M-theory and gauged supergravities

Roest, Diederik

Appendix A

Conventions

Generalities

We use mostly plus signature $(-+\cdots+)$. Greek indices $\mu,\nu,\rho\ldots$ denote world coordinates and Latin indices $a,b,c\ldots$ represent tangent space-time. The different indices are related by the Vielbeins $e_a{}^\mu$ and inverse Vielbeins $e_\mu{}^a$, that satisfy

$$e_a{}^\mu e_b{}^\nu g_{\mu\nu}=\eta_{ab}\,,\qquad e_\mu{}^a e_\nu{}^b\eta_{ab}=g_{\mu\nu}\,. \tag{A.1}$$

Here η_{ab} is the Minkowski space-time metric and the space-time metric is $g_{\mu\nu}$. Underlined explicit indices $0,\ldots,D-1$ refer to the tangent space-time coordinates.

The covariant derivative on fermions is given by $D_\mu=\partial_\mu+\omega_\mu$ with the spin connection $\omega_\mu=\frac{1}{4}\omega_\mu{}^{ab}\Gamma_{ab}$, where

$$\omega_{abc}=-\Omega_{abc}+\Omega_{bca}-\Omega_{cab}\,,\qquad \Omega_{ab}{}^c=e_a{}^\mu e_b{}^\nu\partial_{[\mu}e^c{}_{\nu]}\,. \tag{A.2}$$

The Riemann curvature tensor is given in terms of the spin connection by

$$R_{\mu\nu a}{}^b=2\partial_{[\mu}\,\omega_{\nu]a}{}^b-2\omega_{[\mu|a}{}^c\,\omega_{|\nu]c}{}^b\,. \tag{A.3}$$

We symmetrise and anti-symmetrise with weight one.

Gauge potentials of rank d are denoted by $C^{(d)}$ with field strength $G^{(d+1)}$. For notational compactness, we sometimes omit the superscript label and denote gauge potentials of rank 0 up to 3 by χ, A or V, B and C respectively. The corresponding field strengths are given by the symbol $G^{(1)},F,H$ and G, respectively.

Our conventions in form notation in D dimensions are as follows:

$$\begin{aligned}
P^{(p)}&=\frac{1}{p!}P^{(p)}_{\mu_1\cdots\mu_p}\mathrm{d}x^{\mu_1}\wedge\cdots\wedge\mathrm{d}x^{\mu_p}\,,\\
P^{(p)}\cdot Q^{(p)}&=\frac{1}{p!}P^{(p)}_{\mu_1\cdots\mu_p}Q^{(p)\,\mu_1\cdots\mu_p}\,,
\end{aligned}$$

$$
\begin{aligned}
P^{(p)} \wedge Q^{(q)} &= \frac{1}{p!q!} P^{(p)}_{\mu_1 \cdots \mu_p} Q^{(q)}_{\mu_{p+1} \cdots \mu_{p+q}} \mathrm{d}x^{\mu_1} \wedge \cdots \wedge \mathrm{d}x^{\mu_{p+q}} \,, \\
\star P^{(p)} &= \frac{1}{(D-p)!p!} \sqrt{-g} \varepsilon^{(D)}_{\mu_0 \cdots \mu_{D-1}} P^{(p)\,\mu_{D-p} \cdots \mu_{D-1}} \mathrm{d}x^{\mu_0} \wedge \cdots \wedge \mathrm{d}x^{\mu_{D-p-1}} \,, \\
&\quad \text{with } \varepsilon^{(D)}_{0123 \cdots D-1} = -\varepsilon^{0123 \ldots D-1} = 1 \,, \\
\star\star\, P^{(p)} &= (-)^{p(D-p)+1} P^{(p)} \,, \\
d &= \partial_\mu \mathrm{d}x^\mu \,,
\end{aligned}
\tag{A.4}
$$

where the last line is the exterior derivative, acting from the left.

In the case of dimensional reduction, we will always be reducing a D-dimensional theory to a D-dimensional one, over an internal manifold of $n = D - D$ dimensions. The higher-dimensional fields will be hatted and the lower-dimensional ones unhatted. The corresponding split in the coordinates reads $x^\mu = (x^\mu, z^m)$, with indices μ and μ ranging from 0 to $D-1$ and $D-1$, respectively, while $m = 1, \ldots, n$.

Spinors and Γ-matrices in Various Dimensions

We will denote the Γ-matrices by Γ_μ (of dimensions 32) in eleven and ten dimensions and by γ_μ (of dimensions 16) in nine dimensions. They can be chosen to satisfy

$$
\Gamma^\dagger_\mu = \eta_{\mu\mu} \Gamma_\mu \qquad \text{and} \qquad \gamma^\dagger_\mu = \eta_{\mu\mu} \gamma_\mu \,, \tag{A.5}
$$

respectively. We can also choose the Γ-matrices to be real, while in nine dimensions they will be purely imaginary, which implies that

$$
\Gamma^T_\mu = \eta_{\mu\mu} \Gamma_\mu \qquad \text{and} \qquad \gamma^T_\mu = -\eta_{\mu\mu} \gamma_\mu \,. \tag{A.6}
$$

The following notation is used to denote the antisymmetric product of n Γ-matrices:

$$
\Gamma_{\mu_1 \cdots \mu_n} = \Gamma_{[\mu_1} \cdots \Gamma_{\mu_n]} \,. \tag{A.7}
$$

Slashes are used to contract Γ-matrices and field strengths in the following sense:

$$
\not{H} = H^{\mu\nu\rho} \Gamma_{\mu\nu\rho} \,, \qquad \not{H}_\mu = H_{\mu\nu\rho} \Gamma^{\nu\rho} \,, \tag{A.8}
$$

with similar formulae for other field strengths. In nine dimensions the same notation is used with Γ replaced by γ.

In eleven and ten dimensions we use the 32-dimensional spinor representation, with Γ-matrices Γ_μ (and Γ_{11} in 10D). Upon reduction to nine dimensions we will split this into 16-dimensional representations, with Γ-matrices γ_μ. This will be discussed below. In contrast, upon reduction to eight dimensions we will use the corresponding spinor representation; rather, we preserve the 32-dimensional representation, with Γ-matrices Γ_μ and Γ_i with $i = 1, 2, 3$.

In ten dimensions the minimal spinor is a 32-component Majorana-Weyl spinor with 16 (real) degrees of freedom. With the choice

$$\Gamma_{11} = -\Gamma_{\underline{0}\cdots\underline{9}}\,, \qquad \Gamma_{11} = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}, \tag{A.9}$$

we can write a ten-dimensional Majorana-Weyl spinor as being composed of nine-dimensional, 16 component, Majorana-Weyl spinors according to

$$\psi_+^{MW} = \begin{pmatrix} \psi_1 \\ 0 \end{pmatrix}, \qquad \psi_-^{MW} = \begin{pmatrix} 0 \\ \psi_2 \end{pmatrix}, \tag{A.10}$$

where ψ_i are nine-dimensional Majorana-Weyl spinors and $+$ or $-$ denotes the chirality of the ten-dimensional spinor. The split of an arbitrary ten-dimensional spinor into two Majorana-Weyl spinors of opposite chirality can of course be done without reference to nine dimensions (through the specific choice of Γ_{11}), but each ten-dimensional Majorana-Weyl spinor will then in general have 32 non-zero components even though it only has 16 degrees of freedom. In order to reduce to nine dimensions we use

