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Measuring industry productivity and cross-country convergence

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ABSTRACT

This paper introduces a new method for simultaneously comparing industry productivity across countries and over time. The new method is similar to the method for making multilateral comparisons of Caves, Christensen and Diewert (1982b) but their method can only compare gross outputs across production units and not compare real value added of production units across time and space. The present paper uses the translog GDP methodology for measuring productivity levels across time that was pioneered by Diewert and Morrison (1986) and adapts it to the multilateral context. The new method is illustrated using an industry level data set and shows that productivity dispersion across 38 countries between 1995 and 2011 has decreased faster in the traded sector than in the non-traded sector. In both sectors, there is little evidence of decreasing distance to the productivity frontier.

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1. Introduction

Determining whether and how fast productivity is converging across countries is a question of enduring interest, and for good reasons.[1] Most importantly, it tells us if lower-income countries are catching up to high-income countries and, if so, how fast. Furthermore, it can help shed light on the circumstances under which countries would benefit from an 'advantage of backwardness', which is helpful information for designing development policies.[2]

A sectoral perspective on convergence is particularly valuable as it can provide clearer policy targets. For instance, if – as found

by Rodrik (2013) – convergence in manufacturing is unconditional, i.e. it occurs regardless of country circumstances, it could be helpful to gear policies towards building and strengthening this sector. Alternatively, if the finding for OECD countries by Bernard and Jones (1996) of convergence in services but not in manufacturing would hold more broadly, the argument for support of the manufacturing sector would be much weaker.

Despite the interest in the results, the methods used in compiling the productivity measures used in these studies are not well-suited for analyzing productivity convergence. Obviously, convergence is a topic that requires a simultaneous comparison of productivity levels across countries *and* over time.[3] Instead, the typical analysis uses measures that are comparable across

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[1] For a recent study and overview, see Barro (2012).
[2] See e.g. Aghion et al. (2014).

[3] See Hill (2004) and Diewert and Fox (2015) for more general discussions of consistency of price indexes across countries and over time. See also Lichtenberg (1994) on σ-convergence, a more direct and robust concept than β-convergence.

countries only in a single year or national growth rates that are comparable only over time.

A major contribution of this paper is to propose a new method for measuring industry productivity levels that are comparable across both countries and over time. The proposed approach resolves the comparability problem through an extension of the work of Caves et al. (1982b, CCD henceforth), who showed how to compare productivity across countries at a point in time. Their approach was based on the use of distance functions to construct output and input aggregates. Unfortunately, their approach cannot be used to compare real value added, since distance functions are, in general, not well-defined when accounting for intermediate inputs. Thus, we will use the GDP function, or value added function, approach pioneered by Diewert and Morrison (1986) as a basic building block in our new approach to replace the distance function approach used by CCD.

Section 2 shows how outputs, inputs and productivity levels for an industry (or sector of an economy) can be compared across countries and time in a consistent manner. In Section 3, we show how the analysis of Section 2 can be extended to construct measures of "world" productivity at time t, Γ_t, that are consistent across time and space. We also define the relative efficiency of the industry (or production unit) in country k at time t, Γ_{kt}, with the most efficient production unit across all countries and time periods up to time t, $\Gamma_{t,max}$.

Section 4 defines two measures of industry convergence. The first measure, E_t, is the ratio of actual world productivity, Γ_t, to the maximum possible value of world productivity $\Gamma_{t,max}$, all at time t. If all countries produce at the maximum possible level of productivity at time t, then E_t will equal unity. Thus if E_t increases over time, this indicates a movement towards productivity convergence. The second measure of convergence at time t, σ_t, is an input-weighted average of the dispersion of the country productivity levels relative to the world average productivity level, both at time t. If all country productivity levels are the same in period t, then σ_t will equal 0, so if σ_t declines over time, productivity levels across countries are converging towards the mean level of productivity.

Section 5 gives a brief description of the data used in this study. The data set covers 38 economies across two sectors of each economy for the period 1995–2011. The two sectors are the *traded sector* and the *non-traded sector*. A third sector is the *market sector* for each economy, which is an aggregate of the traded and non-traded sectors. This setting is of interest as these 38 economies include most advanced economies as well as major emerging economies, like China and India. Moreover, the period since 1995 has seen rapid growth across many of these emerging economies, raising the question whether aggregate productivity levels converged and, if so, which sectors contributed most. There is also interest in determining whether the global financial crisis affected convergence. The data are constructed mainly using the World Input–Output Database[4]; see Section 5 and Inklaar and Diewert (2015) for additional details.[5]

In Section 6, we show that convergence of productivity levels towards the mean has indeed been strong over this period, with the weighted standard deviation of market sector productivity levels (the dispersion measure) decreasing by 23% over the sample

period. Based on the literature on the Harrod–Balassa–Samuelson (HBS) model, productivity dispersion should be larger and productivity growth should be faster in the traded sector than in the non-traded sector.[6] We confirm that dispersion in the traded sector is about 50% greater than in the aggregate market economy and a new finding is that aggregate convergence is almost entirely due to convergence in the traded sector of the economy.[7] However, we find that there is no evidence that countries are converging towards the productivity frontier over our sample period. We also find that the global financial crisis did not decrease the rate of growth of the productivity frontier but it did decrease realized "world" productivity growth substantially from an average of 1.1% per year over the years 1995–2007 to 0.6% per year over the years 2007–2011.[8] Section 7 concludes.

