Abstract: The paper proposes a nonlinear robust controller for steam governor control in power systems. Based on dissipation theory, an innovative recursive design method is presented to construct the storage function of single machine infinite bus (SMIB) and multi-machine power systems. Furthermore, the nonlinear $L_2$-gain disturbance attenuation control strategy is achieved for speed governor control of turbo-generators without requiring the solution of HJI inequality. With multi-machine power system control, all the variables in the control law are only relevant to the state variables of the local generator, thus a decentralized control strategy is achieved. Simulation results of a 4-machine system show that the controller can enhance power transient stability and dynamic performances.

Keywords: Recursive design; Nonlinear disturbance attenuation control; Decentralized control; Multi-machine power system

1. Introduction

Modern power systems have developed into large, distributed and highly nonlinear systems with complicated and random disturbances. To design robust nonlinear decentralized controller is under heavy investigation for improving power system transient stability and dynamic performances. Speed governor control for large-scale turbo-generators is one of the most efficient and effective methods to enhance system stability when large disturbances occur. With the development of modern control theory in the recent 20 years, various control strategies are applied to the speed governor control systems, including linear optimal control, robust control and nonlinear control. With the application of differential geometric approach $[1]$, decentralized stabilization problem for multi-machine power systems is successfully resolved by exact feedback linearization. However, its requirement for a precise mathematical model limits its application into practical electric engineering, in which various uncertainties exist, such as internal and exogenous disturbances and measurement errors. Ref $[2]$ introduces nonlinear robust control method into speed governor controller design. However the $H_{\infty}$ control law is achieved from the system after feedback linearization, so it loses the strict robustness for the original nonlinear system.

This paper adopts the recursive design technique $[3][4]$ to design nonlinear disturbance attenuation controller for turbo-generators without any linearization and the design process does not require the solution of HJI inequality. A single machine infinite bus system is firstly studied to illustrate the approach, and then multi-machine system is discussed. All the variables in the control law are locally measurable, hence the goal of decentralized control is achieved. Simulation results are given to demonstrate the effectiveness of the proposed controller.

2. Nonlinear disturbance attenuation control

Consider the following nonlinear system with uncertainty,
\[
\begin{align*}
\dot{x}_1 &= f_1(x) + S_1(x)w_1 \\
\dot{x}_2 &= f_2(x) + S_2(x)w_2 \\
\vdots & \quad \vdots \\
\dot{x}_{k-1} &= f_{k-1}(x) + S_{k-1}(x)w_{k-1} \\
\dot{x}_k &= f_k(x) + g(x)u + S_k(x)w_k \\
\dot{z} &= h(x)
\end{align*}
\] 

where \( x = (x_1, x_2, \ldots, x_k) \in \mathbb{R}^k \) is the state variable, \( u \in \mathbb{R} \) is the control variable, \( w = (w_1, w_2, \ldots, w_k) \in \mathbb{R}^k \) is the disturbance, \( z \) is the regulation output, and \( f, g, S \) are smooth mappings.

The nonlinear disturbance attenuation problem for system (1) is to construct a \( C^1 \) state feedback controller \( u = u(x) \) such that the corresponding close-loop system satisfies the following \( L^2 \)-gain dissipative inequality for a given positive constant \( \gamma \)

\[
\int_0^T \gamma \| z(t) \|^2 + V(x(0)) \, dt \leq \int_0^T \| u(t) \|^2 \, dt < \infty \quad \forall T \geq 0 \tag{2}
\]

where \( V(\cdot) \) is a non-negative storage function to be constructed, \( x(0) \) is the initial state, \( \| \cdot \| \) denotes the Euclidian norm of a vector, and \( L^2(0, T) = \{ \| w \| : [0, T] \to \mathbb{R}^k, \int_0^T \| w(t) \|^2 \, dt < \infty \} \).

To solve the nonlinear disturbance attenuation problem above for system (1) is formidable difficult especially when the system scale is very large. The paper will construct the storage function via the recursive design method to obtain its solution for speed governor control systems.

