Instantons and cosmologies in string theory

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Chapter 4

Introduction to Cosmology

4.1 FLRW cosmology

To begin our studies of cosmology, we must first introduce a bit of formalism and terminology that is now part of what is called *the standard cosmology*. The language and formulae in which we will state facts about cosmology are deceitfully simple. They hide the massive amounts of observational data and research required to arrive at them. Doing justice to the topic of modern cosmology would obviously require a lot more than one chapter. For a proper introduction to standard cosmology and cosmology in the context of string theory, the reader is referred to the lecture notes [71–73], on which this chapter is mainly based. Often in physics one tries to reproduce or model complicated phenomena by defining a fundamental[1] theory that is simple to begin with, but requires all kinds of approximations and truncations in order to describe realistic physics. In cosmology, one does the exact opposite. One tries to model complicated phenomena with simple models, which are not really *derived* from a fundamental theory. They can ultimately be seen as large scale gross approximations of some unknown fundamental theory. When discussing inflation, F. Quevedo describes it as "a scenario in search of an underlying theory" [72]. A fundamental theory that could account for cosmology would also have to explain the Big Bang. General Relativity breaks down for highly curved spacetimes, where quantum effects become important. String theory is a current candidate as an underlying theory of cosmology because it is a theory of quantum gravity.

4.1.1 The FLRW Anstatz: Motivation and definition

We begin by defining the FLRW, or *Friedmann-Lemaître-Robertson-Walker* spacetime metric. It is actually a class of metrics defined by two properties as follows: a metric is FLRW if there exists a frame (i.e. a family of geodesic observers), in which it is *spatially* homogeneous and isotropic (see appendix C for definitions and examples). These two properties that are imposed are based on the observations that the universe "looks the same" at every point in space, and it

[1] Of course, the concept of a *fundamental* theory is only relative. So far there is no such thing as a fundamental theory that is valid in all regimes.

"looks the same" in every direction about a point. Of course this is only true on a very very large scale, a cosmological scale. Our lives would be pretty difficult if we were not capable of telling the difference between our boss' office and our bathroom, and driving would be impossible if we couldn't make a distinction between the right and the wrong way of a one-way street. But we as humans are looking too closely at things and what we see are only tiny fluctuations from homogeneity and isotropy.

So the Ansatz for an FLRW metric is the following:

$$ds^2 = -f^2(t)\,dt^2 + g^2(t)\,d\Sigma_3^2\,, \tag{4.1}$$

where $f(t)$ an $g(t)$ are two undetermined functions of time, and $d\Sigma_3$ is the line element of some homogeneous and isotropic spatial manifold. It can be shown that in three dimensions there are only three possible metrics that satisfy the requirement of homogeneity and isotropy :

$$d\Sigma_3^2 = \frac{dr^2}{1-k\,r^2} + r^2\left(d\theta^2 + \sin(\theta)^2\,d\phi^2\right) \qquad \text{with} \quad k = +1, 0, 1\,. \tag{4.2}$$

This can also be written as follows:

$$d\Sigma_3^2 = d\rho^2 + f^2(\rho)\left(d\theta^2 + \sin(\theta)^2\,d\phi^2\right)\,, \tag{4.3}$$

where

$$f(\rho) = \begin{cases} \sin(\rho) & \text{if } k = +1 \\ \rho & \text{if } k = 0 \\ \sinh(\rho) & \text{if } k = -1 \end{cases}\,. \tag{4.4}$$

The parameter k lables the curvature of the spatial section of the metric. A spatial section of (4.1) with line element

$$ds^2_{spatial} = g^2(t)\,d\Sigma_3^2 \tag{4.5}$$

has the following Ricci scalar:

$$R_\Sigma = \frac{6\,k}{g^2(t)}\,. \tag{4.6}$$

We easily recognize the three spatial metrics as those of the 3-sphere, 3-plane and 3-hyperboloid respectively. But we must be careful not to confuse local with global statements about a manifold. The three spatial metrics in (4.2) contain only local information and do not imply anything about the topologies of their respective manifolds. For instance, the $k = 0$ metric may be defined on the 3-plane R^3 as well as on the 3-torus T^3. Similarly, the 3-hyperboloid H^3 can be compactified by means of discrete group identifications that do not affect curvature. So what does the metric $g^2(t)\,d\Sigma_3^2$ tell us about a spatial manifold? Any physically meaningful statement in General Relativity must be expressible in terms of "clock and rods", and in this case specifically, in terms of "rods".

Let us start with the spatially flat ($k = 0$) case. We place an observer at time $t = t_0$ at the origin of our coordinate system ($\rho = 0$) and at rest w.r.t. it ($\dot{\rho} = 0$). Let the observer pick a plane passing through him (without loss of generality the $\theta = \pi/2$ plane), and draw a circle on it around himself of radius

$$R = g(t_0)\rho' \quad \text{for some} \quad \rho'\,, \tag{4.7}$$

If the observer measures the circumference L of this circle instantaneously, or fast enough so that $g(t)$ does not change significantly, the metric (4.5) tells us that he will find it to be

$$L = 2\pi\, g(t_0)\,\rho' = 2\pi R\,, \tag{4.8}$$

as expected. For general k this will change. If we conduct the same experiment, (4.5) tells us that the circumference of a circle of radius $R = g(t_0)\,\rho'$ will be

$$L = 2\pi\, g(t_0)\, f(\rho') = \begin{cases} 2\pi\, g(t_0)\, \sin[R/g(t_0)] & \text{if } k = +1 \\ 2\pi R & \text{if } k = 0 \\ 2\pi\, g(t_0)\, \sinh[R/g(t_0)] & \text{if } k = -1 \end{cases} . \tag{4.9}$$

The first thing to notice about this result is that $g(t_0)$ completely drops out for the $k = 0$ case, making its value at any given time physically meaningless. The other thing to notice is that if we take R to be very small and expand $f(\rho)$, we see that, to leading order, the circumferences become $2\pi R$ for the $k \neq 0$ cases. If this were not the case, we would have what is called a *conical singularity* on our spatial manifold. Hence, the $k = +1$ case tells us that circles have smaller circumferences than we are used to, and the $k = -1$ tells us that they are larger than normal.

