The evolution of productivity gaps and specialization patterns
Los, Bart; Verspagen, Bart

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ABSTRACT: The literature on catching-up based growth has developed mainly in isolation of other theories on technology and economic growth. This paper presents an industry-level technology gap model in which evolving specialization patterns are the endogenous result of innovation, international technology diffusion, learning-by-doing and balance-of-payments restricted growth. Differences between sectors with regard to their share in consumption are shown to reinforce or mitigate the effects of specialization on aggregate productivity convergence patterns, depending on other parameters. The implications of the model are studied by means of simulation analyses for a wide range of parameter configurations.
The Evolution of Productivity Gaps and Specialization Patterns

1. Introduction

The productivity growth performance of countries has always been of central interest to economists. Over time, many theories about the sources of growth have been proposed, tested and refined. In most theories, technological progress emerges as the engine of growth. These theories vary widely, however, with regard to the specific ways in which technology affects economic growth. In general, three sources of technological progress at the national level are discerned: innovation, catching-up and learning-by-doing.

Most theoretical models focus on a single source of growth, which renders their outcomes of limited relevance. The vast majority of empirical studies also provide analyses that relate differences in growth performances between countries to differences in their opportunities to benefit from one or at most two of these sources. Fagerberg reviews many of these studies and concludes, “probably, the estimation of a single-equation model – with GDP per capita and other variables included- is an activity to which there are now sharply diminishing returns” (Fagerberg, 1994, p. 1171). This quote could be read as a plea for more empirical work that takes simultaneous interactions between important variables into account. In our view, a similar argument applies to theoretical models. We think that a model that presents a more unified framework is asked for, because many complications blur the comparability of the major lessons to be learnt from more partial models.

One of the complications is that outcomes often differ due to particular choices concerning the level of aggregation. This holds for both theoretical and empirical studies. For instance, studies for single sectors mostly yield different outcomes than studies that suppose that economies consist of one commodity market. Differences between sectors with regard to their technological progress from whatever source and changes of the sectoral structure can together have important effects on aggregate productivity growth rates. Baumol’s disease (Baumol, 1967) illustrates this issue quite convincingly.

Further, results are likely to be affected by the perspective chosen with respect to international trade. Especially the effects of learning-by-doing could well be highly dependent on the export performance of countries. Moreover, the international competitiveness of a country’s products is generally thought to be affected by this country’s technological standards vis-à-vis its rivals. Consequently, a process of virtuous (or vicious) circles emerges, which also affects the sectoral structure. Thus, studies in which countries are seen as parts of an international system of
interdependent countries are likely to yield different outcomes than studies that consider countries as autarkic entities.

In this paper, we propose a very general model. It describes the evolution of a system of two trading economies that consist of multiple sectors. The productivity growth rates of sectors, as well as the specialization patterns of countries, are the outcome of the interplay between innovation, catching-up and learning-by-doing. We will present outcomes of the model for a rather comprehensive set of parameter values that indicate the relative strength of these sources of technological progress. This allows us to draw some broad conclusions about the characteristics (in terms of parameter values) of technological progress for which aggregate labor productivity levels of countries tend to be relatively equal and to describe the corresponding specialization patterns.

The paper is organized as follows. In the next section, we will discuss the sources of productivity growth and their impact on the convergence process according to several growth theories. Special attention will be devoted to the interactions of these sources with changes in the sectoral structure and international trade. Sections 3 and 4 deal with the model equations. In Section 5, we will analyze the behavior of the model, mainly by means of simulation analysis. Section 6 concludes.

2. **Growth and technology: theoretical perspectives**

Most economists will agree that in the long run, technology is one of the main drivers of economic growth. The relationship between growth and technology, however, takes many different forms and can hardly be envisaged as a simple process that is steady through time. Looking at the literature on growth and technology, one may distinguish four main ways in which technology and innovation have their impact on growth.

The first, and perhaps simplest, way is found in the traditional neo-classical way of considering the growth process. This views economic growth as a steady-state process in which GDP per capita grows at a fixed rate over time. This rate, which is equal to the rate of technological progress, may be exogenously determined, as it was in the first version of the model (Solow, 1956). In the more recent so-called endogenous growth theory, (e.g. Romer, 1986, and Lucas, 1988) the rate of productivity growth is determined endogenously through, for example, research and development efforts. Viewed from this perspective, the answer to the question “why growth rates differ” can take two forms. The first answer argues about differences in the steady states between countries. For the old (Solow) model, this poses a logical difficulty, especially in relation to the assumption that knowledge and technology are public goods, which are freely available for all countries in the world. Under this assumption, differences in the steady state between countries cannot exist. The new growth models solve this logical difficulty by modeling the steady state growth rate itself as a function of economic variables, such as, for example, the
amount of human capital available in a country. In this case, individual countries converge to their own steady state, which will generally differ from that of other countries.

The neoclassical approach offers a second explanation of different productivity growth rates. Countries may not be on their steady state growth path, but may instead be converging to it. Thus, both below and above steady state growth rates are possible, depending on which side of the steady state the country starts from. Traditionally, the capital labor ratio has been taken as an indication of this, where the natural path for a country is to start from a relatively low capital labor ratio. This is associated to high growth rates through the mechanism of decreasing marginal returns. The speed of the convergence process towards the steady state has been the subject of a recent debate, in which the overall conclusion seems to be that this speed is rather low (e.g., Mankiw, Romer & Weil, 1992). Thus, on any practical time scale (say, decades), convergence may be a more dominant factor behind growth rate differentials than differences in steady states.

The idea of convergence is also the crucial factor in the second notion of technological change found in the literature. This is the literature that builds on the arguments of Gerschenkron (1962) and Abramovitz (1979), and to which we will refer as the catching-up literature. The main idea is that technological knowledge may diffuse from advanced countries to less advanced countries, i.e., that relatively backward countries may learn from more advanced countries. This view is inspired by the notion that knowledge is to a certain degree a public good. Imitating may be cheaper than developing brand new knowledge. In this way, there is actually an advantage to being relatively backward, because rapid growth may be achieved at lower costs.\(^1\)

The main line of argument in the catching-up literature is quite different from the neoclassical perspective. The difference lies in the degree to which it is assumed that knowledge is actually a public good. The argument in the catching-up literature is that in order to benefit from a catch-up bonus, several conditions need to be fulfilled. Abramovitz (1979) has labeled these conditions “social capability” and “technological congruence”. Social capability refers to the notion that in order to apply knowledge developed in a more advanced country, the catching-up country needs certain skills, equipment or institutions. Social capability is thus a broad concept that includes widely differing factors such as a stable financial system, a sufficient level of human capital, a sufficient level of infrastructure (transport, power generation), and so on. Without sufficient social capability, falling behind may be a more realistic outcome than catching-up.