3. Mathematical model for turbo-generators

(1) SMIB system

Suppose the steam turbine is of reheat type. Suppose that \( E'q \) in generator keeps constant in transient dynamics, and the dynamic model is as follows

\[
\begin{align*}
\dot{\delta} &= \omega - \omega_s \\
\dot{\omega} &= \frac{\omega_s}{M} P_m - \frac{D}{M} (\omega - \omega_s) - \frac{\omega_s}{M} P_m + \frac{\omega_s}{M} e_i \\
\dot{P}_m &= -\frac{1}{T_s} (P_m - P_m^*) + \frac{1}{T_s} u_s + \frac{1}{T_s} e_i \\
z &= \begin{bmatrix} \eta_1 (\delta - \delta_0) \\ \eta_2 (\omega - \omega_0) \end{bmatrix}
\end{align*}
\]

where \( \delta \) is rotor angle (rad); \( \omega \) is the rotor speed (p.u.); \( \omega_s = 2\pi f_s \) is synchronous speed (rad/s); \( P_m \) is mechanical power produced by the boiler (p.u.); \( P_m^* \) is the output active power of the generator set; parameters \( D, M \) and \( T_s \) are damping constant, inertia constant and time constant of the HP stage with its valving system respectively. \( u_s \) is speed governor control. \( e_i \) denotes torque disturbance acting on rotating shaft of the generator set; \( e_i^* \) denotes the disturbance in the mechanical input; \( z \) is the regulation output. \( \eta_1, \eta_2 \) are positive weighing constants determined by designer. \( (\delta_0, \omega_0, P_m^*) \) is the initial operation point.

(2) Multi-machine system

Taking the same assumption as the SIMB system above, the dynamic model for multimachine power system is

\[
\begin{align*}
\dot{\omega}_i &= \frac{\omega_s}{M_i} P_m - \frac{D}{M_i} (\omega_i - \omega_s) - \frac{\omega_s}{M_i} P_m + \frac{\omega_s}{M_i} e_i \\
\dot{P}_m &= -\frac{1}{T_s} (P_m - P_m^*) + \frac{1}{T_s} u_s + \frac{1}{T_s} e_i \\
z &= \begin{bmatrix} \eta_1 (\delta - \delta_0) \\ \eta_2 (\omega - \omega_0) \end{bmatrix}
\end{align*}
\]

where \( P_m^* = E'_q \sum_{j=1} E'_q (B_e \sin(\delta_i - \delta_j) + G_e \cos(\delta_i - \delta_j)) \), the subscript \( i \) identifies the \( i \)th unit and other notations are the same as system (3).

4. Recursive design for nonlinear disturbance attenuation controller

(1) SMIB system
Firstly, set a pre-feedback as
\[ u_s = P_n - P_m + \nu T, \quad \text{(5)} \]
where \( \nu \) is the new input of the system. Define the following state variables as
\[ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} S - \Delta c_0 \\ \alpha - \alpha q \\ P_n \end{bmatrix}, \quad \text{(6)} \]
Denote \( a_1 = \frac{\alpha q}{M}, a_2 = -\frac{D}{M}, a_3 = \frac{1}{T}, \) and introduce the following coordinate transformation
\[ \begin{align*}
\tilde{x}_1 &= x_1 \\
\tilde{x}_2 &= k x_1 + x_2 \\
\tilde{x}_3 &= \phi(x_1, x_2, x_3)
\end{align*} \quad \text{(7)} \]
where \( k \) is a given positive number and \( \phi \) is a smooth function to be designed.

Then in the new coordinate, system (3) can be written as
\[ \begin{align*}
\dot{\tilde{x}}_1 &= -k x_1 + \tilde{x}_2 \\
\dot{\tilde{x}}_2 &= k \tilde{x}_1 + \tilde{x}_2 = (k + a_2) x_2 + a_1(x_3 - P_a) + a_2 e_1 \\
\dot{\tilde{x}}_3 &= \frac{\partial \phi}{\partial x_1} x_1 + \frac{\partial \phi}{\partial x_2} x_2 + \frac{\partial \phi}{\partial x_3} x_3
\end{align*} \quad \text{(8)} \]
Now we will seek for a storage function via recursive design and find \( v = \beta(x_1, x_2, x_3, \phi(x_1, x_2, x_3)) \) to make the close-loop system satisfy dissipative inequality (2).

