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TEXTURE ANALYSIS USING RÉNYI'S GENERALIZED ENTROPIES

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ABSTRACT
We propose a texture analysis method based on Rényi’s generalized entropies. The method aims at identifying texels in regular textures by searching for the smallest window through which the minimum number of different visual patterns is observed when moving the window over a given texture. The results show that any of Rényi’s entropies can be used for texel identification. However, the second order entropy, due to its robust estimation, is the most reliable. The main advantages of the proposed method are its robustness and its flexibility. We illustrate the usefulness and the effectiveness of the method in a texture synthesis application.

1. INTRODUCTION
Only rarely have real world objects surfaces of uniform intensity; most of the time they are textured. The word texture is used to refer to a number of commonly encountered visual patterns in natural scenes, such as foliage, grass, pebbles, clouds, etc. While there is no proper definition of texture, it is widely accepted that the term generally refers to a repetition of basic elements, sometimes called texels. The textures exhibiting such a repetition of basic primitives are commonly referred to as structural textures. Although the structural approach to texture analysis is an old research direction in the field of texture analysis, it was less productive than other areas in the same field, due to various difficulties encountered [1, 2].

Structural texture analysis focuses primarily on identifying periodicity in texture or on identifying texels and their placement rules. Such methods are mainly based on Fourier analysis [3], coocurrence matrices [4], and autocorrelation functions [1, 2]. While widely used, these methods prove to be fragile when it comes to natural texture analysis due to the imperfection in the regularity of these textures [1, 4].

Whether they deal directly with texel identification or they search for periodicity in texture, the structural texture analysis methods try to find the basic texture unit - the texel. This step is important for texture analysis not only in the context of structural approaches, it can also be used when other methods are employed for texture study. Next to being a tool for texture analysis [2, 4], texel identification is also useful in other fields of image processing, computer vision, and computer graphics such as texture synthesis [4], texture compression [4], image database retrieval [1], and 3D vision [5].

In this paper, we propose a texel identification method based on a concept borrowed from information theory - Rényi’s generalized entropies. Rényi’s entropy family is one of the most popular generalizations of Shannon entropy [6]. Similar to the Shannon entropy, it measures the amount of uncertainty in predicting the output of a probabilistic event [6]: when the uncertainty is reduced, the entropy decreases. This property is used in the method proposed here. More precisely, the texel identification method described in the following sections searches for the smallest window through which the minimum number of different visual patterns – different in a given sense – is observed when moving the window over a given texture. Such a window has the property of minimizing Rényi’s generalized entropies.

2. RÉNYI’S GENERALIZED ENTROPIES
Rényi’s generalized entropies were introduced in [7] as a family of measures that characterize the distribution of a random variable. They are defined as follows: if a random variable $\xi$ takes the values $[x_1]_{i=1}^N$ with probabilities $[p_i = P(\xi = x_i)]_{i=1}^N$, then the generalized entropy of order $q$ of $\xi$ is defined as

$$ H_q = \begin{cases} \frac{\log \sum_{i=1}^{N} p_i^q}{1-q} & \text{for } q \neq 1, \\ \lim_{q \to 1} \left( \frac{\log \sum_{i=1}^{N} p_i^q}{1-q} \right) = -\sum_{i=1}^{N} p_i \log p_i & \text{for } q = 1, \\ \log \max_{i=1}^{N} (p_i) & \text{for } q \to \infty. \end{cases} \tag{1} $$

The zero order entropy counts the number of values $x_i$ for which $p_i$ is nonzero. The entropy of order one is the Shannon entropy. The entropy of order two, also called the quadratic entropy [8], gives the probability that $\xi$ takes twice the same value. The order of the entropy can be seen as a weight of the contribution of each $x_i$ to the value of the entropy. For the limit case $q \to \infty$, the entropy $H_\infty$, also called the min-entropy [9], depends only on the probability of occurrence of the most frequent $x_i$.

