M-theory and gauged supergravities

Roest, Diederik

Chapter 1

Introduction

This thesis deals with the construction of different gauged supergravities that can arise as low-energy effective descriptions of string and M-theory. The latter are thought to be consistent theories of quantum gravity, which unify the four different forces and thus merge quantum field theory (QFT) and general relativity (GR). To fully appreciate their emergence and merits, we will first sketch the historical development of particle and high-energy physics during the last century.

The inception of particle physics spanned the larger part of the twentieth century, starting with Thompson's discovery of the electron in 1897. We now know that all elementary particles are either bosons or fermions, depending on whether their spin is integer or half-integer. The fermionic sector contains all matter and consists of three generations, comprising two quarks (which cluster into hadrons) and an electron and a neutrino (known as leptons). The lightest of these three generations makes up for nearly all known matter. The matter particles interact by exchanging bosons: the electromagnetic, weak and strong force are described by the exchange of photons, $W_{\pm}$ and Z_0 intermediate vector bosons and gluons, respectively.

All these particles are described very elegantly by the Standard Model (SM) of particle physics, which was completed in the 1970s. It is a particular quantum field theory (of infinitely many possible ones), unifying the early twentieth century theories of quantum mechanics and special relativity. Such QFTs can be plagued by non-sensible infinities, in which case they are said to be non-renormalisable. It was shown by 't Hooft and Veltman [1] that such problems do not occur for QFTs whose interactions are based on internal gauge symmetries. The particular gauge group of the SM is $SU(3) \times SU(2) \times U(1)$. In such theories, all particles are a priori massless, in contradiction with experiment. This is resolved by the introduction of the Higgs boson, whose interaction with the matter particles gives them their masses.

Note that the Standard Model does not contain the fourth fundamental force, gravity. In many situations (with low enough energy) this is a very reasonable approximation since the strengths of the three SM forces are much stronger than gravity, see figure 1.1.

The experimental confirmation of the SM up to the scale of 10^2 GeV is excellent[1]. However, the Higgs sector has eluded discovery so far; this is one of the primary goals of the new LHC accelerator of CERN, which raises the experimental scale up to $\sim 10^4$ GeV. Also, there are compelling theoretical arguments to consider possible extensions. Firstly, one would not expect a fundamental theory to contain the nineteen parameters that are present in the Standard Model. In addition, the reason why there are three generations of matter particles is not understood. Also, it is difficult to explain the smallness of the Higgs mass (with $m_{\rm H} \lesssim 1$ TeV$/c^2$), which goes under the name of the hierarchy problem. In addition, it turns out that the strengths of the three forces become equal at very high energy (the GUT scale of $\sim 10^{14}$ GeV), see figure 1.1, which suggests a unification of these forces. However, for the Standard Model, the match is not exact. Moreover, perhaps the most troublesome feature from our point of view is that the SM only describes three of the four forces and does not include gravity.

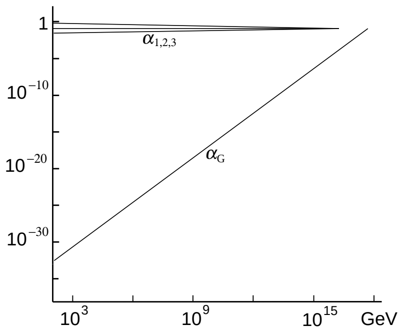

Figure 1.1: *A sketch of the different strengths $a_{1,2,3}$ of the three SM forces and α_G of gravity at different energies. From [3].*

One theoretical improvement of the Standard Model is to introduce a different type of symmetry, called supersymmetry. The introduction of supersymmetry to the Standard Model solves the hierarchy problem and yields perfect agreement of $\alpha_{1,2,3}$ at 10^{16} GeV, see figure 1.1. However, it also introduces many new particles, the supersymmetric partners of the SM particles (the so-called sparticles), which have not been observed. If supersymmetry exists, it must therefore be spontaneously broken, yielding sparticles of higher mass. It is strongly hoped that these will be discovered at the LHC.

