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**Archiv der Mathematik**

# Mumford coverings of the projective line

By

MARIUS VAN DER PUT and HARM H. VOSKUIL

**Abstract.** A Mumford covering of the projective line over a complete non-archimedean valued field $K$ is a Galois covering $X \to \mathbf{P}^1_K$ such that $X$ is a Mumford curve over $K$. The question which finite groups do occur as Galois group is answered in this paper. This result is extended to the case where $\mathbf{P}^1_K$ is replaced by any Mumford curve over $K$.

**1. Introduction.** The field $K$ is supposed to be complete with respect to a non-archimedean valuation. In [4], D. Harbater has proved that any finite group $G$ is the Galois group of a Galois covering $X \to \mathbf{P}^1_K$, where $X$ is an absolutely irreducible, smooth, projective curve over $K$. We will exploit here the simplified proof given by Q. Liu [6].

We recall that a *Schottky group* (over $K$) is a free, finitely generated subgroup $\Gamma$ of $\mathrm{PGL}(2, K)$ which acts discontinuously on the analytic space $\mathbf{P}^{1,an}_K$ associated to the projective line over $K$. The complement $\Omega$ of the compact set of the limit points of $\Gamma$ is an open subspace of $\mathbf{P}^{1,an}_K$. The *Mumford curve* $Y$ (over $K$) associated to $\Gamma$ is defined as $Y := \Omega/\Gamma$. This curve is absolutely irreducible, smooth and projective over $K$. The genus of $Y$ is equal to the number of free generators of $\Gamma$. The projective line over $K$ will also be considered as a Mumford curve over $K$ (corresponding to the trivial group $\Gamma = \{1\}$). For details on Mumford curves we refer to [3].

Let $Y$ be a Mumford curve over $K$. A *Mumford covering of* $Y$ is a (in general ramified) Galois covering $X \to Y$ such that $X$ is a Mumford curve over $K$. The question, that we consider in this paper, is:

*What are the Galois groups for Mumford coverings of* $\mathbf{P}^1_K$*?*

Examples of Mumford coverings of $\mathbf{P}^1_K$, unramified outside $\{0, 1, \infty\}$ and for a field $K$ of characteristic 0, have been given by Y. André [1] and F. Kato [5]. The first author has studied this question in connection with $p$-adic orbifolds and $p$-adic hypergeometric differential equations. The second author has produced many examples in an attempt to classify all of them.

The answer to the above question (allowing any type of ramification) is the following:

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*Mathematics Subject Classification* (2000):

**Theorem 1.1.** (a) *Suppose that $K$ has characteristic* 0. *Then any finite group is a Galois group of a Mumford covering of* $\mathbf{P}^1_K$.

(b) *Suppose that $K$ has characteristic $p > 0$. A finite group is a Galois group of a Mumford covering of $\mathbf{P}^1_K$, if and only if this group is generated by its elements of order not divisible by $p^2$.*

*We thank the referee for excellent suggestions* which lead to a complete answer for the corresponding question where $\mathbf{P}^1_K$ is replaced by any Mumford curve $Y$ over $K$. The results are:

**Theorem 1.2.** *Let $Y$ be a Mumford curve over $K$ of genus $g$ with $Y(K) \neq \emptyset$.*

(a) *If the characteristic of $K$ is* 0, *then every finite group is the Galois group of a Mumford covering of $Y$.*

(b) *Suppose that the characteristic of $K$ is $p > 0$. Let $G$ be a finite group and denote by $N$ the subgroup of $G$ generated by the elements having order not divisible by $p^2$. Then $G$ is the Galois group of a Mumford covering $X \to Y$, if and only if the $p$-group $G/N$ is generated by at most $g$ elements.*

We note that part (b) of Theorem 1.2 (and of 1.1) is somewhat surprising. The proofs the theorems will be given in Sectioins 2–4. A Mumford covering of $\mathbf{P}^1_K$ corresponds to a finitely generated discontinuous subgroup $\Gamma$ (in fact generated by elements of finite order) of PGL(2, $K$) such that $\Omega/\Gamma \cong \mathbf{P}^1_K$. In Theorem 1.2 one has described the finite groups $G \cong \Gamma/\Gamma_0$, where $\Gamma_0$ is a normal subgroup of finite index which is a Schottky group. A much more involved problem is to classify these groups $\Gamma$. This is the theme of the sequel [7] of the present paper.