3. RÉNYI’S GENERALIZED ENTROPIES OF TEXTURE
Let us consider an artificial texture image (Fig. 1) comprising a checkerboard pattern whose black and white blocks are squares $k \times k$ pixels wide. Such a texture can be described as consisting of square texels of size $2k \times 2k$. If we observe this texture locally, through a square shaped window of size $2k \times 2k$, we will see pieces of the texture that are, in fact, circularly shifted versions of the same pattern. A similar thing will happen if the size of the observation window is a multiple of $2k \times 2k$, i.e. $4k \times 4k$.

\[\text{When using the standard convention that } 0^0 = 0.\]
Fig. 2. Rényi’s entropies of the texture in Fig. 1, for \( k = 32 \). The values on the abscissa represent the window size \( w \), while the
values on the ordinate represent \( H_k(w) \).

For a given texture, the texture entropies depend on the size of
the observation window. We illustrate this property by considering a simple case, the synthetic texture from Fig. 1. We compute the
distribution of visual events observed in a square window of size
\( w \times w \) and we plot its corresponding \( H_0(w) \) as a function of \( w \).

While in a general context the classification of the patterns in
the third column of Fig. 3 in the same group could be a mistake, in
the context of texture analysis such a classification is not necessarily
wrong since all these patterns could be regarded as translated or rotated versions of the same pattern. When classifying textures,
assigning rotated or translated version of the same texture to one
class is desirable. However, for segmentation purposes the lack
of discrimination between texture patches that are rotated versions of each other is undesirable. Nonetheless, in the current context
of texture entropy computation, the rotation and translation invari-
ance can be added or removed by choosing the right type of pattern
comparison method for grouping different patterns in one event.

The zero order entropy reaches its minimum value when the
observation window has a size multiple of \( 2k \times 2k \). Similarly, the
other entropies plotted in Fig. 2 reach their minima for windows
whose sizes are multiples of \( 2k \times 2k \). In other words, the window
sizes for which the texture entropies reach their minima depend on
the size of the texels of the texture and do not depend on the ent-
ropy’s order. The same thing cannot be said about the maximum.

4. RÉNYI’S ENTROPIES OF NATURAL TEXTURES

The results displayed in Fig. 2 show that Rényi’s entropies of a
texture could be used to find the texel size in a regular texture.
They also show that Rényi’s entropies of any order can be used
for this purpose. The next question is how such entropies behave for natural textures. Unlike artificial textures, natural textures do not consist of identical texels, even when they are highly regular. In order to see how this difference affects Rényi’s entropy behavior, in the following, we present experiments with textures from Brodatz album [10] and from VisTex texture database [11]. We selected a number of regular textures containing square texels of various sizes (Fig. 4). The test images are of size 256 × 256 pixels.

(a) D20
(b) D101
(c) D21
(d) Tile9

Fig. 4. Natural textures consisting of square shaped texels.

To account for the variability of the texels in real textures, we compare the histograms of the patterns observed in two windows of the same size by means of the two-sample Kolmogorov-Smirnov statistical test [12]. Applying Kolmogorov-Smirnov test to histogram comparison is a natural choice if one takes in consideration that histograms are estimates of the probability distribution functions of the gray levels. In such a case, computing the $p_i$ in (1) is done using the box-counting schemes described in [13].

In Fig. 5, we display the results obtained for the textures in Fig. 4. For conciseness reasons, we present here only the plots obtained for the quadratic entropy. Similar plots were obtained for the other entropies. These results were obtained using the Kolmogorov-Smirnov test at a significance level 0.01, a significance level often used in statistics. Similar results were obtained at significance levels 0.05 and 0.10. We computed the texture entropies for window sizes between $4 \times 4$ and $133 \times 133$. We used window sizes greater than $4 \times 4$ pixels in order to have enough samples when performing the Kolmogorov-Smirnov test. We limited the size of the windows to $133 \times 133$ because using bigger windows would reduce the number of observed events in the image making the entropy computation less robust while it would not make the plots more informative.

In Section 3, we saw that in the case of a simple synthetic texture, Rényi’s entropies can be used to determine the size of its texels, when the texels were square shaped. The entropies of such a texture are minimal around the window size that is equal to the size of the texel. Moreover, the entropy plots have deep valleys around such points.