2. An economic approach to the measurement of productivity over time and space

To analyze the degree of convergence towards the productivity frontier, it is necessary to measure output and input levels that are comparable across countries and over time. It is also useful to have measures that are invariant to the choice of a reference point—i.e. a single country and year that acts as a basis for comparison for all other countries and years. Finally, it is useful to have a methodology that is based on an economic approach to production theory. Such an approach was developed by CCD but their approach has a significant limitation. Their approach relies on the distance function methodology for aggregating inputs and outputs that can be traced back to Malmquist (1953) and further developed by Caves et al. (1982a). The problem is that this distance function methodology does not allow us to compare real GDP or real value added across countries as that methodology requires a strict separation of outputs and inputs. Net output aggregates based on distance function techniques do not work if the output aggregate includes intermediate inputs or imports. In this section, we show how this problem can be addressed in a production theory framework by using the methodology that was developed by Diewert and Morrison (1986).[9] Our suggested methodology also draws on the techniques used by CCD.

We give a brief explanation of the methodology developed by Diewert and Morrison (1986) for a comparison of real outputs, inputs and productivity levels across two time periods or two production units in the same industry.[10] Consider a set of production units that produce a vector of M net outputs,[11] $y \equiv [y_1, \ldots, y_M]$, using a nonnegative vector of N primary inputs, $x \equiv [x_i, \ldots, x_N]$. Let the feasible set of net outputs and primary inputs for production unit i be denoted by S^i for $i = 1, \ldots, I$. It is assumed

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[4] See Timmer et al. (2015) for an overview of this database.

[5] We draw on the World Bank's PPPs for 1996, 2005 and 2011 as a starting point for developing PPPs for our industry data. A full set of industry PPPs covering the years 1995–2011 is required for our purposes so the World Bank PPPs for the three benchmark years are interpolated using the method that makes use of national growth rates that was suggested by Diewert and Fox (2015). Our full set of PPPs does not make use of country exchange rates.

[6] See Asea and Corden (1994) for an overview of the model, Hsieh and Klenow (2007) and Herrendorf and Valentinyi (2012) on productivity dispersion and De Gregorio et al. (1994) and Ricci et al. (2013) on relative productivity growth. We find that realized "world" productivity growth over 1995–2011 was 1.3% per year for the traded sector and 0.6% per year for the nontraded sector.

[7] When we decomposed the traded sector into additional sectors, we found that the manufacturing sector is the main contributor to convergence, confirming a result of Rodrik (2013).

[8] The data used in this paper are listed in Inklaar and Diewert (2015).

[9] A similar methodology was independently developed by Kohli (1990). Shiu (2003) applied the Diewert/Morrison/Kohli methodology in a multilateral context. The difference is that we use the *averaging approach*, pioneered by Gini (1931), to obtaining base-country invariant multilateral comparisons, whereas Shiu (2003) uses *the similarity linking approach*, pioneered by Hill (1999).

[10] We interpret the "same industry" to comprise the production units that produce the same outputs using the same set of inputs.

[11] If $y_m > 0$, then net output m is an output and y_m denotes the production of this commodity; if $y_m < 0$, then net output m is an intermediate input and y_m denotes the negative of the amount of this input that is used by the production unit.

that each S^i is a closed convex cone in R^{M+N} so that production is subject to constant returns to scale for each production unit.[12] For each strictly positive net output price vector $p \equiv [p_1, \ldots, p_M] \gg 0_M$ and each strictly positive primary input vector $x \gg 0_N$, define the *value added function* or *GDP function* for production unit k, $g^i(p, x)$, as follows:

$$g^i(p, x) \equiv \max_y \left\{ \sum_{m=1}^M p_m y_m : (y, x) \in S^i \right\}; \quad i = 1, \ldots, I. \quad (1)$$

These value added functions g^i provide a dual representation of the technology sets S^i under our assumptions on the technology sets.[13] Finally, Diewert and Morrison assumespecific functional forms for the value added functions g^i defined by (1): they assumed that each value added function has a translog functional form with some restrictions on the parameters that define these functional forms.[14] Armed with these assumptions, Diewert and Morrison (1986, 661–665) were able to construct output, input and productivity levels between any two production units using the economic approach to index number theory and Theil (1967, 136–137) output price and input quantity indexes.[15]

We can now address our specific problem, which is to develop a methodology for constructing aggregate output, input and productivity levels for a panel data set on comparable production units in different countries. We assume that the data set is organized in four sets of basic data, each for $k = 1, \ldots, K$ countries and $t = 1, \ldots, T$ years.[16] (i) The *value of net output* m in country k in domestic currency during period t is v_{ktm} for $m = 1, \ldots, M$. Thus there are M net output commodities and if $v_{ktm} < 0$, commodity m is used as an input by country k in period t. (ii) The *price or purchasing power parity (PPP, in domestic currency)* for net output m in country k for time period t is $p_{ktm} > 0$. These output prices or PPPs are prices that use *the same unit of measurement* for the same commodity across countries. (iii) The *value of primary input* n in country k in domestic currency during period t is $V_{ktn} > 0$ for $n = 1, \ldots, N$. (iv) The *price or PPP (in domestic currency)* for primary input n in country k for time period t is $w_{ktn} > 0$ for $n = 1, \ldots, N$. These input prices or PPPs are prices that use *the same unit of measurement* for the same input across countries.