**2. Reduction of the problem to the cyclic case.** A finite group $G$ is called *realizable* if there exists a Mumford covering $X \to \mathbf{P}^1_K$ with group $G$ and such that the points of $X$ above $\infty$ are unramified and rational over $K$.

**Lemma 2.1.** *Suppose that the finite group $G$ is generated by two subgroups $G_1, G_2$ which are realizable. Then $G$ is realizable.*

Proof. We follow rather closely Q. Liu's paper [6] and refer to this for more details. Let $G_i$, for $i = 1, 2$, be realized by $f_i : X_i \to \mathbf{P}^1_K$. Let $D_i$ be a large enough closed disk, say $D_i = \{z \in \mathbf{A}^1_K \mid |z| \leqq R\}$ with $R \in |K^*|$. Put $U = \{z \in \mathbf{P}^1_K \mid R \leqq |z| \leqq \infty\}$. We require that the restriction of $f_i$ to $f_i^{-1}U \to U$ is trivial covering, i.e., $f_i^{-1}U$ is the disjoint union of copies of $U$. One considers now a new projective line $\mathbf{P}^1_K$ and disjoint closed disks, say $D_1^* = \{z \in \mathbf{A}^1_K \mid |z| \leqq r_1\}$ and $D_2^* = \{z \in \mathbf{A}^1_K \mid |z-1| \leqq r_2\}$, satisfying $r_1, r_2 \in |K^*|$ and $r_1, r_2 < 1$. Let $C$ denote the affinoid subset of $\mathbf{P}^1_K$ given by the inequalities $|z| \geqq r_1,\ |z-1| \geqq r_2$. Above each of the three affinoids $D_1^*, D_2^*, C$ we will consider a (possibly ramified and in general not connected) affinoid covering $A_1 \to D_1^*,\ A_2 \to D_2^*,\ A \to C$ with $G$ as Galois group. The first covering $A_1 \to D_1^*$ is obtained by considering an isomorphism

$D_1^* \to D_1$ and taking the pull back $E_1$ of the covering $f_1^{-1}(D_1) \to D_1$. Then $E_1 \to D_1^*$ is a connected Galois covering with group $G_1$. The induced covering $\mathrm{Ind}_{G_1}^G E_1 \to D_1^*$ is a Galois covering with group $G$. This is the covering $A_1 \to D_1^*$. The covering $A_2 \to D_2^*$ is constructed in the same way. The covering $A \to C$ is simply the trivial covering with group $G$. The three affinoid coverings are glued together in an $G$-invariant way to a rigid analytic Galois covering $f : X \to \mathbf{P}_K^1$ with group $G$. This covering is connected since $G$ is generated by $G_1$ and $G_2$. By GAGA, one concludes that $X$ has a unique structure of an irreducible smooth, projective algebraic curve over $K$. The points above $\infty$ in the covering $f$ are unramified and $K$-rational since $A \to C$ is a trivial covering. A criterion for a curve $Y$ to be a Mumford curve is that the analytification $Y^{an}$ has a finite admissible covering by affinoids sets which are isomorphic to affinoid subsets of $\mathbf{P}_K^1$. By assumption $X_1, X_2$ satisfy this property. Thus $A_1$ and $A_2$ are finite unions of affinoids which can be embedded in $\mathbf{P}_K^1$. The same holds obviously for $A$. Thus $X$ is a Mumford curve. $\square$

Clearly the lemma reduces the general problem to the case of cyclic groups of order a prime power, which we will study in the next section.

**3. The existence of cyclic Mumford coverings.** Let $q$ be a power of a prime number. We have to produce a cyclic covering $X \to \mathbf{P}_K^1$ such that $X$ is a Mumford curve and the points above $\infty$ are unramified and $K$-rational. One has to distinguish several cases.

(a) Suppose that the field $K$ contains a primitive $q^{\text{th}}$ root of unity. The equation $y^q = x^{q-1}(x-1)$ provides such a covering.