The second prerequisite for catching-up identified by Abramovitz is technological congruence. This refers to the notion that technologies developed in advanced countries may not always be appropriate for less advanced countries. An important factor in this is the sectoral structure of the economy. For example, many less advanced economies depend to a large extent on agriculture, while technologies developed by advanced countries may be aimed at other sectors such as high-tech manufacturing. Catching-up then becomes more than just applying foreign technologies.

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\(^1\) If the level of technological knowledge is related somehow to the capital labor ratio (e.g., technological knowledge is embodied in machines, or through a technical progress function as in Kaldor, 1957), the neoclassical story of convergence may be observationally equivalent to this catching-up effect.
technology, and issues like cross-sectoral transfer of resources enter the picture. Again, the conclusion is that with a lack of technological congruence, falling behind is the realistic outcome.

The third source of technological change that we will discuss can be characterized as learning-by-doing or cumulative learning. This is an important topic both in the early literature (e.g., Verdoorn, 1949, Arrow, 1962) and some of the new growth models (e.g., Young, 1991).\textsuperscript{2} Technological change is generated as a result of experience in production or use. Such a mechanism generates positive feedback, i.e., high rates of technological progress result from high activity levels, or high growth of activity levels. A variety of mechanisms can be used to explain this phenomenon at the microeconomic level. Most of these explanations argue that ideas and procedures for small (incremental) innovations related to implementation may emerge as a result of using a technology. Although every single incremental innovation may be small, their joint cumulative effect is often large, as is for example shown in the historical studies of Habakkuk (1962).

The consequences of cumulative learning run against convergence. If using a technology enhances learning, the ones who are most successful at this will also learn most. That is, success breeds success. This implies a tendency for divergence of technology levels between countries, rather than convergence. Kaldor (1966) and Dixon & Thirlwall (1975) use this argument to make a point about persistence of growth rate differentials between regions. In a similar fashion, Lucas (1988) presents a model about persistence of growth rate differentials between countries.

The final view on the role of technology in economic growth is the structural view. Here it is argued that there are important differences between sectors in terms of technology. These differences may be directly related to technology, through variables such as technological opportunity or age of the technology. But also more indirect variables, such as income elasticities may play a role here, although only in combination with the more direct differences in technology between sectors. If sectors are characterized by different rates of productivity growth, or by different rates of demand growth (through, for example income elasticities), economic structure becomes important for aggregate economic growth. This is a topic that is mostly explored in the post-Keynesian literature, such as Pasinetti (1981) and Thirlwall (1979). Whereas Pasinetti (1981) considers this topic mostly in the context of a closed economy, Thirlwall (1979) brings in foreign demand. In the latter approach, the composition of exports and imports, and especially their respective income elasticities determine the growth potential of nations through balance-of-payments constraints.

Each of these different ways of looking at economic growth and technology has its own internal logic, as well as empirical support. Between some of the approaches, there are obvious links, such as, for example, the notion of technological congruence from the catch-up literature and the structural view of the growth process, or between the neo-classical argument about

\textsuperscript{2} Interestingly, cumulative learning plays an important role in both neoclassical-oriented growth models and in more 'heterodox' contributions, for example the Post-Keynesian literature on growth.
convergence and the idea of catching-up. But still, it is fair to say that major differences of insight remain. These differences relate to fundamental questions like whether convergence or divergence is the natural outcome of the growth process, or, more broadly, what is behind empirically observed growth rate differentials.

The model developed in this paper is able to unite all the different views on the technology-economic growth relationship that were briefly discussed in this section. This model will be used to analyze how the mix of relative importance of the various mechanisms will affect the likelihood of convergence or divergence between countries in terms of their productivity levels. Thus our model will take the form of a generic skeleton that encompasses both the steady state view of economic growth and the structural view on economic growth. On top of this skeleton, three different forms of technological change may be imposed: an (exogenous) steady state exponential growth rate of productivity, international technology diffusion between sectors potentially leading to catching-up, and productivity growth in the form of cumulative learning.

3. Model Description: Short-run Equations

In order to keep the model relatively simple, we abstract from any other production factor than labor and further assume that commodities are produced for consumption purposes only. The world economy consists of two countries, called North (N) and South (S). In this world, commodities are produced. Both countries are assumed to be able to produce all commodities, but nothing precludes a situation in which some of the commodities appear to be produced in just one country. The characteristics of a specific commodity are assumed to be independent of its country of origin. Consumption demand can be met by domestically produced and/or by imported commodities.

In each period, the output and employment levels for both countries are the result of the interplay of a number of state variables, presented in Table 1:

<table>
<thead>
<tr>
<th>Table 1: State variables.</th>
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<tbody>
<tr>
<td>1.</td>
</tr>
<tr>
<td>2.</td>
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<tr>
<td>3.</td>
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<tr>
<td>4.</td>
</tr>
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Note: indices N and S refer to North and South, respectively.

The first set of state variables represents the technologies in use. The variables indicate the physical amounts of labor required to produce physical units of output, in each of the industries.

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3 See Los & Verspagen (2000) for a more elaborate model with intermediate input structures. In a qualitative sense, all conclusions are robust to the inclusion of input-output relations between sectors.
in North and South. Labor is supposed to be homogeneous, in the sense that workers can immediately and freely move from one industry to another. On the other hand, we assume that labor is immobile in a geographical sense, i.e. that workers do not migrate from North to South or vice versa. The second set of variables indicates the fractions of domestically produced outputs in consumption, for each of the commodities and for both countries. The third set reflects the preferences of consumers in both countries. These variables are defined as the shares of total consumption in a country devoted to a particular good, measured in constant prices. Finally, the fourth set of state variables indicates the prices of the goods, expressed in a common currency.