$$\Gamma_{11} = \sigma_3 \otimes \mathbb{1}\,, \qquad \Gamma_{\underline{z}} = \sigma_1 \otimes \mathbb{1}\,, \qquad \Gamma_a = \sigma_2 \otimes \gamma_a\,, \tag{A.11}$$

where z is the reduction coordinate and the Pauli matrices are defined as

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.12}$$

As mentioned above the nine dimensional γ-matrices are purely imaginary. If we work with a reduction of type IIB, where the two spinors have the same chirality, it may be convenient to introduce complex, nine-dimensional, Weyl spinors according to

$$\psi_c = \psi_1 + i\psi_2\,, \qquad \lambda_c = \lambda_2 + i\lambda_1\,, \tag{A.13}$$

$$\epsilon_c = \epsilon_1 + i\epsilon_2\,, \qquad \tilde{\lambda}_c = \tilde{\lambda}_2 + i\tilde{\lambda}_1\,, \tag{A.14}$$

which in ten-dimensional notation can be written as, e.g.,

$$\psi_+^W = \begin{pmatrix} \psi_1 \\ 0 \end{pmatrix} + i \begin{pmatrix} \psi_2 \\ 0 \end{pmatrix}. \tag{A.15}$$

If we instead work with a reduction of type IIA the two spinors will have opposite chirality, and can thus be composed into a ten-dimensional Majorana spinor according to

$$\psi^M = \begin{pmatrix} \psi_1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \psi_2 \end{pmatrix}. \tag{A.16}$$

$$
\begin{tikzcd}
 & \psi^{M}_{32\mathbb{R}} \arrow[dr, "Weyl"] & \\
\psi^{D}_{32\mathbb{C}} \arrow[ur, "Majorana"] \arrow[dr, "Weyl"'] & & \psi^{MW}_{16\mathbb{R}} \\
 & \psi^{W}_{16\mathbb{C}} \arrow[ur, "Majorana"'] &
\end{tikzcd}
$$

Figure A.1: Schematic view of how a ten-dimensional Dirac spinor can be projected down to a Majorana-Weyl spinor along two different routes. The number of real or complex degrees of freedom for each spinor is also indicated. The relation between the spinors at the intermediate stage (in nine dimensions) is given by (A.17).

When working with these non-minimal spinors, which are either just Majorana (ψ^M_μ) or just Weyl (ψ^W_μ) [150], the two formulations are (in nine dimensions) related via

$$
\begin{aligned}
\tfrac{1}{2}(1+\Gamma_{11})\psi^M_\mu &= \mathrm{Re}(\psi^W_\mu)\,, & \tfrac{1}{2}(1-\Gamma_{11})\psi^M_\mu &= \mathrm{Im}(\Gamma_{\underline{z}}\psi^W_\mu)\,,\\
\tfrac{1}{2}(1+\Gamma_{11})\lambda^M &= \mathrm{Im}(\Gamma_{\underline{z}}\lambda^W)\,, & \tfrac{1}{2}(1-\Gamma_{11})\lambda^M &= \mathrm{Re}(\lambda^W)\,,\\
\tfrac{1}{2}(1+\Gamma_{11})\tilde{\lambda}^M &= \mathrm{Im}(\Gamma_{\underline{z}}\tilde{\lambda}^W)\,, & \tfrac{1}{2}(1-\Gamma_{11})\tilde{\lambda}^M &= \mathrm{Re}(\tilde{\lambda}^W)\,,\\
\tfrac{1}{2}(1+\Gamma_{11})\epsilon^M &= \mathrm{Re}(\epsilon^W)\,, & \tfrac{1}{2}(1-\Gamma_{11})\epsilon^M &= \mathrm{Im}(\Gamma_{\underline{z}}\epsilon^W)\,,
\end{aligned}
\tag{A.17}
$$

for positive (ψ^W_μ, ϵ^W) and negative ($\lambda^W, \tilde{\lambda}^W$) chirality Weyl fermions. With the above mentioned decomposition into nine-dimensional Majorana-Weyl spinors we can write

$$
\psi^M_\mu = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \epsilon^M = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}, \quad \lambda^M = \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}, \quad \tilde{\lambda}^M = \begin{pmatrix} \tilde{\lambda}_1 \\ \tilde{\lambda}_2 \end{pmatrix} \tag{A.18}
$$

and

$$
\psi^W_\mu = \begin{pmatrix} \psi_1 + i\psi_2 \\ 0 \end{pmatrix}, \quad \epsilon^W_\mu = \begin{pmatrix} \epsilon_1 + i\epsilon_2 \\ 0 \end{pmatrix}, \tag{A.19}
$$

$$
\lambda^W = \begin{pmatrix} 0 \\ \lambda_2 + i\lambda_1 \end{pmatrix}, \quad \tilde{\lambda}^W = \begin{pmatrix} 0 \\ \tilde{\lambda}_2 + i\tilde{\lambda}_1 \end{pmatrix}, \tag{A.20}
$$

where the spinors without an M or W superscript are Majorana-Weyl spinors. The two different routes to obtain Majorana-Weyl spinors are illustrated in figure A.1. Note also that it follows from the Clifford algebra and the choice of Γ_{11} that $\Gamma_{\underline{z}}$ is off-diagonal, which is crucial for this construction.

Appendix B

Supergravity and Reductions

B.1 11D Supergravity

11D Supersymmetry Transformations and Field Equations

The supersymmetry transformation rules of $N = 1$ eleven-dimensional supergravity read

$$\begin{aligned} \delta e_\mu{}^a &= \bar{\epsilon}\Gamma^a\psi_\mu\,, \\ \delta C_{\mu\nu\rho} &= -3\,\bar{\epsilon}\Gamma_{[\mu\nu}\psi_{\rho]}\,, \\ \delta\psi_\mu &= D_\mu\epsilon + \tfrac{1}{192}(\not{G}\Gamma_\mu - \tfrac{1}{3}\Gamma_\mu\not{G})\epsilon\,, \end{aligned} \tag{B.1}$$

with the field strengths $G = \mathrm{d}C$ and $D_\mu\epsilon = (\partial_\mu + \omega_\mu)\epsilon$. The 11D fermionic field content consists solely of a 32–component gravitino, whose field equation reads

$$X_0(\psi^\mu) = \Gamma^{\mu\nu\rho}D_\nu\psi_\rho = 0\,, \tag{B.2}$$

with $D_\nu = \partial_\nu + \omega_\nu$ and where we have set the three-form equal to zero. Under supersymmetry this fermionic field equations transforms into

$$\delta_0 X_0(\psi^\mu) = \tfrac{1}{2}\Gamma^\nu\epsilon\,[R^\mu{}_\nu - \tfrac{1}{2}Rg^\mu{}_\nu]\,, \tag{B.3}$$

which implies the bosonic Einstein equation for the metric.