In addition to the development of QFT and the SM in particle physics, the previous century has also provided us with the framework to describe gravity: Einstein proposed his theory of general relativity in 1914, which accommodates for gravity by the curvature of

[1]The recently claimed non-zero neutrino masses [2] can be accommodated within the Standard Model without too drastic changes.

space-time. It describes the gravitational attraction at cosmological scales, where gravity is the dominant force since the other three forces are either short-ranged or their effects cancel out (e.g. for electrically neutral cosmological objects). In contrast, the gravitational force has infinite range and cannot cancel out since all matter attracts and never repulses.

Again, the experimental confirmation of GR is overwhelming. Among other tests, it has skillfully explained the anomalous precession of Mercury's perihelion and the deflection of light by massive objects like the sun. From a theoretical point of view, however, GR cannot be the final word. It is only understood classically and so far has resisted all attempts to quantisation: gravity is non-renormalisable. The reason for these difficulties is due to its coupling constant G_N, which is not dimensionless. Rather, the dimensionless combination is $G_N E^2$, which becomes arbitrarily strong at high enough energies (see figure 1.1).

A theory of quantum gravity is only called for in extreme conditions, but these do arise in the universe. For example, GR predicts the generic occurrence of space-time singularities [4] inside black holes. Due to the infinitely strong gravitational field, the negligence of gravity by the SM is then no longer a valid approximation. One can infer the energy scale of this situation: for a particle to qualify as a black hole, its Schwarzschild radius $R_S = 2mG_N/c^2$ must be larger than its Compton wavelength $\lambda_C = h/mc$. The saturating case $R_S = \lambda_C$ defines the Planck mass or energy

$$E_{\text{Pl}} = m_{\text{Pl}}c^2 = \left(\frac{hc}{2G_N}\right)^{1/2} \sim 10^{19}\text{GeV}\,. \tag{1.1}$$

The corresponding length scale is the Planck length $\sim 10^{-33}$ cm (whereas typical hadron sizes are of the order of 10^{-13} cm). Such high energies also occurred in the very early universe (10^{-43} seconds after the Big Bang). Therefore, a quantum theory of gravity is also of great relevance to cosmology.

In addition to its non-renormalisability, another problem of GR is provided by the vacuum energy of the Standard Model. This would yield a cosmological constant of at least 50 orders of magnitude larger than its experimental bound [5]. To reconcile this, one would need to introduce a bare cosmological constant in GR with enormous finetuning.

Despite great differences, there are also many similarities between the Standard Model and general relativity. Of importance here is that both are based on gauge groups: the internal symmetries $SU(3) \times SU(2) \times U(1)$ and the space-time symmetries of general coordinate transformations, respectively. Both transform bosons to bosons and fermions to fermions but do not mix the two species [6]. The previously introduced supersymmetry (susy) is an interesting generalisation hereof: it is neither an internal symmetry nor an ordinary space-time symmetry but can rather be viewed as a space-time symmetry of fermionic coordinates. It relates fermions and bosons [7] via

$$\delta(\text{boson}) = \text{fermion}\,, \qquad \delta(\text{fermion}) = d(\text{boson})\,, \tag{1.2}$$

where d represents the translation operation. No field is invariant under this operation, which generically implies that all fields transform under susy. Thus, susy requires equal amounts of bosonic and fermionic degrees of freedom.

Due to the success of gauge theories like the SM and GR, it seems natural to consider theories based on local supersymmetry. From (1.2) one infers that local susy also introduces general coordinate transformations (with local translations being the infinitesimal form of these) and therefore gravity. Such theories are called supergravities, the first one of which was constructed in 1976 [8, 9]. They were met enthusiastically when it was found that the inclusion of local susy softens the divergences of perturbative quantum gravity; indeed, up to the 1980s it was thought that supergravities could well be renormalisable. Despite the past fervour, it is now generally believed that this is not the case; there seems to be no reason to expect cancellation of certain non-renormalisable infinities.

A more radical idea than supergravity seems to be required for quantum gravity. At the moment, the most promising candidate is string theory. It first surfaced in the 1960s in the context of certain scattering amplitudes of hadrons [10] (where the string was a flux tube between two quarks), but only in 1974 it was interpreted as a theory of quantum gravity [11].