Given these primary data sets, we can construct implicit output and input quantities for each country and each time period. Define the *implicit quantity* (or volume) y_{ktm} of net output m in country k and time period t as $y_{ktm} \equiv v_{ktm}/p_{ktm}$ for $m = 1, \ldots, M$; $k = 1, \ldots, K$ and $t = 1, \ldots, T$. Define the *implicit quantity* (or volume) x_{ktn} of primary input n in country k and time period t as $x_{ktn} \equiv V_{ktn}/w_{ktn}$ for $n = 1, \ldots, N; k = 1, \ldots, K$ and $t = 1, \ldots, T$. Define the *total value added* in domestic currency for country k in

period t, v_{kt}, and the *total value of primary inputs* for country k in period t, V_{kt}, by summing over net outputs and inputs[17]:

$$v_{kt} \equiv \sum_{m=1}^M v_{ktm};$$

$$V_{kt} \equiv \sum_{n=1}^N V_{ktn}; \quad k = 1, \ldots, K; \ t = 1, \ldots, T. \quad (2)$$

In what follows, we will make use of the *value added output shares* s_{ktm} and the *primary input cost shares* S_{ktn} defined as:

$$s_{ktm} \equiv v_{ktm}/v_{kt}; \quad m = 1, \ldots, M;$$
$$k = 1, \ldots, K; \ t = 1, \ldots, T;$$
$$S_{ktm} \equiv V_{ktn}/V_{kt}; \quad n = 1, \ldots, M;$$
$$k = 1, \ldots, K; \ t = 1, \ldots, T. \quad (4)$$

Define the (strictly positive) *net output price vector for country k in period t* as $p_{kt} \equiv [p_{kt1}, \ldots, p_{ktM}]$ and the corresponding *net output quantity vector* as $y_{kt} \equiv [y_{kt1}, \ldots, y_{ktM}]$ for $k = 1, \ldots, K$ and $t = 1, \ldots, T$. Then under our assumptions on technology and behavior, Diewert and Morrison (1986, 665) showed that the aggregate price of real value added in country k in period t relative to the aggregate price of real value added in country j in period s, $P_{kt/js}$, is equal to the Törnqvist–Theil output price index $P_T(p_{js}, p_{kt}, y_{js}, y_{kt})$; i.e., we have[18]:

$$P_{kt/js} \equiv P_T(p_{js}, p_{kt}, y_{js}, y_{kt})$$
$$\equiv \exp \left[\sum_{m=1}^M \frac{1}{2} (s_{jsm} + s_{ktm}) \ln (p_{ktm}/p_{jsm}) \right];$$
$$k, j = 1, \ldots, K; \ t, s = 1, \ldots, T. \quad (5)$$

Diewert and Morrison (1986, 665) also indicated that the corresponding implicit quantity index, $Y_{kt/js}$, provides a good estimator of the ratio of real value added in country k in period t relative to the real value added of country j in period s; i.e., we have:

$$Y_{kt/js} \equiv [v_{kt}/v_{js}]/P_T(p_{js}, p_{kt}, y_{js}, y_{kt});$$
$$k, j = 1, \ldots, K; \ t, s = 1, \ldots, T. \quad (6)$$

Obviously, we could pick a country and a time period (say period 1 and country 1) and treat this production unit as a numeraire unit and measure the GDP output prices and quantities of other observations relative to this numeraire unit. This would lead to a sequence of aggregate prices, $P_{kt/11}$, and quantities, $Y_{kt/11}$, for $k = 1, \ldots, K$ and $t = 1, \ldots, T$. However, we could just as easily pick country 2 in period 1 as the numeraire country and this would lead to the sequence of country PPPs and real value added of $P_{kt/21}$ and $Y_{kt/21}$. Unfortunately, $P_{kt/21}$ will not, in general, be proportional to $P_{kt/11}$ and $Y_{kt/21}$ will not be proportional to $Y_{kt/11}$; i.e., the results will depend on the choice of the numeraire country. CCD solved this numeraire dependence problem by averaging over all possible choices of the numeraire observation.[19] We will

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[12] There are some additional regularity conditions on these production possibilities sets that are listed in Diewert and Morrison (1986) and in Diewert (1973).

[13] Note that working with the dual functions implies that the production units are competitive price takers, a common (but debatable) assumption in the economic approach to index number theory.

[14] The logarithm of g^k is assumed to have the following functional form: $\ln g^i(p, x) \equiv \alpha_0^i + \sum_{m=1}^M \alpha_m^i \ln p_m + \frac{1}{2} \sum_{m=1}^M \sum_{k=1}^M \alpha_{mk}^i \ln p_m \ln p_k + \sum_{n=1}^N \beta_n^i \ln x_n + \frac{1}{2} \sum_{n=1}^N \sum_{j=1}^N \beta_{nj}^i \ln x_n \ln x_j + \sum_{m=1}^M \sum_{j=1}^N \gamma_{mn}^i \ln p_m \ln x_n$. Parameter restrictions to impose constant returns to scale are in Diewert (1974, 139). The α_0^i, α_m^i and β_n^i parameters depend on i and can thus vary across the I production units. The translog functional form for a single output is due to Christensen et al. (1971); Diewert (1974, 139) generalized this functional form to the value added function context.

[15] We will adapt their bilateral results to the present multilateral context; see Eqs. (5), (6), (14) and (21) for the Diewert and Morrison bilateral results that we will use here.

[16] Note that the four data sets do not involve exchange rates!

[17] We assume that $v_{kt} = V_{kt}$ for $k = 1, \ldots, K$ and $t = 1, \ldots, T$ so that our data are consistent with the constant returns to scale assumption required for implementing the Diewert–Morrison methodology.

[18] $P_{kt/js}$ can also be interpreted as the ratio of GDP PPPs; i.e., $P_{kt/js}$ is equal to the GDP PPP for country k in period t divided by the GDP PPP for country j in period s.