The balance of payments plays an important role in our model. Arbitrary consumption vectors would only by chance yield a balance-of-payments (current account) equilibrium. As McCombie & Thirlwall (1994) argue, nothing prevents a country from being caught in a situation of current account deficits for a short time. In the long run, however, such a state of affairs is unsustainable unless capital inflows keep coming in infinitely and in steadily increasing amounts. Moreover, infinitely cumulating stocks of Southern (Northern) currency would not improve North’s (South’s) welfare.

Following Thirlwall (1979) and Verspagen (2001), we assume that total consumption levels in North and South must always correspond to balance-of-payments equilibrium. As we consider a closed system without exports and imports to and from third countries, we could write North’s current account surplus (or deficit) as the difference between the value of the goods imported by South and the value of goods imported by North (see McCombie, 1993, for a similar approach). Hence, current account equilibrium prevails if and only if the following holds:

\[
\sum_{i=1}^{n} p_{Si} (1 - z_{Ni}) b_{Ni} f_{N} = \sum_{i=1}^{n} p_{Si} (1 - z_{Si}) b_{Si} f_{S},
\]

in which \( f_{N} \) and \( f_{S} \) denote the total consumption levels (in constant prices) in North and South, respectively. This equation can be rewritten in a way that expresses the current account-equilibrium consumption level in South in some of the state variables in Table 1 and the level of consumption in North:

\[
f_{S} = \frac{\sum_{i=1}^{n} p_{Si} (1 - z_{Ni}) b_{Ni}}{\sum_{i=1}^{n} p_{Si} (1 - z_{Si}) b_{Si}} f_{N} \equiv \frac{r_{N}}{r_{S}} f_{N} \tag{2}
\]

Equation (2) shows that the solutions that represent current account equilibrium together constitute a line in the plane \( f_{N} \cdot f_{S} \). Since prices and consumption shares are always positive and trade shares cannot exceed 1, the slope of the line is positive. The interpretation of this result is
rather straightforward. Increased consumption in North induces more imports from South, which enables South to import more from North. Given current account equilibrium, these imports increase South’s consumption. The value of the slope depends on the relative prices (exporting a cheap good does not earn much money for South to import expensive goods from North), the trade shares (South does not gain much in terms of consumption from increased Northern consumption if North produces its own goods, or if South imports virtually everything) and the consumption shares (South does not gain much by exporting a good that is hardly demanded by North).

In order to determine a unique short-run solution for $f_N$ and $f_S$ (i.e., to choose a specific point on the current account equilibrium line) we have to impose an additional equation. We assume that one of the countries is constrained by available labor resources. In each period, either employment in North equals its labor supply ($1^N_{sup}$) or employment in South equals its labor supply ($1^S_{sup}$), depending on which country reaches its constraint first. Labor supply is exogenous in every period. Since trade implies that employment levels in North do not only depend on North’s own consumption but also on consumption in South (and vice versa), the full employment assumptions yield another two relations between the two total consumption levels $f_N$ and $f_S$.

Full employment prevails in North if and only if

$$\frac{1^N_{sup}}{\lambda_N} = \frac{\sum_{i=1}^n [1 - z_{Ni}] b_{Ni} f_N + [1 - z_{Si}] b_{Si} f_S}{\sum_{i=1}^n [1 - z_{Si}] b_{Si}}$$

This expression can be rewritten as

$$f_S = \frac{1^S_{sup}}{\lambda_S} - \left( \frac{\sum_{i=1}^n [1 - z_{Si}] b_{si} f_N}{\sum_{i=1}^n [1 - z_{Si}] b_{Si}} \right) f_N \equiv \frac{1^S_{sup}}{\lambda_S} \frac{\lambda_N}{\lambda_S} f_N$$

(4)

Since both the numerator and the denominator of the slope coefficient are positive, the line described by equation (4) is downward sloping. If the consumption level in North would be reduced, the consumption level in South would have to increase in order to keep the Northern labor previously employed in the production of domestically demanded output at work. The slope itself is determined by productivity coefficients, trade coefficients and consumer preferences. Analogously, we can find a relation corresponding to full employment in South:

$$f_N = \frac{1^N_{sup}}{\mu_N} - \left( \frac{\sum_{i=1}^n [1 - z_{Ni}] b_{Ni}}{\sum_{i=1}^n [1 - z_{Ni}] b_{Ni}} \right) f_S \equiv \frac{1^N_{sup}}{\mu_N} \frac{\mu_S}{\mu_N} f_S$$

(5)
Solving equations (2) and (4) yields the situation in which North’s consumption is both balance-of-payments-constrained and resource-constrained, but South is only balance of payments-constrained:

\[ f_N = \frac{r_s}{\left( r_N \lambda_N + r_s \lambda_N \right)} l_N^{\sup} \quad \text{and} \quad f_S = \left( \frac{r_N}{r_N \lambda_N + r_s \lambda_N} \right) l_S^{\sup} \quad (6) \]

Of course, we can also find the intersection of the upward sloping balance-of-payments line (2) and the downward sloping line (5), which corresponds to resources and balance-of-payments constraints on South’s consumption and balance of payments-constrained consumption in North:

\[ f_N = \frac{r_s}{\left( r_s \mu_N + r_N \mu_N \right)} l_N^{\sup} \quad \text{and} \quad f_S = \left( \frac{r_N}{r_s \mu_N + r_N \mu_N} \right) l_S^{\sup} \quad (7) \]

The short-run equilibrium point in the \(( f_N, f_S )\)-plane is now defined as the minimum of the points given by equations (6) and (7), since one country would produce above its capacity in the other intersection point. Given these consumption levels and their composition with regard to industries and countries of origin, the output levels and its sectoral compositions in North and South can be determined for the period under consideration:

\[ q_{NI} = z_{NI} b_{NI} f_N + (1 - z_{SI}) b_{SI} f_S \quad \text{and} \quad q_{SI} = (1 - z_{NI}) b_{NI} f_N + z_{SI} b_{SI} f_S \quad (i = 1, ..., n) \quad (8) \]

Finally, commodity-specific prices (expressed in a common currency, assuming an exchange rate which is known at the beginning of each period) are assumed to reflect labor costs:

\[ p_{NI} = w_N l_N^{c} \quad \text{and} \quad p_{SI} = x w_S l_S^{c} \quad (i = 1, ..., n) \quad (9) \]

The symbols \( w_N \) and \( w_S \) denote the nominal wage rates (in North and South, expressed in the national currencies), which we assume to be given at the beginning of each period. The exchange rate (also given at the beginning of each period) is denoted by \( x \). This implies that we adopt the rule to express both Northern and Southern prices in Northern currency.