11D Reduction Ansätze to IIA

We use the following reduction Ansätze (where hatted quantities are 11D and unhatted are IIA)

$$
\begin{aligned}
\hat{e}_{\hat{\mu}}{}^{\hat{a}} &= e^{m_{11}z} \begin{pmatrix} e^{-\phi/12}e_\mu{}^a & -e^{2\phi/3}C_\mu^{(1)} \\ 0 & e^{2\phi/3} \end{pmatrix}, \\
\hat{\psi}_a &= e^{-m_{11}z/2}e^{\phi/24}[\psi_a - \tfrac{1}{24}\Gamma_a\lambda]\,, \\
\hat{\psi}_{\underline{z}} &= \tfrac{1}{3}e^{-m_{11}z/2}e^{\phi/24}\Gamma_{\underline{z}}\lambda\,, \\
\hat{\epsilon} &= e^{m_{11}z/2}e^{-\phi/24}\epsilon\,, \\
\hat{C}_{\mu\nu\rho} &= e^{3m_{11}z}C^{(3)}_{\mu\nu\rho}\,, \\
\hat{C}_{\mu\nu z} &= -e^{3m_{11}z}B_{\mu\nu}\,,
\end{aligned} \tag{B.4}
$$

to arrive at the IIA supersymmetry transformations in ten dimensions, where

- $m_{11} = 0$ for toroidal reduction and
- $m_{11} \neq 0$ for twisted reduction using the trombone symmetry of 11D supergravity.

B.2 IIA Supergravity

IIA Supersymmetry Transformations and Field Equations

The supersymmetry transformation rules of ten-dimensional massless or ungauged IIA supergravity read

$$
\begin{aligned}
\delta_0 e_\mu{}^a &= \bar{\epsilon}\Gamma^a\psi_\mu\,, \\
\delta_0\psi_\mu &= \big(D_\mu + \tfrac{1}{48}e^{-\phi/2}(\not{H}\Gamma_\mu + \tfrac{1}{2}\Gamma_\mu\not{H})\Gamma_{11} + \tfrac{1}{16}e^{3\phi/4}(\not{G}^{(2)}\Gamma_\mu - \tfrac{3}{4}\Gamma_\mu\not{G}^{(2)})\Gamma_{11} + \\
&\quad + \tfrac{1}{192}e^{\phi/4}(\not{G}^{(4)}\Gamma_\mu - \tfrac{1}{4}\Gamma_\mu\not{G}^{(4)})\big)\epsilon\,, \\
\delta_0 B_{\mu\nu} &= 2e^{\phi/2}\bar{\epsilon}\Gamma_{11}\Gamma_{[\mu}(\psi_{\nu]} + \tfrac{1}{8}\Gamma_{\nu]}\lambda)\,, \\
\delta_0 C^{(1)}_\mu &= -e^{-3\phi/4}\bar{\epsilon}\Gamma_{11}(\psi_\mu - \tfrac{3}{8}\Gamma_\mu\lambda)\,, \\
\delta_0 C^{(3)}_{\mu\nu\rho} &= -3e^{-\phi/4}\bar{\epsilon}\Gamma_{[\mu\nu}(\psi_{\rho]} - \tfrac{1}{24}\Gamma_{\rho]}\lambda) + 3C^{(1)}_{[\mu}\delta_0 B_{\nu\rho]}\,, \\
\delta_0\lambda &= \big(\not{\partial}\phi + \tfrac{1}{12}e^{-\phi/2}\not{H}\Gamma_{11} + \tfrac{3}{8}e^{3\phi/4}\not{G}^{(2)}\Gamma_{11} + \tfrac{1}{96}e^{\phi/4}\not{G}^{(4)}\big)\epsilon\,, \\
\delta_0\phi &= \tfrac{1}{2}\bar{\epsilon}\lambda\,,
\end{aligned} \tag{B.5}
$$

with the following field strengths:

$$
G^{(2)} = \mathrm{d}C^{(1)}\,, \qquad H = \mathrm{d}B\,, \qquad G^{(4)} = \mathrm{d}C^{(3)} + C^{(1)}\wedge H\,, \tag{B.6}
$$

and $D_\mu\epsilon = (\partial_\mu + \omega_\mu)\epsilon$. Upon (massless) reduction with our Ansätze the 11D field equation splits up in two field equations for the 10D IIA fermionic field content, a gravitino and a dilatino:

$$X_0(\psi^\mu) = \Gamma^{\mu\nu\rho} D_\nu \psi_\rho - \tfrac{1}{8}(\partial\!\!\!/\phi)\Gamma^\mu\lambda = 0\,, \qquad X_0(\lambda) = \Gamma^\nu D_\nu\lambda - \Gamma^\nu(\partial\!\!\!/\phi)\psi_\nu = 0\,, \tag{B.7}$$

with $D_\nu = (\partial_\nu + \omega_\nu)$ and where we have set the vector, two- and three-form equal to zero. Under supersymmetry these fermionic field equations transform into

$$\begin{aligned} \delta_0 X_0(\psi^\mu) =& \tfrac{1}{2}\Gamma^\nu\epsilon\,[R^\mu{}_\nu - \tfrac{1}{2}Rg^\mu{}_\nu - \tfrac{1}{2}(\partial^\mu\phi)(\partial_\nu\phi) + \tfrac{1}{4}(\partial\phi)^2 g^\mu{}_\nu]\,, \\ \delta_0 X_0(\lambda) =& \epsilon\,[\Box\phi]\,, \end{aligned} \tag{B.8}$$

which imply the usual graviton-dilaton field equations.

IIA Reduction Ansätze to 9D

We use the following reduction Ansatz with z-dependence implied by the $SO(1,1)$-symmetries (where hatted quantities are IIA and unhatted are 9D):

$$\begin{aligned}
\hat{e}_{\hat\mu}{}^{\hat a} &= e^{9m_{\rm IIA}z/8}\begin{pmatrix} e^{\phi/16-3\varphi/16\sqrt{7}}e_\mu{}^a & e^{-7\phi/16+3\sqrt{7}\varphi/16}A^1_\mu \\ 0 & e^{-7\phi/16+3\sqrt{7}\varphi/16}\end{pmatrix}\,, \\
\hat\psi_a &= e^{-9m_{\rm IIA}z/16}e^{-\phi/32+3\varphi/32\sqrt{7}}[\psi_a + \tfrac{1}{32}\Gamma_a(\lambda - \tfrac{3}{\sqrt{7}}\tilde\lambda)]\,, \\
\hat\psi_{\underline{z}} &= -\tfrac{7}{32}e^{-9m_{\rm IIA}z/16}e^{-\phi/32+3\varphi/32\sqrt{7}}\Gamma_{\underline{z}}(\lambda - \tfrac{3}{\sqrt{7}}\tilde\lambda)\,, \\
\hat B_{\mu\nu} &= -e^{3m_{\rm IIA}z+m_4z/2}(B^1_{\mu\nu} - 2A^1_{[\mu}A_{\nu]})\,, \\
\hat B_{\mu z} &= -e^{3m_{\rm IIA}z+m_4z/2}A_\mu\,, \\
\hat C^{(1)}_\mu &= -e^{-3m_4z/4}(A^2_\mu + \chi A^1_\mu)\,, \\
\hat C^{(1)}_z &= -e^{-3m_4z/4}\chi\,, \\
\hat C^{(3)}_{\mu\nu\rho} &= e^{3m_{\rm IIA}z-m_4z/4}(C_{\mu\nu\rho} - 3A_{i\,[\mu}B^i_{\nu\rho]} + 6A^1_{[\mu}A^2_\nu A_{\rho]})\,, \\
\hat C^{(3)}_{\mu\nu z} &= -e^{3m_{\rm IIA}z-m_4z/4}(B^2_{\mu\nu} - 2A^2_{[\mu}A_{\nu]})\,, \\
\hat\lambda &= \tfrac{1}{4}e^{-9m_{\rm IIA}z/16}e^{-\phi/32+3\varphi/32\sqrt{7}}(3\lambda + \sqrt{7}\tilde\lambda)\,, \\
\hat\epsilon &= e^{9m_{\rm IIA}z/16}e^{\phi/32-3\varphi/32\sqrt{7}}\epsilon\,, \\
\hat\phi &= \tfrac{1}{4}(3\phi + \sqrt{7}\varphi) + \left(\tfrac{3}{2}m_{\rm IIA} + m_4\right)z\,,
\end{aligned} \tag{B.9}$$

where the mass parameters are given by

- $m_{\rm IIA} = 0$ and $m_4 = 0$ for toroidal reduction,
- $m_{\rm IIA} = 0$ and $m_4 \neq 0$ for twisted reduction using the scale symmetry α,