\textbf{a.} Let \( V_1(\tilde{x}_1, \tilde{x}_2) = \frac{\sigma}{2} \tilde{x}_1^2 + \frac{1}{2} \tilde{x}_2^2, \quad \sigma > 0 \quad \text{(9)} \]
Introduce a function \( H_1 \) defined as
\[ H_1 = V_1(\tilde{x}_1, \tilde{x}_2) + \frac{1}{2} \left( \|f\|_2^2 - \gamma^2 \|e_1\|_2^2 \right) \quad \text{(10)} \]
where
\[ \dot{V}_1(\tilde{x}_1, \tilde{x}_2) = \sigma \tilde{x}_1^2 + \frac{1}{2} \tilde{x}_2^2 + \tilde{x}_2 ((k + a_2) x_2 + a_1(x_3 - P_a)) + \tilde{x}_2 a_1 e_1 \]
denotes the differential of function \( V_1(\tilde{x}_1, \tilde{x}_2) \) with respect to time \( t \) along the state equation of system (8).

From (9),
\[ \frac{1}{2} \|f\|_2^2 = \frac{1}{2}((q_1^2 + q_2^2 k^2) \tilde{x}_1^2 + q_1^2 \tilde{x}_2^2 - q_2^2 k \tilde{x}_2^2). \]

Then from (10), we can have
\[ H_1 = -e \tilde{x}_1^2 - \frac{1}{4} \left( (q_1^2 - \frac{2}{\gamma} a_1 \tilde{x}_1^2 \right) - \frac{\gamma^2}{4} e_1^2 + \tilde{x}_2 (\mu_1 x_1 + \mu_2 x_2 + a_1 x_3 - a_1 P_a) \quad \text{(11)} \]
where
\[ e = \sigma k - \frac{1}{2} q_1^2 - \frac{1}{2} q_2^2 k^2, \quad \mu_1 = \frac{1}{\gamma^2} ka_1^2 + \sigma - \frac{1}{2} k q_2^2 \]
\[ \mu_2 = \frac{a_1^2}{\gamma^2} + a_2 + k + \frac{1}{2} q_2^2. \]
Set \( \tilde{x}_3 = \phi(x_1, x_2, x_3) = \mu_1 x_1 + \mu_2 x_2 + x_3 \quad \text{(12)} \]
Since the second term on the right side of (11) \( \leq 0 \), hence
\[ H_1 \leq -e \tilde{x}_1^2 + \tilde{x}_2 (\mu_1 x_1 + \mu_2 x_2 + x_3) \quad \text{(13)} \]
\[ e \leq \gamma^2 e_1^2 + \tilde{x}_2 (\mu_1 x_1 + \mu_2 x_2 + x_3) \]
Introduce a function \( v_2 = \frac{1}{2} \|f\|_2^2 - \gamma^2 \|e_1\|_2^2 \)
\[ = (V_1(\tilde{x}_1, \tilde{x}_2) + \frac{1}{2} \left( \|f\|_2^2 - \gamma^2 \|e_1\|_2^2 \right) - \frac{1}{2} \gamma^2 e_1^2 + \tilde{x}_2 (\mu_1 x_1 + \mu_2 x_2 + x_3) \quad \text{(15)} \]
where \( \dot{V}_1(\tilde{x}_1, \tilde{x}_2) \) denotes the differential of function \( V_1(\tilde{x}_1, \tilde{x}_2) \) with respect to time \( t \) along the state equations of system (8).