In string theory, elementary particles are replaced by small strings of Planck size. These strings can vibrate in different ways. However, since the strings are so small, we cannot see their extended character and their vibration: the different vibrational modes are interpreted as different elementary particles. Amongst the massless modes is a graviton, which is why string theory includes gravity. In addition, string theory provides a discrete but infinite tower of massive vibration modes. Their mass scale is the only parameter of string theory, which is of the order of the Planck mass.

The presence of the graviton was the first surprise of string theory. In supersymmetric versions of string theory[2], the graviton is (at the massless level) accompanied by the supergravity field content. Indeed, the low-energy approximation of string theory yields supergravity. The status of supergravity within string theory is often compared to Fermi's theory of the four-fermion interaction as an approximation of the weak force. Fermi's theory is correct at low energies, where the fermions seemingly interact at a point; however, its dimensionless coupling strength is $G_F E^2$, yielding infinite strength at high enough energies and causing non-renormalisability. A resolution of this divergence is to smear out the interaction by the exchange of an intermediate vector boson. The situation is much alike in supergravity, where the coupling strength is $G_N E^2$; at high energies, this is replaced by an interaction of two strings, see figure 1.2. Indeed, string theory has finite scattering amplitudes. The massive modes of string theory (or, equivalently, the extended nature of the string) are responsible for the alteration of the ultraviolet behaviour of supergravity.

Another amazing property of string theory is its critical dimension: all superstring theories necessarily live in ten rather than in four dimensions. One could take this as a virtue rather than a vice. It has been known for a long time that higher-dimensional theories have a number of attractive features. Already in the 1920s, Kaluza [12] and Klein [13] tried to unify Einstein's and Maxwell's theories by embedding 4D gravity and electromagnetism in a 5D space-time. Similarly, in string theory, we take the internal six coordinates to be very

[2] Bosonic string theory is plagued by an unphysical particle called the tachyon, see section 2.1.

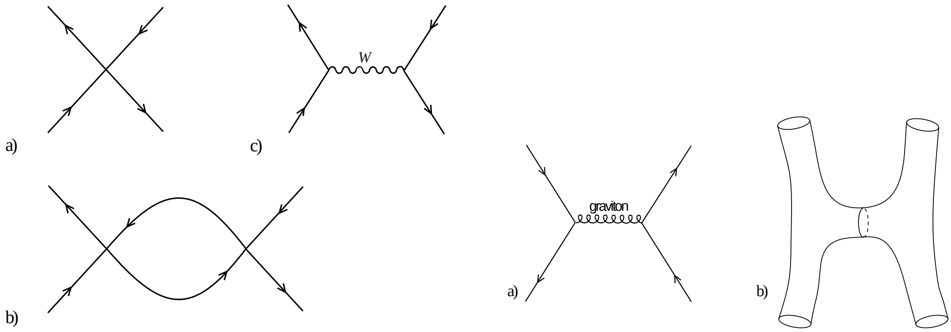

Figure 1.2: *The resolutions of the point-like interactions: on the left,* a) *and* b) *represent the tree-level and one-loop diagrams of Fermi's four-fermion theory, while* c) *is the resolution of* a) *in the weak theory. On the right,* a) *is the tree-level diagram of supergravity, while* b) *is its resolution in string theory. From [3].*

small[3] and therefore invisible to present-day experiments. This procedure is called Kaluza-Klein theory or dimensional reduction. The idea of such small dimensions is perhaps less mystifying than it sounds at first: it also appears in the cosmological Big Bang paradigm for the closed universe, where all spatial dimensions are of Planck size in the very early universe and gradually expand into the present-day 4D universe.

The idea of string theory as quantum gravity was not picked up directly. This changed during (what is now known as) the first superstring revolution in the mid 1980s, in which three appealing features were unveiled. Firstly, it was found that 10D supergravity (with 16 supercharges, see section 3.2) is chiral and free of anomalies provided it is coupled to a Yang-Mills sector with gauge groups $SO(32)$ or $E_8 \times E_8$ [15]. Secondly, heterotic string theories with exactly these gauge groups were constructed [16], whose low-energy limit reduces to the corresponding supergravity. Thirdly, a class of compactification models (so-called Calabi-Yau manifolds) were found to yield a 4D effective description that was reasonably close to the SM [17].