The short-run model we introduced in this section yields output and employment levels by industry and country, as well as consumption levels for each of the commodities produced in North and South. These levels are completely determined by the production technologies, trade relations, consumer preferences, labor supply, prices, the exchange rate and the nominal wage rates prevailing during a period. Since we are primarily interested in the long-run dynamics of the
endogenous variables mentioned, we now turn to the specification of the equations that describe the intertemporal behavior of the variables that we have assumed so far to be fully exogenous.

4. Model Description: Intertemporal Relations

In this section we present the intertemporal equations that allow us to study long-run issues like output growth, convergence and structural change. Unlike the previous section, this section does not include the derivation of solutions. Due to the mathematical complexity caused by our explicit focus on economies consisting of multiple industries, analytical solutions are hard to derive. We will first deal with the equations describing technological progress and subsequently discuss the relations we specified with regard to trade share dynamics and exchange rate movements.

4.a Technology Dynamics

Following our earlier intersectoral models of technological change (Los, 2001, and Verspagen, 2001), we model innovation as changes of input coefficients. We define the (industry-specific) rates of technological progress as the proportional changes of the inverses of the $c_i$ coefficients, i.e. the ratios between output and labor input. We write the dynamics of labor inputs per unit of output as:

$$l_{N_i}[t + 1] = \gamma_{N_i}[t + 1] s_{i}[t] \quad \text{and} \quad l_{S_i}[t + 1] = \gamma_{S_i}[t + 1] s_{i}[t] \quad (10)$$

We allow both for differences in productivity growth rates between industries and between countries. Following the catching-up literature discussed in Section 2, we model two regimes. In the first regime, the industry under consideration has lower labor requirements per unit of output than its foreign competitor. In this case, we call the industry in this country the leader industry, and the corresponding industry in the other country the lagging industry.

We assume that leader industries experience technological progress through two mechanisms. First, a constant exogenous productivity growth rate is assumed, which reflects the laborsaving effects of innovation. This reflects the (neo-classical) idea of steady state growth at a fixed rate. Second, a mechanism that we label the Kaldor-Verdoorn mechanism represents cumulative learning: if the output of an industry grows, it is assumed that learning-by-doing and opportunities for specialization of workers lead to increased productivity.

In the lagging-behind regime, we assume that the Kaldor-Verdoorn mechanism is still functioning, but that the steady-state fixed rate of technological progress is absent. In other words, we assume that this source of technological progress is generated only by the leading

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4 For simplicity, we do not include stochastic arrivals of innovations, nor do we incorporate explicit search for innovations (R&D) in the model. See Los (2001) for an intersectoral model with R&D and stochastic innovation in a single-country context.
country, as would be consistent with an endogenous growth model in which R&D can only be performed by the advanced firms. Instead, in the lagging country, the technological gap to the leader industry is assumed to give opportunities for catching-up, for instance by imitation of the competitor’s production technology.

Summarizing, the reductions in labor coefficients under the two regimes can be represented by

Leader regime: \[ \gamma_i[t + 1] = \frac{1}{1 + \kappa \left[ \frac{q_i[t] - q_i[t-1]}{q_i[t-1]} \right]^{\sigma_i} + \rho_i} \]  
\[ \theta_i[t + 1] = \frac{1}{1 + \kappa \left[ \frac{q_i[t] - q_i[t-1]}{q_i[t-1]} \right]^{\alpha_i} + \kappa_i \ln \left[ \frac{\lambda_i[t]}{\lambda_i[t-1]} \right]} \]

Lagging regime: \[ \gamma_i[t + 1] = \frac{1}{1 + \kappa \left[ \frac{q_i[t] - q_i[t-1]}{q_i[t-1]} \right]^{\sigma_i} + \rho_i} \]  
\[ \theta_i[t + 1] = \frac{1}{1 + \kappa \left[ \frac{q_i[t] - q_i[t-1]}{q_i[t-1]} \right]^{\alpha_i} + \kappa_i \ln \left[ \frac{\lambda_i[t]}{\lambda_i[t-1]} \right]} \]

in which \( \kappa (>0), \sigma (>1), \rho (>0) \) and \( \alpha (0<\alpha<1) \) are industry-specific parameters and \( 1_{i,*c} \) indicates the unit labor requirements in the corresponding leader industry. \( \kappa \) and \( \sigma \) relate to the Kaldor-Verdoorn mechanism taken from Verspagen (1993) and reflect the rate of dynamic learning. The mechanism brings a positive feedback into the dynamic model, since high output growth leads to high productivity growth (although at a decreasing marginal rate). \( \rho \) is the steady-state rate of technological progress in the leader industry, \( \alpha \) reflects the catching-up mechanism in the lagging country. The larger the productivity gap in terms of labor inputs per unit of gross output, the more the lagging industry will catch up. The value of \( \alpha \) is assumed to reflect social capability, which means this is taken as an exogenous factor. Note, however, that our multi-industry specification implicitly takes account of the fact that incompatibility of output structures plays an important role, i.e., technological congruence is endogenous. For example, if South would lag North in one industry but produces only a very small part of its output in this industry, aggregate labor productivity would not benefit much from this industry-specific technology gap.

4.b Trade share dynamics

Trade share dynamics are assumed to be ruled by changes in relative competitiveness. According to traditional theory, the competitiveness of countries is determined by costs per unit of output. In such a situation, the interplay of two developments would determine changes in a country’s competitiveness: reductions in the amounts of labor required per unit of output and changes in the relative wage rates.\(^5\) These factors are summarized by prices (equation 9), so that we assume

\(^5\) These unit labor costs are generally converted to a common currency. Hence, exchange rate changes may alter the competitiveness structure as well.
that changes in a country’s shares on the world markets are dependent on price differentials (only).