(b) Suppose that $q$ is not divisible by the characteristic of $K$ and that $\zeta_q$, the primitive $q^{\text{th}}$ root of unity, is not present in $K$. One considers the extension $K \subset K' = K(\zeta_q)$ with a generator $\tau$ of the Galois group. Put $\tau\zeta_q = \zeta_q^m$, $k = \frac{m^s-1}{q}$ where $s = [K' : K]$ is the order of $\tau$. The action of $\tau$ is extended to the field $K'(x)$ by putting $\tau(x) = x$. Further one considers the extension $K(x) \subset K'(x)[Y]/(Y^q - a)$ for a suitable element $a \in K'(x)$. This element should be such that it gives rise to a $q$-cyclic extension $L$ of $K(x)$ which defines the required Mumford covering $X \to \mathbf{P}_K^1$.

Consider $b = \frac{1+\zeta_q x}{1+\zeta_q x+\pi}$, where $0 \neq \pi \in K$ is an element with small enough absolute value. One defines $a$ by the formula

$$a = b^{m^{s-1}} \tau(b)^{m^{s-2}} \cdots \tau^{s-2}(b)^m \tau^{s-1}(b).$$

The automorphism $\tau$ of $K'(x)$ extends to an automorphism $\tilde{\tau}$ of $K'(x)(y) = K'(x)[Y]/(Y^q - a)$ by $\tilde{\tau}(y) = y^m b^{-k}$. Let $\sigma$ denote the automorphism of $K'(x)(y)/K'(x)$ given by $\sigma(y) = \zeta_q y$. A straightforward calculation yields that $\sigma$ and $\tilde{\tau}$ commute, that the Galois group of $K'(x)(y)$ is generated by $\sigma$ and $\tau$ and that $L := K'(x)(y)^{\langle\tilde{\tau}\rangle}$ is a $q$-cyclic extension of $K(x)$. We are left with proving that the curve $X$ corresponding to the field $L$ is a Mumford curve. In proving this we may replace $X$ by $X \otimes K''$, where $K''$ is the completion of the algebraic closure of $K$. We have to show that the equation $Y^q = a$ defines a Mumford covering of $\mathbf{P}_{K''}^1$. One observes that the pole and the zero of $b = \frac{1+\zeta_q x}{1+\zeta_q x+\pi}$ are very close together. The same holds for $a$, i.e., the divisor of $a$ has the form $\sum_{i=1}^{d} m_i([x_i]-[y_i])$, where $x_i$ and $y_i$ are very close together. For notational convenience we suppose that $\infty$ is not a pole or zero of $a$. For $i=1,\dots,d$ one considers a small disk $D_i=\{z|\ |z-a_i|\leqq\epsilon\}$ containing $x_i, y_i$. Let $C$ be the affinoid subset of $\mathbf{P}^1_{K''}$ defined by the inequalities $|z-a_i|\geqq\epsilon$ for $i=1,\dots,d$. We note that $\epsilon$ can be made as small as required by changing the absolute value of $\pi$. The equation $Y^q=a$ above $D_i$ is equivalent to $Y^q=(\frac{z-x_1}{z-y_i})^{m_i}$. The affinoid covering $E_i\to D_i$ defined by the latter equation is clearly an affinoid subset of $\mathbf{P}^1_{K''}$. The equation $Y^q=a$ above the affinoid $C$ gives a trivial covering since the function $a$ on $C$ has the form $c\cdot(1+r)$, with $c\in(K'')^*$ and where the norm of $r$ is very small (depending again on the absolute values of $\pi$). Thus $X$ satisfies the criterion for a Mumford curve, used in the proof of Lemma 2.1.

(c) Suppose that $K$ has characteristic $p>0$. The equation $y^p-y=x$ defines a $p$-cyclic Mumford covering of $\mathbf{P}^1_K$.

Conclusions. The above proves part (a) of Theorem 1.1 and the implication "if" of part (b) of that theorem.