[19] This averaging strategy was used by Gini (1931), Elteö and Köves (1964) and Szulc (1964) in the literature on multilateral international comparisons at a point in time. Their method, using the Fisher index for the bilateral index number formula, is known as the EKS or GEKS method.

follow their strategy but using the Diewert–Morrison PPPs as the basic bilateral building blocks rather than the CCD bilateral choice of index number formula which did not allow for negative net outputs. Thus define the *geometric mean of all the PPP parities* for country k in time period t relative to all possible choices j, s of the base country, P_{kt*} as follows:

$$P_{kt*} \equiv \left[\prod_{j=1}^{K} \prod_{s=1}^{T} P_{kt/js} \right]^{\frac{1}{KT}}; \quad k = 1, \ldots, K; \ t = 1, \ldots, T. \quad (7)$$

It turns out that the base invariant PPPs, P_{kt*}, can be written in a very simple form. First define the *M sample average value added shares* $s_{..m}$ and the *sample arithmetic average of the log output prices* $\ln p_{..m}$ over all countries and all time periods as follows:

$$s_{..m} \equiv \left(\frac{1}{KT} \right) \sum_{k=1}^{K} \sum_{t=1}^{T} s_{ktm};$$

$$\ln p_{..m} \equiv \left(\frac{1}{KT} \right) \sum_{k=1}^{K} \sum_{t=1}^{T} \ln p_{ktm}; \quad m = 1, \ldots, M. \quad (8)$$

Now take logarithms of both sides of (7) and use definitions (5) in order to obtain the following expression for $\ln P_{kt*}$:

$$\ln P_{kt*} = \left(\frac{1}{KT} \right) \left[\sum_{j=1}^{K} \sum_{s=1}^{T} \sum_{m=1}^{M} \frac{1}{2} \left(s_{jsm} + s_{ktm} \right) \ln \left(p_{ktm} / p_{jsm} \right) \right]$$

$$= \ln P_{kt**} + \alpha \quad (9)$$

where α and the logarithm of an alternative PPP for country k in period t, $\ln P_{kt**}$, are defined as follows:

$$\alpha \equiv \sum_{m=1}^{M} \frac{1}{2} s_{..m} \ln p_{..m} - \left(\frac{1}{KT} \right) \sum_{j=1}^{K} \sum_{s=1}^{T} \sum_{m=1}^{M} \frac{1}{2} s_{jsm} \ln p_{jsm}; \quad (10)$$

$$\ln P_{kt**} \equiv \sum_{m=1}^{M} \frac{1}{2} \left(s_{..m} + s_{ktm} \right) \ln \left(p_{ktm} / p_{..m} \right). \quad (11)$$

Note that α does not depend on k or t; i.e., it is a constant. Note further that $\ln P_{kt**}$ is the Törnqvist–Theil output price index for country k in period t relative to an artificial "world" country that has net output shares equal to the sample average net output shares $s_{..m}$ and has log prices equal to the sample average log prices, $\ln p_{..m}$, for $m = 1, \ldots, M$.[20] It is much easier numerically to compute $\ln P_{kt**}$ defined by (11) (a single summation) than it is to compute $\ln P_{kt*}$ defined by the first equation in (9) (a triple summation).

It is usually convenient to pick out the country with the largest economy (say country 1) in period 1 and form a set of normalized aggregate output PPPs that compare the PPPs defined by (9) or (11) to the PPP for country 1 in period 1. Thus we define our final set of *value added output deflators*, P_{kt}, as follows:

$$P_{kt} \equiv P_{kt*} / P_{11*} = P_{kt**} / P_{11**}; \quad k = 1, \ldots, K; \ t = 1, \ldots, T \quad (12)$$

where we used the fact that $P_{kt*} = e^{\alpha} P_{kt**}$ for $k = 1, \ldots, K$ and $t = 1, \ldots, T$ to derive the second set of equations in (12). Thus our final set of net output PPPs is the same whether we use the country PPPs defined by (9) or by (11).

Our final set of *real value added estimates* Y_{kt} that are comparable across time and space is defined by deflating each country's nominal value added by the PPPs defined by (12)[21]:

$$Y_{kt} \equiv [v_{kt} / P_{kt}]; \quad k = 1, \ldots, K; \ t = 1, \ldots, T. \quad (13)$$

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[20] Our decomposition (9) is analogous to a similar decomposition obtained by CCD (p. 78).

[21] Note that Eqs. (12) and (13) imply that $P_{11} = 1$ and $Y_{11} = v_{11}$.

We next turn our attention to the problems associated with measuring real primary input across countries. Define the (strictly positive) *input quantity vector for country k in period t* as $x_{kt} \equiv [x_{kt1}, \ldots, x_{ktN}]$ and the corresponding *input price vector* as $w_{kt} \equiv [w_{kt1}, \ldots, w_{ktN}]$ for $k = 1, \ldots, K$ and $t = 1, \ldots, T$. Then under our assumptions on technology and behavior, Diewert and Morrison (1986, 665) showed that the aggregate quantity of primary input in country k in period t relative to the aggregate quantity of primary input in country j in period s, $X_{kt/js}$, is equal to the Törnqvist–Theil input quantity index $Q_T \left(w_{js}, w_{kt}, x_{js}, x_{kt} \right)$; i.e., we have:

$$X_{kt/js} \equiv Q_T \left(w_{js}, w_{kt}, x_{js}, x_{kt} \right)$$

$$\equiv \exp \left[\sum_{n=1}^{N} \frac{1}{2} \left(S_{jsn} + S_{ktn} \right) \ln \left(x_{ktn} / x_{jsn} \right) \right];$$

$$k, j = 1, \ldots, K; \ t, s = 1, \ldots, T. \quad (14)$$

As was the case with the construction of output aggregates, there are KT different choices of a base country and so we follow the same strategy of taking a geometric average of these alternative choices of a base observation. Thus define X_{kt*} as follows:

$$X_{kt*} \equiv \left[\prod_{j=1}^{K} \prod_{s=1}^{T} X_{kt/js} \right]^{\frac{1}{KT}}; \quad k = 1, \ldots, K; \ t = 1, \ldots, T. \quad (15)$$