To determine the commodity-specific trade shares, we use the concept of an inverted logistic curve, as illustrated in Figure 1. In mathematical terms, this relation between the price ratio and the ‘equilibrium’ market share of North’s industry i on its domestic market for consumption goods is:

\[
\begin{align*}
z^*_{Ni} &= \frac{1}{1 + e^{-\left(\ln \frac{p_{Ni}}{p_{Si}} - \varepsilon\right)/\phi_i}} \quad \text{and} \quad z^*_{Si} = \frac{1}{1 + e^{-\left(\ln \frac{p_{Si}}{p_{Ni}} - \varepsilon\right)/\phi_i}}
\end{align*}
\]

(13)

The parameter \(\varepsilon\) can be interpreted as the value of the logarithm of the price ratio for which the market shares are equally divided. For perfectly tradable goods this parameter will equal zero, but for most goods the price ratio corresponding to equal market shares will have a positive value. In this case, the market shares of domestically produced inputs will be larger than fifty percent when prices are equal. The extent to which the market share at the unit price ratio differs from fifty percent is also dependent on the parameter \(\phi_i > 0\), which represents the commodity-specific sensitivity of trade shares to changes in the price ratio. The lower \(\phi_i\), the more sensitive the trade shares are. The parameters \(\phi_i\) may be affected by a number of things, such as costs of transportation, geographical distance and non-tariff trade barriers.

![Figure 1: Trade Shares](image)

Trade shares are not assumed to adjust immediately to changes in prices. We model an adjustment process, in which the gap between the actual trade shares \(z\) and the equilibrium trade shares \(z^*\) vanish gradually in the absence of shocks:

\[
z[t + 1] = z[t] - \eta \left( z[t] - z^*[t + 1] \right)
\]

(14)

with \(\eta\) (0 < \(\eta\) < 1) denoting the speed of adjustment.
4.c Wage rates, the exchange rate and remaining variables

The two country-specific nominal wage rates are assumed to be constant, at least if expressed in national currencies. When technological progress reduces the unit labor requirements, prices will consequently be reduced and the real wage rate will grow. One caveat applies, however. In a system of flexible exchange rates, the purchasing power of a given amount of Northern currency relative to an identical amount of Southern currency may change.

In the modeling of exchange rate dynamics we use a specification similar to that of trade shares. The exchange rate is a weighted average of the actual rate and an equilibrium rate based on purchasing power parity of the two currencies:

\[ x_{t+1} = x_t - \eta_t (x_t - x_t^*) \]  

(15)

The smaller \( \eta_t (0 \leq \eta_t \leq 1) \), the more a system of fixed exchange rates is approximated. \( x_t^* \) is determined according to:

\[ x_t^* = \frac{\sum_{i=1}^{n} p_{Ni}^{nc}[t](q_{Ni}[t] + q_{Si}[t])}{\sum_{i=1}^{n} p_{Si}^{nc}[t](q_{Ni}[t] + q_{Si}[t])} \]  

(16)

The superindices \( \text{nat} \) indicate prices expressed in national currencies.

Finally, we assume that the consumption shares of the sectors are fixed in each country. Moreover, we assume that these variables do not differ between countries. Thus, consumer preferences are assumed to be identical.⁶

5. Model Analysis

As mentioned earlier, the long-run behavior of the two national economies in our model can best be studied by means of simulation analysis. Before we turn to a discussion of some of the simulation results we obtained, we illustrate the basic effects of an innovation by a diagram that depicts some highly simplified comparative statics.

In Figure 2, the axes indicate the total consumption levels for North and South. The solid upward sloping line (I) reflects all pairs of consumption levels for which the balance of payments is in equilibrium in the initial situation, see equation (7). The downward sloping solid lines (IIN and IIS) indicate all pairs of consumption levels for which full employment initially prevails in North and in South, respectively, see equations (10) and (11). The slopes of these lines will generally differ, e.g. due to different compositions of consumption bundles with respect to

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⁶ Los & Verspagen (2000) study the consequences of income-specific consumer preferences for productivity growth differentials, by means of a model that resembles the one introduced here.
countries of origin. In the situation depicted, consumption levels are given by the intersection point $F$, in which North experiences full employment and South faces excess labor supply.

**Figure 2: Consumption effects of innovation.**

![Diagram](attachment:image.png)

Now, suppose that North generates a labor saving innovation. Consequently, the labor constraint will be less tight, and for given consumption levels in North higher consumption levels in South are attainable, since more ‘effective’ labor is available to produce exports. This is reflected by the upward shift of the Northern full employment line (see dashed line $II_N'$). In general, the slope will change as well, but the sign of this change depends on the interplay of many parameter values. Nevertheless, the ‘first order’ effect of technological progress in the labor-constrained country is an increase in both North’s and South’s consumption levels.

The supposed innovation in North also has some ‘second order’ effects. The net effects of these are ambiguous, as will become clear after studying the dashed lines $I$ and $I'$. The lines represent two important opposite effects. First, innovation yields a lower price per unit of output for North. This implies that for a given amount of exports, North can buy fewer imports and must reduce its consumption, while South can increase its import volume at constant costs ($I'$). This is a terms-of-trade effect. Second, North’s reduced price enhances its competitiveness relative to South, which will cause higher market shares. Consequently, innovating North requires fewer imports per unit of output, which enables it to produce more consumption goods. For South the opposite holds and its attainable consumption level will fall. This effect is documented by curve $I''$.

Which of the two second-order effects dominates depends on a number of parameters and variables, of which the sensitivity of market shares to price changes is an important one. The bottom line of this simple comparative statics analysis is that it is impossible to tell in advance which effect will dominate, and hence what the ultimate effect of an innovation on relative growth rates will be.
The interplay between the various forms of technological progress in the model may also yield rather interesting effects. Especially the combination of the Kaldor-Verdoorn and catching-up effects is noteworthy. Since this mechanism, which we dub as the ‘catching-up-cumulative-learning cycle’ is rather crucial for the overall analysis of the model, we discuss it here in some detail. Note that the ‘catching-up-cumulative-learning cycle’ is in some respects similar to the properties of the model proposed by Landesmann & Stehrer (2000).

**Figure 3: The ‘catching-up-cumulative-learning cycle’**

A typical situation is depicted in Figure 3. The vertical axis of the upper diagram indicates the so-called technology gap, which is defined as the logarithm of the ratio between the labor requirements per unit of output in a Northern industry to such requirements in its Southern counterpart. Hence, a negative value points towards a technological advantage of a Northern industry. In the lower panel, the shares of North and South in the world production of the two goods are indicated on the vertical axis (e.g., SH-N1 stands for the share of North in the world production of good 1).