Now that we understand the spatial geometry of the FLRW metric, let us study the spacetime geometry. Once k is fixed, the only undetermined parts of the metric (4.1) are the time-dependent functions $f(t)$ and $g(t)$. However, these two functions are not independent of each other. If we perform the following simple coordinate transformation:

$$\tau(t') \equiv \int_0^{t'} f(t)\, dt\,, \tag{4.10}$$

we end up with the following metric:

$$ds^2 = -d\tau^2 + a^2(\tau)\, d\Sigma_3^2\,, \tag{4.11}$$

where we are now left with only one undetermined function $a(\tau)$, usually called the *scale factor*. The time coordinate τ as defined in (4.11) is called *cosmic time*. In the standard cosmology jargon, if the scale factor is an increasing or decreasing function of time we say that the universe is "expanding" or "contracting" respectively. Similarly, if its second time derivative is positive, we say that the universe is "accelerating". But these words can be misleading. If the spatial topology of the universe is compact, one can define a volume of the universe, and then it makes sense to talk about expansion or contraction. But if the universe has a non-compact spatial topology, such as R^3 or H^3, then this does not make sense. So what does the scale factor really tell us about the universe? Again, the only meaningful thing to do is to revert to our "clocks and rods". The only information we can and should infer from a metric is what geodesic observers see. So let us define two geodesic trajectories $x_1(t)$ and $x_2(t)$ as follows:

$$x^0(t) = \tau(t) = t, \quad x^i(t) = x^i(\tau) = a^i \tag{4.12}$$

$$y^0(t) = \tau(t) = t, \quad y^i(t) = y^i(\tau) = b^i\,, \tag{4.13}$$

where a^i and b^i are constants. Such geodesics are called *comoving*. Notice that for comoving observers the time coordinate τ in (4.11) measures their proper time, so all comoving observers can keep their clocks synchronized. The spatial separation of x_1 and x_2 in the comoving frame is given by:

$$d^2 = d^i\, d^j\, g_{ij}\,, \qquad \text{where} \quad d^i \equiv a^i - b^i\,. \tag{4.14}$$

Differentiating this w.r.t. time we find that

$$\dot{d} = H\, d\,, \tag{4.15}$$

where $H \equiv \dot{a}/a$ is called the *Hubble parameter*. Therefore, the scale factor tells us that two comoving observers will notice a relative velocity between them that is proportional to their separation, and the Hubble parameter. In a universe with accelerated expansion (i.e $H > 0$ and $\ddot{a} > 0$), this means that this relative velocity will eventually exceed the speed of light! Although this may seem like a violation of causality, it is not. No information is travelling from one point to another acausally. What this does mean, however, is that the two observers will eventually cease to be in causal contact, as no signal sent from one can ever catch up with the other.

4.1.2 The right-hand side of the Einstein equation

Having studied the general form of an FLRW cosmological metric, we should now study the kind of matter or energy that can coexist with or drive such a metric. The assumption of spatial isotropy leads us to consider perfect fluids as unique candidates. They have the property (which can be taken as a defining property [18]) of looking isotropic in their rest frames. The stress-energy tensor of a perfect fluid has the following form:

$$T_{\mu\nu} = (\rho + p)\, U_\mu\, U_\nu + p\, g_{\mu\nu}\,, \tag{4.16}$$

where $U^\mu(x)$ is the velocity field of the fluid, ρ is the energy density of the fluid in its rest frame, and p its pressure in its rest frame. This is the stress-energy tensor that will be on the right-hand side of the Einstein equation. In order for the fluid to coexist in equilibrium, or be consistent with the FLRW metric, its elements must be comoving. In other words, in comoving coordinates the velocity field of the fluid must be

$$U^\mu = (1, 0, 0, 0)\,. \tag{4.17}$$

Note that if the fluid is made of photons then U^μ cannot be interpreted as the velocity of the individual photons, but must be interpreted as an average displacement of energy. Using these assumptions we can write the Einstein equations and cleverly rearrange them into the following two equations:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}\,, \tag{4.18}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)\,, \tag{4.19}$$

where H is the Hubble parameter. The first equation is called the *Friedmann equation* and the second is called the *acceleration equation*. Note that if we want to include several species of fluid we can simply add up the ρ's and p's. The equations of motion for the fluid follow from the conservation laws of the stress-energy tensor:

$$\nabla_\mu T^{\mu\nu} = 0\,. \tag{4.20}$$

They imply the continuity equation for the fluid:

$$\dot{\rho} + 3\,H\,(\rho + p) = 0\,. \tag{4.21}$$

This equation can actually also be obtained by differentiating the Friedmann equation (4.18) w.r.t. time and combining it with the acceleration equation (4.19).

To be able to solve for $a(t)$, $\rho(t)$ and $p(t)$, we need to make one more assumption about the fluid, namely, that it obeys an equation of state. In other words, that the pressure is a function of density, $p = p(\rho)$. For ordinary matter, we can approximate the equation of state by the following instantaneous relation:

$$p = \omega\,\rho\,, \tag{4.22}$$

where ω is a constant that depends on the kind of matter that makes the fluid. For pressureless dust (i.e. non-interacting particles) $\omega = 0$. For radiation, meaning either photons or highly relativistic particles, $\omega = 1/3$. In the case of radiation, one can see this by writing the stress-energy tensor of the Maxwell field:

$$T_{\mu\nu} = -\frac{1}{4\,\pi}\,(F_{\mu\alpha}\,F_\nu{}^\alpha - \frac{1}{4}\,g_{\mu\nu}\,F^2)\,, \tag{4.23}$$

which is manifestly traceless in four dimensions. Our assumptions about comoving perfect fluids tell us that the trace of this tensor is $T_\mu{}^\mu = 3\,p - \rho$. Combining these two facts gives us $\omega = 1/3$.

Dust and radiation are part of a larger class of possible forms of "matter" called *ordinary matter*. Another form of matter is *dark matter*, which is essentially non-baryonic matter. There is another important kind of energy that can drive an FLRW metric, a cosmological constant Λ. It cannot be viewed as matter, it is regarded as a vacuum energy. The cosmological constant also satisfies an equation of state (4.22), with $\omega = -1$, and its energy density is equal to itself, $\rho = \Lambda$. It is part of a class of possible forms of energy called *dark energy*, which characteristically have equations of state with $\omega < -1/3$.

Observations show that our universe is not made of just one kind of fluid, but it is a combination of different kinds of fluids. Also, throughout the history of the universe, the different kinds of matter and energy have swapped the roles of dominance and subdominance. Therefore, a convenient notation for comparing the energy densities of the fluids has been developped. From the Friedmann equation (4.18) we see that the energy density required to have a spatially flat universe is

$$\rho_c = \frac{3\,H}{8\,\pi\,G}\,. \tag{4.24}$$

This is called the *critical* density. By computing the ratio of the actual energy density of a fluid to the critical density

$$\Omega \equiv \frac{\rho}{\rho_c}\,, \tag{4.25}$$

we can easily relate the matter content and observed Hubble parameter of the universe to its spatial geometry as follows:

$$\begin{aligned} \Omega > 1 &\iff k = 1 \\ \Omega = 1 &\iff k = 0 \\ \Omega < 1 &\iff k = -1\,. \end{aligned} \tag{4.26}$$

In a universe with coexisting fluids Ω is simply decomposed into the fractional contributions of each species to the total ratio:

$$\Omega_{\text{total}} = \sum_i \Omega_i\,. \tag{4.27}$$

Observations indicate that our current universe is spatially flat, and it is composed of ordinary (baryonic) matter, dark matter, and dark energy in the following respective ratios:

$$\begin{aligned} \Omega_B &= 0.04 \\ \Omega_{DM} &= 0.26 \\ \Omega_\Lambda &= 0.7\,. \end{aligned} \tag{4.28}$$

A statement of modern cosmology is that the early universe (shortly after the Big Bang) would have been radiation dominated. It is puzzling that, presently, the energy densities of all three forms of matter and energy are of the same order (i.e. $\propto 1$). This puzzle is known as the *cosmic coincidence problem*.