The base-invariant quantity indexes, X_{kt*}, can also be written more simply. Define the *N sample average input cost shares* $S_{..n}$ and the *sample arithmetic average of the log input quantities* $\ln x_{..n}$ over all countries and all time periods as:

$$S_{..n} \equiv \left(\frac{1}{KT} \right) \sum_{k=1}^{K} \sum_{t=1}^{T} S_{ktn};$$

$$\ln x_{..n} \equiv \left(\frac{1}{KT} \right) \sum_{k=1}^{K} \sum_{t=1}^{T} \ln x_{ktn}; \quad n = 1, \ldots, N. \quad (16)$$

Now take logarithms of both sides of (15) and use definitions (14) in order to obtain the following expression for $\ln X_{kt*}$:

$$\ln X_{kt*} = \left(\frac{1}{KT} \right) \left[\sum_{s=1}^{K} \sum_{s=1}^{T} \sum_{n=1}^{N} \frac{1}{2} \left(S_{jsn} + S_{ktn} \right) \ln \left(x_{ktn} / x_{jsn} \right) \right]$$

$$= \ln X_{kt**} + \beta \quad (17)$$

where β and the logarithm of an alternative input index for country k in period t, X_{kt**}, are defined as follows:

$$\beta \equiv \sum_{n=1}^{N} \frac{1}{2} S_{..n} \ln x_{..n} - \left(\frac{1}{KT} \right) \sum_{j=1}^{K} \sum_{s=1}^{T} \sum_{n=1}^{N} \frac{1}{2} s_{jsn} \ln x_{jsn}; \quad (18)$$

$$\ln X_{kt**} \equiv \sum_{n=1}^{N} \frac{1}{2} \left(S_{..n} + S_{ktn} \right) \ln \left(x_{ktn} / x_{..n} \right). \quad (19)$$

β is a constant since it does not depend on k or t. Note further that X_{kt**} is the Törnqvist–Theil input quantity index for country k in period t relative to an artificial "world" country that has primary input cost shares equal to the sample average primary input cost shares $S_{..n}$ and has log input quantities equal to the sample average log input quantities, $\ln x_{..n}$, for $n = 1, \ldots, N$. As above, it is much easier numerically to compute $\ln X_{kt**}$ defined by (19) (a single summation) than it is to compute $\ln X_{kt*}$ defined by the first equation in (17) (a triple summation).

We follow the same convention as on the output side to define a set of *input quantity aggregates*, X_{kt} relative to country 1 in year 1

as[22]:

$$X_{kt} \equiv V_{11}X_{kt*}/X_{11*} = V_{11}X_{kt**}/X_{11**};$$
$$k = 1, \ldots, K; \quad t = 1, \ldots, T \qquad (20)$$

where we used the fact that $X_{kt*} = e^{\beta}X_{kt**}$ for $k = 1, \ldots, K$ and $t = 1, \ldots, T$ to derive the second set of equations in (20). Thus our final set of primary input aggregates is the same whether we use the country input indexes defined by (17) or by (19).

Diewert and Morrison (1986, 663) showed that under their assumptions, a theoretical productivity index[23] between the production unit k at period t relative to the production unit j at period s, $\Gamma_{kt/js}$, was equal to the output ratio $Y_{kt/js}$ defined by (6) divided by the input ratio $X_{kt/js}$ defined by (14); i.e., we have:

$$\Gamma_{kt/js} \equiv Y_{kt/js}/X_{kt/js}; \quad k, j = 1, \ldots, K; \quad t, s = 1, \ldots, T. \qquad (21)$$

As before, the bilateral TFP indexes defined by (21) are not transitive and so they are made transitive by defining the ratio of the productivity of country k in period t to the geometric mean of all country TFP levels over all years, Γ_{kt*}, as follows[24]:

$$\Gamma_{kt*} \equiv \left[\prod_{j=1}^{K} \prod_{s=1}^{T} \Gamma_{kt/js} \right]^{\frac{1}{KT}} = Y_{kt*}/X_{kt*};$$
$$k = 1, \ldots, K; \quad t = 1, \ldots, T, \qquad (22)$$

where the last expression in (22) follows from definitions (6), (7), (13), (14) and (15). The Γ_{kt*} are analogues to the *translog multilateral productivity indexes* defined by CCD (p. 81). Again, for ease of interpretation, we replace the productivity levels defined by (22) by the following *normalized productivity levels* Γ_{kt}:

$$\Gamma_{kt} \equiv [Y_{kt*}/X_{kt*}] / [Y_{11*}/X_{11*}] = Y_{kt}/X_{kt};$$
$$k = 1, \ldots, K; \quad t = 1, \ldots, T, \qquad (23)$$

where Y_{kt} is defined by (13) and X_{kt} is defined by (20). Thus the KT normalized TFP levels for production unit k in time period t, Γ_{kt}, defined by (23) is equal to the corresponding normalized output level Y_{kt} divided by the corresponding normalized input level X_{kt}. The Y_{kt} are comparable across time and space as are the X_{kt}.

This completes the exposition of our methodology for making cross-country comparisons of output, input and productivity using the economic approach to index number theory when the output aggregate contains intermediate inputs. In general, these productivity levels are likely to differ across countries, i.e. not all production units operate on the world production frontier. In order to allow for the effects of inefficiency in a pragmatic way, we will use the output and input aggregates that we defined in this section to form estimates of "world" productivity and to estimate the "world" production frontier at each point in time in the next section.