- $m_{\text{IIA}} = 0$ and $m_4 \neq 0$ for twisted reduction using the trombone symmetry β and
- $m_{\text{IIA}} \neq 0$ and $m_4 \neq 0$ for a combination of the latter two.

B.3 IIB Supergravity

IIB Supersymmetry Transformations and Field Equations

The supersymmetry transformation rules of ten-dimensional IIB supergravity read (in complex notation)

$$\begin{aligned}
\delta e_\mu{}^a &= \tfrac{1}{2}\bar{\epsilon}\,\Gamma^a\psi_\mu + \text{h.c.}\,,\\
\delta\psi_\mu &= D_\mu\epsilon - \tfrac{i}{16\cdot 5!}\not{G}^{(5)}\Gamma_\mu\epsilon\\
&\quad + \tfrac{i}{16\cdot 3!}e^{\phi/2}\left(\Gamma_\mu\Gamma^{(3)} + 2\Gamma^{(3)}\Gamma_\mu\right)\left(H^2+\tau H^1\right)_{(3)}\epsilon^*\,,\\
\delta\lambda &= -e^{\phi}\not{\partial}\tau\epsilon^* - \tfrac{1}{2\cdot 3!}e^{\phi/2}\Gamma^{(3)}\left(H^2+\tau H^1\right)_{(3)}\epsilon\,,\\
\delta B^1_{\mu\nu} &= e^{\phi/2}\left(\bar{\epsilon}^*\,\Gamma_{[\mu}\psi_{\nu]} - \tfrac{i}{8}\bar{\epsilon}\,\Gamma_{\mu\nu}\lambda\right) + \text{h.c.}\,,\\
\delta B^2_{\mu\nu} &= -e^{\phi/2}\tau^*\left(\bar{\epsilon}^*\Gamma_{[\mu}\psi_{\nu]} - \tfrac{i}{8}\bar{\epsilon}\,\Gamma_{\mu\nu}\lambda\right) + \text{h.c.}\,,\\
\delta C^{(4)}_{\mu\nu\lambda\rho} &= 2i\,\bar{\epsilon}\,\Gamma_{[\mu\nu\lambda}\psi_{\rho]} - \tfrac{3}{2}\,B_{i\,[\mu\nu}\delta B^i_{\lambda\rho]} + \text{h.c.}\,,\\
\delta\chi &= -\tfrac{1}{4}e^{-\phi}\bar{\epsilon}\lambda^* + \text{h.c.}\,,\\
\delta\phi &= \tfrac{i}{4}\bar{\epsilon}\lambda^* + \text{h.c.}\,,
\end{aligned} \tag{B.10}$$

with the complex scalar $\tau = \chi + ie^{-\phi}$, the axion $\chi = C^{(0)}$, the doublet $B^i = (-B, C^{(2)})$ and the field strengths[1]

$$H^i = \mathrm{d}B^i\,, \qquad G^{(5)} = \mathrm{d}C^{(4)} + \tfrac{1}{2}B_i\wedge H^i\,. \tag{B.11}$$

Indices i,j of $SL(2,\mathbb{R})$ are contracted with $\varepsilon_{ij} = -\varepsilon^{ij}$ with $\varepsilon_{12} = -\varepsilon_{21} = 1$. The covariant derivative of the IIB Killing spinor reads

$$D_\mu\epsilon = (\partial_\mu + \omega_\mu + \tfrac{i}{4}e^{\phi}\partial_\mu\chi)\epsilon\,. \tag{B.12}$$

When truncating to the metric, scalars and fermions, the massless 9D fermionic field equations read

$$\begin{aligned}
X_0(\psi^\mu) &= \Gamma^{\mu\nu\rho}(\partial_\nu + \omega_\nu + \tfrac{1}{4}ie^{\phi}\partial_\nu\chi)\psi_\rho + \tfrac{1}{8}e^{\phi}(\not{\partial}\tau)\Gamma^\mu\lambda^* = 0\,,\\
X_0(\lambda) &= \Gamma^\mu(\partial_\mu + \omega_\mu + \tfrac{3}{4}ie^{\phi}\partial_\mu\chi)\lambda + e^{\phi}\Gamma^\mu(\not{\partial}\tau)\psi^*_\mu = 0\,.
\end{aligned} \tag{B.13}$$

[1] Note that $G^{(5)}$ is not of the canonical form (3.12); the difference amounts to a field redefinition.

IIB Reduction Ansätze to 9D

The reduction Ansätze we used for reducing the above rules are (where hatted quantities are IIB and unhatted are 9D)

$$
\begin{aligned}
\hat{e}_{\hat{\mu}}{}^{\hat{a}} &= e^{m_{\mathrm{IIB}} z} \begin{pmatrix} e^{\sqrt{7}\varphi/28} e_{\mu}{}^{a} & -e^{-\sqrt{7}\varphi/4} A_{\mu} \\ 0 & e^{-\sqrt{7}\varphi/4} \end{pmatrix} , \\
\hat{\psi}_{a} &= e^{-m_{\mathrm{IIB}} z/2} e^{-\sqrt{7}\varphi/56} \left(\frac{c\tau^* + d}{c\tau + d}\right)^{1/4} (\psi_a + \tfrac{1}{8\sqrt{7}} \Gamma_a \tilde{\lambda}^*) , \\
\hat{\psi}_{\underline{z}} &= -\tfrac{\sqrt{7}}{8} e^{-m_{\mathrm{IIB}} z/2} e^{-\sqrt{7}\varphi/56} \left(\frac{c\tau^* + d}{c\tau + d}\right)^{1/4} \Gamma_{\underline{z}} \tilde{\lambda}^* , \\
\hat{\lambda} &= i e^{-m_{\mathrm{IIB}} z/2} e^{-\sqrt{7}\varphi/56} \left(\frac{c\tau^* + d}{c\tau + d}\right)^{3/4} \lambda , \\
\hat{\epsilon} &= e^{m_{\mathrm{IIB}} z/2} e^{\sqrt{7}\varphi/56} \left(\frac{c\tau^* + d}{c\tau + d}\right)^{1/4} \epsilon , \\
\hat{\tau} &= \frac{a\tau + b}{c\tau + d} , \\
\hat{B}^{i}_{\mu\nu} &= e^{2m_{\mathrm{IIB}} z} (\Omega(z)^{-1})_{j}{}^{i} B^{j}_{\mu\nu} , \qquad \hat{B}^{i}_{\mu z} = -e^{2m_{\mathrm{IIB}} z} (\Omega(z)^{-1})_{j}{}^{i} A^{j}_{\mu} , \\
\hat{C}^{(4)}_{\mu\nu\lambda\rho} &= e^{4m_{\mathrm{IIB}} z} D_{\mu\nu\lambda\rho} , \qquad \hat{C}^{(4)}_{\mu\nu\lambda z} = e^{4m_{\mathrm{IIB}} z} (-C_{\mu\nu\lambda} + \tfrac{3}{2} A_{i\,[\mu} B^{i}_{\nu\rho]}) ,
\end{aligned}
\tag{B.14}
$$