4.1.3 Solutions

Given the matter or energy content of the universe one is trying to model, it is easy to solve for the scale factor by combining the Friedmann and acceleration equations (4.18) (4.19) with the proper equations of state. Since observations show that our current universe is spatially flat to a high degree of precision, we will focus on the $k = 0$ case. The solutions are the following:

$$\begin{aligned} a(t) &= a_0 \left(\frac{t}{t_0}\right)^{2/3\,(1+\omega)} && \text{for} \quad \omega \neq -1 \\ a(t) &\propto e^{H\,t} && \text{for} \quad \omega = -1 \end{aligned} \tag{4.29}$$

where H is now constant. The first solution is called *power law* solution. It is mainly used to model pre- and post-inflationary cosmology. Note that for $t = 0$ such a metric has a singularity, namely all spatial distances are zero. This is called the *Big Bang* singularity. The second metric is a solution to the Einstein equation with a *positive* cosmological constant. It is called *de Sitter* space, after Willem de Sitter, the great mathematician, physicist, and astronomer who studied at the University of Groningen. Solutions for $k \neq 0$ can also easily be found.

At this point a word of caution would be in order. Specifying FLRW metrics in terms of k and $a(t)$ is, as we said before, only a local statement about the spacetime manifold. For instance, we noted earlier that division by a discrete group can related two different manifolds with the same local geometry. More generally, we have to remember that a manifold is defined as a

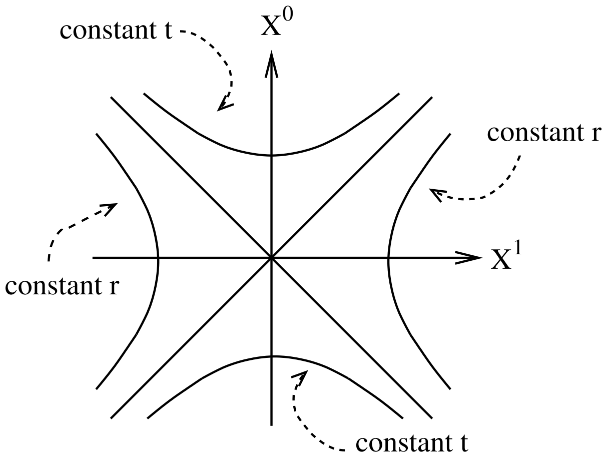

Figure 4.1: *Minkowski spacetime with two suppressed dimensions. A three-dimensional picture can be obtained by rotating everything about the X^0-axis. The constant t surfaces are the two-sheeted Euclidean hyperboloids covering the Milne patch. The constant r surfaces are the one-sheeted Lorentzian hyperboloids (dS) covering the Rindler patch.*

collection of *patches* (i.e. open sets of the underlying space) with *charts* (i.e. coordinates) and *transition functions* relating the charts of intersecting patches. In many cases a single patch may cover the whole space minus a finite set of points. For instance, polar coordinates cover the whole sphere except for the two poles. In such cases, that one patch is all we need. However, some coordinate systems cover only half of a space. So any metric that we write down may just represent one patch of a manifold.

Let us illustrate this with a familiar manifold, Minkowski spacetime. Minkowski spacetime is defined as the manifold $\mathbb{R}^4$ with a flat Lorentzian metric (i.e. Riemann tensor is zero). In cartesian coordinates we write this as follows:

$$ds^2 = -d(X^0)^2 + d(X^1)^2 + d(X^2)^2 + d(X^3)^2 \,. \tag{4.30}$$

So far so good. Now let us introduce the so-called *Milne* coordinates.

$$X^0 = t\ \cosh(\psi)\,,$$
$$X^1 = t\ \sinh(\psi)\ \sin(\theta)\ \sin(\phi)\,,$$
$$X^2 = t\ \sinh(\psi)\ \sin(\theta)\ \cos(\phi)\,, \tag{4.31}$$
$$X^3 = t\ \sinh(\psi)\ \cos(\theta)\,. \tag{4.32}$$

These coordinates don't cover all of Minkowski spacetime. They only cover the regions within the future and past light-cones of the origin of Minkowski spacetime:

$$(X^0)^2 - ||\vec{X}||^2 = t^2 > 0\,. \tag{4.33}$$

Milne coordinates slice up the space with a one-parameter family of two-sheeted Euclidean hyperboloids, parametrized by t, see figure 4.1. In these coordinates, the flat metric (4.30) becomes

$$ds^2 = -dt^2 + t^2 \left(d\psi^2 + \sinh^2(\psi)\, d\Omega_{S^2}^2 \right) . \tag{4.34}$$

In other words, the FLRW metric with $a(t) = t$ and $k = -1$ is nothing other than a patch of Minkowski spacetime in disguise!

For completness, and because it will come in handy in chapter 7, let us study the *Rindler* coordinates, which cover the complement of the region covered by the Milne coordinates, i.e. $(X^0)^2 - \|\vec{X}\|^2 < 0$. Define the following parametrization of Minkowski spacetime:

$$\begin{aligned} X^0 &= r\, \sinh(t)\,, \\ X^1 &= r\, \cosh(t)\, \sin(\theta)\, \sin(\phi)\,, \\ X^2 &= r\, \cosh(t)\, \sin(\theta)\, \cos(\phi)\,, && (4.35) \\ X^3 &= r\, \cosh(t)\, \cos(\theta)\,. && (4.36) \end{aligned}$$

These coordinates slice up the spacetime with a one-paramteter family of one-sheeted Lorentzian hyperboloids, where the parameter is r, see figure 4.1. The metric (4.30) takes the following form:

$$ds^2 = dr^2 + r^2 \left(-dt^2 + \cosh^2(t)\, d\Omega_{S^2}{}^2 \right) . \tag{4.37}$$

Although they are hyperboloids, the constant-r subspaces have Lorentzian signature and are positively curved. In fact, they are three-dimensional de Sitter spacetimes, as we will see next.

