for Wilson fermions up to a further linear dependence on $N_f$ in the numerator of the exponent. In a nutshell, the difference originates from the fact that we are fixing a bare reference coupling $\beta_L^ref$, which will be specified later. Notice that by construction $\Lambda_{ref}$ reproduces the lattice Lambda-parameter $\Lambda_L$ in the limit $\Lambda_{ref}(\beta_L^ref \to 0) = \Lambda_L(1 + \mathcal{O}(1/\beta_L^c))$. 
Let us consider first \( R(N_f) \big|_{\beta_L^{\text{ref}} = 0} = T_c/\Lambda_L \). The values of \( T_c/\Lambda_L \) are found to be 600 ± 34, 620 ± 28, 1023 ± 117, and 2098 ± 191 for \( N_f = 0, 4, 6, \) and 8, respectively, and represented as circles in the right panel of Fig. 2 (the vertical axis is normalized by \( R(0) = (T_c/\Lambda_L)(N_f = 0) \) for each \( \beta_L^{\text{ref}} \)). The ratio does not show a significant \( N_f \) dependence in the region \( 0 \leq N_f \leq 4 \), it starts increasing at \( N_f = 6 \), and undergoes a rapid rise around \( N_f = 8 \). The nearly constant nature of \( T_c/\Lambda_L \) in the region \( N_f \leq 4 \) indicates that the role of such energy scale is not significantly changed by the variation of \( N_f \) (see [25] for a detailed discussion of this point.) In turn, the increase of \( T_c/\Lambda_L \) in the region \( N_f \geq 6 \) might well imply that the chiral dynamics becomes different from the one for \( N_f \leq 4 \). Indeed, a recent lattice study [17] indicates that \( N_f = 6 \) might well be close to the threshold for preconformal dynamics.

We now consider \( T_c/\Lambda_{\text{ref}} \) with finite \( \beta_L^{\text{ref}} \). The \( N_f \) dependence of the ratio \( R(N_f) \equiv (T_c/\Lambda_{\text{ref}})(N_f) \) is shown for several \( \beta_L^{\text{ref}} \) in the right panel of Fig. 2 (with normalization by \( R(0) = (T_c/\Lambda_{\text{ref}})(N_f = 0) \) for each \( \beta_L^{\text{ref}} \)). \( T_c/\Lambda_{\text{ref}} \) is now a decreasing function of \( N_f \) for a larger \( \beta_L^{\text{ref}} \). The \( \Lambda_{\text{ref}} \) associated with a \( \beta_L^{\text{ref}} \gg \beta_s = 2N_c/g_{\text{imp}}^2 \) would be less sensitive to the IR or chiral dynamics. Assuming \( N^c_s \simeq 12 \), the two-loop beta-function leads to \( \beta_s = -2N_c b_1/b_0 \simeq 0.63 \). The decreasing nature of \( (T_c/\Lambda_{\text{ref}})(N_f) \) is found to start around \( \beta_L^{\text{ref}} = 1.0 \gtrsim \beta_s \). Thus, the use of a UV reference scale leads to the decreasing \( (T_c/\Lambda_{\text{ref}})(N_f) \). This trend is consistent with the FRG study [8], where the decreasing \( T_c(N_f) \) has been obtained by using the \( \tau \)-lepton mass \( m_\tau \) as a common UV reference scale with a common coupling \( \alpha_s(m_\tau) \). We note that we have constrained our analyses \( \beta_L^{\text{ref}} < \beta_{\text{UV}} = \beta_L^c(N_f) \leq 4.1125 \pm 0.0125 \).

With the use of a UV reference scale, we should observe the predicted critical behavior [3].

\[
T_c(N_f) = K|N_f - N^c_f|^{-1/\theta}.
\]

By choosing the critical exponent \( \theta \) in the range predicted by FRG: \( 1.1 < 1/|\theta| < 2.5 \), our data are consistent with the values \( N^c_f = 9(1) \) for \( \beta_L^{\text{ref}} = 4.0 \) and \( N^c_f = 11(2) \) for \( \beta_L^{\text{ref}} = 2 \). We plan to extend and refine this analysis in the future, and here we only notice a reasonable qualitative behaviour.

4. Summary

We have studied the chiral phase transition at finite \( T \) for colour SU(3) QCD with \( N_f = 6 \) by using lattice QCD Monte-Carlo simulations with improved staggered fermions [1]. We have determined the critical lattice coupling \( \beta_L^c \) for several lattice temporal extensions \( N_t \), and extracted the dimensionless ratio \( T_c/\Lambda_L \) (\( \Lambda_L = \text{Lattice Lambda-parameter} \)) by using two-loop asymptotic scaling. The analogous result for \( N_f = 8 \) has been extracted from Ref. [10]. \( T_c/\Lambda_L \) for \( N_f = 0 \) and \( N_f = 4 \) has been measured at fixed \( N_t = 6 \), barring asymptotic scaling violations. Then we have discussed the \( N_f \) dependence of the ratios \( T_c/\Lambda_L \) and \( T_s/\Lambda_{\text{ref}} \), where \( \Lambda_{\text{ref}} \) is a UV reference energy scale, related to \( \Lambda_L \) via \( \Lambda_{\text{ref}}/\Lambda_L \approx \exp[\beta_L^{\text{ref}}/(4N_c b_0)] \). We have observed that \( T_c/\Lambda_L \) shows an increase in the region \( N_f = 6 \) – 8, while it is approximately constant in the region \( N_f \leq 4 \). We have discussed this qualitative change for \( N_f \geq 6 \) and a possible relation with a preconformal phase. The ratio \( T_c/\Lambda_{\text{ref}} \) is a decreasing function of \( N_f \). This behaviour is consistent with the result obtained in the functional renormalization group analysis [3]. Next steps of the current project involve a scale setting at zero temperature by measuring a common UV observable.
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References

[4] See Ref. [3], where the analytic results are summarized with references.
[16] See reviews in Ref. [7], and references are therein.