where we take the Ω to be z-dependent:

$$
\begin{aligned}
\Omega(z)_{i}{}^{j} &= \exp \begin{pmatrix} \frac{1}{2} m_1 z & \frac{1}{2}(m_2 + m_3) z \\ \frac{1}{2}(m_2 - m_3) z & -\frac{1}{2} m_1 z \end{pmatrix} , \\
&= \begin{pmatrix} \cosh(\alpha z) + \frac{m_1}{2\alpha} \sinh(\alpha z) & \frac{1}{2\alpha}(m_2 + m_3) \sinh(\alpha z) \\ \frac{1}{2\alpha}(m_2 - m_3) \sinh(\alpha z) & \cosh(\alpha z) - \frac{m_1}{2\alpha} \sinh(\alpha z) \end{pmatrix} ,
\end{aligned}
\tag{B.15}
$$

where $\alpha^2 = \frac{1}{4}({m_1}^2 + {m_2}^2 - {m_3}^2)$. The mass parameters $\vec{m} = (m_1, m_2, m_3)$ and m_{IIB} take the following values in the different reduction schemes

- $\vec{m} = 0$ and $m_{\mathrm{IIB}} = 0$ for toroidal reduction,
- $\vec{m} \neq$ and $m_{\mathrm{IIB}} = 0$ for twisted reduction with the $SL(2, \mathbb{R})$ symmetry,
- $\vec{m} = 0$ and $m_{\mathrm{IIB}} \neq 0$ for twisted reduction with the trombone symmetry and
- $\vec{m} \neq 0$ and $m_{\mathrm{IIB}} \neq 0$ for a combination of the latter two.

B.4 9D Maximal Supergravity

9D Supersymmetry Transformations and Field Equations

The unique nine-dimensional $N = 2$ supergravity theory has the following supersymmetry transformations:

$$
\begin{aligned}
\delta_0 e_\mu{}^a &= \tfrac{1}{2}\bar\epsilon\gamma^a\psi_\mu + \text{h.c.}\,, \\
\delta_0\psi_\mu &= D_\mu\epsilon + \tfrac{i}{16}e^{-2\varphi/\sqrt{7}}\left(\tfrac{5}{7}\gamma_\mu\gamma^{(2)} - \gamma^{(2)}\gamma_\mu\right)F_{(2)}\epsilon \\
&\quad - \tfrac{1}{8\cdot 2!}e^{3\varphi/2\sqrt{7}}\left(\tfrac{5}{7}\gamma_\mu\gamma^{(2)} - \gamma^{(2)}\gamma_\mu\right)\,e^{\phi/2}\left(F^2 + \tau F^1\right)_{(2)}\epsilon^* \\
&\quad + \tfrac{i}{8\cdot 3!}e^{-\varphi/2\sqrt{7}}\left(\tfrac{3}{7}\gamma_\mu\gamma^{(3)} + \gamma^{(3)}\gamma_\mu\right)e^{\phi/2}\left(H^2 + \tau H^1\right)_{(3)}\epsilon^* \\
&\quad - \tfrac{1}{8\cdot 4!}e^{\varphi/\sqrt{7}}\left(\tfrac{1}{7}\gamma_\mu\gamma^{(4)} - \gamma^{(4)}\gamma_\mu\right)G_{(4)}\epsilon\,, \\
\delta_0\tilde\lambda &= i\partial\!\!\!/\varphi\,\epsilon^* - \tfrac{1}{\sqrt{7}}e^{-2\varphi/\sqrt{7}}F\!\!\!/\epsilon^* - \tfrac{3i}{2\cdot 2!\sqrt{7}}e^{3\varphi/2\sqrt{7}}e^{\phi/2}\gamma^{(2)}\left(F^2 + \tau^* F^1\right)_{(2)}\epsilon \\
&\quad + \tfrac{1}{2\cdot 3!\sqrt{7}}e^{-\varphi/2\sqrt{7}}e^{\phi/2}\gamma^{(3)}\left(H^2 + \tau^* H^1\right)_{(3)}\epsilon \\
&\quad + \tfrac{i}{4!\sqrt{7}}e^{\varphi/\sqrt{7}}G\!\!\!/\epsilon^*\,, \\
\delta_0\lambda &= i\partial\!\!\!/\phi\,\epsilon^* - e^{\phi}\partial\!\!\!/\chi\,\epsilon^* - \tfrac{i}{2\cdot 2!}e^{3\sqrt{7}\varphi/14}e^{\phi/2}\gamma^{(2)}\left(F^2 + \tau F^1\right)_{(2)}\epsilon \\
&\quad - \tfrac{1}{2\cdot 3!}e^{-\sqrt{7}\varphi/14}e^{\phi/2}\gamma^{(3)}\left(H^2 + \tau H^1\right)_{(3)}\epsilon\,, \\
\delta_0 A_\mu &= \tfrac{i}{2}e^{2\varphi/\sqrt{7}}\bar\epsilon(\psi_\mu - \tfrac{i}{\sqrt{7}}\gamma_\mu\tilde\lambda^*) + \text{h.c.}\,, \\
\delta_0 A^1_\mu &= \tfrac{i}{2}e^{\phi/2}e^{-3\varphi/2\sqrt{7}}\left(\bar\epsilon^*\psi_\mu + \tfrac{i}{4}\bar\epsilon\gamma_\mu\lambda + \tfrac{3i}{4\sqrt{7}}\bar\epsilon^*\gamma_\mu\tilde\lambda^*\right) + \text{h.c.}\,, \\
\delta_0 A^2_\mu &= -\tfrac{i}{2}e^{\phi/2}\tau^* e^{-3\varphi/2\sqrt{7}}\left(\bar\epsilon^*\psi_\mu + \tfrac{i}{4}\bar\epsilon\gamma_\mu\lambda + \tfrac{3i}{4\sqrt{7}}\bar\epsilon^*\gamma_\mu\tilde\lambda^*\right) + \text{h.c.}\,, \\
\delta_0 B^1_{\mu\nu} &= -e^{\phi/2}e^{\varphi/2\sqrt{7}}\left(\bar\epsilon^*\gamma_{[\mu}\psi_{\nu]} + \tfrac{i}{8}\bar\epsilon\gamma_{\mu\nu}\lambda - \tfrac{i}{8\sqrt{7}}\bar\epsilon^*\gamma_{\mu\nu}\tilde\lambda^*\right) - A_{[\mu}\delta_0 A^1_{\nu]} + \text{h.c.}\,, \\
\delta_0 B^2_{\mu\nu} &= e^{\phi/2}\tau^* e^{\varphi/2\sqrt{7}}\left(\bar\epsilon^*\gamma_{[\mu}\psi_{\nu]} + \tfrac{i}{8}\bar\epsilon\gamma_{\mu\nu}\lambda - \tfrac{i}{8\sqrt{7}}\bar\epsilon^*\gamma_{\mu\nu}\tilde\lambda^*\right) - A_{[\mu}\delta_0 A^2_{\nu]} + \text{h.c.}\,, \\
\delta_0 C_{\mu\nu\rho} &= \tfrac{3}{2}e^{-\varphi/\sqrt{7}}\bar\epsilon\gamma_{[\mu\nu}\left(\psi_{\rho]} + \tfrac{i}{6\sqrt{7}}\gamma_{\rho]}\tilde\lambda^*\right) - \tfrac{3}{2}B_{i\,[\mu\nu}\,\delta_0 A^i_{\rho]} + \text{h.c.}\,, \\
\delta_0\varphi &= -\tfrac{i}{4}\bar\epsilon\tilde\lambda^* + \text{h.c.}\,, \\
\delta_0\chi &= \tfrac{1}{4}e^{-\phi}\bar\epsilon\lambda^* + \text{h.c.}\,, \\
\delta_0\phi &= -\tfrac{i}{4}\bar\epsilon\lambda^* + \text{h.c.}\,,
\end{aligned}
\tag{B.16}
$$