Having seen this familiar example, let us study de Sitter spacetime. It can be defined as a four-dimensional hyperboloid embedded in five-dimensional Minkowski spacetime:

$$-(X^0)^2 + (X^1)^2 + (X^2)^2 + (X^3)^2 + (X^4)^2 = \ell^2 \tag{4.38}$$

$$ds^2 = -d(X^0)^2 + d(X^1)^2 + d(X^2)^2 + d(X^3)^2 + d(X^4)^2\,, \tag{4.39}$$

where the first equation defines the hyperboloid, and the second defines the metric in the embedding space. The radius ℓ is related to the cosmological constant Λ in the Einstein equation as $\ell^2 = 3/\Lambda$. There are several coordinate systems that can be used to parametrize de Sitter spacetime, or at least a patch of it. In fact, it can be viewed as three different FLRW cosmologies with $k = 1, 0$, and -1 respectively. Let us start with the $k = 1$ form. Define the following coordinates:

$$\begin{aligned} X^0 &= \ell\, \sinh(t/\ell)\,, \\ X^1 &= \ell\, \cosh(t/\ell)\, \sin(\psi)\, \sin(\theta)\, \sin(\phi)\,, \\ X^2 &= \ell\, \cosh(t/\ell)\, \sin(\psi)\, \sin(\theta)\, \cos(\phi)\,, \\ X^3 &= \ell\, \cosh(t/\ell)\, \sin(\psi)\, \cos(\theta)\,, \\ X^4 &= \ell\, \cosh(t/\ell)\, \cos(\psi)\,. \end{aligned} \tag{4.40}$$

These coordinates solve the constraint (4.38) on the whole hyperboloid. The resulting four-dimensional metric is

$$ds^2 = -dt^2 + \ell^2\, \cosh^2(t/\ell)\, d\Omega_{S^3}^2\,. \tag{4.41}$$

This is called the de Sitter metric in *global* coordinates. It represents spacetime as a spacelike sphere that contracts from an infinite to a minimal radius ℓ (at $t = 0$), and then enters an eternal phase of accelerated expansion. The acceleration rate is constant at $\ddot{a}/a = 1$. This will cause causally connected spatial regions to become causally disconnected in the future. In other words, any two spatially separated observers will eventually become causally disconnected. To see this, we only need to look at null geodesics in de Sitter space. For simplicity, let us study a 'radial' geodesic emitted from the origin at time t_0:

$$-dt + \ell \cosh(t/\ell)\, d\psi = 0\,. \tag{4.42}$$

The solution is

$$\psi(t) = 2\Big(\arctan[\tanh(t/2\,\ell)] - \arctan[\tanh(t_0/2\,\ell)]\Big)\,. \tag{4.43}$$

If the light ray is emitted at time $t = 0$, it will asymptotically reach $\psi = \pi/2$ for $t \to \infty$. However, the later it is emitted the less it will travel as can be seen from the solution. This means that if we place a comoving observer at position $\psi = \epsilon$, it will at first be capable of receiving light rays emitted from the origin; however after a certain time (for $t > 2\,\mathrm{arctanh}[\tan(\pi/4 - \epsilon)]$) it will be causally disconnected from the origin. This feature of de Sitter spacetime poses a serious problem in modern physics. One cannot define asymptotic states for a quantum field theory, or conservation laws for general relativity in the usual way.

Now, let us write down the $k = 0$ form of de Sitter spacetime. Once again, we implicitly define four-dimensional coordinates by solving the five-dimensional constraint (4.38):

$$\begin{aligned} X^0 + X^1 &= \ell \exp(t/\ell)\,, \\ X^i &= \ell \exp(t/\ell)\, x^i\,, \quad \text{for} \quad i = 2,3,4\,, \\ X^0 - X^1 &= \ell \exp(t/\ell)\left(\sum_{i=2}^{4}(x^i)^2 - \exp(-2\,t/\ell)\right), \end{aligned} \tag{4.44}$$

where the first equation defines a light-cone coordinate $\exp(t)$, the second equation defines cartesian coordinates x^i, and the third equation follows from the hyperboloid constraint (4.38). Note that the light cone coordinate is defined to be positive, which means that we are only covering half of the de Sitter manifold. Plugging this into (4.39) yields the following metric:

$$ds^2 = -dt^2 + \ell^2 \exp(2\,t/\ell) \sum_i (dx^i)^2\,. \tag{4.45}$$

These are the de Sitter equivalent of Poincaré coordinates for anti-de Sitter spacetime. This form of de Sitter is the one used to model inflation because it has $k = 0$ and it is expanding for all t, unlike the global form (4.42). Finally, let us write down the $k = -1$ form. The trick is to put X^4 on the right-hand side of the constraint equation (4.38) and view the space as a one-parameter family of hyperboloids of radius $(X^4)^2 - \ell^2$, with the assumption that $|X^4| > \ell$:

$$\begin{aligned}
X^4 &= \ell \cosh(t/\ell)\,, \\
X^0 &= \ell \sinh(t/\ell) \cosh(\psi)\,, \\
X^1 &= \ell \sinh(t/\ell) \sinh(\psi) \sin(\theta) \sin(\phi)\,, \\
X^2 &= \ell \sinh(t/\ell) \sinh(\psi) \sin(\theta) \cos(\phi)\,, \\
X^3 &= \ell \sinh(t/\ell) \sinh(\psi) \cos(\theta)\,,
\end{aligned} \tag{4.46}$$

(4.47)

which yields the following metric:

$$ds^2 = -dt^2 + \ell^2 \sinh^2(t/\ell) \left(d\psi^2 + \sinh^2(\theta)\, d\Omega_{S^2}\right) . \tag{4.48}$$

The Anstatz for X^4 implies that this parametrization only cover half of the manifold. Note that this metric has a Big Bang singularity at $t = 0$.

Finally, we should briefly discuss anti-de Sitter spacetime or AdS. This is a solution to the Einstein equation with a negative cosmological constant. It can also be defined as a hyperboloid embedded in a higher dimensional spacetime, and many coordinate systems are available to cover it or at least partly cover it. However, AdS admits only one coordinate system such that its metric is in the FLRW form. The metric looks as follows:

$$ds^2 = -dt^2 + \ell^2 \sin^2(t/\ell) \left(d\psi^2 + \sinh^2(\theta)\, d\Omega_{S^2}\right) , \tag{4.49}$$

where ℓ is defined analogously to the de Sitter case. This is a $k = -1$ cosmology with a Big Bang singularity at $t = 0$ and a *big crunch* singularity at $t = \pi\,\ell$.

4.2 Physics of FLRW cosmologies

Having laid the foundations of cosmology we are ready to study the phenomena that drive the field of modern cosmology. The standard cosmology is a model of our universe that has been developed over decades by fitting observations from innumerably many experiments to theoretical models that rely upon the foundations of different fields such as general relativity, quantum field theory, thermodynamics, astrophysics, spectroscopy, etc.. Again, I would like to post my disclaimer here, and reiterate how extremely rich and complicated standard cosmology is, and that I in no way pretend to do justice to it. I will, however, try to give a condensed account of the history of our universe. Then, I will present three issues that arise in the standard cosmology, namely the *horizon problem*, the *flatness problem*, and the *relics problem*; and I will briefly explain the concept of inflation and show how it solves all three problems. I will then mention the presently observed acceleration of the universe, and finally, I will motivate the need for scalar cosmology models.