Substitute \( \tilde{x}_3 = \phi(x_1, x_2, x_3) = \mu_1 x_1 + \mu_2 x_2 + x_3 \)
into (15), we have
\[ H_2 = \dot{V}_1(\tilde{x}_1, \tilde{x}_2) - \frac{1}{2} \|f\|_2^2 - \gamma^2 \|e_1\|_2^2 - \frac{1}{2} \gamma^2 e_1^2 + \tilde{x}_2 (\mu_1 x_1 + \mu_2 x_2 + x_3) \quad \text{(16)} \]
Similarly to step a., it can be proved that
\[ H_2 \leq -e \tilde{x}_1^2 - \frac{1}{4} \left( (q_1^2 - \frac{2}{\gamma} a_1 \tilde{x}_1^2 \right) - \frac{1}{2} \left( (q_2^2 - \tilde{x}_2 a_2 k^2) \right) + \tilde{x}_2 (\mu_1 x_1 + \mu_2 x_2 + x_3) \quad \text{(17)} \]
Since the second and third terms on the right side of (17) \( \leq 0 \), hence
\[ \dot{H}_1 \leq -e \tilde{x}_1^2 - \frac{1}{4} \left( (q_1^2 - \frac{2}{\gamma} a_1 \tilde{x}_1^2 \right) - \frac{1}{2} \left( (q_2^2 - \tilde{x}_2 a_2 k^2) \right) + \tilde{x}_2 (\mu_1 x_1 + \mu_2 x_2 + x_3) \quad \text{(18)} \]
Since the second and third terms on the right side of (17) \( \leq 0 \), hence
\[ \dot{H}_1 \leq -e \tilde{x}_1^2 - \frac{1}{4} \left( (q_1^2 - \frac{2}{\gamma} a_1 \tilde{x}_1^2 \right) - \frac{1}{2} \left( (q_2^2 - \tilde{x}_2 a_2 k^2) \right) + \tilde{x}_2 (\mu_1 x_1 + \mu_2 x_2 + x_3) \quad \text{(19)} \]
\[ H_2 \leq -e_1 \dot{x}_1^2 + \dot{x}_1 (a_1 x_1 - a_1 P_e - x_1) + \dot{x}_3 (a_1 x_1 + (1 + \mu_1 + \mu_2 a_2) x_1 + x_3) + \frac{\dot{x}_1}{\gamma^2} (\mu_1^2 a_1^2 + \frac{a_1^2}{2}) + \mu_2 a_1 (x_3 - P_e) + v \]

Then we define
\[ v = -k x_1 - (1 + \mu_2 a_2) x_2 - \mu_2 a_1 (x_3 - P_e) - \frac{\dot{x}_1}{\gamma^2} (\mu_1^2 a_1^2 + \frac{a_1^2}{2}) - \frac{\dot{x}_1}{x_3} (a_1 x_1 - a_1 P_e - x_1) \]

and if take \( \sigma > \frac{1}{2k} (q_1^2 + q_2^2 k^2) \), then \( e_1 \geq 0 \)

Now from (18), we have
\[ H_2 \leq -e_1 \dot{x}_1^2 \leq 0 \]

From equations (15) and (20), we can get
\[ \dot{V}_1 + \frac{1}{2} (\|e\|^2 - \gamma^2 \|e\|^2) \leq 0 \]

implying
\[ 2V_1 (x(T)) - 2V_1 (x(0)) \leq \int_0^T (\gamma^2 \|e\|^2 - \|e\|^2) dt \]

Because the storage function \( V_1 (x(T)) \) is non-negative, we have
\[ \int_0^T \|e\|^2 dt \leq \gamma^2 \int_0^T \|e\|^2 dt + 2V_1 (x(0)) \]

Inequality (23) is called the dissipative inequality of system (3).

Now the disturbance attenuation problem for SMIB system (3) has been resolved, the control strategy is
\[ u_g = P_m - P_{me} + vT \]
\[ v = -k \Delta \dot{\delta} - (1 + \mu_1 + \mu_2 a_2) \Delta \omega - \mu_2 a_1 (P_m - P_{me}) - \mu_2 a_1 (P_m - P_e) - \frac{k \Delta \dot{\delta} + \Delta \omega}{\gamma^2} - \frac{k \Delta \dot{\delta} + \Delta \omega}{\gamma^2} \]

where \( \Delta \dot{\delta} = \dot{\delta} - \dot{\delta}_0, \Delta \omega = \omega - \omega_0 \) and \( k, \mu_1, \mu_2, a_1, a_2 \) are defined as equations (7) and (11).