with the complex scalar $\tau = \chi + ie^{-\phi}$. The field strengths read

$$G^{(1)} = \mathrm{d}\chi\,, \quad F = \mathrm{d}A\,, \quad F^i = \mathrm{d}A^i\,, \quad H^i = \mathrm{d}B^i - A\wedge F^i\,, \quad G = \mathrm{d}C + B_i\wedge F^i\,. \tag{B.17}$$

The covariant derivative of the Killing spinor reads

$$D_\mu\epsilon = (\partial_\mu + \omega_\mu + \tfrac{i}{4}e^\phi\partial_\mu\chi)\epsilon\,. \tag{B.18}$$

When truncating to the metric, scalars and fermions, the massless 9D fermionic field equations read

$$\begin{aligned}
X_0(\psi^\mu) &= \gamma^{\mu\nu\rho}(\partial_\nu + \omega_\nu + \tfrac{1}{4}ie^\phi\partial_\nu\chi)\psi_\rho - \tfrac{1}{8}e^\phi(\partial\!\!\!/\tau)\gamma^\mu\lambda^* + \tfrac{1}{8}i(\partial\!\!\!/\varphi)\gamma^\mu\tilde{\lambda}^* = 0\,,\\
X_0(\lambda) &= \gamma^\mu(\partial_\mu + \omega_\mu + \tfrac{3}{4}ie^\phi\partial_\mu\chi)\lambda + e^\phi\gamma^\mu(\partial\!\!\!/\tau)\psi^*_\mu = 0\,,\\
X_0(\tilde{\lambda}) &= \gamma^\mu(\partial_\mu + \omega_\mu - \tfrac{1}{4}ie^\phi\partial_\mu\chi)\tilde{\lambda} - i\gamma^\mu(\partial\!\!\!/\varphi)\psi^*_\mu = 0\,.
\end{aligned} \tag{B.19}$$

Under supersymmetry these yield the variation

$$\begin{aligned}
\delta_0 X_0(\psi^\mu) &= \tfrac{1}{2}\gamma^\nu\epsilon\,[R^\mu{}_\nu - \tfrac{1}{2}Rg^\mu{}_\nu - \tfrac{1}{2}((\partial^\mu\phi)(\partial_\nu\phi) - \tfrac{1}{2}(\partial\phi)^2 g^\mu{}_\nu) +\\
&\quad - \tfrac{1}{2}e^{2\phi}((\partial^\mu\chi)(\partial_\nu\chi) - \tfrac{1}{2}(\partial\chi)^2 g^\mu{}_\nu) - \tfrac{1}{2}((\partial^\mu\varphi)(\partial_\nu\varphi) - \tfrac{1}{2}(\partial\varphi)^2 g^\mu{}_\nu)]\,,\\
\delta_0 X_0(\lambda) &= \epsilon^*[-e^\phi(\Box\chi + 2(\partial_\mu\phi)(\partial^\mu\chi))] + i\epsilon^*[\Box\phi - e^{2\phi}(\partial\chi)^2]\,,\\
\delta_0 X_0(\tilde{\lambda}) &= i\epsilon^*[\Box\varphi]\,,
\end{aligned} \tag{B.20}$$

which are the massless bosonic field equations for the metric and the scalars.

Twisted Reduction of IIA using β

The reduction of massless IIA supergravity using the scale symmetry β of table 5.1 for twisting, with reduction Ansätze (B.9) with $m_{\mathrm{IIA}} = 0$, leads to a massive deformation with mass parameter m_4. Only the supersymmetry variations of the dilatini receive explicit massive deformations:

$$\delta_{m_4}\lambda = \tfrac{3}{4}m_4 e^{\phi/2 - 3\varphi/2\sqrt{7}}\epsilon\,, \qquad \delta_{m_4}\tilde{\lambda} = \tfrac{\sqrt{7}}{4}m_4 e^{\phi/2 - 3\varphi/2\sqrt{7}}\epsilon\,. \tag{B.21}$$

The implicit massive deformations read:

$$\begin{aligned}
&\mathrm{D}\phi = e^{-\phi}\mathrm{D}e^\phi\,, \qquad \mathrm{D}\varphi = e^{-\varphi}\mathrm{D}e^\varphi\,, \qquad G^{(1)} = \mathrm{D}\chi + \tfrac{3}{4}m_4 A^2\,, \qquad G = \mathrm{D}C + B_i\wedge F^i\,,\\
&F = \mathrm{D}A + \tfrac{1}{2}m_4 B^1\,, \qquad F^1 = \mathrm{d}A^1\,, \qquad F^2 = \mathrm{D}A^2\,,\\
&H^1 = \mathrm{D}B^1 - A\wedge F^1\,, \qquad H^2 = \mathrm{D}B^2 - A\wedge F^2 - \tfrac{1}{4}m_4(C + 3A^2\wedge B^1)\,.
\end{aligned} \tag{B.22}$$

The $\mathbb{R}^+$ covariant derivative is defined by $\mathrm{D} = \mathrm{d} - w_\beta\, m_4 A^1$ with w_β the β scale weight of the field it acts on, as given in the table 5.2, and $\mathrm{DD} = -w_\beta\, m_4 F^1$. The covariant derivative of the supersymmetry parameter has no massive deformation.