4.2.1 An ephemerally brief history of time

Let us start with an extremely brief history of the universe. In the beginning was the Big Bang. There are singularity theorems by Hawking and Penrose [74] that predict that any universe occupied by matter with $\rho > 0$ and $p > 0$ must have a Big Bang singularity. Since observations show that our early universe was mainly radiation dominated, the theorems would imply that our universe started with such a singularity. So what is a Big Bang singularity? A power law FLRW metric (4.29) provides us with a good metaphor for the Big Bang. At $t = 0$ the scale factor vanishes and the spatial section has 'zero size'. This is the 'beginning of time'. All the matter in the universe is condensed to a 'point', and thus ρ is really high. One must, however, realize that at the time of the Big Bang $t = 0$ the solution has a curvature singularity and the laws of General Relativity break down. No one knows, whether the singularity is a physical event, or a mere mathematical extrapolation from GR into uncharted territory. At this point a new theory is needed, namely one that can combine gravity and quantum mechanics. String theory is a strong candidate for this. For the time being, we must use GR within its regime of validity. This means that we cannot take $t = 0$ and $a = 0$ too literally. The standard cosmology is only meant to describe what happened after the first millisecond (or less) of the classically describable universe. So, although we cannot say that the universe 'started out' with 'zero size', or 'small' (unless $k = 1$, in which case a size can be defined), we can certainly say that it was occupied by very dense matter or radiation.

Since shortly after the Big Bang the universe was hot, dense and in thermal equilibrium, it started emitting light in every direction like a perfect blackbody. This radiation is observable today, especially its microwave component. This is the famous *Cosmic Microwave Background Radiation* or CMBR (or just CMB), which was almost accidentally discovered in 1965 by two radio astronomers, Arno Penzias and Robert Wilson. Its spectrum is so close to that of a perfect blackbody, that the CMBR is considered to be the strongest existing evidence of the Big Bang scenario. While the light was constantly scattering off of the rest of the matter constituents of the universe, the latter kept expanding. Expansion not only means that matter is driven apart at a rate proportional to the Hubble parameter, as we saw before, but it also means that the wavelengths of photons stretch. They get *redshifted*. Around 300,000 years after the Big Bang, the photons were so redshifted, that they no longer scattered off of particles. They decoupled, and simply went through everything. This is why the CMBR we observe today gives us such a perfectly undistorted picture of the universe as it was 300,000 years after the Big Bang. Before that, matter was constantly being ionized into plasma due to the constant scattering of photons. After that decoupling, the average temperature of the universe was low enough that atoms were able to form. This is called *recombination*. That is when galaxies and other structures started to form, leading to our present universe, at $t \sim 10^{10}$ years.

End of the schematic history of the universe.

4.2.2 Three problems

Like any great discovery in Physics, the CMBR has not only brought us answers, but also questions. It turns out that this radiation background has a remarkable property, it is almost perfectly isotropic. In any direction we look in the sky, this radiation has the same temperature to within 0.01%, about $2.7K$. Most of this variation by 0.01% is nowadays interpreted as proof that the Earth has a non-zero speed relative to the cosmological frame. We are not quite comoving. Taking this into account, the CMBR is ridiculously isotropic. This is puzzling from a causality point of view for the following reason: if one assumes that the universe has gone through a power-law expansion from the Big Bang until recently due to radiation and matter domination, then a calculation shows that the CMBR light that we see in the sky must have been emitted at recombination time ($t_{\text{CMBR}} \sim 3 \times 10^5$ years) by points that could not have been in causal contact with each other. In other words, if we observe the light coming from two completely opposite directions in the sky, and we take the power-law expansion into account, we conclude that the two sources of light we are looking at were so distant from each other when they emitted it, that they had not had enough time to communicate since the Big Bang 300,000 years earlier. But why is the CMBR so isotropic, then? Why would causally disconnected regions of space emit such perfectly coordinated radiation? This is called the *horizon problem*.

The reader may find this paradox itself, paradoxical. One could ask the following question: "If the universe started with the Big Bang, and all spatial distances were (close to) zero in the beginning, then why couldn't all points in the universe simply have communicated back then, when they were so close to each other? How could 300,000 years not be enough for points that were at an initial distance of zero to communicate? As was pointed out before, no one knows, whether the universe really had 'zero size' in the past. The only trustworthy predictions of the standard cosmology are those regarding the history of the universe, beginning moments after the Big Bang. So, in this text, I will abandon the notion of a universe of 'zero size'. At most, one might say that a $k = 1$ model has an initially 'small' spatial section, in which case the above-mentioned paradox within the paradox becomes a valid one. Fortunately, it can be solved. Wald's book [75] discusses this very clearly. I will try, however, to explain this here. Let us start by defining the word *horizon*, or in this case *particle horizon*.

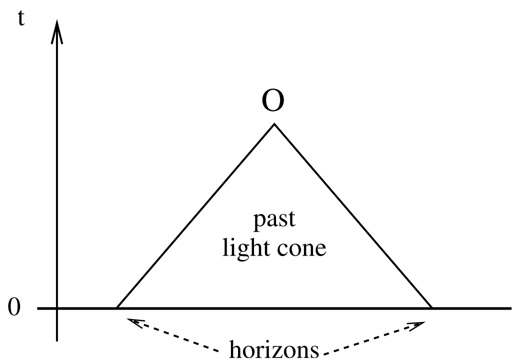

Figure 4.2: *The observer at event O can only see information emitted within the horizons.* $t = 0$ *marks the beginning of time.*

As observers, we can only see information coming from events that are within our past light-cones. We cannot, for instance, see something that happened one second ago in a galaxy that's three light-years away. When we look into the sky, the light that we see comes to us from the past. The farther the source is spatially, the older the information. But what if there was a 'beginning of time' such as in the Big Bang scenario? Then we would only be able to see information coming from a restricted area around our location. If spacetime were flat, but with a beginning of time, then only events that were within a distance d = (speed of light) $\times$ (age of the universe) of us at the time of emission could influence us. The spatial area that we can see is delimited by what is called a *particle horizon*. See figure 4.2. Now let us take a $k = 0$

FLRW metric with a power-law scale factor, and impose a cutoff minimal time t_i, which we will effectively treat as the beginning of time. Do horizons form? To see what happens, we must look at what light rays do. In comoving coordinates, a null geodesic has the following velocity:

$$\frac{dx}{dt} = \frac{1}{a(t)}\,, \tag{4.50}$$

where we use just one spatial axis for simplicity. This velocity is infinite at first, but decays more or less rapidly depending on the scale factor. We need to calculate how much comoving distance the geodesic can cover if it is emitted right after the Big Bang, at our cutoff time t_i, and observed at t_o:

$$\Delta x = x(t_o) - x(t_i) = \int_{\tau_i}^{t_o} \frac{dt}{a(t)}\,. \tag{4.51}$$

We can easily see that, for $a \propto t^\alpha$, this integral diverges as $t_i \to 0$, if $\alpha \geq 1$. In that case, there is a particle horizon, but the smaller t_i is, the bigger it gets. In other words, light coming from any point in the universe can reach the observer if it was emitted early enough. In the case where $\alpha < 1$, however, there is a particle horizon, and it is present even as $t_i \to 0$. Translating this into statements about matter

$$\alpha = \frac{2}{3\,(1+\omega)}\,, \tag{4.52}$$

we see that a radiation or matter dominated universe will generate horizons. Dark energy (i.e. $\omega < -1/3$), however, will generate horizons that are large at early time.