After this period, one was left with five different superstring theories: four of closed strings (the two type II and heterotic theories) and one of open and closed strings (the type I theory). The five-fold possibility for the supposedly ultimate theory was considered an embarrassment of riches. In addition, only perturbative descriptions of these theories were available, corresponding to the first quantisation[4] of the particle (where its motion is treated quantum-mechanically in a classical background of force fields). In the perturbative expansion, the sum of Feynman diagrams is replaced by a sum of Riemann surfaces due to the extended nature of the string, see figure 1.2. For this reason, it is impossible to go to finite

[3]An alternative idea is the brane-world scenario [14], in which we are thought to be living on a hypersurface in higher-dimensional space-time. We will mainly consider traditional compactification, however.

[4]The string analogue of second quantisation, leading to string field theory instead of QFT, is not well understood.

coupling.

This situation changed due to the discovery of string dualities, culminating in the second superstring revolution of the mid 1990s. It was found that the different string theories are related to each other, for different values of certain parameters. For example, the strong coupling limit of one theory yields another theory at weak coupling (called S-duality). In addition, string theories on different backgrounds were found to be equivalent (called T-duality). The upshot was that the five string theories could be unified in a single eleven-dimensional theory, which was named M-theory [18]. The different string theories are thus understood as perturbative expansions in different limits of the parameter space of M-theory. This appreciation is known as U-duality [19] and has spectacularly changed our understanding of string theory and the distinction between perturbative and non-perturbative effects.

Another important development is the AdS/CFT correspondence, which was proposed in 1997 by Maldacena [20]. It explicitly realises earlier ideas about certain limits of gauge theories [21] and holography [22,23], which connects quantum gravity to a QFT in one dimension less. The correspondence basically relates a string theory in a particular background (IIB on $\mathrm{AdS}_5 \times \mathrm{S}^5$) to a particular and supersymmetric QFT ($N = 4$ SYM in $D = 4$). However, the link contains a non-perturbative duality: where the quantum field theory is well-behaved we have little control over the string theory and vice versa. Due to the large amount of supersymmetry one can still extract information from the non-perturbative domain and thus check the conjectured correspondence. The hope is to learn something about non-perturbative effects in QFT, such as quark confinement, from this approach.

Of central importance for the different dualities are certain ingredients of string theory called D-branes [24], which are extended objects of p spatial dimensions[5]. These branes are required to fill out the multiplets of string dualities, e.g. the fundamental string is mapped onto the D-brane with $p = 1$ under S-duality. In addition, different descriptions of D-branes play a crucial role in the string theory calculation [25] of the Bekenstein-Hawking entropy of a black hole [26, 27]

$$S = \frac{A}{4G_N\hbar}\,, \tag{1.3}$$

which relates the area A of the event horizon to the black hole entropy S. The latter is a (classically) non-decreasing quantity, very much like the thermodynamic entropy. The AdS/CFT correspondence can be seen as an extension of this idea for the D-brane with $p = 3$.

In addition, it is hoped that string theory will solve the information paradox of black holes [28]. In a semi-classical approximation, black holes emit Hawking radiation [29] of a black-body spectrum. Therefore, different infalling matter seems to end up as the same radiation: information has been lost, which is forbidden in quantum theory. A possible resolution would lie in certain correlations of the outcoming radiation [30]. This requires a subtle non-locality, which string theory seems to possess. A rigorous proof, requiring to go beyond the semi-classical approximation, is still lacking.

[5] The nomenclature is particle, string, membrane, . . ., p-brane for objects of spatial dimension $0, 1, 2, \ldots, p$.