3. The measurement of world productivity and country efficiency levels

We now consider how to measure the level of "world" productivity[25] in each time period t. We define the world productivity level at time period t as the ratio of world output

to world input, thus requiring a definition of world output and input. The multilateral output indexes, Y_{kt} defined by (13), are comparable across countries and time periods.[26] Hence, it is meaningful to add them up to obtain aggregate measures of real output. Thus define *world output* at time t, Y_t, as follows:

$$Y_t \equiv \sum_{k=1}^{K} Y_{kt}; \quad t = 1, \ldots, T, \qquad (24)$$

where the Y_{kt} are defined by (13). In a similar fashion, *world input* for time period t, X_t, is defined as the sum of the country k multilateral input aggregates X_{kt} defined by (20) for each time period t:

$$X_t \equiv \sum_{k=1}^{K} X_{kt}; \quad t = 1, \ldots, T. \qquad (25)$$

Define the country k share of world real input during period t, ω_{kt}, as:

$$\omega_{kt} \equiv X_{kt} \Big/ \sum_{j=1}^{K} X_{jt}; \quad k = 1, \ldots, K; \quad t = 1, \ldots, T. \qquad (26)$$

Finally, the level of *world productivity* at time t, Γ_t, is defined as the ratio of world output to input at time t. Using definitions (23)–(26), it is straightforward to show that Γ_t is equal to an input-share-weighted average of the multilateral productivity indexes Γ_{kt} over all countries k for time period t:

$$\Gamma_t \equiv Y_t/X_t = \sum_{k=1}^{K} \omega_{kt} \Gamma_{kt}; \quad t = 1, \ldots, T. \qquad (27)$$

It is useful to define the *efficiency* of each country relative to the best practice frontier that exists in the world economy at each point in time. At each time period t, define the *maximum productivity level* across all production units and all time periods including time period t and the periods prior to it, $\Gamma_{t,max}$, as follows:

$$\Gamma_{t,max} \equiv \max_{s,k} \{\Gamma_{ks} : s \leq t, \ k = 1, \ldots, K\}. \qquad (28)$$

The relative efficiency of country k in time period t is defined as follows:

$$E_{kt} \equiv \Gamma_{kt}/\Gamma_{t,max}; \quad t = 1, \ldots, T; \quad k = 1, \ldots, K. \qquad (29)$$

The E_{kt} satisfy the bounds $0 < E_{kt} \leq 1$; if $E_{kt} = 1$, then country k is efficient at time period t. This measure of efficiency can be traced back to Debreu (1951) and Farrell (1957), but has also more recent applications as the 'distance to the productivity frontier' in Schumpeterian growth theory (Aghion et al., 2014).

4. Measures of productivity convergence

To assess the degree of convergence we will consider two measures. The first is "world" efficiency, following the definition from Eq. (29) and using Γ_t from Eq. (27):

$$E_t = \Gamma_t/\Gamma_{t,max}; \quad t = 1, \ldots, T. \qquad (30)$$

If all production in the world used frontier productivity levels, world efficiency would be equal to 1. The actual degree of world efficiency thus tells us how efficiently the global stock of primary inputs is used to produce worldwide value added or, equivalently,

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[22] Note that our normalizations will imply that $Y_{11} = X_{11} = v_{11} = V_{11}$.

[23] Index number methods for computing productivity go back to Jorgenson and Griliches (1967).

[24] $Y_{kt*} \equiv v_{kt} \Big/ \left\{ P_{kt*} \left[\Pi_{j=1}^{K} \Pi_{s=1}^{T} v_{js} \right]^{\frac{1}{KT}} \right\}$.

[25] "World" productivity here means the productivity of the aggregate of the productivity levels of the K countries in the sample for each time period t.

[26] Note that these output indexes do not depend on exchange rates, which are often subject to large short-run fluctuations. The accuracy of the output indexes does depend on the quality of the PPPs that have been used to convert national expenditures into comparable units.

by how much global value added would increase if country productivity levels were increased to frontier productivity levels.

The second measure is the dispersion of country productivity levels around world productivity levels. This is more commonly known as σ-convergence, see Lichtenberg (1994) and Barro (2012). This can be seen as the productivity counterpart to measures of cross-country income inequality (Milanovic, 2012), showing to what extent productivity levels are becoming more similar over time. We define the following input-weighted measure of productivity dispersion as:

$$\sigma_t \equiv \left[\sum_{k=1}^{K} \omega_{kt} \ln \left(\Gamma_{kt}/\Gamma_t \right)^2 \right]^{1/2}; \quad t = 1, \ldots, T. \tag{31}$$

Note that Γ_{kt}/Γ_t is the ratio of the productivity level of country k in period t to world average level of productivity in period t. If all country productivity levels are the same in period t, each Γ_{kt} will be equal to Γ_t and σ_t will be equal to 0; i.e. there is complete productivity convergence.

5. Data

To illustrate the general method proposed in the previous section and more specifically assess the degree of convergence using Eqs. (30) and (31), we assemble a data set covering 38 economies between 1995 and 2011 across three main sectors of the economy. These are the *traded sector*, covering agriculture, mining and manufacturing; the *non-traded sector*, covering utilities, construction and (market) services. The *market sector* is a third sector, which combines the traded and non-traded sectors.[27]

Recall that our method requires four sets of data, namely the value of net outputs and of primary inputs (the v_{ktm} and V_{ktn}), and prices (PPPs) corresponding to those net outputs and primary inputs (the p_{ktm} and w_{ktn}). In the online appendix to this paper, we detail the construction of these four sets of data and provide the basic data.[28] We note here that the value of net output and of factor inputs are drawn from the harmonized national supply and use tables and socio-economic accounts of the World Input–Output Database (WIOD, see Timmer et al., 2015). The PPP data on net outputs are mostly from the International Comparison Program, see e.g. World Bank (2014) and Feenstra et al. (2015). The PPP data on factor inputs are primarily based on WIOD. In terms of country coverage, we cover many advanced economies (e.g. the US, the countries of the EU, Japan) and major emerging economies, such as Brazil, China and India.