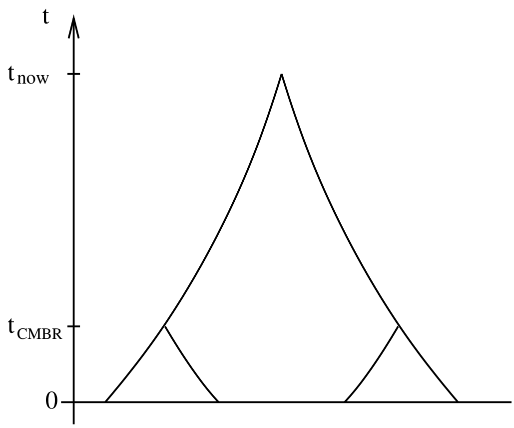

Figure 4.3: *The two sources of CMBR that we see today could not have been in causal contact.*

We are now ready to restate the horizon problem in the following oversimplified way:
At the present time, we can observe highly uniform CMBR rays. Choosing two widely separated CMBR sources in the sky will be separated by a comoving distance $\Delta_s \approx 4\,H_0$, where H_0 is the current Hubble parameter. The beams were emitted at t_{CMBR}. Assuming radiation domination ($a \propto t^{2/3}$), a null geodesic emitted at the Big Bang and observed at t_{CMBR} will travel a distance

$\Delta_l \approx 6 \times 10^{-2} H_0$. Hence we see that $\Delta_l \ll \Delta_s$, so the outermost sources of the CMBR could have never communicated, see figure 4.3. This is the horizon problem.

Another problem in the standard cosmology before inflation was known is the so-called *flatness problem*. Observations indicate that currently $\Omega \sim 1$ to a high degree of precision. However, in order for the universe to be so spatially flat in the present, it needs to have been extremely spatially flat from the get-go. This requires a high degree of fine-tuning that would have no apparent explanation. To understand how this comes about, let us start by rewriting the Friedmann equation as follows:

$$\Omega - 1 = \frac{k}{H^2 a^2} . \tag{4.53}$$

Differentiating this w.r.t. time this yields:

$$\dot{\Omega} = H (1 + 3\omega) \Omega (\Omega - 1) . \tag{4.54}$$

Note that, since $a(t)$ is a strictly monotonic function of t, we can treat the scale factor as a time parameter. This does not represent the proper time of any particular observer, but it allows us to look at the equations from the point of view of dynamical systems. We will do extensively in the next two chapters. Using $dt = da/H$ we rewrite the evolution equation for Ω as follows:

$$\frac{d\Omega}{da} = (1 + 3\omega) \frac{\Omega (\Omega - 1)}{a} . \tag{4.55}$$

We immediately see that $\Omega = 1$ is a critical point of this system (4.55), i.e. a point where $d\Omega/da = 0$. However, *assuming* the universe is dominated by ordinary matter or radiation (i.e. $\omega > -1/3$), this critical point is not an attractor, but a repellor or unstable critical point:

$$\frac{d}{d\Omega} \left(\frac{d\Omega}{da} \right) \Bigg|_{\Omega = 1} > 0 . \tag{4.56}$$

This means that, in order for Ω to be one today, it must have been incredibly close to one in the early universe. In fact, by looking at (4.55) we see that any slight deviation from the value one is magnified by the small scale factor (early universe) in the denominator. The fine tuning required to keep the rate of change of Ω small enough so that Ω is close to one today cannot be explained without inflation.

Finally, there is one more problem that arises in the standard cosmology, which is also solved by inflation. It is called the *unwanted relics problem*. I will not treat this problem in any detail whatsoever, but will merely state it. In spontaneously broken gauge theories, topologically non-trivial objects such as monopoles, strings, or textures naturally arise. The gauge theory that describes the matter in the universe is a GUT (Grand Unified Theory), and it has a gauge group, which is spontaneously broken to the standard model gauge group $SU(3) \times SU(2) \times U(1)$. It is possible to predict the density of monopoles that should be present in our universe today, by standard calculations using the assumptions about cosmology that we have been using so far. The result turns out to be far too big. The abundant number of monopoles as predicted by the standard cosmology is very generous, however, not one monopole has ever been observed.

4.2.3 Inflation saves the day

Inflation is a scenario for the evolution of the universe, which was created in the 80's [76–78] to solve a number of problems, among which are the three that were mentioned in the previous section. The idea is to have the universe go through a period of accelerated expansion (i.e. $\ddot{a} > 0$) starting $10^{-12}s$ after the Big Bang, and lasting long enough for the scale factor to increase by a factor of 10^{60}. Let us start by looking at how this could solve the horizon problem.

As was mentioned in the previous subsection, solving the horizon problem consists in explaining how regions that seem causally disconnected at $t = t_{\text{CMBR}}$ under the assumption of power-law expansion could have actually been in causal contact at earlier times. As shown earlier, if the scale factor is a power law function with exponent $\alpha < 1$, then there is a finite horizon, no matter how early we take time to begin. However, if $1/a(t)$ blows up faster than $1/t$ for $t \to 0$, then the horizon can be made large (in comoving coordinates). By choosing a function that blows up fast enough, we can enlarge the horizons of the CMBR sources such that they will include each other, thereby solving the horizon problem. Note that this applied not only to power-law solutions with $\alpha \geq 1$, but also to the de Sitter solution, $a \propto \exp(H\,t)$. As mentioned before, in terms of matter or energy content, this requires $\omega < -1/3$. This can be a cosmological constant or some other form of dark energy.

Another way to see how this solves the problem is the following: take two comoving points separated initially by a distance $s = a(t_i)\,\Delta x$. Their proper relative speed is $\dot{s} = \dot{a}\,\Delta x$. If $\ddot{a} > 0$, this relative speed will increase with time, eventually exceeding the proper speed of light, which is

$$a\,\frac{dx}{dt} = a\,\frac{1}{a} = 1\,. \tag{4.57}$$

So regions that are initially causally connected can become causally disconnected by moving away from each other faster than the speed of light.

The flatness problem is also solved by inflation. Intuitively speaking, the period of accelerated expansion blows up small regions of space into huge ones in a short time, thereby flattening out any initial spatial curvature. This explains why the present universe is spatially flat without resorting to fine-tuning at early times. There are two ways to see how this works mathematically:

From the Friedmann equation, which I rewrite for the reader's convenience,

$$\Omega - 1 = \frac{k}{H^2\,a^2}\,, \tag{4.58}$$

we see that the right-hand side decreases with time if $\ddot{a} > 1$, leading to a spatially flat universe, even if the spatial curvature k/a was initially huge. We can also understand this in the language of critical points. From the acceleration equation (4.19) we read off that an accelerating universe requires $\omega < -1/3$. Analyzing (4.55) as we previously did, with this assumption about ω, we see that $\Omega = 1$ is now a stable critical point.