The most prominent phenomenon emerging from Figure 3 is that so-called ‘taking-over’ occurs at the level of industries: initially each country is leader in either one of the industries, but after an interval of about 65 periods the leadership pattern is reversed. This taking-over happens almost simultaneously in both industries. Specialization patterns also evolve in a cyclical fashion. What drives these cycles?
The catch-up part of the dynamics consists of technological imitation that drives the technology gap towards an equilibrium value. It is important to note that this equilibrium value can change endogenously over the simulation, and this is exactly what happens when the technology gaps alternate between positive and negative ‘attractors’. Whether the equilibrium value is positive or negative depends on $\rho$ and the Kaldor-Verdoorn effect. The first of these two factors ($\rho$) always gives the leading country an advantage, since $\rho=0$ in the lagging country.

At the upper turning point of either one of the technology gaps in Figure 3, the lagging country has been experiencing a fall of its market share, because the increasing gap has led to a competitive disadvantage of the lagging country. The technology gap approaches a short-run equilibrium value, because the catch-up bonus increases, and thus offsets the effect due to $\rho$ and Kaldor-Verdoorn. Because the technology gap now stops growing, the trend of a declining market share also stops. The laggard is now able to benefit from the generally expanding world market and realize positive production growth. This sets in motion the Kaldor-Verdoorn learning effect. A slowdown of production growth in the leading country is induced because the lagging country captures more of the market. Through the Kaldor-Verdoorn effect, this induces a slowdown of the rate of productivity growth in the leading country. This changes the short-run equilibrium value of the technology gap, which becomes smaller. The result is that the gap now starts to decline.

When the gap becomes zero, the catching-up bonus disappears, but now the country becomes the leading country, and the effect due to a positive $\rho$ sets in. The other (formerly leading) country now loses the bonus due to $\rho$, but starts to experience a catch-up bonus. In Figure 3, the parameter values have been chosen such that the technology gap actually reverses sign, but with different parameter values, this does not need to happen. In particular, for higher values of $\rho$, the leader country has such a strong advantage that the gap does not reverse sign, and the lower turning point occurs on the same side of the horizontal zero-level as the upper turning point. At the lower turning point, the same logic as for the upper turning point applies, but now with the roles (leader or laggard) of the two countries reversed. The dynamics are symmetric.

In order to get insight into the behavior of the model beyond these simple comparative statics, we resort to simulation analysis. We concentrate on the set of parameters related to technological progress, i.e. $\rho$, $\kappa$, $\sigma$ and $\alpha$. The number of sectors is set to two in all simulations. To keep the analysis as tractable as possible, we choose our basic parameter configuration in a way that both goods and both countries are (initially) as similar as possible. Exact parameter values are documented in an appendix, which shows that the production processes of both goods in North and South are initially the same, except for a very small difference in labor requirements to induce differences between the two countries in terms of the regimes of technological progress. Each country is assumed to be the technological leader in one industry. Initial trade shares are all equal. The only difference between sectors lies in the consumption shares: good 1 is consumed four
times as much as good 2. With respect to the other parameters that govern the dynamic processes, the two goods are perfectly identical.

The main variable that we are interested in is the aggregate level of inequality between countries, which we measure as the absolute value of the logarithm of the ratio between the macroeconomic labor productivity levels in North and South, averaged over the simulation period of a single run. All results are based on 250 periods, of which the first 100 were not used because they show traces of the choice of initial conditions. To illustrate some of the general tendencies of the model, we show in Figure 4 the results for a range of values of $\rho$, while simultaneously switching on or off the other forms of technological progress. In the figure, the inequality indicator is depicted on the vertical axis (higher values indicate more inequality). Values on the horizontal axis denote the value of $\rho$. The values for $\sigma$ and $\alpha$ are 6.0 and 0.2, respectively. We pick the value of $\kappa$ such that the Kaldor-Verdoorn mechanism yields a 1% growth rate of labor productivity for a growth rate of production equal to 1%.

**Figure 4: Effects of innovation rates on productivity gaps**

The figure contains four lines. The first one is denoted by “none”, which indicates that both the catch-up parameters and the growth rate implied by the Kaldor-Verdoorn mechanism (cumulative learning) are set equal to zero. We see that this curve rises steeply in the segment of very low rates of technological progress in the leading industries, reaches a maximum average aggregate technology gap for $\rho=0.001$ and approaches zero for larger values of $\rho$. This tendency towards zero is perhaps easiest to grasp. South is the leader in industry 1 and will thus also innovate in this industry. North is the leader and innovator in industry 2. Without any other ways
of productivity increase, there is no way for the follower to catch-up to the leader, and the price gap between the two countries for each good will grow. Complete specialization is the result. Since both the rates of technological progress and the initial labor productivity levels are equal for both industries in this set of simulations, this will lead to equal macroeconomic levels of labor productivity.\footnote{It should be noted that this does not lead to equal GDP levels: the unequal consumption shares cause unemployment in the country that produces the good that is relatively unpopular with the consumers.}

However, at low rates of innovation, we see the level of disparity between the countries rising with $\rho$. The reason for this is that specialization is less pronounced in this case, and both countries remain active in the industry in which they have a competitive disadvantage. This most heavily affects North, since it attracts the same share of the world production of good 1 as South attracts of the world production of good 2, but the market for good 1 is much larger. Consequently, more labor is allocated to low-productivity activities in North. This effect is obviously absent for $\rho=0$, but increases for small positive values of $\rho$. This is reflected in the rising trend of disparity. When complete specialization starts to emerge, the curve peaks and starts to fall towards zero in the way already described.

Introduction of the Kaldor-Verdoorn cumulative causation mechanism (line “KV”) leaves these dynamics essentially unchanged, but increases the intensity. Specialization in an industry causes output growth, which in turn leads to productivity growth and even more specialization. If the annual long-run growth rate implied by the Kaldor-Verdoorn mechanism is set at 0.01, very low levels of $\rho$ (in the order of magnitude of 0.001) are already sufficient to reach almost full specialization before period 100. Hence the “KV” line jumps up immediately after $\rho=0$, but falls directly after that.