Proof of Theorem 1.2, part (a). Let $Y$ and the finite group $G$ be given. One chooses an affinoid subset $D$ in $Y$, such that there exists an isomorphism $\phi:\{z\in\mathbf{P}^1_K|\ |z|\leqq1\}\to D$. Put $D^o=\phi(\{z\in\mathbf{P}^1_K|\ |z|<1\})$ and $\partial D=\phi(\{z\in\mathbf{P}^1_K|\ |z|=1\})$. Then $\{Y\setminus D^o, D\}$ is an admissible (affinoid) covering of $Y$. By 1.1, there exists a connected Galois covering $U\to D$ with group $G$ which has the properties:

(i) $U$ is embedded in a Mumford covering of $\mathbf{P}^1_K$ with group $G$.
(ii) The induced covering above $\partial D$ is trivial.

Now one glues this covering $U\to D$ to the trivial covering of $Y\setminus D^o$ (with group $G$). The result is the required Mumford covering of $Y$ with group $G$.

Remarks. The "only if" implication of Theorem 1.1, part (b) would follow from the statement that for any Galois covering $X\to\mathbf{P}^1_K$ (with $X$ absolutely irreducible, smooth, projective over $K$) with group $\mathbf{Z}/p^n\mathbf{Z}$ and $n\geqq2$ (or $n=2$), the curve $X$ is *not* a Mumford curve. Using Witt vectors one can describe these cyclic coverings explicitly and derive that $X$ is indeed not a Mumford curve. The *referee* remarks that any $X$, thus obtained, has a reduction with rational curves over the residue field of $K$ as irreducible components. However this reduction has bad singularities which prevents $X$ from being a Mumford curve.

In the next section, we will present a more *conceptional proof*, using the uniformization of a Mumford curve, of the statement that $\mathbf{Z}/p^n\mathbf{Z}$ does not occur as Galois group of a Mumford covering of $\mathbf{P}^1_K$ for $n\geqq2$.

**4. Coverings in positive characteristic.** We consider the situation: $K$ has characteristic $p>0$, $X\to\mathbf{P}^1_K$ a Galois covering with group $G$ and $X$ a Mumford curve. Let $N$ denote the subgroup of $G$ generated by its elements of order not divisible by $p^2$. The proof of Theorem 1.1 is complete when we have shown that $N = G$. In the sequel we may enlarge the field $K$. For convenience we will suppose that $K$ is algebraically closed.

Proof of the "only if" part of Theorem 1.1 (b). Let $u : \Omega \to X$ denote the universal rigid analytic covering of $X$. According to [3], $\Omega$ is isomorphic to the complement in $\mathbf{P}^1_K$ of some compact subset. Consider a point $x_0 \in X$ which is ramified over $\mathbf{P}^1_K$ and a point $\omega \in \Omega$ with $u(\omega) = x_0$. The point $\omega$ can be identified with $\infty \in \mathbf{P}^1_K$. Let $\mathrm{St}_{x_0} \subset G$ denote the stabilizer of $x_0$. Any $\sigma \in \mathrm{St}_{x_0}$ can uniquely be lifted to an automorphism $\tilde{\sigma}$ of $\Omega$ such that $\tilde{\sigma}(\omega) = \omega$. The map $\sigma \mapsto \tilde{\sigma}$ is therefore a group homomorphism and $\mathrm{St}_{x_0}$ can be identified with a subgroup of the automorphisms of $\Omega$ having $\omega$ as fixed point. According to [2] or [3] the automorphisms of $\Omega$ extend to automorphisms of $\mathbf{P}^1_K$. This implies that $\mathrm{St}_{x_0}$ can be identified with a subgroup of the Borel group of $\mathrm{PGL}(2, K)$. In particular, $\mathrm{St}_{x_0}$ does not contain elements of order $p^2$. Thus $\mathrm{St}_{x_0} \subset N$.

The group $N$ is a normal subgroup of $G$. The covering $X/N \to \mathbf{P}^1_K$ with group $G/N$ is unramified since $N$ contains all the ramification groups of $X \to \mathbf{P}^1_K$. We conclude that the covering $X/N \to \mathbf{P}^1_K$ is trivial and that $G = N$. This finishes the proof of Theorem 1.1.