6. Results

Fig. 1 shows convergence results based on the world efficiency measure defined in Eq. (30) for the traded, non-traded and market sectors. All three sectors show declining efficiency, which means that the world average set of primary inputs in 2011 is used less efficiently than the 1995 set. In 1995, market sector efficiency was 51% of the productivity level of the country with the maximum productivity level, which is the United States in these data. By 2011, world efficiency had decreased to 46%. This change is primarily due to a compositional shift. In 1995 the US accounted for 15% of world factor inputs (ω_{kt} from Eq. (26)) but in 2011 this share

had declined to 10%. Conversely, China's share increased from 27% to 32% and India's from 9% to 16%. Since the US defines the productivity frontier for the market sector, while China and India have efficiency levels lower than world efficiency, this drags down world efficiency. Indeed, a counterfactual world efficiency level that combines productivity levels in 2011 with factor input shares from 1995 (i.e. $E^*_{2011} = \sum_{k=1}^{K} \omega_{k1995} \Gamma_{k2011}/\Gamma_{2011,max}$) is equal to 50.5%, barely lower than the 51% in 1995.

That said, 21 of the 38 countries show a decline in efficiency levels and these are predominantly the countries with higher efficiency levels. In other words, US productivity levels have increased relative to those of other advanced economies since 1995, a trend that has been documented before (e.g. Timmer et al., 2010).

Our findings may also have some bearing on the recent discussions on secular stagnation and concerns about the rate of technological progress.[29] We find that frontier productivity, $\Gamma_{t,max}$ from Eq. (28), in the market sector has increased at a rate of 1.7% per year. On the other hand, 'world' market sector productivity, Γ_t only grew by 1.0% per year, indicating that world production moved further from the best-practice production frontier. These aggregate results mask some notable differences between the two sectors. In the traded sector, the frontier moved outwards at a rate of 2.6% per year while in the non-traded sector, the rate was only 1.3% per year. In both sectors, world average productivity, Γ_t from Eq. (27), grew much slower at 1.3% in the traded sector and 0.6% in the non-traded sector. The frontier growth rates still provide grounds for optimism about the rate of technological progress, but the slow growth of world average productivity suggests greater cause for concern.[30] Over time, market sector productivity growth slowed down after the financial crisis of 2007, from a rate of 1.1% in 1995–2007 to 0.6% for 2007–2011. This slowdown is entirely due to slower growth in the traded sector, where average growth of 2.1% before 2007 changed to an average annual growth rate of −0.9% after 2007. In the non-traded sector, average annual productivity growth increased from 0.4% to 1.2%.

The other notable feature of Fig. 1 is that world efficiency in the non-traded sector is higher (between 0.61 and 0.55) than in the market sector or the traded sector (between 0.35 and 0.28). This fits with the Harrod–Balassa–Samuelson hypothesis that productivity differences in the non-traded sector are smaller than in the traded sector (see Asea and Corden, 1994) and is in line with earlier results of Hsieh and Klenow (2007) and Herrendorf and Valentinyi (2012). The downward trend in world efficiency is very similar in the traded and the non-traded sector, declining by approximately five percentage points over the period, again mostly due to compositional shifts. In the non-traded sector, the US also defines the productivity frontier, but Denmark, Ireland and Sweden alternate in defining the productivity frontier in the traded sector; see also the online appendix for detailed country results.

The results for the second measure of convergence, which captures productivity dispersion as defined in Eq. (31), are shown in Fig. 2. There is a pronounced downward trend in this dispersion measure for the market sector, declining from 0.66 in 1995 to 0.41 in 2011. The dispersion in the traded sector is larger than in the market and non-traded sectors – as was also implied by Fig. 1 – but this sector also shows a more rapid decline in dispersion, from 1.00 to 0.64. In comparison, productivity dispersion in the non-traded sector changes much less, from 0.38 to 0.35.[31] This evidence

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[27] Excluded from the data set are government, health and education, as there is no data about relative output prices. Also excluded is the real estate industry, since this industry mostly consists of (imputed) rents of residential buildings (and hence input will equal output for this industry).

[28] The working paper version of this paper, Inklaar and Diewert (2015) has a more extensive appendix detailing further features of the data (see Appendix A).

[29] See e.g. Gordon (2012, 2014) and Mokyr et al. (2015).

[30] The appendix to Inklaar and Diewert (2015) provides a more detailed exposition of results by period and country.