With only technological imitation of leaders by laggards being present, the nature of the dynamics is again similar (the “alfa”-line). A positive rate of innovation for leaders and a positive rate of catch-up for laggards imply convergence towards a constant technology gap, and therefore a constant degree of specialization. The higher $\rho$ for the given value of $\alpha$, the stronger is the long-run degree of specialization. Hence, for high values of $\rho$, specialization is almost complete and both countries perform equally well according to the macroeconomic labor productivity yardstick. For low $\rho$, we observe the same kind of dynamics as was seen for the “none” line. Now, however, labor productivity levels of both industries in a country deviate much less, because of the tendency to converge to a fixed gap. Hence incomplete specialization is relatively harmless in terms of aggregate technology gaps. This is why the upward-sloping part of the curve covers a wider span of $\rho$.

The by far most dramatic effects of changes in the rate of innovation in the leading industries are found if Kaldor-Verdoorn effects and abilities to catch-up are simultaneously taken into account (line “alfa+KV”). In this case, the type of dynamics that was described in the discussion of Figure 3 start to play a role. In particular, the steep rise and decline of labor productivity
differentials in the range of $\rho$ after the value $\rho=0.009$ deserves special attention. It turns out that this sudden change of behavior of the aggregate productivity gap is due to a qualitative change in the ‘catching-up-cumulative-learning cycle’.

To the left of the point $\rho=0.009$, the ‘catching-up-cumulative-learning cycle’ unfolds as in Figure 3, i.e., leadership periodically switches between the countries and the sign of the technology gaps reverses. The amplitude of the technology gap cycles is relatively small, i.e., the upper and lower turner points occur at relatively small (absolute) values of the gaps. Specialization patterns co-evolve with the technology gaps, and this yields inequality as described in the discussion of Figure 3.

To the right of the ‘bifurcation point’ at $\rho=0.009$, the technology gaps no longer change signs, and technological leadership always remains with one country. This is due to the fact that the bonus for the technological leader ($\rho$) is now so high that it prevents taking-over. As a result, trade specialization does not change sign over time, and this leads to a higher aggregate productivity gap because it is now always just one of the countries benefiting from the larger market for good 1. In fact, at values of $\rho$ just slightly larger than 0.009, the technology gaps converge to an equilibrium value that corresponds to a fixed trade specialization pattern.

The sharp drop in the “alfa+KV” line at $\rho=0.011$ corresponds to a situation in which South is the technological leader in both sectors. The technology gap for North is equal in the two sectors, and hence no specialization occurs: the share of North (South) in both sectors is equal to each other. Specialization does not induce inequality in this case. To the right of the point $\rho=0.011$, the behavior of the model does not change in a qualitative sense. Specialization becomes more pronounced, however, and hence inequality grows.

Around the point $\rho=0.022$, specialization becomes almost complete in both sectors. The tendency for the equilibrium technology gaps to grow does not stop, due to the increasing $\rho$. Consequently, prices between the two countries keep diverging. Because of the complete specialization, the trade shares of both countries for the goods in which they are specialized cannot grow further, and the only effect of the price differentials is on the terms of trade. In terms of the discussion of Figure 2 above, this means that one of the two second-order effects (associated to a shift of the balance-of-payments-restriction line in Figure 2) disappears. This causes more equality, which is reflected the huge drop of the “alfa+KV”-curve in Figure 4 for innovation rate values that exceed $\rho=0.024$. For very high values of $\rho$, the “alfa+KV”-curve and the “alfa”-curve coincide, which reflects that the strong effects of exogenous innovations dominate the Kaldor-Verdoorn effect.

The point of the model is not so much to explain specific figures like Figure 4. After all, by choosing different values of the parameters, or even by plotting values of a different parameter on the horizontal axis, it would be possible to generate quite different figures. Thus, Figure 4 should only be considered as an illustration of the type of mechanisms that exist in the model,
and the way they interact. In order to paint a broader picture of the range of possible outcomes, we proceed to vary the technological change parameters in a systematic way against each other.

In Figure 5, we plot the macroeconomic productivity gap indicator that featured in Figure 5 against two technology-related parameters, the innovation parameter $\rho$ and the catch-up parameter $\alpha$. The Kaldor-Verdoorn parameter $\sigma$ is held constant at 3.0. Dark (light) regions indicate parameter configurations for which the inequality indicator is high (low). The picture looks rather chaotic for some ranges of parameter values, but some tendencies emerge immediately. For very low values of $\rho$ and very high values of $\alpha$, the gaps are small. This is due to the well-known logic from technology-gap theories of growth that equilibrium gaps are low for such parameter configurations. The macroeconomic gap is thus small in both sectors.

**Figure 5: Joint effects of innovation and catch-up on productivity gaps**

At first sight, the finding that the gaps are small for the opposite configuration, in which exogenous innovation is high and catching-up is virtually absent, may appear less straightforward. In this case, complete specialization prevails. Further, the terms-of-trade effect discussed with respect to the right-hand-side range of $\rho$-values in Figure 4 applies. The diagonally oriented combinations of $\rho$ and $\alpha$ values that produce large productivity gaps correspond to the situation in which sectoral gaps are large but specialization is not yet complete. The steep decline “to the
southeast” also corresponds to the steep decline found in Figure 4. Finally, no clear-cut pattern can be detected for the range characterized by low values of $\rho$ and low to medium values of $\alpha$. This is partly due to the presence of situations in which one country becomes the productivity leader in both sectors and specialization hardly occurs (note in particular the “narrow valley” for $\rho$s approximately equal to 0.02 and $\alpha$s higher than about 0.15).

The relationship between the aggregate productivity gap on the one hand and the innovation rate $\rho$ and cumulative learning mechanism $\sigma$ on the other is depicted in Figure 6. The value of the catching-up parameter $\alpha$ is held constant at 0.2.