As for the field equations, the explicit deformations in the bosonic sector are given by the scalar potential

$$V = \tfrac{1}{2} e^{\phi - 3\varphi/\sqrt{7}} {m_4}^2 \,, \tag{B.23}$$

which can not be written in terms of a superpotential as (5.1). The explicit deformations of the fermionic field equations read

$$\begin{aligned} X_{m_4}(\psi^\mu) &= i m_4 e^{\phi/2 - 3\varphi/2\sqrt{7}} \gamma^{\mu\nu} [-i\tfrac{3}{256}\gamma_\nu \lambda - i\tfrac{\sqrt{7}}{256}\gamma_\nu \tilde{\lambda}] \,, \\ X_{m_4}(\lambda) &= -m_4 e^{\phi/2 - 3\varphi/2\sqrt{7}} \gamma^\nu [\tfrac{3}{4}\psi_\nu + \tfrac{2}{9\sqrt{7}} i \gamma_\nu \tilde{\lambda}^*] \,, \\ X_{m_4}(\tilde{\lambda}) &= -m_4 e^{\phi/2 - 3\varphi/2\sqrt{7}} \gamma^\nu [\tfrac{\sqrt{7}}{4}\psi_\nu - \tfrac{2}{9\sqrt{7}} i \gamma_\nu \lambda^*] \,. \end{aligned} \tag{B.24}$$

These massive deformations can be seen as a gauging of the scale symmetry β with gauge field transformation

$$\Lambda = e^{-w_\beta m_4 \lambda^1} \,, \qquad A^1 \to A^1 - \mathrm{d}\lambda^1 \,, \tag{B.25}$$

with gauge vector A^1 and parameter λ^1. In addition, we find that the parabolic $\mathbb{R}$ subgroup of $SL(2,\mathbb{R})$ is gauged:

$$\chi \to \chi + \tfrac{3}{4} m_4 \lambda^2 \,, \quad B^2 \to B^2 - \tfrac{3}{4} m_4 \lambda^2 B^1 \,, \quad A^2 \to A^2 - d\lambda^2 - \tfrac{3}{4} m_4 \lambda^2 A^1 \,, \tag{B.26}$$

with gauge vector A^2 and parameter λ^2. These two symmetries do not commute but rather form the two–dimensional non-Abelian Lie group, consisting of scalings and translations in one dimension (so-called collinear transformations [222]). The algebra reads

$$[T_1, T_2] = T_2 \,, \tag{B.27}$$

which is non–semi–simple. The emergence of this non-Abelian group is an example of the enhanced gaugings of section 4.3 and can be understood by the scaling of the 10D vector A_μ under β, see table 5.1.

Twisted Reduction of IIA using α

The twisted reduction of massless IIA supergravity based on the α symmetry of table 5.1, with reduction Ansätze (B.9) with $m_4 = 0$, leads to a gauged supergravity with mass parameter m_{IIA}. The explicit massive deformations in this case appear in the variation of the gravitino and one of the dilatini:

$$\delta_{m_{\mathrm{IIA}}} \psi_\mu = -\tfrac{9}{14} i m_{\mathrm{IIA}} e^{\phi/2 - 3\varphi/2\sqrt{7}} \gamma_\mu \epsilon^* \,, \qquad \delta_{m_{\mathrm{IIA}}} \tilde{\lambda} = \tfrac{6}{\sqrt{7}} m_{\mathrm{IIA}} e^{\phi/2 - 3\varphi/2\sqrt{7}} \epsilon \,. \tag{B.28}$$

The implicit massive deformations are given by

$$
\begin{aligned}
&\mathrm{D}\phi = e^{-\phi}\mathrm{d}e^{\phi}\,, \qquad \mathrm{D}\varphi = e^{-\varphi}\mathrm{D}e^{\varphi}\,, \qquad G^{(1)} = \mathrm{d}\chi\,, \qquad G = \mathrm{D}C + B_i\wedge F^i\,,\\
&F = \mathrm{D}A + 3m_{\mathrm{IIA}}B^1\,, \qquad F^1 = \mathrm{d}A^1\,, \qquad F^2 = \mathrm{d}A^2\,,\\
&H^1 = \mathrm{D}B^1 - A\wedge F^1\,, \qquad H^2 = \mathrm{D}B^2 - A\wedge F^2 + 3m_{\mathrm{IIA}}C\,.
\end{aligned}
\tag{B.29}
$$

The $\mathbb{R}^+$ covariant derivative is defined by $\mathrm{D} = \mathrm{d} - w_\alpha\, m_{\mathrm{IIA}}A^1$ with w_α the scale weight under α of the field it acts on, as given in the table 5.2, and $\mathrm{DD} = -w_\alpha\, m_{\mathrm{IIA}}F^1$. The covariant derivative of the supersymmetry parameter is given by

$$
D_\mu\epsilon = (\partial_\mu + \omega_\mu + \tfrac{i}{4}e^{\phi}\partial_\mu\chi - \tfrac{9}{14}m_{\mathrm{IIA}}\Gamma_\mu \not{A}^1)\epsilon\,. \tag{B.30}
$$

The 9D fermionic field equations have the following explicit massive deformations:

$$
\begin{aligned}
X_{m_{\mathrm{IIA}}}(\psi^\mu) &= i m_{\mathrm{IIA}} e^{\phi/2-3\varphi/2\sqrt{7}}\gamma^{\mu\nu}[\tfrac{9}{2}\psi^*_\nu - i\tfrac{9}{32}\gamma_\nu\lambda + i\tfrac{3}{4\sqrt{7}}\gamma_\nu\tilde{\lambda}]\,,\\
X_{m_{\mathrm{IIA}}}(\lambda) &= -m_{\mathrm{IIA}} e^{\phi/2-3\varphi/2\sqrt{7}}\gamma^{\nu}[-i\tfrac{\sqrt{7}}{6}\gamma_\nu\tilde{\lambda}^*]\,,\\
X_{m_{\mathrm{IIA}}}(\tilde{\lambda}) &= -m_{\mathrm{IIA}} e^{\phi/2-3\varphi/2\sqrt{7}}\gamma^{\nu}[\tfrac{6}{\sqrt{7}}\psi_\nu - \tfrac{11}{6\sqrt{7}}i\gamma_\nu\lambda^* + \tfrac{1}{7}i\gamma_\nu\tilde{\lambda}^*]\,.
\end{aligned}
\tag{B.31}
$$

This massive deformation is a gauging of the scale symmetry α with transformation:

$$
\Lambda = e^{-w_\alpha m_{\mathrm{IIA}}\lambda^1}\,, \qquad A^1 \to A^1 - \mathrm{d}\lambda^1\,, \tag{B.32}
$$

with gauge vector A^1 and parameter λ^1.

Twisted Reduction of IIB using δ

The other possibility for twisted reduction of $D = 10$ IIB supergravity involves the trombone symmetry of IIB supergravity. We use the reduction Ansätze given in (B.14) with $m_1 = m_2 = m_3 = 0$, yielding a massive deformation with parameter m_{IIB}. The explicit deformations of the supersymmetry rules read

$$
\delta_{m_{\mathrm{IIB}}}\psi_\mu = -\tfrac{4}{7}i m_{\mathrm{IIB}} e^{2\varphi/\sqrt{7}}\gamma_\mu\epsilon\,, \qquad \delta_{m_{\mathrm{IIB}}}\tilde{\lambda} = -\tfrac{4}{\sqrt{7}}m_{\mathrm{IIB}}e^{2\varphi/\sqrt{7}}\epsilon^*\,. \tag{B.33}
$$