[31] The T^3-test of Carree and Klomp (1997) indicates significant convergence (at the 5-percent level) in the market sector and the traded sector.
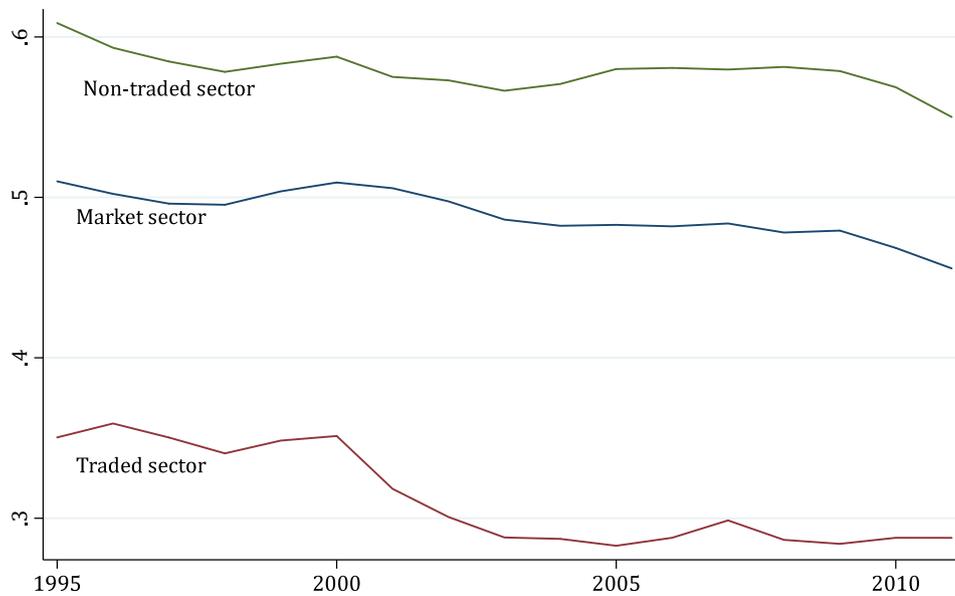
Fig. 1. World efficiency across sectors, 1995–2011. Note: the figure shows world efficiency E_t, defined in Eq. (30) in the traded sector, the non-traded sector and the market sector.
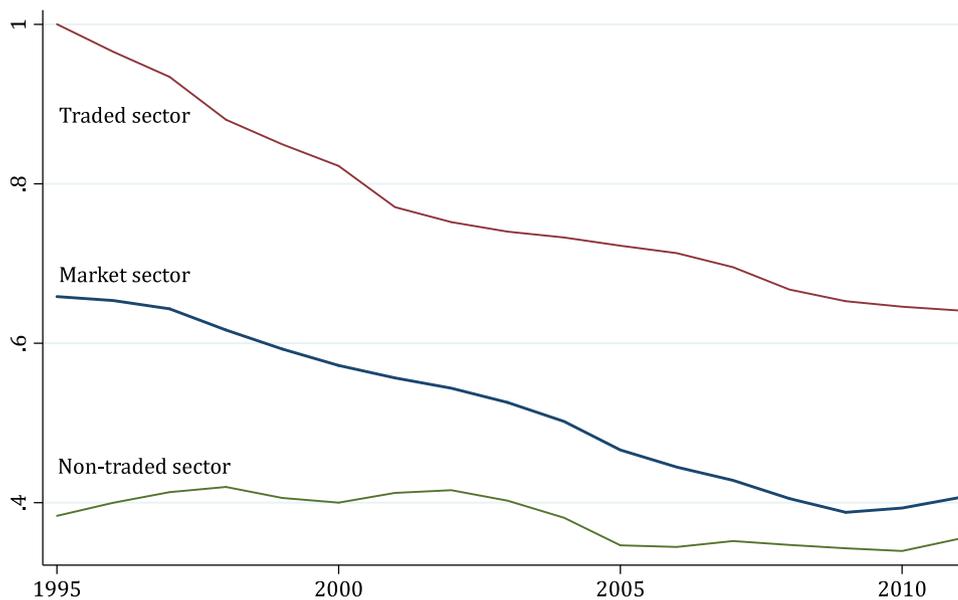
Fig. 2. Productivity dispersion across sectors, 1995–2011. Note: the figure shows the input-weighted dispersion of log productivity levels, σ_t, from Eq. (31).

extends the literature on the Harrod–Balassa–Samuelson theory, which has found larger dispersion and faster productivity growth in the traded sector than in the non-traded sector.[32]

7. Conclusions

Measuring the pace of productivity convergence across countries requires measures of relative productivity that are comparable across both countries and over time. Extending the theory of cross-country productivity comparisons, this paper has proposed a new method for constructing relative productivity levels that are well-suited for convergence analysis. We have illustrated the new

method by constructing relative aggregate and sectoral productivity levels for a set of 38 economies over the period 1995–2011. Some of our findings are as follows:

- Dispersion of country productivity levels decreased over the sample period, but the convergence of productivity levels to the average level of productivity was accompanied by a decline in the average level of productivity relative to the maximum possible level of productivity.
- The rate of growth of the maximum possible productivity level for the market sector grew at about 1.7% per year over the sample period but actual "world" productivity grew at only about 1.0% per year.
- The productivity frontier for the traded sector expanded at about 2.6% per year while the productivity frontier for the non-traded sector expanded at only 1.3% per year. Actual TFP growth for the traded sector was 1.3% per year and 0.6% per year for the non-traded sector.

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[32] See Hsieh and Klenow (2007) and Herrendorf and Valentinyi (2012) on differences in dispersion and De Gregorio et al. (1994) and Ricci et al. (2013) on differences in growth rates. This pattern also holds if an unweighted measure of productivity dispersion had been used.

- The global financial crisis was associated with a market sector slowdown in actual world TFP growth from a 1.1% per year growth rate over 1995–2007 to a 0.6% per year over 2007–2011. However, the productivity slowdown was entirely concentrated in the traded sector.
- The productivity frontier for the nontraded sector expanded at 2.6% per year over 2007–2011. Since the nontraded sector is roughly twice as big as the traded sector, this rapid expansion in the nontraded production frontier offers some hope for future improvements in global productivity growth.

Obviously, it would be very useful if the WIOD data base could be extended beyond 2011. Hopefully, the World Bank, the OECD and the IMF will work together with national statistical agencies to develop productivity accounts at the national level with some industry detail along with the production of timely industry level PPPs.

Acknowledgments

Appendix A. Supplementary data

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.jeconom.2015.12.013.

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