Finally, inflation also solves the problem of unwanted relics. The precise argument is beyond the scope of this chapter, so I will just state the intuitive one. Basically, inflation blows up small regions in space into huge ones, however the amount of monopoles and other topological relics does not increase. The consequences is that the latter are diluted in our universe, which provides us with a plausible explanation for why we have not detected them yet.

4.2.4 Present day acceleration

Another important piece of information about the physics of cosmology concerns the present. By measuring the redshift of light coming from supernovae, two independent teams [79, 80] have concluded that our universe is currently undergoing a period of accelerated expansion. From the acceleration equation (4.19), we see that this implies the presence of dark energy. In fact, these measurements imply that dark energy is the dominant form of energy in the universe today, providing us with the estimate $\Omega_\Lambda \sim 0.7$, mentioned in (4.29).

In this section, we described the history of the universe from moments after the Big Bang until the present day. We have seen that in order to solve the horizon, flatness, and relics problems, the early universe must have gone through a period of inflation lasting long enough to generate 60 e-foldings (i.e. $\log(a_{\mathrm{now}}/a_i) = 60$). Inflation actually also solves a number of problems that I have not even mentioned here. Therefore, inflation is definitely a necessary scenario for modern cosmology. However, it is a 'passing the buck' solution to those problems. It merely merges several problems into one big problem: What drives inflation? Even though we know that dark energy is required for it, there is no known mechanism in physics to *derive* inflation from a fundamental theory. Similarly, there is no *derivation* of the current period of acceleration we are going through. To repeat the quote by Quevedo, inflation is "a scenario in search of an underlying theory." So is present acceleration. In recent years, new hope has arisen that string theory may be used to derive realistic cosmological scenarios. Especially, the latest very precise measurements of CMBR anisotropies have given theorists the hope of finding observational signatures of stringy or transplanckian physics. On one hand cosmology poses a challenge for string theory to come up with a mechanism to drive inflation and present day acceleration, on the other hand, it may provide string theorists with their first lab in which to test string theory ideas.

4.3 New challenges lead to new ideas

If string theory truly is the *theory of everything*, and especially if it is a theory of quantum gravity, then it must ultimately explain the Big Bang, inflation, and current acceleration. In this section we will be looking at some candidate mechanisms by means of which string theory might induce those two cosmological events. I will begin by introducing a new form of dark energy as a possible source for acceleration: the scalar field. Then, I will briefly introduce how gravity-scalar models with accelerating cosmological solutions can arise from string or M-theory. Consider this as an introduction for the next two chapters, which will be based on two articles about scalar cosmologies and their possible string/M-theory origins.

4.3.1 Scalar models for cosmology

As we pointed out before, in order to have accelerated expansion, be it for inflation or present day acceleration, we must have a perfect fluid with $\omega < -1/3$ in the universe. Having $\omega = -1$, a positive cosmological constant will do. It will source a de Sitter spacetime. However, it does have some drawbacks: being a constant by definition, it is non-dynamical. This means that the universe would be in a state of eternal inflation at a constant rate of acceleration, which is not quite consistent with observations. A more flexible and more interesting approach would be to have a form of dark energy that mimics a cosmological constant and is yet dynamical. This has two advantages: firstly, it could induce a de Sitter-like universe with a slowly varying acceleration rate, which would be more consistent with observations of current acceleration. Secondly, it would in principle allow for a dynamical start and end of inflation and current acceleration, and also for a dynamical resolution of the cosmic coincidence problem, which is more appealing from a theoretician's point of view.

Let us write down a simple gravity-scalar model, namely gravity with one scalar field and some potential for it:

$$\mathcal{L} = \sqrt{-g}\,\left(R - \tfrac{1}{2}\,(\partial\phi)^2 - V(\phi)\right) . \tag{4.59}$$

The equations of motion for a $k = 0$ FLRW Ansatz are the following:

$$H^2 = \tfrac{1}{12}\,\dot{\phi}^2 + \tfrac{1}{6}\,V\,, \tag{4.60}$$

$$\frac{\ddot{a}}{a} = \tfrac{1}{6}\,\left(-\dot{\phi}^2 + V\right)\,, \tag{4.61}$$

$$\ddot{\phi} + 3\,H\,\dot{\phi} + \frac{\partial V}{\partial \phi} = 0\,, \tag{4.62}$$

where we recognize the first two equations as the Friedmann and acceleration equations, respectively, and the third one is the equation of motion of the scalar field. To be consistent with homogeneity, we have assumed that ϕ depends only on time. Comparing this to (4.18) and (4.19), we see that

$$\rho = \frac{1}{16\,\pi\,G}\,\left(\tfrac{1}{2}\,\dot{\phi}^2 + V\right)\,, \tag{4.63}$$

$$p = \frac{1}{16\,\pi\,G}\,\left(\tfrac{1}{2}\,\dot{\phi}^2 - V\right)\,. \tag{4.64}$$

So, if ϕ varies slowly in time, its equation of motion approaches that of a cosmological constant, i.e. $\omega \sim -1$. We also see from the acceleration equation (4.61) that V acts in favor of acceleration like a cosmological constant, and the kinetic energy acts against it. This is why in scalar models for inflation such as *chaotic inflation*, one requires that the field be slowly varying, i.e. $\dot{\phi} \ll 1$, by restricting the form of the potential. However, a realistic model for inflation must have an inflationary period of at least 60 e-foldings. A scalar field will naturally roll down its potential until it reaches a minimum, and its kinetic energy will only increase in the meantime, leading to a non-accelerating or even decelerating cosmology. Therefore, in order to prevent a premature end of inflation one must also require that $\ddot{\phi} \ll 1$. These two conditions, $\dot{\phi} \ll 1$ and $\ddot{\phi} \ll 1$ are called the *slow roll* conditions. Of course, in a specific model, one usually parametrizes these constraints to obtain controlled results.

Introducing the scalar field allows for cosmologies that are more complicated than just power-law or de Sitter solutions. Because it is dynamical, it can source solutions that interpolate in time between those two basic solutions. Cosmological solutions that interpolate in time between two non-accecelerating regimes, but are separated by one or several periods of transient acceleration are of special interest. We will see a specific example of this in the next section, and in the next two chapters we will be looking at more general examples where we introduce several scalar fields with intricate potentials that couple them to each other.

4.3.2 Acceleration from string/M-theory

In principle one can obtain all kinds of interesting geometries to model inflation and current acceleration from scalar field models by having several scalar fields and the right potential, as we will see in the next two chapters. However, even if one can write down such a model the question remains: where do these fields and their potential come from? Often one refers to such scalar fields as *inflatons* and to their potentials as *quintessence*, meaning they are a fifth force in nature that drives acceleration. However, as string theorists, we do not like to invoke new forces unless we can derive them from a unified theory. In the past few years, string theorists have made numerous attempts to derive scalar cosmology models by dimensionally reducing 10-dimensional supergravities and making appropriate truncations leaving only scalar fields and scalar potentials in the four-dimensional spacetime. In chapter 6, we will look at what happens when one reduces supergravities on three-dimensional *group manifolds*. However, before jumping into that, I will attempt to give a brief review of what happens when one considers simpler schemes, such as reducing over Einstein spaces[2].