Thus we have gained a better understanding of perturbative and non-perturbative string theory and their unification in M-theory. However, there are still many interesting open issues. An outstanding obstacle to make contact with our 4D world is the problem of vacuum selection: there are infinitely many ways to compactify the internal directions, yielding different four-dimensional descriptions. The expectation is that string theory itself will prefer one of these vacua, but this seems impossible to determine from the perturbative theory. Another avenue to be explored is cosmology: due to the enormous energies in the remote past, this is a laboratory to directly test string theory predictions. String theory should shed light on inflation, the expansion of the universe and, if recent astronomical observations [31, 32] are right, the acceleration of this expansion. Recently, the cosmological implications of string theory have gained an enormous interest.

The low-energy limit of string theory, supergravity, remains an important tool to study the different phenomena in string theory. Many features of string and M-theory are also present in its supergravity limit, such as D-branes and U-duality, and therefore it is interesting to study this effective description. In particular, one can extract effective lower-dimensional descriptions by considering string or M-theory on a compact internal manifold, which is taken to be very small (i.e. dimensional reduction). Different reductions give rise to different lower-dimensional supergravities. Thus it is clearly very desirable to have a proper understanding of the different reduction procedures and their resulting lower-dimensional descriptions. In particular, we will be interested in gauged supergravities as the lower-dimensional theories.

Ungauged supergravities have a global symmetry group which is called the U-duality group (and indeed is a consequence of the U-duality of M-theory). In gauged supergravities a subgroup of this global group is turned into a gauge symmetry by the introduction of mass parameters. The combination of a gauge group and local supersymmetry implies the appearance of a scalar potential, which is quadratic in the mass parameters.

It is the scalar potential which makes gauged supergravities interesting since it generically breaks the Minkowski vacuum to solutions like (Anti-)de Sitter space-time (AdS or dS), domain walls or cosmological solutions. These play important roles in the AdS/CFT correspondence[6] and its generalisation, the DW/QFT correspondence [33, 34], brane-world scenarios [14, 35] and accelerating cosmologies [36, 37]. From various points of view, it would therefore be highly advantageous to have a classification of gauged supergravities in the different dimensions.

We are only interested, however, in gauged supergravities with a higher-dimensional origin in string or M-theory: the lower-dimensional theory must be obtainable via dimensional reduction. Our approach consists of the dimensional reduction of eleven- and ten-dimensional maximal supergravities and the investigation of the resulting gauged supergravity. We have applied two reduction methods, both preserving supersymmetry: reduction with a twist and reduction on a group manifold. In the twisted reduction one employs a global symmetry of the parent theory to induce a gauging of one of its subgroups in the lower dimension. In the

[6]Indeed, the effective description of string theory on the particular background $\mathrm{AdS}_5 \times \mathrm{S}^5$ is a gauged supergravity: the $N = 4$ $SO(5)$ theory in $D = 5$, see also section 5.5.

group manifold reduction one reduces over a number of isometries that do not commute and form the algebra of a Lie group. This results in the gauging of this group in the lower dimension. The consistency of both reductions is guaranteed by symmetry, as proven by Scherk and Schwarz in 1979 [38, 39] and as opposed to reduction on a coset manifold, whose consistency remains to be understood in generality.

The outline of this thesis is as follows. In chapter 2 string and M-theory are introduced. Chapter 3 is devoted to their low-energy limits: we discuss the different supergravity theories, their solutions and their relations. Next, in chapter 4 we describe a number of techniques to generate lower-dimensional gauged supergravities. In particular, reduction with a twist, over a group manifold and over a coset manifold are explained in sections 4.3 to 4.5, respectively. One finds their application in chapter 5, where different gauged theories are constructed. By applying reductions with a twist and over a group manifold, we have generated a number of mass parameters in nine and eight dimensions, as can be found in sections 5.3 and 5.4. We also discuss a family of mass parameters in lower dimensions, which are obtainable only by reduction over a coset or other manifolds in section 5.5. Finally, in chapter 6 we construct and discuss half-supersymmetric domain wall solutions for the different gauged supergravities. The D8-brane is the topic of section 6.1, while lower-dimensional domain walls and their relation to higher-dimensional branes are disclosed in section 6.2. The particular cases of 9D and 8D are emphasised in sections 6.3 and 6.4. To end with, we discuss intersections of domain walls and strings in section 6.5.