(2) Multimachine system

An \( n \)-generator power system can be considered as \( n \) interconnected sub-systems each of which can construct a function \( V_{st} \), \( 1 \leq i \leq n \) taking the form of equation (14). Then for the whole multimachine system (4), we define the storage function as
\[ V (x_1, x_2, \cdots, x_n) = \sum_{i=1}^{n} V_{si} \]

then
\[ \dot{V} = \sum_{i=1}^{n} \dot{V}_{si} \leq \sum_{i=1}^{n} \gamma^2 \|e_{i}\|^2 - \|e_i\|^2 = \frac{1}{2} (\gamma^2 \|e\|^2 - \|e\|^2) \]

where \( \gamma = \max \{ \gamma_i \} \).

Hence
\[ 2V (x(T)) - 2V (x(0)) \leq \int_0^T (\gamma^2 \|e\|^2 - \|e\|^2) dt \]

Similar to inequality (23), we get the dissipative inequality for system (4) as follows
\[ \|e\|^2 dt \leq \gamma^2 \|e\|^2 dt + 2V (x(0)) \]

Then the disturbance attenuation control law for multimachine system of turbo-generators is
\[ u_{g1} = P_{m1} - P_{me1} + v_{T1} \]
\[ v_1 = -k_1 \Delta \dot{\delta}_1 - (1 + \mu_1 + \mu_2 a_2) \Delta \omega_1 - \mu_2 a_1 (P_{m1} - P_{e1}) - \mu_2 a_1 (P_{m1} - P_{e1}) - \frac{k_1 \Delta \dot{\delta}_1 + \Delta \omega_1}{\gamma^2} - \frac{k_1 \Delta \dot{\delta}_1 + \Delta \omega_1}{\gamma^2} \]

where \( \Delta \dot{\delta}_1 = \dot{\delta}_1 - \dot{\delta}_0, \Delta \omega_1 = \omega_1 - \omega_0 \) and \( P_{me1} \) are the state variables of the \( i \)-th generator, and other notations have the same expression as those for the SMIB system.

In practical engineering, \( \Delta \dot{\delta}_1 \) is usually not measured directly, so it is replaced by the integral of rotor speed, namely \( \Delta \dot{\delta}_1 = \int_0^T \Delta \omega_1 dt \), and the final control strategy is
\[ u_{di} = P_m - P_{cm} + T_{di} - k_1 \int T \omega_i d\tau - (1 + \mu_1 + \mu_2 \sigma_i) \Delta \omega_i - \\
\frac{\mu_2 R^2}{\gamma^2} + \frac{a_i^2}{\gamma^2} (\mu_1 \int T \omega_i d\tau + \mu_2 \Delta \omega_i - P_{cm}) - \\
k_1 \int T \omega_i d\tau + \Delta \omega_i - \\
\mu_1 \int T \omega_i d\tau + \mu_2 \Delta \omega_i - P_{cm} \]

(30)

where all the variables are locally measurable. Since the control law is only relevant to the local variables, it is decentralized.

5. Simulation results

A 4-machine power system, as shown in Fig.1, is studied in this paper, and the system data are listed in reference [6].

The speed governor control system designed according to the strategy (30) are installed on G1 to G4. In simulation, we set the disturbance attenuation level as \( \gamma_1 = 1 \), weighting constants \( g_y = 0.5 \), \( g_{y_1} = 1 \) and \( k_1 = 10 \), \( \sigma_1 = 6 \). In order to investigate the effectiveness of the proposed controller in improving transient stability, comparisons are made with nonlinear robust speed governor controller from reference [2], which is achieved through exact feedback linearization. System transients are stimulated by a three-phase short circuit fault occurred on line 7-8 close to bus 7, and cleared by tripping the faulted line in 0.15s. The simulation results are shown in Fig.2 to Fig.4, where the solid lines represent response of the proposed decentralized controller, the dash lines represent that of the controller from reference [2]. It can be easily seen that the proposed controller performs better.

6. Conclusions

The paper proposes nonlinear disturbance attenuation controller for SMIB and multimachine systems of
turbo-generators respectively via a recursive design method. The design procedure does not require the solution of HJI inequality and is based on the original system without any linearization treatment. The nonlinear robust control strategy obtained for multimachine system is only relevant to the local measurement, so the decentralized controller is realized. Simulation results for a 4-machine system clearly illustrate the effectiveness of the proposed controller.

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