The standard torroidal Kaluza-Klein reduction scheme provides us with an easy way of going from ten dimensions to four *and* generating scalar fields (i.e. Kaluza-Klein modes) with potentials. However, the potentials it yields will not generate an accelerating four-dimensional universal. To make things worse, there is a *no-go* theorem [81, 82] that essentially states that compactifications of ten or eleven dimensional supergravities of string/M-theory over compact, non-singular, spaces without boundaries and with time-independent volume[3] never lead to accelerating universes. To circumvent the theorem, one must allow for time-dependent volume of the internal space. P.K. Townsend and M. Wohlfarth [83] showed that reducing gravity over a six or seven-dimensional hyperboloid with time-dependent volume yields a universe with a limited period of acceleration. The solution interpolates in time between two decelerating power-law periods at $t \to 0$ and $t \to \infty$, which are joined by an accelerating epoch. The Ansatz in $4 + n$ dimensions has the following form:

$$ds^2 = \delta^{-n}(t)\, ds_E^2 + \delta^2(t)\, dH_n^2\,, \tag{4.65}$$

where ds_E^2 is the four-dimensional cosmological spacetime that will result in Einstein frame after the reduction, dH_n is the n-dimensional hyperbolic space, and $\delta(t)$ is the *warp factor*, which will act as a time-dependent 'volume' of the internal space. The dimension n of the internal space is left arbitrary, but for string/M-theory we need $n = 6, 7$. I will not write down the actual solutions for $\delta(t)$ and ds_E^2, for I want to stress the qualitative information. The $(4+n)$-dimensional Ansatz is itself flat, i.e. it is Minkowski spacetime with some identifications that do not affect curvature. However, the reduction Ansatz we have chosen, yields a non-trivial four-dimensional spacetime with interpolating behavior. In the early universe it has $a \sim t^{1/3}$; in the future it has $a \sim t^{n/(n+2)}$; and in between it has an epoch of transient acceleration. This is in principle what we are looking for, as this scenario has its own mechanism to begin and end inflation. Unfortunately, the acceleration period generated by this scheme is not long enough

[2] An Einstein space is a manifold with a metric that solves the Einstein equations *in vacuo* or in the presence of a cosmological constant. As a consequence of that, it has the highest possible degree of symmetry.

[3] You may wonder what I mean by 'volume' if the internal space is hyperbolic. In this case one must always make the space compact by topological identifications. Otherwise, one must face the undesirable physical consequences of a so-called *non-compactification*.

to yield the so much needed 60 e-foldings of inflation. But the result may still apply to current acceleration.

The Townsend-Wohlfarth solution turns out to be a special case of a larger class of supergravity solutions called *S-branes*, found in [84]. These are essentially solutions of supergravity, which look like p-brane solutions, except that time is *transverse* to their world-volume as opposed to being in it. These solutions are sourced by the dilaton and some antisymmetric tensor, just like p-brane solutions:

$$\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2\,(p+2)!}\, e^{a_p\,\phi}\, F^2_{p+2}\,, \tag{4.66}$$

where F_{p+2} is the field-strength, and a_p is determined by the supergravity in question. This differs from the previous Ansatz in that the latter was a solution to Einstein's equation *in vacuo*, whereas the S-brane is carried by the dilaton and has a flux from the $p+2$-form field-strength. The Ansatz for the metric is similar to the previous one, except that the internal space no longer needs to be hyperbolic; it can be flat or spherical. The Ansatz for an SD2-branes looks roughly as follows:

$$ds^2 = -f(t)^2\, dt^2 + g(t)^2\, d\mathbf{x}_3^2 + h(t)^2\, d\Sigma^2_{k,6}\,, \tag{4.67}$$

where the three boldfaced spatial coordinates correspond to our space, and to the world-volume of the SD2-brane, the six-dimensional internal space can now be positively curved, flat, or negatively curved ($k = 1, 0, -1$ respectively), and $f(t), g(t)$ and $h(t)$ are determined by the equations of motion. This solution is no longer flat in ten dimensions, since it now solves the Einstein equations with RR flux turned on, but interpolates in time between a flat metric and a horizon-like geometry. In four dimensions, it yields an interpolating solution with a transient accelerating epoch regardless of the kind of internal space we pick (i.e. $k = 1, 0, -1$). For a more detailed review on the subject of S-branes and their status, the reader is referred to [85].

The schemes I have mentioned so far are all based on the assumption that the supergravity approximation is a valid one, allowing one to treat string theory as field theory. However, this assumption is not necessarily justified. One uses it only because it is very difficult to deal with the full string theory. For instance, in a scenario where the dilaton grows large over time, string perturbation theory will break down. That is why attempts are being made to take non-perturbative string theory effects into account in compactification schemes. Another problem posed by these compactifications is that the volume and shape of the internal space, being dynamical by construction, are not always stable. For instance, in many solutions, the volume will tend to blow up in time. This is known as *spontaneous decompactification*. If one takes such models seriously, then one should expect to be able to observe these extra dimensions in the present, or assume that we live in a special moment in the history of the universe, when the extra dimensions happen to be small. These compactification schemes should, therefore, not be regarded as phenomenologically realistic models, but merely as evidence that demonstrates that it is possible to circumvent the Maldacena-Nuñez no-go theorem [82].

Currently, string theorists are trying to create realistic models that can stabilize all of the *moduli* of the internal compactification manifold. A couple of years ago, the authors of [33] came up with a string compactification scenario that exploits non-perturbative string theory effects to stabilize the internal moduli. The idea relies on non-perturbative instanton effects induced by wrapping a Euclidean D3-brane around a 4-cycle of the internal Calabi-Yau space.

The authors of the paper, however, did not find an explicit choice for the required Calabi-Yau space to carry out this idea. String theorists have only recently been able to write down concrete realizations of this scenario. For instance, while this thesis was being written, an article was published [86], in which not only the moduli stabilization problem was dealt with, but also the problem of breaking supersymmetry softly for particle phenomenological purposes.

All of the schemes to obtain acceleration from string/M-theory that I have mentioned so far have one thing in common: from the four-dimensional point of view, they all reduce to an effective field theory with Einstein gravity and scalar fields with potentials. This is true even for models that take non-perturbative string theory effects into account. Therefore, although one would like to be able to derive the ultimate string theory mechanism or scenario that leads to inflation and present day acceleration right away, it is useful and wise to also study which four-dimensional scalar models are capable of driving those two cosmological phenomena at all. After all, most of the conceivable reduction schemes will reduce to four-dimensional scalar-gravity field theories. Should one find a class of models that drive a realistic cosmology, one could then investigate how to obtain it from string theory. In the next two chapters, we will be doing a bit of both. We will study scalar-gravity models with exponential potentials in general, but will also pay attention to potentials obtained from some specific dimensional reduction schemes.