Similar to the synthetic texture case, the generalized entropies of natural textures have deep valleys around the window size equal to the size of the texel. However, the texture entropies are not zero, neither are they minimal at such points. Typically, they do not reach the zero value at all and they are minimal for the smallest window size. Nonetheless, the sizes of the texels can be determined if one looks at the position of the deep valleys in the entropy plots. More precisely, the texel size is given by the window size for which the smallest local minimum

$$H_q(w) \leq H_q(w - 1) \leq H_q(w + 1)$$

is reached. The captions of each of the plots in Fig. 5 give the size of the texel as computed according to this rule.

5. APPLICATION TO TEXTURE SYNTHESIS

In this section, we demonstrate an application of the texel size determination in texture synthesis by tiling. Such a synthesis method has the advantages of being fast and able to preserve the regularity of the texture sample. In Fig. 6, we present artificially generated counterparts of the textures in Fig. 4. Each artificial texture was generated by tiling with a square window randomly selected from the sample image. The size of that window was computed according to the method presented in Section 4.

$$H_q(w)$$

is considered to have a local minimum for window size $w$ if $H_q(w) \leq H_q(w - 1)$ and $H_q(w) \leq H_q(w + 1)$. 
its computational simplicity, while being a good discriminator be-
a choice are the invariance of the histogram to circular shift and
togram of a window as a descriptor. The main reasons for such
statistical approach to texture analysis. It uses the gray level his-
pared. For illustration, we compare here the gray level histograms
visual patterns observed through two different windows are com-
derent texture models by simply adapting the way in which two
method is its flexibility. The method can easily incorporate dif-
noise in the analyzed texture. Another advantage of the proposed
Method is its flexibility. The method can easily incorporate dif-
ions and noise and (ii) high computational demands. Our method,
method proposed here does not use cooccurrence matrices, Fourier
window was inspired by the time delay embedding technique used
in dynamic systems for time series analysis [6]. Using an exten-
sion of the time delay embedding for texture analysis is a suitable
approach because texture is a neighborhood property. However,
advantages of 1D techniques to 2D situations proves to be a difficult
problem with no optimal solution. In this paper, we used the sim-
plest way of extending a 1D technique to a 2D problem. Studying
other extension alternatives remains a task for future work.

6. SUMMARY AND CONCLUSIONS

We study here an application of Rényi’s generalized entropies to
texture analysis. More precisely, we show that these entropies can
be used to identify square shaped texels in regular textures. The
proposed method is based on the property of the generalized en-
tropies of the distribution of a stochastic variable of being small
when the uncertainty in the outcome of that stochastic variable is
small. In the case of texture analysis, the stochastic variable is the
histogram of the pattern observed through a square window of a
given size. We show that, for regular textures with square shaped
texels, Rényi’s entropies depend on the size of the observation win-
dow and that they are small when the size of the observation win-
dow matches the size of the texels in the analyzed texture.

Unlike most of the structural texture analysis methods, the
method proposed here does not use cooccurrence matrices, Fourier
analysis, or autocorrelation functions. These classical methods
exhibit some drawbacks, such as (i) sensitivity to slight distor-
tions and noise and (ii) high computational demands. Our method,
while being computationally tractable, is robust to distortions and
noise in the analyzed texture. Another advantage of the proposed
method is its flexibility. The method can easily incorporate dif-
ferent texture models by simply adapting the way in which two
visual patterns observed through two different windows are com-
pared. For illustration, we compare here the gray level histograms
in the considered windows.

The pattern comparison method used in our experiments is a
statistical approach to texture analysis. It uses the gray level his-
togram of a window as a descriptor. The main reasons for such
a choice are the invariance of the histogram to circular shift and
its computational simplicity, while being a good discriminator be-
tween different textures. We are well aware of the limitations of
such descriptors [14], however, in the current context such limita-
tions are not felt.

The proposed texel identification method is able to identify
only square texels. Texels with different shapes can be identified
if windows with different profiles are used. Our choice of a square
window was inspired by the time delay embedding technique used
in dynamic systems for time series analysis [6]. Using an exten-
sion of the time delay embedding for texture analysis is a suitable
approach because texture is a neighborhood property. However,
extending 1D techniques to 2D situations proves to be a difficult
problem with no optimal solution. In this paper, we used the sim-
plest way of extending a 1D technique to a 2D problem. Studying
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