The implicit deformations read

$$
\begin{aligned}
&F = \mathrm{d}A\,, \qquad F^i = \mathrm{d}A^i - 2m_{\mathrm{IIB}}B^i\,, \qquad H^i = \mathrm{d}B^i - A\wedge F^i\,,\\
&G = \mathrm{d}C + B_i\wedge F^i + m_{\mathrm{IIB}}B_i\wedge B^i\,, \qquad D\varphi = \mathrm{d}\varphi - \tfrac{4}{\sqrt{7}}m_{\mathrm{IIB}}A\,,
\end{aligned}
\tag{B.34}
$$

for the bosons and

$$
D_\mu\epsilon = (\partial_\mu + \omega_\mu + \tfrac{i}{4}e^{\phi}\partial_\mu\chi + \tfrac{4}{7}m_{\mathrm{IIB}}\Gamma_\mu\not{A})\epsilon \tag{B.35}
$$

for the supersymmetry parameter. The explicit deformations of the fermionic field equations read

$$\begin{aligned} X_{m_{\rm IIB}}(\psi^\mu) &= i m_{\rm IIB} e^{2\varphi/\sqrt{7}} \gamma^{\mu\nu} [4\psi_\nu - \tfrac{15}{16\sqrt{7}} i \gamma_\nu \tilde{\lambda}^*]\,, \\ X_{m_{\rm IIB}}(\lambda) &= m_{\rm IIB} e^{2\varphi/\sqrt{7}} \gamma^\nu [\tfrac{4}{9} i \gamma_\nu \lambda]\,, \\ X_{m_{\rm IIB}}(\tilde{\lambda}) &= m_{\rm IIB} e^{2\varphi/\sqrt{7}} \gamma^\nu [\tfrac{4}{\sqrt{7}} \psi^*_\nu - i \tfrac{4}{7} \gamma_\nu \tilde{\lambda}]\,. \end{aligned} \tag{B.36}$$

This is a supergravity where the scale symmetry δ has been gauged, whose action reads

$$\Lambda = e^{w_\delta m_{\rm IIB} \lambda}\,, \qquad A \to A - {\rm d}\lambda\,, \qquad B^i \to e^{2 m_{\rm IIB} \lambda}(B^i - A^i\,{\rm d}\lambda)\,, \tag{B.37}$$

with gauge vector A and parameter λ.

Toroidal Reduction of Gauged IIA

Finally, one can also consider the toroidal reduction of the $D = 10$ IIA gauged supergravity of section 5.3, again with reduction Ansätze (B.9) with $m_4 = m_{\rm IIA} = 0$. This leads to a $D = 9$ gauged supergravity with the following explicit deformations

$$\delta_{m_{11}} \psi_\mu = \tfrac{9}{14} i m_{11} e^{\phi/2 - 3\varphi/2\sqrt{7}} \tau \gamma_\mu \epsilon^*\,, \qquad \delta_{m_{11}} \tilde{\lambda} = -\tfrac{6}{\sqrt{7}} m_{11} e^{\phi/2 - 3\varphi/2\sqrt{7}} \tau^* \epsilon\,. \tag{B.38}$$

The bosonic implicit deformations read

$$\begin{aligned} &{\rm D}\varphi = {\rm d}\varphi - \tfrac{6}{\sqrt{7}} m_{11} A^2\,, \qquad F = {\rm D}A + 3 m_{11} B^2\,, \qquad G = {\rm D}C + + B_i \wedge F^i\,, \\ &F^i = dA^i\,, \qquad H^1 = {\rm D}B^1 - A \wedge F^{(1)} - 3 m_{11} C\,, \qquad H^2 = {\rm D}B^2 - A \wedge F^{(2)}\,, \end{aligned} \tag{B.39}$$

with the scale covariant derivative of a field with weight w defined by ${\rm D} = {\rm d} - w_\alpha m_{11} A^2$. For the supersymmetry parameter we find

$$D_\mu \epsilon = (\partial_\mu + \omega_\mu + \tfrac{i}{4} e^\phi \partial_\mu \chi + \tfrac{9}{14} m_{11} \Gamma_\mu \not{A}^2)\epsilon\,. \tag{B.40}$$

The fermionic field equations are deformed by the massive contributions

$$\begin{aligned} X_{m_{11}}(\psi^\mu) &= -i m_{11} e^{\phi/2 - 3\varphi/2\sqrt{7}} \gamma^{\mu\nu} [\tfrac{9}{2} \tau \psi^*_\nu - i \tfrac{9}{32} \tau^* \gamma_\nu \lambda + i \tfrac{3}{4\sqrt{7}} \tau \gamma_\nu \tilde{\lambda}]\,, \\ X_{m_{11}}(\lambda) &= m_{11} e^{\phi/2 - 3\varphi/2\sqrt{7}} \gamma^\nu [-i\tau \tfrac{\sqrt{7}}{6} \gamma_\nu \tilde{\lambda}^*]\,, \\ X_{m_{11}}(\tilde{\lambda}) &= m_{11} e^{\phi/2 - 3\varphi/2\sqrt{7}} \gamma^\nu [\tfrac{6}{\sqrt{7}} \tau^* \psi_\nu - \tfrac{11}{6\sqrt{7}} i \tau \gamma_\nu \lambda^* + \tfrac{1}{7} i \tau^* \gamma_\nu \tilde{\lambda}^*]\,. \end{aligned} \tag{B.41}$$

This massive deformation is a gauging of the scale symmetry α, reading

$$\Lambda = e^{-w_\alpha m_{11} \lambda^2}\,, \qquad A^2 \to A^2 - {\rm d}\lambda^2\,, \tag{B.42}$$

with gauge vector A^2 and parameter λ^2.

Publications

1. E. Bergshoeff, R. Kallosh, T. Ortín, D. Roest, A. Van Proeyen,
   New Formulations of D=10 Supersymmetry and D8-O8 Domain Walls,
   Class.Quant.Grav. 18 (2001) 3359-3382, `(hep-th/0103233)`

2. K. Behrndt, E. Bergshoeff, D. Roest, P. Sundell,
   Massive Dualities in Six Dimensions,
   Class.Quant.Grav. 19 (2002) 2171-2200, `(hep-th/0112071)`

3. E. Bergshoeff, U. Gran, D. Roest,
   Type IIB Seven-brane Solutions from Nine-dimensional Domain Walls,
   Class.Quant.Grav. 19 (2002) 4207-4226, `(hep-th/0203202)`

4. E. Bergshoeff, T. de Wit, U. Gran, R. Linares, D. Roest,
   (Non-)Abelian Gauged Supergravities in Nine Dimensions,
   JHEP 0210 (2002) 061, `(hep-th/0209205)`

5. N. Alonso-Alberca, E. Bergshoeff, U. Gran, R. Linares, T. Ortín, D. Roest,
   Domain Walls of D=8 Gauged Supergravities and their D=11 Origin,
   JHEP 0306 (2003) 038, `(hep-th/0303113)`

6. E. Bergshoeff, U. Gran, R. Linares, M. Nielsen, D. Roest,
   Domain Walls and the Creation of Strings,
   Class.Quant.Grav. 20 (2003) 3465-3482, `(hep-th/0303253)`

7. E. Bergshoeff, U. Gran, R. Linares, M. Nielsen, T. Ortín, D. Roest,
   The Bianchi Classification of Maximal D=8 Gauged Supergravities,
   Class.Quant.Grav. 20 (2003) 3997-4014, `(hep-th/0306179)`

8. E. Bergshoeff, A. Collinucci, U. Gran, M. Nielsen, D. Roest,
   Transient Quintessence from Group Manifold Reductions or how all roads lead to Rome,
   Class.Quant.Grav. 21 (2004) 1947-1969, `(hep-th/0312102)`

9. E. Bergshoeff, M. Nielsen, D. Roest,
The domain walls of gauged maximal supergravities and their M-theory origin,
`(hep-th/0404100)`