Proof of the "only if" part of Theorem 1.2 (b). The field $K$ has characteristic $p > 0$ and, for convenience, is supposed to be algebraically closed, $Y$ is a Mumford curve over $K$, $X \to Y$ is a Galois covering with group $G$, $X$ is a Mumford curve and $N$ is the subgroup of $G$ generated by the elements whose orders are not divisible by $p^2$.

As above we conclude that the covering $X/N \to Y$ is unramified. By definition the group $G/N$ is a $p$-group. The "only if" part of 1.2 (b), follows from the next lemma.

**Lemma 4.1.** *Let $Y$ be a Mumford curve over $K$ of genus $g$. The uniformization of $Y$ is denoted by $\Omega/\Gamma$, where $\Gamma$ is a free group on $g$ generators. Let $Z \to Y$ be a Galois étale covering (with irreducible $Z$) and with a Galois group $G$ which is a $p$-group. Then this étale covering is a rigid analytic covering, i.e., there is a surjective homomorphism $\Gamma \to G$ with kernel $\Gamma_0$ such that the given covering is isomorphic to the covering $\Omega/\Gamma_0 \to \Omega/\Gamma = Y$.*

Proof. We consider first the case where $G$ is a $p$-cyclic group. The $p$-cyclic étale coverings of $Y$ correspond to the elements of order $p$ of the Jacobian variety $J$ of $Y$. This group $J[p]$ is isomorphic to $(\mathbf{Z}/p\mathbf{Z})^r$ with $0 \leqq r \leqq g$. Since $Y$ is a Mumford curve, the Jacobian variety has multiplicative reduction and $r = g$. Thus the $p$-cyclic étale extensions correspond to $p$-cyclic subgroups of $(\mathbf{Z}/p\mathbf{Z})^g$. On the other hand the $p$-cyclic rigid analytic coverings of $Y$ are given by $\Omega/\Gamma' \to \Omega/\Gamma$, where $\Gamma'$ is the kernel of a surjective homomorphism $\Gamma \to \mathbf{Z}/p\mathbf{Z}$. We conclude that any $p$-cyclic étale covering is a rigid analytic covering and that the corresponding curve $Z$ is again a Mumford curve.

Now we consider the general situation. Since $G$ is a $p$-group, there exists a normal subgroup $H \subset G$ such that $G/H$ is a $p$-cyclic group. The covering $Z \to Y$ decomposes as $Z \to Z/H \to Y$. We have shown that the $p$-cyclic covering $Z/H \to Y$ is a rigid analytic covering and that $Z/H$ is again a Mumford curve. By induction on the order of the group we may suppose that $Z \to Z/H$ is also a rigid analytic covering. It follows that $Z \to Y$ is a rigid analytic covering. As is well known a rigid analytic covering is given by surjective homomorphisms $\Gamma \to G$ having some kernel $\Gamma_0$. The covering is then isomorphic to the covering $\Omega/\Gamma_0 \to \Omega/\Gamma$. $\square$

The proof of the "if" part of Theorem 1.2 (b). In general $G$ is not a semi-direct product of $N$ and $G/N$. One chooses a subgroup $W$ of $G$ generated by at most $g$ elements which maps surjectively to $G/N$. Consider the semi-direct product $G' = N \rtimes W$, where the multiplication is given by $(n_1, w_1) \cdot (n_2, w_2) = (n_1 w_1 n_2 w_1^{-1}, w_1 w_2)$. There is an obvious surjective morphism $G' \to G$. According to the next lemma, it suffices to produce a Mumford covering of $Y$ with group $G'$.

**Lemma 4.2.** *Let $Z$ be a Mumford curve over $K$. Let $H$ be a finite group of automorphisms of $Z$. Then $Z/H$ is Mumford curve over $K$.*

Proof. Let $g$ be the genus of $Z$. The case $g \leqq 1$ is trivial and we suppose now $g \geqq 2$. Write $Z = \Omega/\Gamma$ with $\Omega \subset \mathbf{P}_K^{1,an}$. Since $\Gamma \subset \mathrm{PGL}(2, K)$, the set of the limit points $\mathcal{L}$ of $\Gamma$ is a compact perfect subset of $\mathbf{P}_K^1(K)$. Let $\Delta \subset \Gamma$ denote the subgroup of $\mathrm{PGL}(2, K)$ consisting of all the lifts of all elements of $H$. Then $\Gamma$ is a normal subgroup of $\Delta$ and $\Delta/\Gamma = H$. It follows that $\Delta$ is also a discontinuous subgroup of $\mathrm{PGL}(2, K)$ and has again $\mathcal{L}$ as its set of limit points.