Figure 6: Joint effects of innovation and learning-by-doing on productivity gaps

For very low values of $\sigma$, the aggregate productivity gap is huge (note the difference in the indicator values associated with a given level of darkness, relative to the Figure 6). This is due to the fact that a modest growth of output leads to a substantial increase in productivity in this range of $\sigma$ values. Consequently, the cycles from Figure 3 have much higher amplitudes (sector-specific gaps will be large) and the frequency of the cycles is much higher. Specialization patterns change quickly, but full specialization occurs often. Hence, aggregate productivity gaps change sign quite often, but are large in the majority of time periods. This effect is reinforced if $\rho$ takes on relatively high values, as is evidenced by the border between the lightest and lightest but one shade.
For higher values of $\sigma$ (exceeding 2.5), another interesting phenomenon emerges from the figure. Both low and high values of $\rho$ yield small gaps on average, whereas intermediate values yield larger gaps. Again, this resembles the hump-shape observed in Figure 4. For low values, gaps and specialization patterns behave cyclically and for high values specialization is complete. Due to negative terms of trade effects, inequality declines for higher $\rho$ values. The inverse relation between $\rho$ and $\sigma$ that characterizes this “ridge of divergence” is due to the strong Kaldor-Verdoorn effect, which promotes cyclical behavior. Thus, for a low value of $\sigma$, which indicates strong cumulative learning effects, a high exogenous innovation parameter value $\rho$ is required to offset this tendency and obtain full specialization.

Figure 7: Joint effects of catching-up and learning-by-doing on productivity gaps

Finally, Figure 7 gives an impression of the model’s behavior for combinations of varying catching-up parameter $\alpha$ and the learning-by-doing parameter $\sigma$. The exogenous innovation rate is held constant at 0.025. Like in Figure 6, the productivity inequality is huge for low values of $\sigma$. Again, this is due to large productivity gaps at the sectoral level and fast reversals of specialization patterns. It should be noted, however, that a relatively modest strength of the cumulative learning mechanism suffices to generate such patterns for low values of $\alpha$. This is due to the fact that weak capabilities to catch-up yield large short-run equilibrium gaps for a given innovation rate. Consequently, leader industries can benefit to a significant extent from learning-by-doing.
Another noticeable feature is the “ridge of divergence” that emerges for configurations with high values of $\alpha$ and $\sigma$. On the left of this ridge, the aggregate gaps are small, due to the terms-of-trade effects that also cause the drop in Figure 4. For low values of $\alpha$, sectoral technology gaps are large. In this sense, this phenomenon also resembles the northeast region of Figure 6. High values of $\rho$ and low values of $\alpha$ both yield large equilibrium technology gaps at the sectoral level. On the right hand side of the ridge, catching-up is so strong that technology gaps at the sectoral level remain very small. Thus, specialization is far from complete, but because labor productivity levels between sectors do not deviate much, this does not have strong effects on macroeconomic productivity gaps.

6. Concluding Remarks

The two-country-two-industry model that was developed in this paper integrates a number of well-known partial determinants of economic growth. First, demand-side and supply-side mechanisms were merged. The post-Keynesian notion that the rate of export expansion limits the attainable output growth rate was combined with the more mainstream argument that output growth is determined by the increasing availability of production factors or the growth of their productivity. In the model, the latter determines the size of the world markets, whereas the former play an important role in the division of the world markets between the two countries.

A second important integration of well-recognized explanations concerns the sources of technological progress. The model distinguishes between two regimes, one in which the industry is on the technological frontier, and one in which the industry lags its foreign competitor in terms of labor productivity. Both regimes have one source of productivity growth in common: the benefits from learning-by-doing, specified as a causal relationship from output growth to productivity growth. Further, exogenous productivity growth is present in the leader regime. In the laggard regime, the industry can benefit from knowledge that cannot be appropriated by the competing industry at the productivity frontier.

The model is set up in a very general way, so that it can be helpful in analyzing a wide range of issues regarding relations between technological change, trade and growth. Consequently, the model cannot be solved by analytical means. This paper presented a set of results from simulation experiments, of which the focus was on the effects of the three sources of technological progress on patterns of convergence in aggregate labor productivity levels and trade specialization. The chosen parameter configurations were such that the two sectors we distinguish are as identical as possible, with the exception of their share in consumption demand. Even then, the simulations showed that small changes in parameter values could have substantial effects on the relative performance of the two countries. In particular, in cases in which the three sources of technological change were assumed to in operation simultaneously, we found several “bifurcation values” that demarcate ranges of parameter values for which sectoral productivity growth, structural change and international specialization interact in quite different ways.
Our results thus show that the dynamics of catching up (falling behind), innovation and learning are far from simple. The parameter space of our model does not have large subspaces in which productivity convergence between the countries is simple and automatic. Areas of parameter space in which this is the case do exist, but they seem to be only islands in a sea with (big) waves of divergence. From this result, we draw two conclusions, one with regard to the literature on convergence and divergence of productivity levels, and one with regard to policy.

With regard to the existing literature, our model is in line with the conclusion (e.g., Abramovitz, 1979, Fagerberg, 1994) that convergence is by no means an automatic process that will unfold itself as a natural traverse towards a steady state. However, while the existing literature mostly reaches this conclusion on the basis of arguments that are mainly related to the complexity of technological change as a process (e.g., difficulties associated to imitating foreign technology), our model adds to this the complexity of economic structure. The (bi-causal) interaction between economic structure and technological change adds complexity to the story, but it may also increase the level of empirical reality of the theory, especially if applied to countries that are relatively close to each other in terms of the general level of technological knowledge. We hope to be able to put (parts of) our model to the empirical test in a future paper.

With regard to policy, our model suggests that catching-up and convergence of productivity levels is not achieved by single issue-oriented approaches. Catching-up is a more complex process than just (for example) enhancing social capability or technological congruence. Productivity increases (convergence) may be achieved by more than one form of technological change, and these interact with the economy in various ways. The outcome of this process may not always be as simple as some theories suggest, and finding the right mix of policies may be more a matter of trial-and-error than of applying a grand design on the basis of the insights of economic theories.

References

Verspagen, B. (1993), Uneven Growth between Interdependent Economies, Avebury, Aldershot UK.

Appendix: Parameter and initial variable configuration

The values below were used to generate the “alfa+KV” line in Figure 4 (and the figures thereafter), for various values of $\rho_1=\rho_2$. Unless stated otherwise, the documented values refer to both North and South, and both to sector 1 and sector 2.

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<tr>
<td>$\varphi$</td>
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These values of both Kaldor-Verdoorn parameters imply a long-run growth rate of 0.01 per period. Note that Figure 4 was generated with $\sigma=6.0$. The implied long-run growth rate was maintained at 0.01.
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