We recall the standard procedure which associates to $\mathcal{L}$ a tree and an admissible affinoid covering of $\Omega$. One identifies $\mathbf{P}_K^1$ with $\mathbf{P}_K(V)$ where $V$ is a 2-dimensional vector space over $K$. Further $K^o$ denotes the valuation ring of $K$. A lattice $M \subset V$ is a free $K^o$-submodule of $V$ of rank two. Two lattices $M_1$ and $M_2$ are called equivalent if $M_1 = \lambda M_2$ for some element $\lambda \in K^*$. Let $[M]$ denote the equivalence class of the lattice $M$. Any three distinct lines $Kv_1, Kv_2, Kv_3$ in $V$ determine a unique lattice class $[M]$, with $M$ generated over $K^o$ by $\lambda_1 v_1, \lambda_2 v_2, \lambda_3 v_3$, where $\lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 v_3 = 0$ is a non-trivial linear relation. In particular, for every triple $(a_0, a_1, a_\infty)$ of distinct points of $\mathcal{L}$, one has an associated lattice class $[M]$. These lattice classes are the vertices of a graph $\mathcal{T}$. Consider a pair $\{[M_1], [M_2]\}$ of distinct vertices, where the representatives $M_1, M_2$ are chosen such that $M_1 \supset M_2$ and $M_1/M_2 = K^o/\pi K^o$ for some $\pi$ with $0 < |\pi| < 1$. This pair is an edge of $\mathcal{T}$ if there is no vertex $[M]$ such that $M_1 \supset M \supset M_2$ and $M \neq M_1, M_2$. It turns out that $\mathcal{T}$ is a tree. Since $\mathcal{L}$ is $\Delta$-invariant there is an action of $\Delta$ on the tree $\mathcal{T}$. The stabilizers of $\Delta$ on this tree are finite because $\Delta$ is a discontinuous group. One can associate to any vertex $v$ and every edge $e$ of $\mathcal{T}$, affinoid subsets $U(v)$ and $U(e)$ of $\Omega$. For a vertex $v$, corresponding to a triple $(a_0, a_1, a_\infty)$, one takes an identification $\mathbf{P}_K(V)$ with $\mathbf{P}_K^1$ such that $a_0, a_1, a_\infty$ are mapped to $0, 1, \infty$. Let $\overline{K}$ denote the residue field of $K$. The obvious reduction map Red : $\mathbf{P}_K^1 \to \mathbf{P}_{\overline{K}}^1$ maps $\mathcal{L}$ to a finite subset $F$ of $\mathbf{P}_{\overline{K}}^1(\overline{K})$. The preimage, under Red of $\mathbf{P}_{\overline{K}}^1 \setminus F$, is by definition the affinoid subset $U(v)$ of $\Omega$. The set $U(e)$, with $e = \{[M_1], [M_2]\}$, is defined in a similar way, using the two reductions maps corresponding to the two triples of elements of $\mathcal{L}$. The covering $\{U_i\} := \{U(v), U(e)\}$ is an admissible affinoid covering of $\Omega$. The sets $U_i$ have the properties:

(a) $\Delta_i := \{\delta \in \Delta \mid \delta U_i \cap U_i \neq \emptyset\}$ is finite.
(b) If $\Delta_i \neq \{1\}$, then $\Delta_i$ is a subgroup of $\Delta$ and $\delta U_i = U_i$ for each $\delta \in \Delta_i$.

Clearly $\Omega/\Delta = Z/H$ and $Z/H$ is obtained by glueing finitely many affinoid sets of the form $V_i := U_i/\Delta_i$. Each of those sets lies in $\mathbf{P}_K^1/\Delta_i \cong \mathbf{P}_K^1$. Therefore $Z/H$ is a Mumford curve over $K$. $\square$

The group $G'$ is a semi direct product $N \rtimes W$ and the curve $Y$ has the form $\Omega/\Gamma$ where $\Gamma$ is a free group on $g$ generators. There is a surjective homomorphism $\Gamma \to W$, since the group $W$ is generated by at most $g$ elements. The kernel $\Gamma_0$ of this homomorphism gives rise to an analytic étale Galois covering $f : Z := \Omega/\Gamma_0 \to Y$ with Galois group $W$. The preimage under $f$ of a small enough closed disk of $Y$ is the disjoint union of isomorphic closed disks $\{D_w | w \in W\}$. The notation is taken such that $w_1 D_{w_2} = D_{w_1 w_2}$ for all $w_1, w_2 \in W$. One may identify $D_1$ with $\{z|\ |z| \leqq 1\}$ and define the "boundary" $\partial D_1$ as the part corresponding to $\{z|\ |z| = 1\}$. The interior $D_1^o$ is the part of $D_1$ corresponding to $\{z|\ |z| < 1\}$. The boundaries $\partial D_w$ and interiors $D_w^o$ are defined by $\partial D_w = w \partial D_1$ and $D_w^o = w D_1^o$.

Using Theorem 1.1 one finds a Galois covering $X_1 \to D_1$ with group $N$, such that $X_1$ is a connected affinoid subset of a Mumford curve and such that the induced covering above $\partial D_1$, which we will denote by $\partial X_1 \to \partial D_1$, is trivial. For any $w \in W$, the covering $X_w \to D_w$ is obtained from $X_1 \to D_1$ by a push forward via the isomorphism $w : D_1 \to D_w$. The obvious isomorphism $X_1 \to X_w$ is also called $w$. There is an induced action of $N$ on the covering $X_w \to D_w$, which is given explicitly by $n(e_w) = w((w^{-1} n w) e_1)$, with $n \in N$, $e_w \in X_w$ and $e_1 \in X_1$ such that $e_w = w(e_1)$. The group $G' = N \rtimes W$ acts on $\bigcup_{w \in W} X_w$ by the formula $n_1 w_1 (e_w) = w_1 w((w_1 w)^{-1} n_1 (w_1 w) e_1)$, where $e_w = w e_1$ with $e_1 \in X_1$ and $n_1 \in N,\ w, w_1 \in W$.

Let $Z_0$ denote $Z \setminus (\bigcup_{w \in W} D_w^o)$. This is an affinoid subset of $Z$. Moreover $\{Z_0, \{D_w\}_{w \in W}\}$ is an admissible affinoid covering of $Z$. The only non-empty intersections for this covering are $Z_0 \cap D_w = \partial D_w$ for $w \in W$. Above $Z_0$ we consider the trivial covering $X_0 \to Z_0$ with Galois group $N$. Thus $X_0$ can be identified with $N \times Z_0$. The action of $G' = N \rtimes W$ on $X_0$ is defined by the formula $n_1 w_1 (n, z) = (n_1 w_1 n w_1^{-1}, w_1 z)$ with $w, w_1 \in W$, $z \in Z_0$ and $w_1 \in W$. The "boundary" $\partial Z_0$ of $Z_0$ is defined as the union $\bigcup_{w \in W} \partial D_w$. The "boundary" $\partial X_0$ of $X_0$ is the preimage of $\partial Z_0$ under $X_0 \to Z_0$. In the construction of the curve $X$ the affinoid space $X_0$ is glued to the affinoid space $\bigcup_{w \in W} X_w$ (this is the disjoint union of the $X_w$) by an obvious $G'$-equivariant isomorphism between their boundaries $\partial X_0$ and $\bigcup_{w \in W} \partial X_w$. By doing so, one first obtains an analytic space $X$ which is a finite (ramified) covering of $Z$. By GAGA, $X$ has the structure of a non-singular projective curve over $K$ and $X \to Z$ is a finite morphism of curves. Using that each $X_w$ and $Z_0$ is connected and by tracing the glueing, one finds that $X$ is a connected curve over $K$. The action of $G'$ on each $X_w$ and $X_0$ induces an action of $G'$ on $X$. The result is a Galois covering $X \to Y$ with group $G'$ such that $X$ is a Mumford curve.

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